

Properties of T -Anti-Fuzzy Ideals of a ℓ -Near-Ring

Chandrasekaran G^{1*}

¹Assistant Professor, Dept. of Mathematics, Sethupathi Govt Arts College,
Ramanathapuram – 623 502, Tamilnadu, India

Chellappa B²

²Principal, Nachiappa Suvamical Arts and Science College,
Karaikudi - 630 003, Tamilnadu, India

Jeyakumar M³

³Assistant Professor, Dept. of Mathematics, Alagappa University Evening College,
Rameswaram – 623 526, Tamilnadu, India

ABSTRACT

In this paper, we define Anti-fuzzy ideal of a ℓ -near-ring in R and T -anti-fuzzy ideal of a ℓ -near-ring in R . we made an attempt to study the properties of T -anti-fuzzy ideal of a ℓ -near-ring, union of T -anti-fuzzy ideals of ℓ -near-ring, join of T -anti-fuzzy ideal of a ℓ -near-ring in R , join of a family of T -anti-fuzzy ideal of a ℓ -near-ring in R and family of union of T -anti-fuzzy ideal of ℓ -near-ring in R .

Keywords: Fuzzy subset, near-ring, ideal, ℓ -ring, ℓ -near-ring, T -fuzzy ideal, anti-fuzzy ideal, anti-fuzzy ideal of ℓ -near-ring, T -anti-fuzzy ideal, join of T -anti-fuzzy ideal.

INTRODUCTION

The concept of fuzzy sets was initiated by Zadeh LA. [25] in 1965. Wang-jin Liu. [24] has studied fuzzy ideals of a ring and many researchers are engaged in extending the concepts. Abou-Zaid S. [1] introduced the notion of a fuzzy subnear-ring, and studied fuzzy ideals of a near-ring, and many followers discussed further properties of fuzzy ideals in near-rings. In Biswas R. [6] introduced the concept of anti fuzzy subgroups of groups, and Kyung Ho KIM, Young Bae JUN. [12] studied the notion of anti fuzzy R -subgroups of near-ring. In Kim KH, Jun YB, Yon YH. [13] introduced an anti-fuzzy ideal in Near-Rings. Triangular norms were introduced by Schweizer and Sklar [18, 19] to model the distances in probabilistic metric spaces. In fuzzy sets theory, triangular norm (t -norm) is extensively used to model the logical connective conjunction (AND). There are many applications of triangular norms in several fields of mathematics and artificial intelligence [26]. Dheena P, Mohanraaj G. [9] have studied several properties of T -fuzzy ideals of rings and T -fuzzy ideals of near-rings. We extended the results of Akram M. [3] to Γ -near-rings. Prakashmanimaran J, Chellappa B, Jeyakumar M. [16] introduced T -anti-fuzzy right ideals of ℓ -ring.

In this paper we define, characterize and study of T -anti-fuzzy right and left ideals. we define anti-fuzzy ideal of a ℓ -near-ring in R and T -anti-fuzzy right ideals of ℓ -near-ring in R . We discuss some of its properties. We have shown that properties of T -anti-fuzzy ideal of a ℓ -near-ring, union of T -anti-fuzzy ideals of ℓ -near-ring, join of T -anti-fuzzy ideal of a ℓ -near-ring in R , family of join of T -anti-fuzzy ideal of a ℓ -near-ring in R and family of union of T -anti-fuzzy ideal of ℓ -near-ring in R .

Definition: 1

A non-empty set R is called a near-ring with two binary operations “+” and “.” satisfying the following axioms: $(R, +)$ is a group ; $(R, .)$ is a semigroup and $(x + y).z = x.z + y.z$, for all x, y, z in R (i.e. Multiplicative is left distributive with respect to addition) We denote $x.y$ by xy .

Definition: 2

A non-empty set R is called lattice ordered near-ring or ℓ -near-ring if it has four binary operations “+”, “.”, $\vee$, $\wedge$ defined on it and satisfy the following axioms: $(R, +)$ is a group ; $(R, .)$ is a semigroup; $(R, \vee, \wedge)$ is a lattice ; and

- (i) $x.(y + z) = x.y + x.z$, for all x, y, z in R

- (ii) $x+(y \vee z)=(x+y) \vee (x+z)$; $x+(y \wedge z)=(x+y) \wedge (x+z)$
 $(y \vee z)+x=(y+x) \vee (z+x)$; $(y \wedge z)+x=(y+x) \wedge (z+x)$
- (iii) $x \cdot (y \vee z)=(xy) \vee (xz)$; $x \cdot (y \wedge z)=(xy) \wedge (xz)$
 $(y \vee z) \cdot x=(yx) \vee (zx)$; $(y \wedge z) \cdot x=(yx) \wedge (zx)$, for all x, y, z in R and $x \geq 0$

Example: 1

$(n\mathbb{Z}, +, \cdot, \vee, \wedge)$ is a ℓ -near-ring, where $\mathbb{Z}$ is the set of all integers and $n \in \mathbb{Z}$

Definition: 3

A mapping from a nonempty set X to $[0, 1]$, $\mu : X \rightarrow [0, 1]$ is called a fuzzy subset of X .

Definition: 4

A fuzzy subset μ of a lattice ordered ring (or ℓ -ring) R is called an anti-fuzzy sub ℓ -ring of R if the following conditions are satisfied

- (i) $\mu(x-y) \leq \max\{\mu(x), \mu(y)\}$
(ii) $\mu(xy) \leq \max\{\mu(x), \mu(y)\}$
(iii) $\mu(x \vee y) \leq \max\{\mu(x), \mu(y)\}$
(iv) $\mu(x \wedge y) \leq \max\{\mu(x), \mu(y)\}$, for all x, y in R .

Example: 2

Consider the fuzzy subset μ of the ℓ -ring $(\mathbb{Z}, +, \cdot, \vee, \wedge)$

$$\mu(x) = \begin{cases} 0.4 & \text{if } x \in \langle 3 \rangle \\ 0.8 & \text{if } x \in \mathbb{Z} - \langle 3 \rangle \end{cases}.$$

Then, μ is an anti-fuzzy ℓ -sub ring.

Example: 3

Consider a fuzzy subset μ of the ℓ -ring $(\mathbb{Z}, +, \cdot, \vee, \wedge)$.

$$\mu(x) = \begin{cases} 0.3 & \text{if } x \in \langle 3 \rangle \\ 0.8 & \text{if } x \in \mathbb{Z} - \langle 3 \rangle \end{cases}.$$

Then, μ is not an anti-fuzzy ℓ -sub ring.

For example, let $x = 3$ and $y = 7$, then $x + y = 10$. Here $\mu(x) = 0.8$ and $\mu(y) = 0.8$.

Therefore, $\max\{\mu(x), \mu(y)\} = \max\{0.8, 0.8\} = 0.8$. But $\mu(x + y) = 0.3$.

Hence, $\mu(x + y) \leq \max\{\mu(x), \mu(y)\}$. Thus, μ is not an anti-fuzzy ℓ -sub ring of R .

Definition: 5

Let R be a ℓ -near-ring. A nonempty subset $(I, +)$ of $(R, +)$ is called a left ideal if $x \cdot (y + i) - x \cdot y \in I$ for all $x, y \in R$ and $i \in I$; a right ideal if $i \cdot x \in I$ for all $x \in R$ and $i \in I$; an ideal, if it is both a left ideal and a right ideal of R .

Definition: 6

A fuzzy set μ in a near-ring R is said to be an anti-fuzzy ideal of R , if the following conditions are satisfied,

- (i) $\mu(x-y) \leq \max\{\mu(x), \mu(y)\}$
(ii) $\mu(y+x-y) \leq \mu(x)$
(iii) $\mu(xy) \leq \mu(y)$; $\mu(xy) \leq \mu(x)$
(iv) $\mu((x+z)y - xy) \leq \mu(z)$ for all $x, y, z \in R$.

Proposition: 1

If μ is an anti-fuzzy ideal of R then, $\mu(0) \leq \mu(x)$ for all $x \in R$.

Definition: 7

A fuzzy subset μ of a ℓ -near-ring R is called an anti-fuzzy ideal, if the following conditions are satisfied,

- (i) $\mu(x - y) \leq \max(\mu(x), \mu(y))$
- (ii) $\mu(y + x - y) \leq \mu(x)$
- (iii) $\mu(xy) \leq \mu(y); \mu(xy) \leq \mu(x)$
- (iv) $\mu((x + z)y - xy) \leq \mu(z)$
- (v) $\mu(x \vee y) \leq \max(\mu(x), \mu(y))$
- (vi) $\mu(x \wedge y) \leq \max(\mu(x), \mu(y))$ for all $x, y, z \in R$.

Example: 4

Consider a near-ring $R = \{a, b, c, d\}$ with the following Cayley's tables:

+	a	b	c	d
a	a	b	c	d
b	b	a	d	c
c	c	d	b	a
d	d	c	a	b

.	a	b	c	d
a	a	a	a	a
b	a	a	a	a
c	a	a	a	a
d	a	a	b	b

We define an anti-fuzzy subset $\mu : R \rightarrow [0,1]$ by $\mu(a) < \mu(b) < \mu(d) = \mu(c)$

Then, μ is an anti-fuzzy right (resp. left) ideal of R .

Definition: 8

A mapping $T : [0, 1] \times [0, 1] \rightarrow [0, 1]$ is called a triangular norm [t -norm], if and only if, it satisfies the following conditions:

- (i). $T(x, 1) = T(1, x) = x$, for all $x \in [0, 1]$
- (ii). $T(x, y) = T(y, x)$, for all $x, y \in [0, 1]$
- (iii). $T(x, T(y, z)) = T(T(x, y), z)$
- (iv). $T(x, y) \leq T(x, z)$, whenever $y \leq z$.

Proposition: 2

The *minimum* T -norm ($\min T$ -norm) is defined by $T(a, b) = \min\{a, b\}$.

Definition: 9

A fuzzy subset μ of a ring R is called T -anti-fuzzy right (resp. left) ideal if

- (i) $\mu(x - y) \leq T(\mu(x), \mu(y))$
- (ii) $\mu(xy) \leq \mu(x)$ (resp. left $\mu(xy) \leq \mu(y)$), for all x, y in R .

Definition: 10

A fuzzy subset μ of a ring R is called T -anti-fuzzy ideal, if it is satisfied both right and left ideals.

Proposition: 3

Every anti-fuzzy right ideal of a ring R is an T -anti-fuzzy right ideal.

Definition: 11

A fuzzy subset μ of a ℓ -ring R is called an T -anti-fuzzy ideal, if the following conditions are satisfied,

- (i) $\mu(x - y) \leq T(\mu(x), \mu(y))$
- (ii) $\mu(xy) \leq \mu(x); \mu(xy) \leq \mu(y)$
- (iii) $\mu(x \vee y) \leq T(\mu(x), \mu(y))$
- (iv) $\mu(x \wedge y) \leq T(\mu(x), \mu(y))$, for all $x, y \in R$.

Definition: 12

A fuzzy subset μ of a near-ring R is called an T -anti-fuzzy ideal, if the following conditions are satisfied,

- (i) $\mu(x - y) \leq T(\mu(x), \mu(y))$
- (ii) $\mu(y + x - y) \leq \mu(x)$
- (iii) $\mu(xy) \leq \mu(y); \mu(xy) \leq \mu(x)$
- (iv) $\mu((x + z)y - xy) \leq \mu(z)$, for all $x, y, z \in R$.

Proposition: 4

Every T -fuzzy ideal of a near-ring R is a T -fuzzy sub near-ring of R .

Converse of Proposition 1 may not be true in general as seen in the following example.

Example: 5

From example: 4

Let $T : [0,1] \times [0,1] \rightarrow [0,1]$ be a function. defined by $T(x, y) = \max\{x + y - 1, 0\}$ which is a t -norm, for all $x, y \in [0,1]$. By routine calculations, it is easy to check that μ is a T -anti-fuzzy sub near-ring of R . It is clear that μ is also left T -anti-fuzzy ideal of R . But μ is not T -anti-fuzzy right ideal of R , since, $\mu((c + d)d - cd) = \mu(d) < \mu(b)$.

Definition: 13

A fuzzy subset μ of a ℓ -near-ring R is called an T -anti-fuzzy ideal, if the following conditions are satisfied,

- (i) $\mu(x - y) \leq T(\mu(x), \mu(y))$
- (ii) $\mu(y + x - y) \leq \mu(x)$
- (iii) $\mu(xy) \leq \mu(y); \mu(xy) \leq \mu(x)$
- (iv) $\mu((x + z)y - xy) \leq \mu(z)$
- (v) $\mu(x \vee y) \leq T(\mu(x), \mu(y))$
- (vi) $\mu(x \wedge y) \leq T(\mu(x), \mu(y))$ for all $x, y, z \in R$.

Example: 6

Let $(R = \{a, b, c\}, +, \cdot, \vee, \wedge)$ be a ℓ -near-ring.

Consider an anti-fuzzy subset μ of the ℓ -ring R

$$\mu(x) = \begin{cases} 0.2 & \text{if } x = a \\ 0.5 & \text{if } x = b. \\ 0.8 & \text{if } x = c \end{cases}$$

Then, μ is an T -anti-fuzzy ideal of ℓ -near-ring R .

Definition: 14

Let μ and λ be the anti-fuzzy subsets of a set X . An anti-fuzzy subset $\mu \cup \lambda$ is defined as $(\mu \cup \lambda)(x) = \max\{\mu(x), \lambda(x)\}$.

Definition: 15

Let μ and λ be the anti-fuzzy subsets of a set X . An anti-fuzzy subset $\mu \vee \lambda$ is defined as $(\mu \vee \lambda)(x) = T(\mu(x), \lambda(x))$.

Example: 7

Let $R = \{0, a, b, c\}$. We have $(\mu \vee \lambda)(x) = T(\mu(x), \lambda(x))$, for all x in R .

Define t -norm T by $T(p, q) = \max(p, q)$, for all p, q in $[0, 1]$.

Define an anti-fuzzy subset

$\mu : R \rightarrow [0, 1]$ by $\mu(0) = 0.9$ and $\mu(a) = \mu(b) = \mu(c) = 0.4$, where $0, a, b, c \in R$.

Then $\mu = \{(0, 0.9), (a, 0.4), (b, 0.4), (c, 0.4)\}$.

Again, define an anti-fuzzy subset

$\lambda : R \rightarrow [0, 1]$ by $\lambda(0) = 0.7$, $\lambda(a) = 0.6$, $\lambda(b) = 0.5$ and $\lambda(c) = 0.4$ where $0, a, b, c \in R$.

Then $\lambda = \{(0, 0.7), (a, 0.6), (b, 0.5), (c, 0.4)\}$.

Let $0, a, b, c$ in R

Then, $(\mu \vee \lambda)(0) = T(\mu(0), \lambda(0)) = T(0.9, 0.7) = \max(0.9, 0.7) = 0.9$

Now, $(\mu \vee \lambda)(a) = T(\mu(a), \lambda(a)) = T(0.4, 0.6) = \max(0.4, 0.6) = 0.6$

$(\mu \vee \lambda)(b) = T(\mu(b), \lambda(b)) = T(0.4, 0.5) = \max(0.4, 0.5) = 0.5$

$(\mu \vee \lambda)(c) = T(\mu(c), \lambda(c)) = T(0.4, 0.4) = \max(0.4, 0.4) = 0.4$

So, $\mu \vee \lambda = \{(0, 0.9), (a, 0.6), (b, 0.5), (c, 0.4)\}$ is the anti-fuzzy subset of R .

That is, $\mu \vee \lambda : R \rightarrow [0, 1]$, defined by $(\mu \vee \lambda)(0_R) = 0.9$.

$$(\mu \vee \lambda)(x) = \begin{cases} a & \text{if } x = 0.6 \\ b & \text{if } x = 0.5 \\ c & \text{if } x = 0.4, \text{ for all } 0_R \neq x \end{cases}$$

Thus, $\mu \vee \lambda$ is an T -anti-fuzzy ideal of a ℓ -near-ring R .

Definition: 16

Let μ and λ be T -anti-fuzzy ideals of a ℓ -near-ring R . Then, $\mu \cup \lambda$, is an T -anti-fuzzy right ideal is defined by $(\mu \cup \lambda)(x - y) = \max(\mu(x - y), \lambda(x - y))$, for all $x, y \in R$.

Definition: 17

A fuzzy set μ of a ℓ -near-ring R has the **supremum** property if for any subset N of R , there exists a $a_0 \in N$ such that $\mu_A(a_0) = \sup_{a \in N} \mu_A(a)$.

Theorem: 1

Every anti-fuzzy ideal of a ℓ -near-ring R is an T -anti-fuzzy ideal in ℓ -near-ring R .

Theorem: 2

If μ and λ are T -anti-fuzzy ideals of a ℓ -near-ring R , then $\mu \vee \lambda$ is an T -anti-fuzzy ideal of a ℓ -near-ring R .

Proof:

Given μ and λ are T -anti-fuzzy ideals of a ℓ -near-ring R ,

Let $x, y, z \in R$

$$\begin{aligned}
(i) \quad (\mu \vee \lambda)(x - y) &= T(\mu(x - y), \lambda(x - y)) \\
&\leq T(T(\mu(x), \mu(y)), T(\lambda(x), \lambda(y))) \\
&= T(T(T(\mu(x), \mu(y)), \lambda(x)), \lambda(y)) \\
&= T(T(T(\mu(x), \lambda(x)), \mu(y)), \lambda(y)) \\
&= T(T(\mu(x), \lambda(x)), T(\mu(y), \lambda(y))) \\
&= T((\mu \vee \lambda)(x), (\mu \vee \lambda)(y))
\end{aligned}$$

Therefore, $(\mu \vee \lambda)(x - y) \leq T((\mu \vee \lambda)(x), (\mu \vee \lambda)(y))$, for all $x, y \in R$.

$$\begin{aligned}
(ii) \quad \text{Since, } \mu(y + x - y) &\leq \mu(x) \text{ and } \lambda(y + x - y) \leq \lambda(x) \\
(\mu \vee \lambda)(y + x - y) &= T(\mu(y + x - y), \lambda(y + x - y)) \\
&\leq T(\mu(x), \lambda(x)) \\
&= (\mu \vee \lambda)(x)
\end{aligned}$$

Therefore, $(\mu \vee \lambda)(y + x - y) \leq (\mu \vee \lambda)(x)$, for all $x, y \in R$.

$$\begin{aligned}
(iii) \quad \text{Since } \mu(xy) &\leq \mu(x) \text{ and } \lambda(xy) \leq \lambda(x) \\
(\mu \vee \lambda)(xy) &\leq T(\mu(xy), \lambda(xy)), \\
&\leq T(\mu(x), \lambda(x)) \leq (\mu \vee \lambda)(x)
\end{aligned}$$

Therefore, $(\mu \vee \lambda)(xy) \leq (\mu \vee \lambda)(x)$, for all $x, y \in R$.

$$\begin{aligned}
(iv) \quad \text{Since, } \mu((x + z)y - xy) &\leq \mu(z) \text{ and } \lambda((x + z)y - xy) \leq \lambda(z) \\
(\mu \vee \lambda)((x + z)y - xy) &= T(\mu((x + z)y - xy), \lambda((x + z)y - xy)) \\
&\leq T(\mu(z), \lambda(z)) \leq (\mu \vee \lambda)(z)
\end{aligned}$$

Therefore, $(\mu \vee \lambda)((x + z)y - xy) \leq (\mu \vee \lambda)(z)$, for all $x, y \in R$.

$$\begin{aligned}
(v) \quad (\mu \vee \lambda)(x \vee y) &= T(\mu(x \vee y), \lambda(x \vee y)) \\
&\leq T(T(\mu(x), \mu(y)), T(\lambda(x), \lambda(y))) \\
&= T(T(T(\mu(x), \mu(y)), \lambda(x)), \lambda(y)) \\
&= T(T(T(\mu(x), \lambda(x)), \mu(y)), \lambda(y)) \\
&= T(T(\mu(x), \lambda(x)), T(\mu(y), \lambda(y))) \\
&= T((\mu \vee \lambda)(x), (\mu \vee \lambda)(y))
\end{aligned}$$

Therefore, $(\mu \vee \lambda)(x \vee y) \leq T((\mu \vee \lambda)(x), (\mu \vee \lambda)(y))$, for all $x, y \in R$.

$$\begin{aligned}
(vi) \quad (\mu \vee \lambda)(x \wedge y) &= T(\mu(x \wedge y), \lambda(x \wedge y)) \\
&\leq T(T(\mu(x), \mu(y)), T(\lambda(x), \lambda(y))) \\
&= T(T(T(\mu(x), \mu(y)), \lambda(x)), \lambda(y)) \\
&= T(T(T(\mu(x), \lambda(x)), \mu(y)), \lambda(y)) \\
&= T(T(\mu(x), \lambda(x)), T(\mu(y), \lambda(y))) \\
&= T((\mu \vee \lambda)(x), (\mu \vee \lambda)(y))
\end{aligned}$$

Therefore, $(\mu \vee \lambda)(x \wedge y) \leq T((\mu \vee \lambda)(x), (\mu \vee \lambda)(y))$, for all $x, y \in R$.

Thus $\mu \vee \lambda$, is an T -anti-fuzzy right ideal of a ℓ -near-ring R .

Theorem: 3

If μ and λ are T -fuzzy ideals of a ℓ -near-ring R , then $\mu \cup \lambda$ is an T -anti-fuzzy ideal of a ℓ -near-ring R .

Proof:

Given μ and λ are T -anti-fuzzy ideals of a ℓ -near-ring R

Let $x, y, z \in R$

$$\begin{aligned}
 \text{(i). } (\mu \cup \lambda)(x - y) &= \max \{ \mu(x - y), \lambda(x - y) \} \\
 &\leq \max \{ \max \{ \mu(x), \mu(y) \}, \max \{ \lambda(x), \lambda(y) \} \} \\
 &= \max \{ \max \{ \max \{ \mu(x), \mu(y) \}, \lambda(x) \}, \lambda(y) \} \\
 &= \max \{ \max \{ \max \{ \mu(x), \lambda(x) \}, \mu(y) \}, \lambda(y) \} \\
 &= \max \{ \max \{ \mu(x), \lambda(x) \}, \max \{ \mu(y), \lambda(y) \} \} \\
 &= \max \{ (\mu \cup \lambda)(x), (\mu \cup \lambda)(y) \}
 \end{aligned}$$

Therefore, $(\mu \cup \lambda)(x - y) \leq \max \{ (\mu \cup \lambda)(x), (\mu \cup \lambda)(y) \}$, for all $x, y \in R$

$$\text{(ii). Since, } \mu(y + x - y) \leq \mu(x) \text{ and } \lambda(y + x - y) \leq \lambda(x)$$

$$\begin{aligned}
 \text{We have, } (\mu \cup \lambda)(y + x - y) &= \max \{ \mu(y + x - y), \lambda(y + x - y) \} \\
 &\leq \max \{ \mu(x), \lambda(x) \} = (\mu \cup \lambda)(x)
 \end{aligned}$$

Therefore, $(\mu \cup \lambda)(y + x - y) \leq (\mu \cup \lambda)(x)$, for all $x, y \in R$

$$\text{(iii). Since, } \mu(xy) \leq \mu(x) \text{ and } \lambda(xy) \leq \lambda(x)$$

$$\begin{aligned}
 \text{We have, } (\mu \cup \lambda)(xy) &\leq \max \{ \mu(xy), \lambda(xy) \} \\
 &\leq \max \{ \mu(x), \lambda(x) \} \\
 &\leq (\mu \cup \lambda)(x)
 \end{aligned}$$

Therefore, $(\mu \cup \lambda)(xy) \leq (\mu \cup \lambda)(x)$, for all $x, y \in R$

$$\text{(iv) Since, } \mu((x + z)y - xy) \leq \mu(z) \text{ and } \lambda((x + z)y - xy) \leq \lambda(z)$$

$$\begin{aligned}
 (\mu \cup \lambda)((x + z)y - xy) &= \max \{ \mu((x + z)y - xy), \lambda((x + z)y - xy) \} \\
 &\leq \max \{ \mu(z), \lambda(z) \} \leq (\mu \cup \lambda)(z)
 \end{aligned}$$

Therefore, $(\mu \cup \lambda)((x + z)y - xy) \leq (\mu \cup \lambda)(z)$, for all $x, y \in R$

$$\begin{aligned}
 \text{(v). } (\mu \cup \lambda)(x \vee y) &= \max \{ \mu(x \vee y), \lambda(x \vee y) \} \\
 &\leq \max \{ \max \{ \mu(x), \mu(y) \}, \max \{ \lambda(x), \lambda(y) \} \} \\
 &= \max \{ \max \{ \max \{ \mu(x), \mu(y) \}, \lambda(x) \}, \lambda(y) \} \\
 &= \max \{ \max \{ \max \{ \mu(x), \lambda(x) \}, \mu(y) \}, \lambda(y) \} \\
 &= \max \{ \max \{ \mu(x), \lambda(x) \}, \max \{ \mu(y), \lambda(y) \} \} \\
 &= \max \{ (\mu \cup \lambda)(x), (\mu \cup \lambda)(y) \}
 \end{aligned}$$

Therefore, $(\mu \cup \lambda)(x \vee y) \leq \max \{ (\mu \cup \lambda)(x), (\mu \cup \lambda)(y) \}$, for all $x, y \in R$

$$\begin{aligned}
 \text{(vi). } (\mu \cup \lambda)(x \wedge y) &= \max \{ \mu(x \wedge y), \lambda(x \wedge y) \} \\
 &\leq \max \{ \max \{ \mu(x), \mu(y) \}, \max \{ \lambda(x), \lambda(y) \} \} \\
 &= \max \{ \max \{ \max \{ \mu(x), \mu(y) \}, \lambda(x) \}, \lambda(y) \} \\
 &= \max \{ \max \{ \max \{ \mu(x), \lambda(x) \}, \mu(y) \}, \lambda(y) \}
 \end{aligned}$$

$$= \max \{ \max \{ \mu(x), \lambda(x) \}, \max \{ \mu(y), \lambda(y) \} \}$$

$$= \max \{ (\mu \cup \lambda)(x), (\mu \cup \lambda)(y) \}$$

Therefore, $(\mu \cup \lambda)(x \wedge y) \leq \max \{ (\mu \cup \lambda)(x), (\mu \cup \lambda)(y) \}$, for all $x, y \in R$

Thus, $\mu \cup \lambda$ is an T -anti-fuzzy ideal of a ℓ -near-ring R .

Theorem: 4

The join of a family of T -anti-fuzzy ideal of ℓ -near-ring R is an T -anti-fuzzy ideal of a ℓ -near-ring R .

Proof:

Let $\{v_\alpha : \alpha \in I\}$ be a family of T -anti-fuzzy ideal of ℓ -near-ring R

Let $V = \bigvee_{\alpha \in I} V_\alpha$ and let x and y in R .

$$(i). \quad \mu_V(x - y) = T(\mu_V(x - y), \mu_V(x - y))$$

$$\leq T(T(\mu_V(x), \mu_V(y)), T(\mu_V(x), \mu_V(y)))$$

$$= T(T(\mu_V(x), \mu_V(y))) = T(\mu_V(x), \mu_V(y))$$

Therefore, $\mu_V(x - y) \leq T(\mu_V(x), \mu_V(y))$, for all $x, y \in R$

$$(ii). \quad \text{Since } \mu(y + x - y) \leq \mu(x)$$

$$\mu_V(y + x - y) = T(\mu_V(y + x - y), \mu_V(y + x - y))$$

$$\leq T(\mu_V(x), \mu_V(x)) = \mu_V(x)$$

Therefore, $\mu_V(y + x - y) \leq \mu_V(x)$, for all $x, y \in R$

$$(iii). \quad \text{Since } \mu(xy) \leq \mu(x) \text{ and } \mu(xy) \leq \mu(y)$$

$$\mu_V(xy) \leq T(\mu_V(xy), \mu_V(xy)) = T(\mu_V(x), \mu_V(y)) = \mu_V(x)$$

Therefore, $\mu_V(xy) \leq \mu_V(x)$, for all $x, y \in R$

$$(iv). \quad \text{Since } \mu((x + z)y - xy) \leq \mu(z)$$

$$\mu_V((x + z)y - xy) = T(\mu_V((x + z)y - xy), \mu_V((x + z)y - xy))$$

$$\leq T(\mu_V(z), \mu_V(z))$$

$$\leq \mu_V(z)$$

Therefore, $\mu_V((x + z)y - xy) \leq \mu_V(z)$, for all $x, y \in R$

$$(v). \quad \mu_V(x \vee y) = T(\mu_V(x \vee y), \mu_V(x \vee y))$$

$$\leq T(T(\mu_V(x), \mu_V(y)), T(\mu_V(x), \mu_V(y)))$$

$$= T(T(\mu_V(x), \mu_V(y))) = T(\mu_V(x), \mu_V(y))$$

Therefore, $\mu_V(x \vee y) \leq T(\mu_V(x), \mu_V(y))$, for all $x, y \in R$

$$(vi). \quad \mu_V(x \wedge y) = T(\mu_V(x \wedge y), \mu_V(x \wedge y))$$

$$\leq T(T(\mu_V(x), \mu_V(y)), T(\mu_V(x), \mu_V(y)))$$

$$= T(T(\mu_V(x), \mu_V(y))) = T(\mu_V(x), \mu_V(y))$$

Therefore, $\mu_V(x \wedge y) \leq T(\mu_V(x), \mu_V(y))$, for all $x, y \in R$

Thus, the join of a family of T -anti-fuzzy ideal of ℓ -near-ring R is an T -anti-fuzzy ideal of a ℓ -near-ring R .

Theorem: 5

The union of a family of T -anti-fuzzy ideal of ℓ -near-ring R is an T -anti-fuzzy ideal of a ℓ -near-ring R .

Proof:

Let $\{U_\alpha : \alpha \in I\}$ be a family of T -anti-fuzzy ideal of R .

Let $A = \bigcup_{\alpha \in I} U_\alpha$ and Let x, y, z in R .

$$\begin{aligned}
 \text{(i)} \quad \mu_A(x - y) &= \max \{ \mu_A(x - y), \mu_A(x - y) \} \\
 &\leq \max \{ \max \{ \mu_A(x), \mu_A(y) \}, \max \{ \mu_A(x), \mu_A(y) \} \} \\
 &= \max \{ \max \{ \mu_A(x), \mu_A(y) \} \} \\
 &= \max \{ \mu_A(x), \mu_A(y) \}
 \end{aligned}$$

Therefore, $\mu_A(x - y) \leq \max \{ \mu_A(x), \mu_A(y) \}$, for all $x, y \in R$.

$$\begin{aligned}
 \text{(ii)} \quad \text{Since } \mu(y + x - y) &\leq \mu(x) \\
 \mu_A(y + x - y) &= \max \{ \mu_A(y + x - y), \mu_A(y + x - y) \} \\
 &\leq \max \{ \mu_A(x), \mu_A(x) \} \\
 &= \mu_A(x)
 \end{aligned}$$

Therefore, $\mu_A(y + x - y) \leq \mu_A(x)$, for all $x, y \in R$.

$$\begin{aligned}
 \text{(iii)} \quad \text{Since } \mu(xy) &\leq \mu(x) \text{ and } \mu(xy) \leq \mu(y) \\
 \mu_A(xy) &\leq \max \{ \mu_A(xy), \mu_A(xy) \} \\
 &\leq \max \{ \mu_A(x), \mu_A(x) \} \leq \mu_A(x)
 \end{aligned}$$

Therefore, $\mu_A(xy) \leq \mu_A(x)$, for all $x, y \in R$.

$$\begin{aligned}
 \text{(iv)} \quad \text{Since } \mu((x + z)y - xy) &\leq \mu(z) \\
 \mu_A((x + z)y - xy) &= \max \{ \mu_A((x + z)y - xy), \mu_A((x + z)y - xy) \} \\
 &\leq \max \{ \mu_A(z), \mu_A(z) \} \leq \mu_A(z)
 \end{aligned}$$

Therefore, $\mu_A((x + z)y - xy) \leq \mu_A(z)$, for all $x, y \in R$.

$$\begin{aligned}
 \text{(v)} \quad \mu_A(x \vee y) &= \max \{ \mu_A(x \vee y), \mu_A(x \vee y) \} \\
 &\leq \max \{ \max \{ \mu_A(x), \mu_A(y) \}, \max \{ \mu_A(x), \mu_A(y) \} \} \\
 &= \max \{ \max \{ \mu_A(x), \mu_A(y) \} \} \\
 &= \max \{ \mu_A(x), \mu_A(y) \}
 \end{aligned}$$

Therefore, $\mu_A(x \vee y) \leq \max \{ \mu_A(x), \mu_A(y) \}$, for all $x, y \in R$.

$$\begin{aligned}
 \text{(vi)} \quad \mu_A(x \wedge y) &= \max \{ \mu_A(x \wedge y), \mu_A(x \wedge y) \} \\
 &\leq \max \{ \max \{ \mu_A(x), \mu_A(y) \}, \max \{ \mu_A(x), \mu_A(y) \} \} \\
 &= \max \{ \max \{ \mu_A(x), \mu_A(y) \} \} \\
 &= \min \{ \mu_A(x), \mu_A(y) \}
 \end{aligned}$$

Therefore, $\mu_A(x \wedge y) \leq \max \{ \mu_A(x), \mu_A(y) \}$, for all $x, y \in R$.

Thus union of a family of T -anti-fuzzy ideal of ℓ -near-ring R is an T -anti-fuzzy ideal of a ℓ -near-ring R .

REFERENCES

Reference to a journal:

1. Abou-Zaid S. On fuzzy subnear-rings and ideals. *Fuzzy Sets and Systems*. 1991; 44:139-146.
2. Abu Osman MT. On some product of fuzzy subgroups. *Fuzzy Sets and Systems*. 1987; 24: 79-86.
3. Akram M. On T-fuzzy ideals in near-rings. *International Journal of Mathematics and Mathematical Sciences*. 2007; 73514: 1-14.
4. Alam MZ. Fuzzy rings and anti-fuzzy rings with operator. *IOSR Journal of Mathematics*. 2015; 2(4): 48-54.
5. Ayyappan M, Natarajan R. Lattice ordered near rings. *Acta Ciencia Indica*. 2012; 1038(4): 727-738.
6. Biswas R. Fuzzy subgroups and anti fuzzy subgroups. *Fuzzy Sets and System*. 1990; 35(1): 121-124.
7. Booth GL. A note on Γ -near-rings. *Studia Scientiarum Mathematicarum Hungarica*. 1988; 23: 471-475.
8. Coppage WE, Luh J. Radicals of gamma-rings. *Journal of the Mathematical Society of Japan*. 1971; 23(1): 40-52.
9. Dheena P, Mohanraaj G. T-fuzzy ideals in rings. *International Journal of Computational Cognition*. 2011; 9(2): 98-101.
10. Dündar E, Talo Ö. I_2 -convergence of double sequences of fuzzy numbers. *Iranian Journal of Fuzzy Systems*. 2013; 10(3): 37-50.
11. Hassan AAM. On Fuzzy rings and fuzzy homomorphism. *The Journal of Fuzzy Mathematics*. 1999; 7(2): 291-304.
12. Kyung Ho KIM, Young Bae JUN. Anti fuzzy R-subgroups of near-rings. *Scientiae Mathematicae*. 1999; 2(2): 147-153.
13. Kim KH, Jun YB, Yon YH. On anti-fuzzy ideals in Near-Rings. *Iranian Journal of Fuzzy Systems*. 2005; 2(2): 71-80.
14. Mydhily D, Natarajan R. Properties of anti-fuzzy ℓ -subsemiring of an ℓ -semiring. *International Journal of Applied Computational Science and Mathematics*. 2014; 4(2): 209-224.
15. Nobusawa, Nobuo. On a generalization of the ring theory. *Osaka Journal of Mathematics*. 1964; 1(1): 81-89.
16. Prakashmanimaran J, Chellappa B, Jeyakumar M. Some theorem on T-anti fuzzy ideal of ℓ -rings. *International Journal: Mathematical Manuscripts*. 2016; 9(1):151-162.
17. Satyanarayana Bhavanari, Syam Prasad Kuncham. On Fuzzy Cosets of Gamma Nerrings. *Turkish Journal of Mathematics*. 2005; 29:11-22.
18. Schweizer B, Sklar A. Statistical metric spaces. *Pacific Journal of Mathematics*. 1960; 10(1): 313-334.
19. Schweizer B, Sklar A. Associative functions and abstract semigroups. *Publicationes Mathematicae. Debrecen*, 1963; 10: 69-81.
20. Shabir M, Hussain M. Fully Fuzzy Idempotent Nerrings. *Southeast Asian Bulletin of Mathematics*. 2010; 34(5): 959-970.
21. Srinivas T, Nagaiah T, Narasimha Swamy P. Anti fuzzy ideals of Γ -near-rings. *Annals of Fuzzy Mathematics and Informatics*. (2012); 3(2): 255-266.
22. Talo Ö. Some properties of limit inferior and limit superior for sequences of fuzzyreal numbers. *Information Sciences*. 2014; 279: 560-568.
23. Talo Ö, Dündar E. I -Limit Superior and I -Limit Inferior for Sequences of Fuzzy Numbers. *Konuralp Journal of Mathematics*. 2016; 4(2): 1643 172.
24. Wang-jin Liu. Fuzzy invariant subgroups and fuzzy ideals. *Fuzzy Sets and Systems*. 1982; 8(2): 133- 139.
25. Zadeh LA. Fuzzy sets. *Information and Control*. 1965; 8: 338-353.

Reference to a book:

26. Klement EP, Mesiar R, Pap E. Triangular Norms, vol. 8 of Trends in Logic-Studia Logica Library, Kluwer Academic Publishers, Dordrecht, The Netherlands, 2000.