

Fixed Point Theorems for Presic Type Mappings in G_p -Metric Spaces

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Abstract

In this paper, we introduce some fixed point theorems in Presic type mappings on G_p -metric spaces. The present results generalizes various known results in the related literature.

Keywords: Fixed point, G_p -metric space, Contractive mapping

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1 Introduction and Preliminaries

In 1922, Banach [1] established famous fundamental fixed point theorem, also known as Banach contraction principle. The Banach contraction principle is the simplest and one of the most versatile elementary results in fixed point theory. Over the years, various extensions and generalizations of this principle have appeared in the literature. Matthews [2], introduced the partial metric spaces and proved a fixed point theorem on this space. After that several fixed point results have been proved in this space, for more details see [3–7]. In 2006, Mustafa and Sims [8] introduced a new structure called G -metric space as a generalization of the usual metric spaces. Afterwards based on the notion of a G -metric space, many fixed point results for different contractive conditions have been presented, for more details see [9–13]. Recently, based on the two above metric spaces, Zand and Nezhad [14] introduced a new generalized metric spaces G_p as a both generalization of the partial metric space and G -metric spaces. Some of these works may be noted in [15–17].

Now, we mention briefly some fundamental definitions.

Definition 1.1. [14] Let X be a nonempty set and let $G_p : X \times X \times X \rightarrow \mathbb{R}^+$ be a function satisfying the following properties:

- (GP1) $0 \leq G_p(x, x, x) \leq G_p(x, x, y) \leq G_p(x, y, z)$, all $x, y, z \in X$;
- (GP2) $G_p(x, y, z) = G_p(x, z, y) = G_p(y, z, x) \dots$, (symmetry in all three variables);
- (GP3) $G_p(x, y, z) \leq G(x, a, a) + G_p(a, y, z) - G_p(a, a, a)$, for any $a, x, y, z \in X$, (rectangle inequality);
- (GP4) $x = y = z$ if $G_p(x, y, z) = G_p(x, x, x) = G_p(y, y, y) = G_p(z, z, z)$;

Then the pair (X, G_p) is called a G_p -metric space.

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Proposition 1.1. [14] Let (X, G_p) be a G_p -metric space. Then for any x, y, z and $a \in X$ the following relations are true.

- (i) $G_p(x, y, z) \leq G_p(x, x, y) + G_p(x, x, z) - G_p(x, x, x)$;
- (ii) $G_p(x, y, y) \leq 2G_p(x, x, y) - G_p(x, x, x)$;
- (iii) $G_p(x, y, z) \leq G_p(x, a, a) + G_p(y, a, a) + G_p(z, a, a) - 2G_p(a, a, a)$;
- (iv) $G_p(x, y, z) \leq G_p(x, a, z) + G_p(a, y, z) - G_p(a, a, a)$.

Definition 1.2. [14] Let (X, G_p) be a G_p -metric space and a sequence $\{x_n\}$ is called a G_p convergent to $x \in X$ if

$$\lim_{n, m \rightarrow \infty} G_p(x, x_n, x_m) = G_p(x, x, x).$$

A point $x \in X$ is said to be limit point of the sequence $\{x_n\}$ and written $x_n \rightarrow x$.

Thus if $x_n \rightarrow x$ in a G_p metric space (X, G_p) , then for any $\epsilon > 0$, there exists $\ell \in \mathbb{N}$ such that $|G_p(x, x_n, x_m) - G_p(x, x, x)| < \epsilon$, for all $n, m > \ell$.

Proposition 1.2. [14] Let (X, G_p) be a G_p -metric space, then for any sequence $\{x_n\}$ in X , the following are equivalent that

- (i) $\{x_n\}$ is G_p convergent to x ;
- (ii) $G_p(x_n, x_n, x) \rightarrow G_p(x, x, x)$ as $n \rightarrow \infty$;
- (iii) $G_p(x_n, x, x) \rightarrow G_p(x, x, x)$ as $n \rightarrow \infty$.

Definition 1.3. [18]

- (1) The sequence $\{x_n\}_{n \in \mathbb{N}}$ in a G_p -metric space (X, G_p) is said to be a G_p Cauchy sequence if there exists $r \in \mathbb{R}$ such that $\lim_{n, m \rightarrow \infty} G_p(x_n, x_m, x_m) = r$.
- (2) (X, G_p) is said to be G_p -complete if for every G_p Cauchy sequence $\{x_n\}_{n \in \mathbb{N}}$ there exists $x \in X$ such that

$$\lim_{n, m \rightarrow \infty} G_p(x_n, x_m, x_m) = \lim_{n, m \rightarrow \infty} G_p(x_n, x_m, x) = G_p(x, x, x).$$

Lemma 1.1. [15] Let (X, G_p) be a G_p -metric space. Then

- (i) If $G_p(x, y, z) = 0$ then $x = y = z$,
- (ii) If $x \neq y$ then $G_p(x, y, y) > 0$.

Proposition 1.3. [14] Every G_p -metric space (X, G_p) defines a metric space (X, d_{G_p}) as follows:
 $d_{G_p(x, y)} = G_p(x, y, y) + G_p(y, x, x) - G_p(x, x, x) - G_p(y, y, y)$, for all $x, y \in X$.

2 Main Results

Considering the convergence of certain sequences S. B. Presic [19] generalized Banach contraction principle as follows:

Theorem 2.1. [19] Let (X, d) be a complete metric space, k a positive integer and $T : X^k \rightarrow X$ a mapping satisfying the following contractive type condition

$$d(T(x_1, x_2, \dots, x_k), T(x_2, x_3, \dots, x_{k+1})) \leq q_1 d(x_1, x_2) + q_2 d(x_2, x_3) + \dots + q_k d(x_k, x_{k+1}) \quad (2.1)$$

for every $x_1, x_2, \dots, x_{k+1}$ in X where $q_1, q_2, \dots, q_k$ are non negative constants such that $q_1 + q_2 + \dots + q_k < 1$. Then there exist a point x in X such that $T(x, x, \dots, x) = x$. Moreover, if $x_1, x_2, \dots, x_k$, are arbitrary points in X and for $n \in N$,

$$x_{n+k} = T(x_n, x_{n+1}, \dots, x_{n+k-1}) \quad (n = 1, 2, \dots)$$

then the sequence $\{x_n\}_{n=1}^{\infty}$ is convergent and

$$\lim x_n = T(\lim x_n, \lim x_n, \dots, \lim x_n).$$

Remark that condition (2.1) in the case $k = 1$ reduces to the well-known Banach contraction mapping principle. So, Theorem 2.1 is a generalization of the Banach fixed point theorem.

Ćirić and Presic [20] generalized Theorem 2.1 as follows:

Theorem 2.2. [20] Let (X, d) be a complete metric space, k a positive integer and $T : X^k \rightarrow X$ a mapping satisfying the following contractive type condition

$$d(T(x_1, x_2, \dots, x_k), T(x_2, x_3, \dots, x_{k+1})) \leq \lambda \max_{1 \leq i \leq k} \{d(x_i, x_{i+1})\} \quad (2.2)$$

where $\lambda \in (0, 1)$ is constant and $x_1, x_2, \dots, x_{k+1}$ in X . Then there exist a point x in X such that $T(x, x, \dots, x) = x$. Moreover, if $x_1, x_2, \dots, x_k$, are arbitrary points in X and for $n \in N$,

$$x_{n+k} = T(x_n, x_{n+1}, \dots, x_{n+k-1}) \quad (n = 1, 2, \dots)$$

then the sequence $\{x_n\}_{n=1}^{\infty}$ is convergent and

$$\lim x_n = T(\lim x_n, \lim x_n, \dots, \lim x_n).$$

If in addition we suppose that on a diagonal $\Delta \subset X^k$

$$d(T(u, u, \dots, u), T(v, v, \dots, v)) < d(u, v) \quad (2.3)$$

holds for all $u, v \in X$, with $u \neq v$, then x is the unique point in X with $T(x, x, \dots, x) = x$.

Nazir and Abbas [21], proved common fixed point theorems of Presic type in partial metric space. Also Dhasmana [22] showed a unique common fixed point theorem is obtained in settings of G -metric spaces by using the concept of Presic fixed point theorem. Further, Gairola and Dhasmana [23] proved common fixed point theorems of Presic type in G -metric space which extends the result of Ćirić-Presic [20], Dhasmana [22] and George-Khan [24].

We will carry this idea to G_p -metric spaces, which is a generalization of partial metric spaces.

Theorem 2.3. Let (X, G_p) be complete G_p -metric spaces, k a positive integer and $T : X^k \rightarrow X$ a mapping satisfying the following contractive type condition

$$G_p(T(x_1, x_2, \dots, x_k), T(x_2, x_3, \dots, x_{k+1}), T(x_3, x_4, \dots, x_{k+2})) \leq \lambda \max_{1 \leq i \leq k} \{G_p(x_i, x_{i+1}, x_{i+2})\} \quad (2.4)$$

where $\lambda \in (0, 1)$ is constant and $x_1, x_2, \dots, x_k$, are arbitrary elements in X . Then there exists a point x in X such that $T(x, x, \dots, x) = x$. Moreover, if $x_1, x_2, \dots, x_{k+2}$ are arbitrary points in X and $n \in N$,

$$x_{n+k} = T(x_n, x_{n+1}, \dots, x_{n+k-1}) \quad (n = 1, 2, \dots) \quad (2.5)$$

then the sequence $\{x_n\}_{n=1}^{\infty}$ is convergent and

$$\lim_{n \rightarrow \infty} x_n = T(\lim x_n, \lim x_n, \dots, \lim x_n).$$

If in addition we suppose that

$$G_p(T(u, u, \dots, u), T(v, v, \dots, v), T(w, w, \dots, w)) < G_p(u, v, w) \quad (2.6)$$

holds for all $u, v, w \in X$, with $u \neq v \neq w$, then x is the unique point in X with $T(x, x, \dots, x) = x$.

Proof. $x_1, x_2, \dots, x_k$, be k arbitrary in X . Using these points define a sequence (x_n) as follows:

$$x_{n+k} = T(x_n, x_{n+1}, \dots, x_{n+k-1}), \quad (n = 1, 2, \dots).$$

For simplicity set $\gamma_n = G_p(x_n, x_{n+1}, x_{n+2})$. We shall prove by induction that for each $n \in \mathbb{N}$;

$$\gamma_n \leq M\theta^n \quad (2.7)$$

where $\theta = \lambda^{\frac{1}{k}}$ and $M = \max\{\frac{\gamma_1}{\theta}, \frac{\gamma_2}{\theta^2}, \dots, \frac{\gamma_k}{\theta^k}\}$. According to the definition of M we can writing for $n = 1, 2, \dots, k$

$$\gamma_n \leq M\theta^n, \quad \gamma_{n+1} \leq M\theta^{n+1}, \dots, \gamma_{n+k-1} \leq M\theta^{n+k-1}.$$

Then we have:

$$\begin{aligned} \gamma_{n+k} &= G_p(x_{n+k}, x_{n+k+1}, x_{n+k+2}) \\ &= G_p(T(x_n, x_{n+1}, \dots, x_{n+k-1}), T(x_{n+1}, x_{n+2}, \dots, x_{n+k}), T(x_{n+2}, x_{n+3}, \dots, x_{n+k+1})). \end{aligned}$$

By (2.4)

$$\begin{aligned} \gamma_{n+k} &= G_p(T(x_n, x_{n+1}, \dots, x_{n+k-1}), T(x_{n+1}, x_{n+2}, \dots, x_{n+k}), T(x_{n+2}, x_{n+3}, \dots, x_{n+k+1})) \\ &\leq \lambda \max\{\gamma_n, \gamma_{n+1}, \gamma_{n+2}, \dots, \gamma_{n+k-1}\} \\ &\leq \lambda \max\{M\theta^n, M\theta^{n+1}, \dots, M\theta^{n+k-1}\} \end{aligned}$$

as $\theta = \lambda^{\frac{1}{k}}$

$$\begin{aligned} \gamma_{n+k} &\leq \lambda M\theta^n \quad (\text{as } 0 < \theta < 1) \\ &= M\theta^{n+k} \end{aligned}$$

and the inductive proof of (2.7) is complete. Next using (2.7) for any $n, m \in \mathbb{N}$ we have the following argument:

$$\begin{aligned} G_p(x_n, x_m, x_m) &\leq G_p(x_n, x_{n+1}, x_{n+1}) + G_p(x_{n+1}, x_{n+2}, x_{n+2}) + \dots \\ &\quad + G_p(x_{m-1}, x_m, x_m) - \{G_p(x_{n+1}, x_{n+1}, x_{n+1}) + G_p(x_{n+2}, x_{n+2}, x_{n+2}) + \dots \\ &\quad \dots + G_p(x_{m-1}, x_{m-1}, x_{m-1})\} \\ &\leq G_p(x_n, x_{n+1}, x_{n+2}) + G_p(x_{n+1}, x_{n+2}, x_{n+3}) + \dots + G_p(x_{m-2}, x_{m-1}, x_m) \\ &= \gamma_n + \gamma_{n+1} + \dots + \gamma_{m-2} \\ &\leq M\theta^n + M\theta^{n+1} + \dots + M\theta^{m-2} \\ &\leq \frac{M\theta^n}{1-\theta} \end{aligned}$$

by which we conclude that (x_n) is a G_p Cauchy sequence. Since (X, G_p) is complete G_p -metric space, there exists $x \in X$ such that $\{x_n\}$ sequence converges $x \in X$. So,

$$\lim_{n, m \rightarrow \infty} G_p(x_n, x_m, x_m) = \lim_{n, m \rightarrow \infty} G_p(x_n, x_m, x) = G_p(x, x, x) = 0.$$

Then for any integer n we have

$$\begin{aligned}
 G_p(x_{n+k}, x_{n+k}, T(x, x, \dots, x)) &= G_p(T(x, x, \dots, x), T(x_n, x_{n+1}, \dots, x_{n+k-1}), T(x_n, x_{n+1}, \dots, x_{n+k-1})) \\
 &\leq G_p(T(x, x, \dots, x), T(x, \dots, x, x_n), T(x, \dots, x, x_n)) + \\
 &\quad G_p(T(x, \dots, x, x_n), T(x, \dots, x, x_n, x_{n+1}), T(x, \dots, x, x_n, x_{n+1})) + \\
 &\quad G_p(T(x, \dots, x, x_n, x_{n+1}), T(x, \dots, x, x_n, x_{n+1}, x_{n+2}), T(x, \dots, x, x_n, x_{n+1}, x_{n+2})) \\
 &\quad + \dots + G_p(T(x, x_n, \dots, x_{n+k-2}), T(x_n, x_{n+1}, \dots, x_{n+k-1}), T(x_n, x_{n+1}, \dots, x_{n+k-1})) \\
 &\quad - \{G_p(T(x, \dots, x, x_n), T(x, \dots, x, x_n), T(x, \dots, x, x_n)) + \\
 &\quad G_p(T(x, \dots, x, x_n, x_{n+1}), T(x, \dots, x, x_n, x_{n+1}), T(x, \dots, x, x_n, x_{n+1})) + \\
 &\quad \dots + G_p(T(x, x_n, \dots, x_{n+k-2}), T(x, x_n, \dots, x_{n+k-2}), T(x, x_n, \dots, x_{n+k-2}))\} \\
 &\leq G_p(T(x, x, \dots, x), T(x, \dots, x, x_n), T(x, \dots, x, x_n)) + \\
 &\quad G_p(T(x, \dots, x, x_n), T(x, \dots, x, x_n, x_{n+1}), T(x, \dots, x, x_n, x_{n+1})) + \\
 &\quad G_p(T(x, \dots, x, x_n, x_{n+1}), T(x, \dots, x, x_n, x_{n+1}, x_{n+2}), T(x, \dots, x, x_n, x_{n+1}, x_{n+2})) \\
 &\quad + \dots + G_p(T(x, x_n, \dots, x_{n+k-2}), T(x_n, x_{n+1}, \dots, x_{n+k-1}), T(x_n, x_{n+1}, \dots, x_{n+k-1})) \\
 &\leq \lambda \max\{G_p(x, x, x), G_p(x, x_n, x_n)\} + \lambda \max\{G_p(x, x, x), G_p(x, x_n, x_n), G_p(x_n, x_{n+1}, x_{n+1})\} + \\
 &\quad + \dots + \lambda \max\{G_p(x, x, x), G_p(x, x_n, x_n), G_p(x_n, x_{n+1}, x_{n+1}), \dots, G_p(x_{n+k-2}, x_{n+k-1}, x_{n+k-1})\}.
 \end{aligned}$$

Taking the limit when n tends to infinity we obtain

$$G_p(x, x, T(x, x, \dots, x)) \leq \lambda G_p(x, x, x)$$

that is,

$$G_p(x, x, T(x, x, \dots, x)) \leq 0$$

which implies

$$T(x, x, \dots, x) = x.$$

Thus we proved that;

$$\lim x_n = T(\lim x_n, \lim x_n, \dots, \lim x_n).$$

Now suppose that (2.6) holds. To prove the uniqueness of the fixed point, let us assume that for some $y, z \in X$, $x \neq y \neq z$ we have

$$T(y, y, \dots, y) = y, \quad T(z, z, \dots, z) = z$$

Then by (2.6),

$$G_p(x, y, z) = G_p(T(x, x, \dots, x), T(y, y, \dots, y), T(z, z, \dots, z)) < G_p(x, y, z), \quad (2.8)$$

which is a contraction. So, x is the unique point in X such that $T(x, x, \dots, x) = x$. $\square$

Example 2.4. Let $X = [0, 2]$ and $G_p : X \times X \times X \rightarrow \mathbb{R}^+$ defined by

$$G_p(x, y, z) = \begin{cases} |x - y| + |y - z| + |x - z|, & \text{if } x, y, z \in [0, 1) \\ \max\{x, y, z\}, & \text{otherwise} \end{cases}$$

(X, G_p) is a complete G_p metric space. Let $k \in \mathbb{Z}^+$ and $T : X^k \rightarrow X$ be the mapping defined by

$$T(x_1, x_2, \dots, x_k) = \begin{cases} \frac{x_1 + x_k}{4^k}, & \text{if } x_1, x_2, \dots, x_k \in [0, 1) \\ 0, & \text{otherwise} \end{cases}$$

Now $x_1, x_2, \dots, x_k, x_{k+1}, x_{k+2} \in [0, 1)$ and $\lambda = \frac{1}{2}$. Thus, we obtain

$$\begin{aligned}
 & G_p(T(x_1, x_2, \dots, x_k), T(x_2, x_3, \dots, x_{k+1}), T(x_3, x_4, \dots, x_{k+2})) \\
 &= \left| \frac{x_1 + x_k}{4k} - \frac{x_2 + x_{k+1}}{4k} \right| + \left| \frac{x_2 + x_{k+1}}{4k} - \frac{x_3 + x_{k+2}}{4k} \right| + \left| \frac{x_1 + x_k}{4k} - \frac{x_3 + x_{k+2}}{4k} \right| \\
 &\leq \frac{1}{4k} [|x_1 - x_2| + |x_2 - x_3| + |x_1 - x_3| + |x_k - x_{k+1}| + |x_{k+1} - x_{k+2}| + |x_k - x_{k+2}|] \\
 &= \frac{1}{4k} |G_p(x_1, x_2, x_3) + G_p(x_k, x_{k+1}, x_{k+2})| \\
 &\leq \frac{1}{2k} \max\{G_p(x_1, x_2, x_3), G_p(x_k, x_{k+1}, x_{k+2})\} \\
 &\leq \frac{1}{2k} \max_{1 \leq i \leq k} \{G_p(x_i, x_{i+1}, x_{i+2})\} \\
 &\leq \frac{1}{2} \max_{1 \leq i \leq k} \{G_p(x_i, x_{i+1}, x_{i+2})\} \\
 &= \lambda \max_{1 \leq i \leq k} \{G_p(x_i, x_{i+1}, x_{i+2})\}.
 \end{aligned}$$

If $x_1, x_2, \dots, x_k \in [0, 1)$ and $x_{k+1}, x_{k+2} \in [1, 2]$ then

$$\begin{aligned}
 G_p(T(x_1, x_2, \dots, x_k), T(x_2, x_3, \dots, x_{k+1}), T(x_3, x_4, \dots, x_{k+2}))) &= \frac{x_1 + x_k}{4k} \\
 &\leq \frac{1}{2k} x_k \\
 &\leq \frac{1}{2} \max_{1 \leq i \leq k} \{G_p(x_i, x_{i+1}, x_{i+2})\} \\
 &= \lambda \max_{1 \leq i \leq k} \{G_p(x_i, x_{i+1}, x_{i+2})\}.
 \end{aligned}$$

If $x_1, x_2, \dots, x_k, x_{k+1} \in [0, 1)$ and $x_{k+2} \in [1, 2]$ then

$$\begin{aligned}
 G_p(T(x_1, x_2, \dots, x_k), T(x_2, x_3, \dots, x_{k+1}), T(x_3, x_4, \dots, x_{k+2}))) &= \left| \frac{x_1 + x_k}{4k} - \frac{x_2 + x_{k+1}}{4k} \right| \\
 &\leq \frac{1}{2k} \max\{x_k, x_{k+1}\} \\
 &\leq \frac{1}{2} \max_{1 \leq i \leq k} \{G_p(x_i, x_{i+1}, x_{i+2})\} \\
 &= \lambda \max_{1 \leq i \leq k} \{G_p(x_i, x_{i+1}, x_{i+2})\}.
 \end{aligned}$$

Similarly, if $x_1, x_2, \dots, x_k, x_{k+2} \in [0, 1)$ and $x_{k+1} \in [1, 2]$ then

$$\begin{aligned}
 G_p(T(x_1, x_2, \dots, x_k), T(x_2, x_3, \dots, x_{k+1}), T(x_3, x_4, \dots, x_{k+2}))) &= \left| \frac{x_1 + x_k}{4k} - \frac{x_3 + x_{k+2}}{4k} \right| \\
 &\leq \frac{1}{2k} \max\{x_k, x_{k+2}\} \\
 &\leq \frac{1}{2} \max_{1 \leq i \leq k} \{G_p(x_i, x_{i+1}, x_{i+2})\} \\
 &= \lambda \max_{1 \leq i \leq k} \{G_p(x_i, x_{i+1}, x_{i+2})\}.
 \end{aligned}$$

When some $x_j \in [1, 2]$ and $x_1, x_2, \dots, x_{j-1}, x_{j+1}, \dots, x_k, x_{k+1}, x_{k+2} \in [0, 1)$ or $x_1, x_2, \dots, x_k, x_{k+1}, x_{k+2} \in [1, 2]$ then we obtain

$$\begin{aligned}
 G_p(T(x_1, x_2, \dots, x_k), T(x_2, x_3, \dots, x_{k+1}), T(x_3, x_4, \dots, x_{k+2}))) &= 0 \\
 &\leq \lambda \max_{1 \leq i \leq k} \{G_p(x_i, x_{i+1}, x_{i+2})\}.
 \end{aligned}$$

Thus T satisfies (2.4) with $\lambda = \frac{1}{2}$ and we have $T(x, x, \dots, x) = x$. Moreover for all $x, y, z \in X$ with $x \neq y \neq z$

$$G_p(T(x, x, \dots, x), T(y, y, \dots, y), T(z, z, \dots, z)) < G_p(x, y, z).$$

Thus all required hypotheses of Theorem (2.3) are satisfied. Furthermore, for any arbitrary points $x_1, x_2, \dots, x_k \in X$, the sequence (x_n) defined by (2.5) converges to $x = 0$, the unique fixed point of T .

Corollary 2.5. Let (X, G_p) be complete G_p -metric spaces, $k \in \mathbb{Z}^+$ and $T : X^k \rightarrow X$ a mapping satisfying the following contractive type condition

$$G_p(T(x_1, x_2, \dots, x_k), T(x_2, x_3, \dots, x_{k+1}), T(x_3, x_4, \dots, x_{k+2})) \leq \sum_{i=1}^k \lambda_i G_p(x_i, x_{i+1}, x_{i+2}) \quad (2.9)$$

where $\lambda_1, \lambda_2, \dots, \lambda_k$ are non-negative constants, $\sum_{i=1}^k \lambda_i \in (0, 1)$ and $x_1, x_2, \dots, x_k$, are arbitrary elements in X . Then there exists a point x in X such that $T(x, x, \dots, x) = x$. Moreover, if $x_1, x_2, \dots, x_{k+2}$ are arbitrary points in X and $n \in \mathbb{N}$,

$$x_{n+k} = T(x_n, x_{n+1}, \dots, x_{n+k-1}) \quad (n = 1, 2, \dots) \quad (2.10)$$

then the sequence $\{x_n\}_{n=1}^\infty$ is convergent and

$$\lim_{n \rightarrow \infty} x_n = T(\lim x_n, \lim x_n, \dots, \lim x_n).$$

If in addition we suppose that

$$G_p(T(u, u, \dots, u), T(v, v, \dots, v), T(w, w, \dots, w)) < G_p(u, v, w) \quad (2.11)$$

holds for all $u, v, w \in X$, with $u \neq v \neq w$, then x is the unique point in X with $T(x, x, \dots, x) = x$.

Remark 2.1. Theorem 2.3 is generalization of corollary 2.5, as the condition (2.9) implies the conditions (2.4) and (2.6). Actually,

$$\lambda_1 G_p(x_1, x_2, x_3) + \lambda_2 G_p(x_2, x_3, x_4) + \dots + \lambda_k G_p(x_i, x_{i+1}, x_{i+2}) \quad (2.12)$$

$$\leq (\lambda_1 + \lambda_2 + \dots + \lambda_k) \max_{1 \leq i \leq k} \{G_p(x_i, x_{i+1}, x_{i+2})\} \quad (2.13)$$

and $\lambda_1 + \lambda_2 + \dots + \lambda_k < 1$. Beside, for any $u, v, w \in X$, with $u \neq v \neq w$, from (2.9) we have

$$\begin{aligned} G_p(T(u, u, \dots, u), T(v, v, \dots, v), T(w, w, \dots, w)) &\leq G_p(T(u, u, \dots, u), T(u, u, \dots, u, v), T(u, u, \dots, u, v)) + \\ &G_p(T(u, u, \dots, u, v), T(u, u, \dots, u, v, v), T(u, u, \dots, u, v, v)) + \dots + \\ &G_p(T(v, w, \dots, w), T(w, w, \dots, w), T(w, w, \dots, w)) - \\ &\{G_p(T(u, u, \dots, u, v), T(u, u, \dots, u, v), T(u, u, \dots, u, v)) + \\ &G_p(T(u, u, \dots, u, v, v), T(u, u, \dots, u, v, v), T(u, u, \dots, u, v, v)) + \dots + \\ &G_p(T(u, w, \dots, w), T(u, w, \dots, w), T(u, w, \dots, w))\} \\ &\leq G_p(T(u, u, \dots, u), T(u, u, \dots, u, v), T(u, u, \dots, u, v)) + \\ &G_p(T(u, u, \dots, u, v), T(u, u, \dots, u, v, v), T(u, u, \dots, u, v, v)) + \dots + \\ &G_p(T(v, w, \dots, w), T(w, w, \dots, w), T(w, w, \dots, w)) \\ &\leq \lambda_1 G_p(u, v, w) + \lambda_2 G_p(u, v, w) + \dots + \lambda_k G_p(u, v, w) \\ &= (\lambda_1 + \lambda_2 + \dots + \lambda_k) G_p(u, v, w) < G_p(u, v, w) \end{aligned}$$

and, as a result, (2.9) implies (2.6).

3 Conclusion

Nazir and Abbas [21], proved common fixed point theorems of Presic type in partial metric space. Further Dhasmana [22] showed fixed point theorem by using Presic type mapping in G -metric spaces. Our works generalizes several similar results in the literature.

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Competing Interest

The authors declare that no competing interests exist.

References

- [1] Banach, S. (1922). Sur les opérations dans les ensembles abstraits et leur application aux équations intégrales. *Fund Math.*, 3, 133-181.
- [2] Matthews, S.G. (1992). Partial metric spaces topology. Research Reports 21, Dept. of Computer Science, University of Warwick.
- [3] Matthews, S.G. (1994). Partial metric topology. in Proc., 8th Summer Conference on General Topology and Applications, *Annals of the New York Academy of Sciences*, 728, 183-197.
- [4] Aydi, H. Abbas and M. Vetro, C. (2012). Partial Hausdorff metric and Nadler's fixed point theorem on partial metric spaces. *Topol. Appl.*, 159, 3234-3242.
- [5] Shatanawia, W. and Nashine, H.K. (2012). A generalization of Banach's contraction principle for nonlinear contraction in a partial metric space. *J. Nonlinear Sci. Appl.*, 5, 37-43.
- [6] Oltra, S. and Valero, O. (2004). Banach's fixed point theorem for partial metric spaces. *Rendiconti dell'Istituto di Matematica dell'Università di Trieste*, 36, (1-2), 17-26.
- [7] Thanh, T.D., Karapinar, E. and Chi, K.P. (2012). A generalized contraction principle in partial metric spaces. *Mathematical and Computer Modelling*, 55, 1673-1681.
- [8] Mustafa, Z. and Sims, B. (2006). A new approach to a generalized metric spaces. *J. Nonlinear Convex Anal.*, 7, (2), 289-297.
- [9] Abbas, M., Khan, A.R. and Nazir, T. (2011). Coupled common fixed point results in two generalized metric spaces. *Applied Mathematics and Computation*, vol. 217, no. 13, pp. 6328-6336.
- [10] Abbas, M., Nazir, T. and Doric, D. (2012). Common fixed point of mappings satisfying (E.A) property in generalized metric spaces. *Applied Mathematics and Computation*, vol. 218, no. 14, pp. 7665-7670.
- [11] Mustafa, Z., Khandaqji, M. and Shatanawi, W. (2011). Fixed point results on complete G -metric spaces. *Studia Scientiarum Mathematicarum Hungarica* 48, 304-319.
- [12] Mustafa, Z., Shatanawi, W. and Bataineh, M. (2009). Existence of fixed point results in G -metric spaces. *Int. J. Math. Math. Sci.*, Article ID 283028, page 10.
- [13] Mustafa, Z. and Obiedat, H. (2010). A fixed point theorem of Reich in G -metric spaces. *CUBO* 12, (1), 83-93.

- [14] Zand, M.R.A. and Nezhad, A.D. (2011). A generalization of partial metric spaces. Journal of Contemporary Applied Mathematics, 24, 86-93.
- [15] Aydi, H., Karapınar, E. and Salimi, P. (2012). Some fixed point results in G_p -metric spaces. Journal of Applied Mathematics., 2012, Article ID 891713, page 15.
- [16] Popa, V. and Patriciu, A. M. (2015). Two general fixed point theorems for a sequence of mappings satisfying implicit relations in G_p -metric spaces. Appl. Gen. Topol. 16, no. (2), 225-231.
- [17] Eke, K.S. (2015). Some fixed and coincidence point results for expansive mappings on G_p -metric spaces. Adv. Fixed Point Theory, 5, no. 4, 369-386.
- [18] Gajic, L., Kadelburg, Z. and Radenovic, S. (2017). G_p -metric spaces-symmetric and asymmetric. Scientific Publications of the State University of Novi Pazar Ser. A: Appl. Math. Inform. and Mech., vol. 9, 1, 37-46.
- [19] Presic, S. B. (1965). Sur une classe din equations aux difference finite et. sur la convergence de certains suites. Publ. de LInst. Math. Belgrade, 5, (19), 75-78.
- [20] Ćirić, L. B. and Presic, S. B. (2007). On presic type generalization of the Banach contracton mapping principle. Acta Math. Univ. Comenianae vol. LXXVI, 2, pp. 143-147.
- [21] Nazır, T. and Abbas, M. (2014). Common fixed point of Presic type contraction mappings in partial metric spaces. Journal of Nonlinear Analysis and Optimization, vol. 5, no. 49-55.
- [22] Dhasmana, N. (2015). A fixed point theorem of Presic type in G-metric spaces. International J. Math. Arcive, 6, (2), 11-14.
- [23] Gairola U.C. and Dhasmana, N. (2015). A fixed point theorem of Presic type for a pair of maps in G-metric spaces. International J. Math. Arcive, 6, (3), 196-200.
- [24] Gairola, U.C. and Khan, M. S. (2011). On Presic type extension of Banach contraction principle. Int. J. Math. Analysis, 5, (21), 1019-1024.