

Original Research Article

A Financial Prey-Predator model with infection in the predator

ABSTRACT

A modified predator-prey model is proposed with logistic growth in both prey and predator populations and an infection in the predator population. This model uses ideas from the original biological Lotka-Volterra model in an attempt to imitate financial predation. Potential investors, the prey, interact with financial experts, the predators, who provide advice on purchasing financial instruments and investments. Some of these experts are honest while others are 'infected', in that, a portion of them attempt to deceive clients into making irrational investments for their own benefit, incurring losses to the client. Stability analysis is performed and the dynamics of the system are investigated using analytical and numerical simulations. The study shows that varying parameter values can affect whether the system remains stable or experiences Hopf bifurcations. This mathematical model is used in an attempt to determine cases where the potential investors can co-exist with financial experts, under various investor persuasion rates.

Aims: To determine regions of stability for the model by varying parameter values in within simulated datasets.

Study design: Stability Analysis – Analytical and Numerical.

Methodology: The model is first studied analytically by solving for the positive, interior equilibrium point (once it exists) when the differential equations are solved simultaneously. Stability conditions are defined based on these results using the Routh – Hurwitz criteria. These results are verified numerically in MATLAB as an initial value problem, integrating with slight perturbation around the equilibrium point. Each parameter in the dataset is varied one at a time. The regions where the model remains stable for a particular parameter are recorded. The Hopf bifurcation points, where there are stability changes, are also noted and graphical simulations are produced using time series and 3 Dimensional plots for particular stable and unstable scenarios. The effect of three different pairs of financial expert - investor persuasion rates on the populations of investors, is also shown.

Results: Hopf bifurcation parameters for three simulated datasets were found to be associated with the investment persuasion rate, investor-expert interaction rates, expert interaction rate, expert interference constant and growth rate of experts. In particular the population of potential investors increased drastically when the investor persuasion rate for dishonest investors was much greater than that for honest investors.

Conclusion: Variation of parameter values in the model allow for the analysis of model stability conditions. These enhance the creation of detection mechanisms to control the 'infection' of investment fraud among financial experts.

Keywords: prey-predator; financial predation; fraud; stability; bifurcations

1. INTRODUCTION

Financial predation plays a large role in the business eco-system. It uses the idea of a traditional prey-predator model in which the predator attacks a prey food source. In the financial world, a predator has the ability to prey on finances in order to gain profit from investments. The predators are invariably multi-million, if not multi-billion dollar companies, and preys are the average investor or financial institutions that represent a group of average investors (Mesly 2012b). The Great Depression of 1929 and the financial recession crisis in 2008 have both inspired financial experts and mathematicians alike to study models which can assist in prediction of such anomaly events.

There are a variety of factors which lead to financial anomalies including mismanagement of funds, as well as fraud. Hellwig (2009) indicated that financial crises are caused by lack of proper risk assessment on the part of financial institutions. In addition to this, the ideas of perceived predation enables these financial players to hide their agendas because they assume that a negative image leads to increased perceived risk (Bergeron and Laroche 2009). This is the technique used by some financial experts to mislead and therefore gain the trust of potential investors.

Consequently individual, as well as corporate investors, can be victims of financial fraud. The social interactions of key players in the financial world are similar to prey and predatory interactions in Biology. The idea of perceived predation also implies that some market investor agents identify key players in the financial world as individuals willing to take advantage of naive investors, in order to serve their own benefit, causing them financial losses, by surprise (Mesly 2012a). This motivates the behavior of financial experts: they can choose to evaluate the market honestly or to be blatantly dishonest in their assessments. Some may even do both, in different situations.

Mathematical models have been used in financial systems to investigate the dynamics within recent times. Mesly (2013) used a mathematical model of financial predation in order to determine whether financial predators can be detected before they commit fraud. He measured four elements: essential resources, nonessential resources, work and knowledge, and used these to generate a predatory curve to detect abnormal behaviors. These would possibly indicate the presence of financial predators. The implication for the average or wealthy investor is obvious: detecting the predator (before he can act) may mean saving thousands if not millions of dollars (Mesly 2013).

As a result, different methods have been employed to measure this form of predation using variables which measure the social behaviors of the financial predators to their victims. Mesly (2012b) used factorial analysis, cluster analysis, structural equation modelling and multiple regression techniques to identify psychological factors, such as trust and wealth, and how these affect the predation process. In addition to these social behaviors, financial predators have also developed and utilized fraudulent models and schemes to gain money from unsuspecting investors.

One such scheme is the Ponzi scheme, devised by Charles Ponzi in 1920. It is an investment operation based on fraud. New investors are lured by unusually high interest rates and the fabrication of plausible investment projects (Zhu, et al. 2017). One of the most popular examples within recent times occurred in 2008 when then chairman of the NASDAQ Stock Exchange, Bernard Madoff headed a multibillion-dollar Ponzi scheme. The principle of the scheme is to entice potential investors with the promise of a high rate of return, which is impossible to deliver, but can only be paid through money from new investors to the ones already in the fund (Artzrouni 2009).

The scheme is one which is spread like a social infection. Rantala (2015) compared the growth of Ponzi schemes to the spread of infectious diseases and showed that social contagion encourages the epidemic spread of investment ideas. This was also previously discussed by (Shiller 2000) who also proposed that these social dynamics inflate asset prices. Economic systems are neither machineries nor mechanistic linear systems but complex non-linear dynamic systems of systems as are ecosystems (Bricage 2015). This allows for the use of biological models in an attempt to partially explain the dynamics of these complex systems.

Applications of the behavior of infectious diseases within a population have been the focus of many biological models. (Anderson and May 1978) were instrumental in formulating a predator-prey model with infection of the prey species. Their work was based on the original Lotka - Volterra (Lotka 1925, Volterra 1926) model for predator-prey competing species, in addition to the model for infectious disease transmission originally studied by (Kermack and Mc Kendrick 1927). The latter model was known as the SIR (Susceptible-Infected-Removed) compartmental model. The idea here was that a species contains individuals susceptible to a disease, infected with the disease and those who were recovered or died from the infection. Over the years there have been variations to the original form.

According to Pada Das (2011), the influence of predation on epidemics has not yet been studied considerably. Therefore, it is worth exploring some of the work in this area. Although Chattopadhyay and Arino (1999), Han et al. (2001), Xiao and Chen (2001) have explored the effects of predation on disease models, the application of this particular modified model to financial predation is a fairly new area. There is very little work done using the Lotka-Volterra predator – prey model, with the presence of infection, to assess the stability of the financial eco-system with the “disease” of fraud. This idea is one which motivates our study.

Biologically, prey populations influence the size of predator populations and vice versa. Wilcox (1999) also discussed the link between biological predator-prey models and the stock market as an eco-system with the idea that the dynamics of the former assists in the understanding of the stability of the market. The analogy between investment approaches and species makes it clear that one does not want the analogies to famines and parasitic diseases caused by overpopulation within a particular investment approach (Wilcox 1999). This is also a motivating factor in our study.

In this work, potential investors are the financial representation of prey. Financial experts, the predators, are divided into two classes: honest and dishonest. Unlike in biology, entrepreneurs can choose whether to enter as predator or prey (Mehlum et al. 2003). Using this idea, financial experts can decide whether to be honest or dishonest. Dishonest experts are ‘diseased’ and involved in fraudulent schemes, which enables them to deceive potential investors. This is analogous to an ‘infection’ according to biological epidemic models. These experts interact amongst themselves spreading the ‘infection’ where possible to other experts. The dynamics of these interactions, as well as those potential investors, are investigated mathematically using a modified prey-predator model.

Hence, the goal of this paper is to assess the stability of the model with respect to the number of potential investors and financial experts. In this way, one can find the optimal parameters in which they either co-exist or become ‘extinct’ in investment situations. Analyzing attractor types and the transitions between them as significant model parameters change, that is, studying the model’s bifurcations, is quite helpful in understanding regime shifts of stability (Colon et al. 2015). Thus, this work is not meant to replace existing financial or social models, but is a way of showing a novel approach to analyzing these behaviors using stability analysis.

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121 The paper first provides a brief introduction to the different models in Section 1. Section 2
122 describes the model with a brief overview of the model assumptions, methodology with an
123 outline of the analytical analysis for obtaining the positive interior equilibrium point. Next, the
124 stability of this point and Hopf bifurcation is examined. Then, in section 3 numerical
125 simulations are provided with graphical interpretations of the results, as well as a discussion
126 of the findings. Finally, section 4 outlines conclusions, future work and recommendations
127 based on this work.

128 129 **2. METHODOLOGY**

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131 The model used is a modified version of the original Lotka- Volterra model. Potential
132 investors and financial experts represent the prey (X) and predator (Y) populations
133 respectively. Financial experts describes brokers, agents, bankers or any relevant persons
134 involved in the investment industry. They are divided into susceptible (Y_1) and infected (Y_2)
135 groups. The infection here is the “disease” of fraud toward the potential investors. The model
136 takes the form of a modified version of the prey-predator model proposed by (Haque 2010)
137 in which there is a transmissible disease among the predator population.

138
139 Anderson and May (1978) first studied the effect of an epidemic on predation via a
140 modification of Lotka-Volterra prey-predator model with infected prey. Hassell and Varley
141 (1969) proposed a trophic function with a functional response of the form:
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$$s(X, Y) = \frac{\alpha X}{Y^\sigma} \quad (2.1)$$

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144 Where α is the searching efficiency of the predator to find prey and σ represents the
145 interference of the predator (Hassley -Valley constant). Costner, et al. (1999) discussed the
146 value of this constant, assuming $\sigma \in [\frac{1}{2}, 1)$, realistically, depending on predators forming a
147 fixed number of groups. If predators do not form groups the functional response is that of a
148 ratio – dependent model.

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150 The form of the trophic function used in our research is:
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$$r(X) = \frac{\alpha X}{1 + \beta X^\sigma} \quad (2.2)$$

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153 This is a prey-dependent functional response. This term, βX^σ , is motivated by an Agent-
154 Based Model containing the function for a simplified economy consisting of a goods and
155 credit market in (Arslan, et al. 2016). The function for the level of a firm’s production (L) is a
156 function of the total capital (K) in the business cycle. It is given by:
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$$L = \tau K^\sigma \quad (2.3)$$

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160 Where $\tau > 1$ and $0 < \sigma < 1$. In our model, we assume τ can take any positive value and that
161 σ can take a value of 1, as previously discussed in the original ratio-dependent form.

162
163 Let X be the money invested by potential investors (prey),
164 Let Y_1 be the money invested by financial experts (susceptible predators) and
165 Let Y_2 be the money invested by dishonest financial experts (infected predators involved in
166 controlled fraud as the infection).

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The system of differential equations to represent this model is as follows:

$$\begin{aligned}\frac{dX}{dt} &= f(X) - a_1 r_1(X) Y_1 - a_2 r_2(X) Y_2 \\ \frac{dY_1}{dt} &= g_1(Y_1) + a_1 r_1(X) Y_1 - c \varphi_1(Y_1, Y_2) \\ \frac{dY_2}{dt} &= g_2(Y_2) + a_2 r_2(X) Y_2 + c \varphi_2(Y_1, Y_2)\end{aligned}\quad (2.4)$$

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where

$$\begin{aligned}f(X) &= rX \left(1 - \frac{X}{M}\right), & g_i(Y_i) &= n_i Y_i \left(1 - \frac{Y_i}{K}\right), & i &= 1, 2 \\ r_i(X) &= \frac{X}{1 + b_i X^{\sigma_i}}, & \varphi_i(Y_1, Y_2) &= Y_1 Y_2, & i &= 1, 2\end{aligned}$$

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and

$a_1, a_2, b_1, b_2, c, K, M, n_1, n_2, r$ are assumed to be positive constants

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The system becomes:

$$\begin{aligned}\frac{dX}{dt} &= rX \left(1 - \frac{X}{M}\right) - \frac{a_1 X Y_1}{1 + b_1 X^{\sigma_1}} - \frac{a_2 X Y_2}{1 + b_2 X^{\sigma_2}} \\ \frac{dY_1}{dt} &= n_1 Y_1 \left(1 - \frac{Y_1}{K}\right) + \frac{a_1 X Y_1}{1 + b_1 X^{\sigma_1}} - c Y_1 Y_2 \\ \frac{dY_2}{dt} &= n_2 Y_2 \left(1 - \frac{Y_2}{K}\right) + \frac{a_2 X Y_2}{1 + b_2 X^{\sigma_2}} + c Y_1 Y_2\end{aligned}\quad (2.5)$$

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Table 1: Definitions of parameters in the model

Parameter	Definition
r	Maximal growth rate of potential investors
M	Carrying capacity of potential investors
K	Carrying capacity of financial experts
a_1	Interaction rate of honest financial experts with potential investors
a_2	Interaction rate of dishonest financial experts with potential investors
b_1	Honest expert - investor persuasion rate
b_2	Dishonest expert - investor persuasion rate
σ_1	Honest financial expert interference constant
σ_2	Dishonest financial expert interference constant
n_1	Maximal growth rate of honest financial experts
n_2	Maximal growth rate of dishonest financial experts
c	Interaction rate of honest and dishonest financial experts

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189 **Model Assumptions:**

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- 191 i) A specific population is assumed to grow logistically (potential investors, honest
192 financial experts, dishonest financial experts) in the absence of the other two
193 populations.
194 ii) Disease among the financial experts is the infection “control fraud” which is
195 accounting fraud used to deceive potential financial investors.
196 iii) The disease model here is the S-I (Susceptible – Infectious) Model where there
197 is no recovery from the infection.
198 iv) Incidence rate, c , is the contact rate for every fraudulent financial expert with an
199 honest one and is of bilinear mass action.
200 v) All populations of investors and experts grow at a logistic rate with carrying
201 capacities M and K respectively.
202 vi) The honest financial experts interact with the potential investors with functional
203 response, $r_i(X) = \frac{X}{1+b_iX^{\sigma_i}}$, $i = 1,2$ but they may become infected by the
204 dishonest investors.
205 vii) The infection is assumed to spread among financial experts only at a rate γ by
206 the mass action law.
207 viii) The total predator population at time t is $Y(t) = Y_1(t) + Y_2(t)$
208 ix) Total number of potential investors, X , is greater than the total number of
209 financial experts, Y
210 x) In contrast to the assumption by Pada Das (2011), which states that the infected
211 predator is less able to hunt or to capture a prey than a susceptible predator, we
212 assume that the honest and dishonest investors have the ability to use the same
213 effort to attract potential investors. Dishonest investors may be more desperate
214 to attract potential investors.
215 xi) For simplicity, assume the model contains minimal noise, that is, very little
216 stochastic effects.

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219 A dimensionless system is created for each of computations. The following new
220 variables and parameters are introduced:
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$$\tilde{t} = rt, \quad \tilde{x} = \frac{X}{M}, \quad \tilde{y}_1 = \frac{Y_1}{K}, \quad \tilde{y}_2 = \frac{Y_2}{K}, \quad \alpha_1 = \frac{a_1 K}{r}, \quad \alpha_2 = \frac{a_2 K}{r},$$

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$$c_1 = b_1 M^{\sigma_1}, \quad c_2 = b_2 M^{\sigma_2}, \quad \rho_1 = \frac{n_1}{r}, \quad \rho_2 = \frac{n_2}{r}, \quad \gamma = \frac{cK}{r}$$

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225 For simplicity, the bar ($\sim$) is dropped in the substitutions and the original system
226 in equation (2.5) becomes:

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$$\begin{aligned} \frac{dx}{dt} &= x(1-x) - \frac{\alpha_1 x y_1}{1+c_1 x^{\sigma_1}} - \frac{\alpha_2 x y_2}{1+c_2 x^{\sigma_2}} \\ \frac{dy_1}{dt} &= \rho_1 y_1(1-y_1) + \frac{\alpha_1 x y_1}{1+c_1 x^{\sigma_1}} - \gamma y_1 y_2 \end{aligned} \quad (2.6)$$

$$\frac{dy_2}{dt} = \rho_2 y_2 (1 - y_2) + \frac{\alpha_2 x y_2}{1 + c_2 x^{\sigma_2}} + \gamma y_1 y_2$$

The initial condition, $(x(0), y_1(0), y_2(0))$, is the positive interior equilibrium point with slight perturbation.

2.1 Existence of Positive, Interior Equilibrium Point

The interior equilibrium point is the non-zero, positive solution obtained using the method outlined in (Dubey and Upadhyay 2004). There exists two intersecting isoclines are found at a unique point, (y_1, y_2) . Firstly, put $\frac{dx}{dt} = \frac{dy_1}{dt} = \frac{dy_2}{dt} = 0$ where x, y_1 and y_2 all not equal to zero. Let the equilibrium point be (x^*, y_1^*, y_2^*) . For simplicity, the asterisk (*) is dropped in calculations.

This gives:

$$x(1 - x) - \frac{\alpha_1 x y_1}{1 + c_1 x^{\sigma_1}} - \frac{\alpha_2 x y_2}{1 + c_2 x^{\sigma_2}} = 0 \quad (2.7)$$

$$\rho_1 y_1 (1 - y_1) + \frac{\alpha_1 x y_1}{1 + c_1 x^{\sigma_1}} - \gamma y_1 y_2 = 0 \quad (2.8)$$

$$\rho_2 y_2 (1 - y_2) + \frac{\alpha_2 x y_2}{1 + c_2 x^{\sigma_2}} + \gamma y_1 y_2 = 0 \quad (2.9)$$

Using equations (2.8) and (2.9):

$$\frac{\alpha_1 x y_1}{1 + c_1 x^{\sigma_1}} = \gamma y_1 y_2 - \rho_1 y_1 (1 - y_1) \quad (2.10)$$

$$\frac{\alpha_2 x y_2}{1 + c_2 x^{\sigma_2}} = -\gamma y_1 y_2 - \rho_2 y_2 (1 - y_2) \quad (2.11)$$

Equations (2.10) and (2.11) into (2.7) give:

$$x(1 - x) + \rho_1 y_1 (1 - y_1) + \rho_2 y_2 (1 - y_2) = 0 \quad (2.12)$$

$$x = \frac{1}{2} \pm \sqrt{1 + 4\rho_1 y_1 (1 - y_1) + 4\rho_2 y_2 (1 - y_2)} \quad (2.13)$$

For simplicity in calculations, we take the value of $\sigma_i = 1, i = 1, 2$

From (2.10):

$$x = \frac{(\gamma y_2 - \rho_1 (1 - y_1))}{\alpha_1 - c_1 (\gamma y_2 - \rho_1 (1 - y_1))} \quad (2.14)$$

From (2.11):

$$x = \frac{-(\gamma y_1 + \rho_2 (1 - y_2))}{\alpha_2 + c_2 (\gamma y_1 + \rho_2 (1 - y_2))} \quad (2.15)$$

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259 For x to be positive, the following conditions must hold:

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$$\gamma y_2 - \rho_1(1 - y_1) > 0 \quad (2.16a)$$

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$$\alpha_1 - c_1(\gamma y_2 - \rho_1(1 - y_1)) > 0 \quad (2.16b)$$

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$$\gamma y_1 + \rho_2(1 - y_2) < 0 \quad (2.17a)$$

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$$\alpha_2 + c_2(\gamma y_1 + \rho_2(1 - y_2)) > 0 \quad (2.17b)$$

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Solving equations (2.14) and (2.15) give:

$$f(y_1, y_2) \equiv \frac{1}{c_1} \left[\frac{\alpha_1}{\alpha_1 - c_1(\gamma y_2 - \rho_1(1 - y_1))} - 1 \right] + \frac{1}{c_2} \left[\frac{\alpha_2}{\alpha_2 + c_2(\gamma y_1 + \rho_2(1 - y_2))} + 1 \right] = 0 \quad (2.18)$$

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270 Using equations (2.12) and (2.14) gives:

$$g(y_1, y_2)$$

$$\begin{aligned} & \equiv \frac{1}{c_1} \left[\frac{\alpha_1}{\alpha_1 - c_1(\gamma y_2 - \rho_1(1 - y_1))} - 1 \right] \left(1 - \frac{1}{c_1} \left[\frac{\alpha_1}{\alpha_1 - c_1(\gamma y_2 - \rho_1(1 - y_1))} - 1 \right] \right) + \rho_1 y_1(1 - y_1) + \rho_2 y_2(1 - y_2) \\ & = 0 \end{aligned} \quad (2.19)$$

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From equation (2.18), when $y_2 \rightarrow 0$, then $y_1 \rightarrow y_{1a}$ and this is defined as the positive solution to the equation:

$$\frac{1}{c_1} \left[\frac{\alpha_1}{\alpha_1 + c_1(\rho_1(1 - y_1))} - 1 \right] + \frac{1}{c_2} \left[\frac{\alpha_2}{\alpha_2 + c_2(\gamma y_1 + \rho_2)} + 1 \right] = 0 \quad (2.20)$$

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Similarly, from equation (2.19), when $y_1 \rightarrow 0$, then $y_2 \rightarrow y_{2a}$ and this is defined as the positive solution to the equation:

$$\frac{1}{c_1} \left[\frac{\alpha_1}{\alpha_1 - c_1(\gamma y_2 - \rho_1)} - 1 \right] \left(1 - \frac{1}{c_1} \left[\frac{\alpha_1}{\alpha_1 - c_1(\gamma y_2 - \rho_1)} - 1 \right] \right) + \rho_2 y_2(1 - y_2) = 0 \quad (2.21)$$

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The two isoclines in equations (2.18) and (2.19) intersect at a unique point (y_1, y_2) . Using these values, the positive value of x can be calculated from equation (2.13). This completes the existence of the positive equilibrium point.

2.2 Stability Analysis

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The stability of the equilibrium point (x, y_1, y_2) is examined, where x, y_1, y_2 are all positive, by finding the Jacobian Matrix. First, the original system of equations in (2.5) is linearized via the following substitutions:

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$$x = x + u,$$

288

$$y_1 = y_1 + v, \quad (2.22)$$

$$y_2 = y_2 + w,$$

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291 where u, v and w are small perturbations about the equilibrium point. All terms are expanded
292 about the equilibrium point using Taylor's theorem, neglecting higher order terms of u, v, w

293 The characteristic polynomial takes the form:

294

$$\lambda^3 + b_1\lambda^2 + b_2\lambda + b_3 = 0 \quad (2.23)$$

295

296 where

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$$b_1 = -(j_{11} + j_{22} + j_{33}),$$

$$b_2 = j_{22}j_{11} - j_{12}j_{21} + j_{33}(j_{11} + j_{22}) - j_{31}j_{13} - j_{32}j_{23}, \quad (2.24)$$

$$b_3 = -j_{11}j_{22}j_{33} + j_{21}j_{12}j_{33} - j_{12}j_{31}j_{23} + j_{11}j_{32}j_{23} - j_{13}j_{21}j_{32} + j_{13}j_{31},$$

301 and

$$j_{11} = 1 - 2x - \frac{\alpha_1 y_1}{1+c_1 x^{\sigma_1}} + \frac{\alpha_1 y_1 c_1 x^{\sigma_1} \sigma_1}{(1+c_1 x^{\sigma_1})^2} - \frac{\alpha_2 y_2}{1+c_2 x^{\sigma_2}} + \frac{\alpha_2 y_2 c_2 x^{\sigma_2} \sigma_2}{(1+c_2 x^{\sigma_2})^2}$$

$$j_{12} = -\frac{\alpha_1 x}{1+c_1 x^{\sigma_1}}$$

$$j_{13} = -\frac{\alpha_2 x}{1+c_2 x^{\sigma_2}}$$

$$j_{21} = \frac{\alpha_1 y_1}{1+c_1 x^{\sigma_1}} - \frac{\alpha_1 y_1 c_1 x^{\sigma_1} \sigma_1}{(1+c_1 x^{\sigma_1})^2} \quad (2.25)$$

$$j_{22} = \rho_1(1 - 2y_1) + \frac{\alpha_1 x}{1+c_1 x^{\sigma_1}} - \gamma y_2$$

$$j_{23} = -\gamma y_1$$

$$j_{31} = \frac{\alpha_2 y_2}{1+c_2 x^{\sigma_2}} - \frac{\alpha_2 y_2 c_2 x^{\sigma_2} \sigma_2}{(1+c_2 x^{\sigma_2})^2}$$

$$j_{32} = \gamma y_2$$

$$j_{33} = \rho_2(1 - 2y_2) + \frac{\alpha_2 x}{1+c_2 x^{\sigma_2}} + \gamma y_1$$

311 For stability, the eigenvalues, λ , in the characteristic polynomial must have negative real
312 parts. The Routh-Hurwitz criteria provides the conditions to satisfy a stable equilibrium which
313 occurs if and only if

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$$b_1 > 0, \quad b_3 > 0, \quad b_1 b_2 - b_3 > 0 \quad (2.26)$$

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316

317 **Theorem 2.1.** Given an equilibrium point (x, y_1, y_2) satisfying the equations of system (2.5),

318 then once Lemma holds and $j_{11}, j_{12}, j_{13}, j_{21}, j_{22}, j_{23}, j_{31}, j_{32}, j_{33}$ are defined by equation (2.25),

319 then the equilibrium point exists and is stable if and only if (2.26) holds where b_1, b_2 and b_3

320 have been defined in (2.24).

321

322 3. Results and Discussion

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324 Different parameters are applied to the system in order to investigate stability changes in
 325 the model. We investigate if the investors and experts can co-exist in terms a steady state
 326 solution (a node or a stable focus) or a stable oscillatory solution (limit cycle). The Hopf
 327 bifurcation indicates the start or end of a periodic solution from an equilibrium point, when a
 328 parameter surpasses a particular value.

329

330 Parameter values in the following dataset are modified and for simplicity, only non-zero,
 331 positive equilibrium points are considered. In the following datasets shown in the table, the
 332 values of the parameters in the model are fixed while c_1 and c_2 are altered in pairs. The
 333 stability intervals for the remaining parameters in the model for each dataset are recorded.

334

335 For each dataset, each parameter was varied individually while keeping the others constant.
 336 The results were recorded where there was a change in stability for specific parameters.

337 Dataset (1) had equal values of c_1 and c_2 , while in dataset (2) and dataset (3), these values
 338 were varied. Regions of instability occurred at values greater than the Hopf Bifurcation (HB)
 339 point and were found both analytically using the Routh-Hurwitz Criteria and then verified
 340 using software. The automated continuation package, MATCONT (version matcont3p4),
 341 available in MATLAB [16] was used to find the bifurcation points numerically.

342

343 **Table 2. Parameter values for dataset (1), dataset (2) and dataset (3)**

344

Parameter	dataset (1)	dataset (2)	dataset (3)
α_1	5.5	5.5	5.5
α_2	0.5	0.5	0.5
c_1	0.01	0.01	5
c_2	0.01	5	0.01
σ_1	1	1	1
σ_2	1	1	1
ρ_1	0.05	0.05	0.05
ρ_2	0.05	0.05	0.05
γ	0.09	0.09	0.09
Equilibrium point	(0.012, 0.067, 1.239)	(0.012, 0.106, 1.271)	(0.027, 0.086, 1.431)
Stability via Theorem 2.1	Stable equilibrium	Unstable equilibrium	Stable equilibrium

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346

347 Table 2 shows the stability regions and Hopf bifurcation points for the three datasets where
 348 the value of c_1 is varied while the other parameters are held constant. In dataset (1), the
 349 initial values for c_1 and c_2 are the equal while in datasets (2) and (3), the value of $c_1 \ll c_2$
 350 and $c_1 \gg c_2$ respectively. Financially, these parameters represent the rate at which financial
 351 experts (honest and dishonest) encourage the investors to make an investment.

352

353 In the first dataset, Hopf bifurcations occur for the parameter c_1 only. Figs. 1 and 3 show
 354 time series graphs for a stable and unstable case for dataset (1), with respect to the variation
 355 of c_1 . The stable case for the model at $c_1 = 0.01$ shows a slight fluctuation in the three
 356 populations initially. However, eventually over time the populations co-exist and remain
 357 stable. The unstable case at $c_1 = 7.5$ shows the continuous fluctuation of all three
 358 populations over time.

Figs. 2 and 4 show the behavior of the orbits of the system close the equilibrium points corresponding to these stable and unstable cases respectively. These graphical simulations enhance the recorded numerical results, since they display how the variations of different parameters over time affect the model stability.

The other two datasets display similar behaviors for additional parameters. In dataset (2), bifurcations occur at parameters α_1 , α_2 and γ . These represent the investor-expert interaction rates, α_i , $i = 1, 2$ and the interaction rate between the experts, γ . Dataset (3) has bifurcations for the parameters α_1 , σ_1 and ρ_2 . These are the rates of investor-honest expert interaction, α_1 , the investor-honest expert interference constant, σ_1 , as well as the maximal growth rate of the dishonest financial experts, ρ_2 . Similar time series graphs can also be plotted to analyze the behavior of the model for the variations of these parameters.

Fig. 5 also displays the changes in the population of potential investors with time for the three pairs of values of financial expert persuasion rates, c_i for $i = 1, 2$, for datasets (1), (2) and (3) respectively. Interesting behavior occurs in dataset (2) where $c_1 = 0.01$, $c_2 = 5$. The number of potential investors increases drastically when the investor persuasion rate for dishonest experts is much greater than that for honest experts. Here, potential investors are influenced more so by fraudulent experts to enter investment schemes, thereby causing their populations to increase drastically in comparison to datasets (1) and (3) where the dishonest expert persuasion rates are lower.

It is evident that this idea supports the use of the simulations to model real-life fraudulent investment scenarios. It is a perfect example of how the infection of fraud in financial experts can influence the number of investors in the system at different points in time. Policy makers can test how different cases of parameters would affect the long – term populations of investors, as well as financial experts in the model. The value of stability and time series analysis provides an interesting detection system to determine and predict the presence any fraudulent investment activities.

As a result, a number of reasons may be adequate to partially explain some of the population fluctuations. Financial experts use different tactics such as social media, word-of-mouth and other advertising means to attract their clients. This may cause the co-existence equilibrium of the investors and experts to fluctuate. There are alternating periods of time where investors are drawn to investment schemes or shy away from them due to prior knowledge of dishonest tactics.

The latter is particularly attractive for an investment that promises high interest rates and returns. As the influence of the agents and brokers increases, the investors become more educated and are more prone to being careful about risky investments and so the number of potential investors falls. Hence, further analysis of such a model and its parameters can increase the knowledge of how these social behaviors affects the number of players in the market.

Table 3: Stability regions for parameters in datasets (1), (2) and (3)

Dataset	Parameter	Stable (S) /Unstable (U) regions	Hopf Bifurcation Points
(1)	c_1	$0 < c_1 \leq 6.26$ (S) $6.26 < c_1 \leq 34.09$ (U) $c_1 > 34.09$ (S)	6.26 34.09
(2)	α_1	$0 < \alpha_1 \leq 0.69$ (S) $\alpha_1 > 0.69$ (U)	0.69
	α_2	$0 < \alpha_2 \leq 0.67$ (S) $0.67 < \alpha_2 \leq 0.82$ (U) $\alpha_2 > 0.83$ (S)	0.67 0.82
	γ	$0 < \gamma < 0.07$ (S) $0.07 \leq \gamma \leq 0.69$ (U) $\gamma > 0.69$ (S)	0.07 0.69
(3)	α_1	$0 < \alpha_1 \leq 12.88$ (S) $\alpha_1 > 12.88$ (U)	12.88
	σ_1	$0 < \sigma_1 \leq 0.51$ (S) $0.51 < \sigma_1 \leq 0.93$ (U) $\sigma_1 > 0.93$ (S)	0.51 0.93
	ρ_2	$0 < \rho_2 \leq 0.37$ (S) $\rho_2 > 0.37$ (U)	0.37

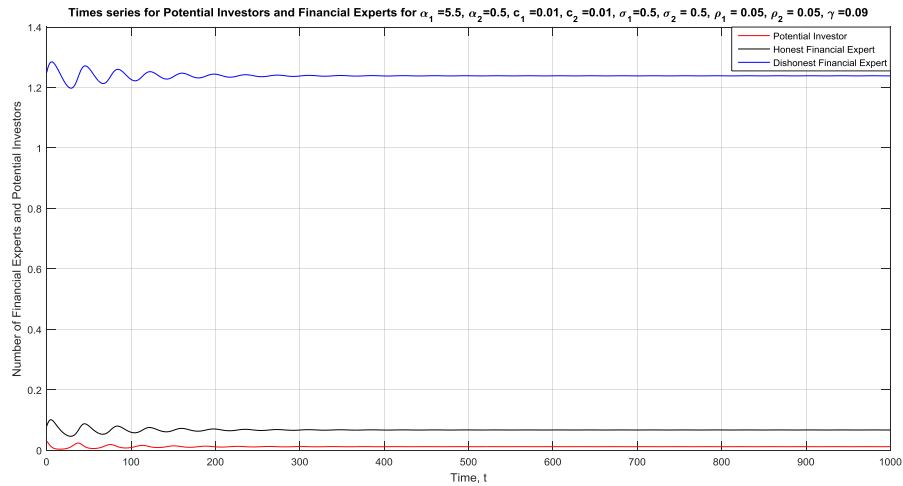


Figure 1: Time series graph showing stable equilibrium where the populations for Investors and Financial experts co-exist over time for dataset (1) where $c_1 = 0.01$

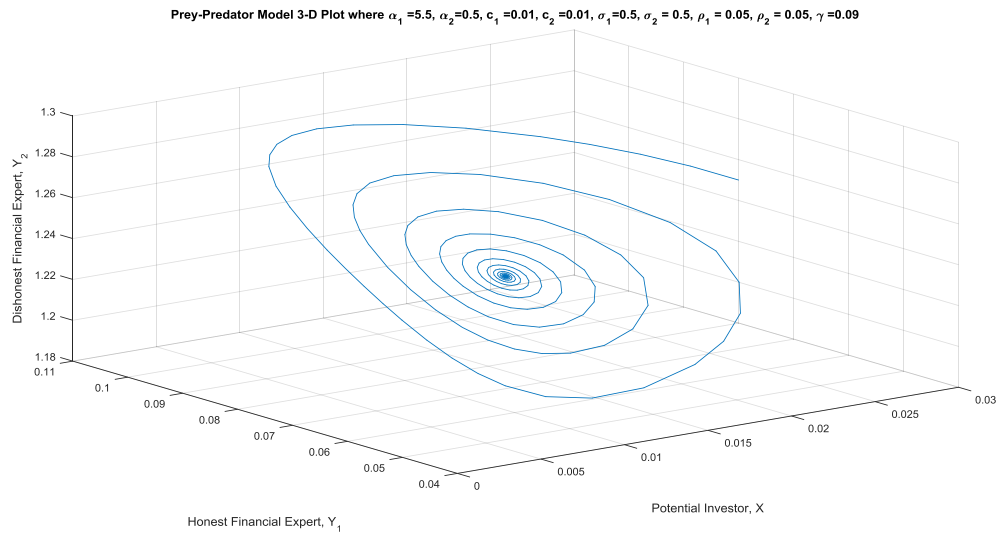


Figure 2: Stable equilibrium for dataset (1) where a small disturbance from the equilibrium initially eventually re-approaches the equilibrium value again

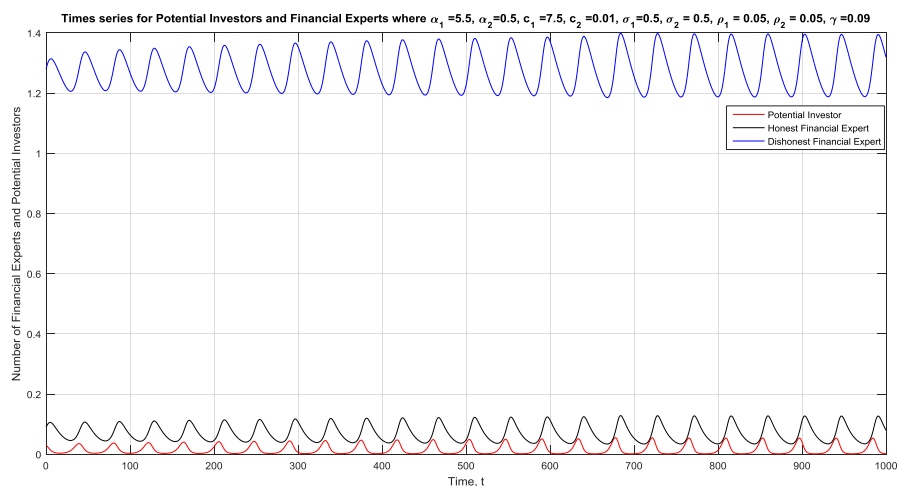


Figure 3: Time series graph showing unstable equilibrium for the populations of Potential investors and Financial experts over time for dataset (1) where $c_1 = 7.5$

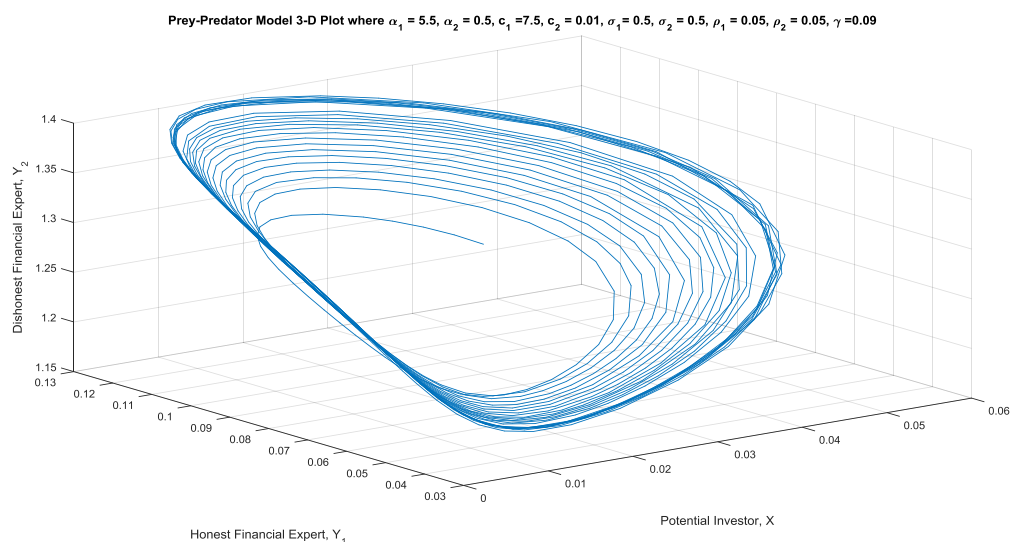


Figure 4: Unstable equilibrium for dataset (1) where $c_1 = 7.5$ where a small disturbance from the equilibrium value cycles away from this point in an oscillatory manner

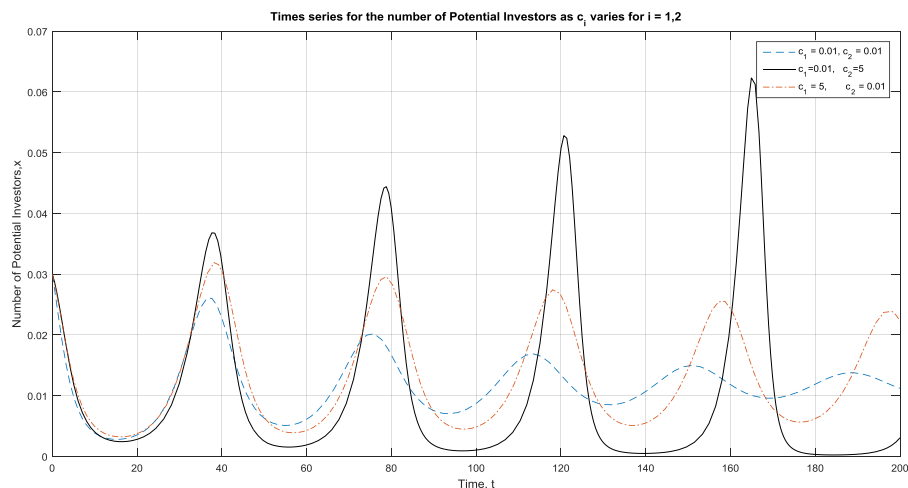


Figure 5: Time series for the number of potential investors in datasets (1), (2) and (3) as the investor persuasion parameter, c_i varies for $i = 1, 2$. Interesting behavior occurs particularly in dataset (2) where $c_1 = 0.01, c_2 = 5$. The potential investor population experiences the largest, increasing fluctuations when the honest expert persuasion rate is much lower than that for dishonest experts. The infection of fraud in experts influences increases the number of potential investors in the system.

4. CONCLUSION

Wilcox (1999) maintained that dynamic markets where individuals buy and sell securities is analogous to organisms competing for information on price prediction. It is evident that ecological ideas assist in the understanding of the complex dynamics which are involved in the financial eco-system. Indeed, the emergence of the cyclic behavior of economy is greatly explained by the financial structure of investment (Abraham-Froi 1998).

Our model is the start of a bigger scheme of financial dynamical analysis. It can be improved by the addition of other financial parameters as well as stochastic effects. This is the direction for future work. These models are not meant to provide an avenue for which persons involved in the investment community can understand when the system is stable or unstable, and hence construct mechanisms and policies to deal with both.

Mehlum et al. (2003) also explained that entrepreneurs are less likely to choose predatory activities if they attach a positive probability on productive businesses being the most profitable choice in the long run. This idea can be used to further improve the model. With the groups of honest and dishonest experts there may be opportunists. These may be described as the financial experts who switch between the states of being honest or dishonest depending on profitability of the investment. A switching mechanism can make the model more realistic since, in reality, agents and brokers have this option.

Hence, the study of such an investment model with investor interactions can provide valuable information to safe-guard the economy from financial crises. Social networks among investors and financial experts are an integral part of the complex dynamics of these

487 systems. Therefore, studying a model predator-prey model with infection is applicable to the
 488 problem at hand as it provides stability analyses which can be utilized in attempt to guide
 489 investors in the realm of financial predation. (Costner, et al. 1999)

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492 **COMPETING INTERESTS**

493 Authors have declared that no competing interests exist in the publication of this work.

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