

Chiral Structure of Free Particles Bound by Magnetic Forces

Abstract

Free particle bound states exist only as microscopic systems in form of atomic and subatomic particles. An interesting class of these objects are particles bound by magnetic forces, which exhibit the particular property of chirality (handedness, which is not parity symmetric). These particles are discussed in quantum field theory based on a QED like Lagrangian with fermion and boson fields, in which about ten boundary conditions can be defined. With four (but effectively two) adjustable parameters only, this leads to a stringent test of the special mathematical structure of the underlying field theory.

A first kind of these particles are leptons, e , μ , τ and neutrinos. With an additional quantum condition the radii of charged leptons can be deduced. Other systems of magnetic structure may be found in the binding of atoms, a first example being a weakly bound H-H system, which could be the origin of gravitation.

Keywords: Free particle bound by magnetic forces described in quantum field theory based on a QED type Lagrangian with fermion and boson fields. Solutions for charged leptons and weakly bound atoms, which violate parity symmetry.

1. Introduction

Bound or stationary systems belong to the most interesting and basic objects in physics, since they are stable over long periods of time. Their average potential and kinetic energies are related by the virial theorem, leading to a well defined frequency or mass. A large fraction of the mass of the universe is composed of free particle bound states in the vacuum in form of atoms and subatomic particle, like hadrons and leptons.

For these systems binding of elementary fermions is not sufficient; accompanying bosons (not only an attractive boson-exchange interaction) are essential to equilibrate the momentum of fermions, see ref. [1]. A special class of these particles are those bound by magnetic forces, which are schematically shown in fig. 1. Since magnetic forces arise from the motion of charge, two charged fermion components are needed, which rotate with relative velocity (v/c) to each other, see the green flashes in fig. 1. The momentum of these fermion components has to be compensated by two boson components, indicated by yellow flashes.

A particularity of these bound states arises from the vector-structure of magnetic forces, in which the directions of spin, relative motion and attraction (and therefore binding) are perpendicular to each other – this is known as the three finger rule. The axis of spin is perpendicular to the rotation axis, so attraction exists only for one direction of rotation (the opposite rotation would give repulsion of the fermions). Therefore, these bound states are not symmetric under parity transformations, which is called chirality or handedness [2]. Leptons and their antiparticles are known to have this property, for leptons only a left-handed type exists, whereas antileptons are right-handed. But so far, leptons have been assumed as point particles, a real bound state description with explicit consideration of the chiral structure has not been performed.

2. Theoretical Description

For the description of these systems a quantum field theory similar to that in ref. [1] has been used, based on a Lagrangian similar to QED, but with fermions dressed by boson fields, which may be written in the form

$$\mathcal{L} = \frac{1}{\tilde{m}^2} (\bar{\Psi} D_\nu) i \gamma^\mu D_\mu (D^\nu \Psi) - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} , \quad (1)$$

where $\tilde{m}$ is the mass parameter and Ψ are charged fermion fields, $\Psi = \Psi^+$ and $\bar{\Psi} = \Psi^-$. Vector boson fields A_μ with coupling g to fermions are contained in the covariant derivatives $D_\mu = \partial_\mu - igA_\mu$. The second term of the Lagrangian represents the Maxwell term of electromagnetism with field strength tensors $F^{\mu\nu}$ given by $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$, which gives rise to both electric and magnetic coupling.

The three covariant derivatives D_μ in the Lagrangian (1) lead to a theory, which includes higher order boson and fermion fields. However, in the past this type of Lagrangian has

not been studied in detail, because in standard (divergent) gauge theory the necessary $1/\tilde{m}^2$ factor gives rise to uncontrolled divergences. Further, Lagrangians with higher order fermion fields can lead to non-physical solutions [3]. None of these arguments are important for the present case: the Lagrangian (1) leads to a finite theory due to a constrained normalization of boson fields, therefore a $1/\tilde{m}^2$ factor is acceptable. Further, it gives rise to a real bound state formalism with negative binding energies, in which non-physical solutions can be excluded by strict geometrical and other basic constraints. By inserting $D^\mu = \partial^\mu - igA^\mu$ and $D_\nu D^\nu = \partial_\nu \partial^\nu - ig(A_\nu \partial^\nu + \partial_\nu A^\nu) - g^2 A_\nu A^\nu$, the first part of the Lagrangian gives rise to a number of terms, which contain boson and fermion fields and/or their derivatives. For stationary solutions only two terms of the Lagrangian contribute

$$\mathcal{L}_{2g} = \frac{-ig^2}{\tilde{m}^2} (\bar{\Psi} A_\nu) \gamma^\mu \partial_\mu (A^\nu \Psi) \quad (2)$$

and

$$\mathcal{L}_{3g} = \frac{-g^3}{\tilde{m}^2} (\bar{\Psi} A_\nu) \gamma^\mu A_\mu (A^\nu \Psi) . \quad (3)$$

From the Lagrangians (2) and (3) fermion-antifermion matrix elements have been derived, see the details in ref. [1], which can be written by

$$\mathcal{M}_{2g} = \frac{\alpha^2}{\tilde{m}^5} \bar{\psi}(p') A_\nu(q'_4) A^\mu(q'_3) \gamma_\mu \gamma^\rho \partial A_\rho(q'_2) \partial A^\sigma(q'_1) \psi(p) \quad (4)$$

and

$$\mathcal{M}_{3g} = \frac{-\alpha^3}{\tilde{m}^5} \bar{\psi}(p') A_\nu(q'_4) A^\mu(q'_3) \gamma_\mu \gamma^\rho A_\rho(q_2) A^\sigma(q_1) A_\sigma(q'_2) A^\tau(q'_1) \psi(p) , \quad (5)$$

in which $\psi(p)$ is a fermion wave function $\psi(p) = \frac{1}{\tilde{m}^{3/2}} \Psi(p_1) \Psi(p_2)$. These matrix elements can be further simplified by assuming also boson (quasi) wave functions $W_\mu^\nu(q') = \frac{1}{\tilde{m}} A_\mu(q'_j) A^\nu(q'_i)$ of scalar ($\mu = \nu$) and vector ($\mu \neq \nu$) structure. By reducing these matrix elements to three dimensions one gets boson wave functions of scalar and vector structure $w_s(q')$ and $w_v(q')$ and a boson-exchange interaction $v_v(q)$. This leads to fermion matrix elements

$$\mathcal{M}_{2g} = \frac{\alpha^2}{2\tilde{m}^3} \bar{\psi}(p') w_s(q') \partial^2 w_s(q') \psi(p) \quad (6)$$

and

$$\mathcal{M}_{3g} = \frac{\alpha^2}{\tilde{m}^2} \bar{\psi}(p') w_{s,v}(q') v_v(q) w_{s,v}(q') \psi(p) , \quad (7)$$

but also to a boson matrix element

$$\mathcal{M}^g = \frac{\alpha^2}{\tilde{m}^2} w_{s,v}(q') v_v(q) w_{s,v}(q') . \quad (8)$$

By going from fermion-antifermion (q^+q^-) matrix elements to $(q^+q^-)^n q^\pm$ structures, which can be bound by magnetic forces, two fermion wave functions are needed, which rotate with relative velocity (v/c). The corresponding matrix elements can be combined to simpler ones, which are of the same forms as those above with additional (v/c) factors. In r-space this leads to matrix elements in the form

$$\mathcal{M}_{ng}^f(r) = \bar{\psi}_{s,v}(r) V_{ng}(r) \psi_{s,v}(r) (v/c)^2, \quad (9)$$

with fermion wave functions $\psi_{s,v}(r)$ of scalar and vector structure and two potentials $V_{2g}(r)$ and $V_{3g}(r)$ of the form

$$V_{2g}(r) = \frac{\alpha^2(\hbar c)^2(2s+1)}{8\tilde{m}} \left(\frac{d^2 w_s(r)}{dr^2} + \frac{2}{r} \frac{dw_s(r)}{dr} \right) \frac{1}{w_s(r)} + E_o \quad (10)$$

with $s=0$ for scalar and $s=1$ for vector states, and

$$V_{3g}(r) = \frac{\alpha^3 \hbar c}{\tilde{m}} \int dr' w_{s,v}(r') v_v(r-r') w_{s,v}(r') \quad (11)$$

where $w_{s,v}(r)$ are boson (quasi) wave functions¹ of scalar and vector structure and $v_v(r) \sim -\hbar c w_v(r)$ a boson-exchange interaction. $\psi_{s,v}^2(r)$ and $w_{s,v}^2(r)$ can be interpreted as densities. It should be emphasized that in the present case of magnetic binding the interaction $v_v(r)$ is of vector structure, which is attractive only for rotation in one particular direction, e.g. left-handed. Bound states for which attraction is obtained for right-handed rotation are of different nature. Further, the boson matrix element is of the form

$$\mathcal{M}^g(r) = \frac{\alpha^3 \hbar c}{\tilde{m}} w_{s,v}(r) v_v(r) w_{s,v}(r) (v/c). \quad (12)$$

Bound states can be generated for different angular momentum L . In the present study we consider only $L=0$, leading to bound states of scalar and vector character of the form $\psi_s(\vec{r}) = \psi_s(r) Y_0(\theta, \Phi)$ and $\psi_v(\vec{r}) = \psi_v(r) Y_1^2(\theta, \Phi)$, respectively.

A coupling to the vacuum is made by assuming $E_o = 0$. This implies that the vacuum is the lowest state with energy $E_{vac} = 0$, consequently all elementary fermions (quantons) have to be massless. The potential $V_{2g}(r)$ is important for a dynamical stabilization of the system: created fermion-antifermion pairs are locked during overlapping boson fields and form a stable system, which cannot decay. As shown below, $V_{2g}(r)$ shows a quite linear

¹with dimension 1/fm.

rise towards larger radii, very similar to the empirical "confinement" potential required in hadron potential models [4].

For a bound state of fermions and bosons the radial form of the fermion and boson wave functions should be similar

$$\psi_{s,v}(r) \sim w_{s,v}(r) . \quad (13)$$

Fermion vector states have normally angular distributions of dipole form $P_1^2(\cos\theta)$, but for a free particle different orientations in space cancel out. However, due to rotation of the system the direction of momentum is fixed, leading to a wave function in q-space of dipole character $\psi_v(q) = 4\pi \int r^2 dr j_1^2(qr) \psi_v(r)$.

Orthogonality of the fermion wave functions leads to

$$\int r^2 dr \psi_s(r) \psi_v(r) = \int r^2 dr w_s(r) w_v(r) = \langle r_{w_s, w_v} \rangle = 0 . \quad (14)$$

This condition requires that the wave functions are finite (with finite radial moments). Condition (14) is satisfied for

$$w_v(r) = w_{v_o} [w_s(r) + \beta R \frac{dw_s(r)}{dr}] , \quad (15)$$

where w_{v_o} is obtained from the normalization of the density $w_v^2(r)$ with $2\pi \int r dr w_v^2(r) = 1$ and $\beta R = - \int r^2 dr w_s(r) / \int r^2 dr [dw_s(r)/dr]$. Because of the derivative structure, $w_v(r)$ has a smaller root mean square radius than $w_s(r)$. A natural condition requires therefore that the interaction for this state takes place inside the volume of the scalar state, leading to the geometrical boundary condition

$$|V_{3g}^v(r)| \simeq c w_s^2(r) . \quad (16)$$

The conditions (14) and (16) are satisfied by assuming the radial part of $w_s(r)$ by

$$w_s(r) \simeq w_{s_o} \exp\{-(r/b)^\kappa\} , \quad (17)$$

where w_{s_o} is fixed by the density normalization $2\pi \int r dr w_s^2(r) = 1$.

The binding energies are given by $E_f^{ng} = 4\pi [\int r^2 dr \psi^2(r) V_{ng}(r) - \frac{1}{2} \int r^3 dr \psi^2(r) \frac{d}{dr} V_{ng}(r)]$ and $E_g = 2\pi [\int r dr w^2(r) v_v(r) - \frac{1}{2} \int r^2 dr w^2(r) \frac{d}{dr} v_v(r)]$. The masses (due to binding) are defined by the sum of absolute binding energies $M_{s,v} = |E_{2g}^{s,v}| + |E_{3g}^{s,v}|$, and the total mass

of the system is given by $M_{tot} = M_{s,v} + m_1 + m_2$, where m_1 and m_2 are the participating fermion masses. As in ref. [1] for all calculations natural units are used.

There are boundary conditions, which allow to determine all four parameters of the model. The first one is momentum conservation, which implies that the average recoil momenta for bosons $\langle q_g^2 \rangle_{rec}^{1/2}$ and fermions $\langle q_f^2 \rangle_{rec}^{1/2}$ cancel each other

$$\langle q_g^2 \rangle_{rec}^{1/2} + \langle q_f^2 \rangle_{rec}^{1/2} = 0 . \quad (18)$$

With normalization $\langle q_g^0 \rangle = \int q dq V_{3g}(q)$ and $\langle q_f^0 \rangle = \int q^2 dq \psi(q) V_{3g}(q)$, this yields $\langle q_g^2 \rangle_{rec} = \int q^3 dq V_{3g}(q) / \langle q_g^0 \rangle$ and $\langle q_f^2 \rangle_{rec} = \int q^4 dq \psi(q) V_{3g}(q) / \langle q_f^0 \rangle$, where the Fourier transformed quantities are given by $(\psi, V_{3g})(q) = 4\pi \int r^2 dr j_0(qr)(\psi, V_{3g})(r)$.

Further, the coupling to the vacuum leads to energy-momentum conservation, which requires that the average momenta of the bound state are compensated by their binding energies

$$[\langle q_g^2 \rangle^{1/2} + \langle q_f^2 \rangle^{1/2}] (v/c) + E_g - x M_f = 0 , \quad (19)$$

where $x = \sqrt{2\tilde{m}/M_f}$ and (v/c) taken as positive. For basic systems energy-momentum conservation can be valid separately for bosons and fermions, this gives rise to four different constraints. For bosons this yields

$$\langle q_g^2 \rangle^{1/2} (v/c) + E_g = 0 \quad (20)$$

and for fermions

$$\langle q_f^2 \rangle^{1/2} (v/c) - x M_f = 0 . \quad (21)$$

The momenta $\langle q_g^2 \rangle = \langle q_g^2 \rangle_{rec}$ and $\langle q_{f_s}^2 \rangle = \langle q_{f_s}^2 \rangle_{rec}$, but for vector particles $\langle q_{f_v}^2 \rangle = \int q^4 dq \psi_v(q) V_{3g}^v(q) / \langle q_{f_v}^0 \rangle$, where the Fourier transformed quantities are given by $(\psi_v, V_{3g}^v)(q) = 4\pi \int r^2 dr j_1^2(qr)(\psi_v, V_{3g}^v)(r)$.

A mass-radius condition derived from the structure of the potential $V_{2g}(r)$ reads

$$Rat_{2g} = \frac{(\hbar c)^2 (v/c)^2}{\tilde{m}(M_s/2) \langle r_{ws}^2 \rangle} = 1 . \quad (22)$$

Finally it is important to note that for magnetic binding the vector state with radial node is not stable. Nevertheless, energy-momentum conservation should be fulfilled also, because this state is part of the system.

3. Bound States of Elementary Fermions (Leptons)

In this case one can require further that the mass parameter $\tilde{m}$ is half of the generated bound state mass

$$\tilde{m} = \frac{1}{2}M_s = \frac{1}{2}(|E_{2g}^s| + |E_{3g}^s|) . \quad (23)$$

By the different boundary conditions (13), (16) and (18) - (23) the parameters of the model, κ , b , α and (v/c) are highly overconstrained. Even more, a value of $\kappa = 1.35 \pm 0.2$ is needed to get $V_{2g}(r)$ correct and $\alpha = 2.14$ is required to satisfy energy-momentum conservation. This leaves only two parameters, b and (v/c) , by which all 10 boundary conditions have to be satisfied. As already stressed in ref. [1] the fulfillment of these conditions is far from trivial: the average momenta are related to the geometry only, whereas E_g and M_f are given by the eigenvalues in the boson and fermion potentials, which are of quite different structure. Nevertheless, for a reasonable determination of the parameters one further condition is needed, a quantum condition on the average radius.

Solutions are discussed for charged leptons, electron, muon and tau-lepton. For the electron a radius smaller than 10^{-9} fm is estimated [5]. The extremely small size explains that the electron could be assumed as point particle in weak interaction theory.

Resulting densities and potentials for a magnetically bound system with a boson root mean square (rms) radius of $2 \cdot 10^{-10}$ fm is shown in fig. 2. In the upper part the potential $V_{2g}(r)$ is shown, which has the typical characteristics of a confinement potential, established empirically in bound state calculations of hadrons [4]. This special potential form represents an inherent property of all bound states of relativistic particles, see refs. [1, 6]. In the second part of the figure the potentials $V_{3g}^{s,v}(r)$ are given together with the boson density $w_s^2(r)$. This shows that the geometrical boundary condition (16) between the density $w_s^2(r)$ and the potential $V_{3g}^v(r)$ is fulfilled. Finally, the Fourier transformed boson density and potentials are shown in the lower part, indicating very similar features of the system in r- and q-space.

The resulting parameters, masses and radii are given in the upper part of table 1; further, average momenta and binding energies are given in the lower part, which indicate that momentum matching and energy-momentum conservation is fulfilled, even for the vector state, which is not stable. The given errors arise mainly from the spacing and cut-off in radius and momentum. These have been simply estimated by changing the momentum

Table 1: Solutions for different systems n with $\kappa = 1.35$ and $\alpha = 2.14$. All dimensional quantities in GeV or fm. Upper limit on the electron radius R_{elec} taken from ref. [5] and rms-radius of p_M from ref. [7].

syst.	n	b	(v/c)	$\langle q_{gs}^2 \rangle^{1/2}$	$\langle r_{gs}^2 \rangle^{1/2}$	R_{exp}	M_s	M_{exp}
e	4	$2.1 \cdot 10^{-10}$	$2.45 \cdot 10^{-13}$	$2.11 \cdot 10^9$	$1.9 \cdot 10^{-10}$	$< 10^{-9}$	$0.51 \cdot 10^{-3}$	$0.51 \cdot 10^{-3}$
μ	3	$3.9 \cdot 10^{-6}$	$9.22 \cdot 10^{-6}$	$1.14 \cdot 10^5$	$3.5 \cdot 10^{-6}$	—	0.105	0.105
τ	2	$9.6 \cdot 10^{-1}$	$4.1 \cdot 10^{-2}$	$4.68 \cdot 10^1$	$8.5 \cdot 10^{-3}$	—	1.97	1.97
p_M	1	0.44	0.29	0.93	0.56	~ 0.74	0.94	0.94
		system	s	$\langle q_g^2 \rangle^{1/2} (v/c)$	E_g	$\langle q_f^2 \rangle^{1/2} (v/c)$	M_f	
e		0		$0.51 \pm 0.03 \cdot 10^{-3}$	$-0.52 \cdot 10^{-3}$	$0.50 \pm 0.05 \cdot 10^{-3}$	$0.51 \cdot 10^{-3}$	
(e)		1		$0.76 \pm 0.1 \cdot 10^{-3}$	$-0.84 \cdot 10^{-3}$	$2.8 \pm 0.3 \cdot 10^{-3}$	$2.5 \cdot 10^{-3}$	
μ		0		0.105 ± 0.01	-0.107	0.10 ± 0.02	0.105	
(μ)		1		0.155 ± 0.02	-0.17	0.42 ± 0.1	0.505	
τ		0		1.98 ± 0.2	-1.96	1.98 ± 0.5	1.97	
(τ)		1		3.06 ± 0.3	-3.12	8.5 ± 2.5	9.10	

cut-off by $\pm 10\%$ at a value of about $7 \langle q_g^2 \rangle_s^{1/2}$.

For other systems, μ and τ , quite similar features of the densities and potentials are expected, which are just scaled by a different radius. However, for a reasonable estimate of their properties a quantum condition on the radius of these systems is needed.

For light atoms such a condition has been established [8] by assuming that the different states are higher harmonics of the strongest bound state. Similarly, the different leptons may be assumed as higher members of a fundamental state (with the same spin and charge $J^{ch} = 1/2^\pm$), which is in this case of the magnetic component [7] of the proton or antiproton $p_M^\pm$ (giving rise to the magnetic form factor). Characterizing $p_M^\pm$ by quantum number $n = 1$ and the different leptons by $n > 1$, their rms-radii should follow the radial form of the density $w_s^2(r)$ or the potential $V_{3g}^s(r)$.

Using this picture the results are shown in fig. 3. By adjusting the radial fall-off of $w_s^2(r)$ and $V_{3g}^s(r)$ between the rms-radius of p_M and the assumed electron radius of $2 \cdot 10^{-10}$ fm,

the dashed and solid lines are obtained, from which an estimate of the rms-radii of τ and μ is obtained, as given in table 1. In the lower part resulting values of $\langle q_{g,f}^2 \rangle^{1/2} (v/c)$ are compared to E_g and M_f , which shows that in all cases (also for the nonstable vector states) momentum matching and energy-momentum conservation is fulfilled. It should be noted that the electron radius [5] is quite uncertain; from the estimates in fig. 3 the radii of the other leptons have the same relative uncertainties.

The chiral structure is manifested in the handedness of these bound states. For leptons of $(q^+ q^-)^n q^-$ structure ($q^\pm$ are charged massless quanta) the motion is dominated by negative charge, leading to left-handedness; for $(q^+ q^-)^n q^+$ antileptons the motion is dominated by positive charge, giving rise to right-handedness.

A similar description should be possible for neutral leptons (neutrinos). But different from charged leptons, only the elementary fermion spins (different for the neutral quanta q^o and $\bar{q}^o$) can give rise to a tiny binding energy.

4. Bound States of Atoms (H-H)

Another type of magnetic bound state may exist in form of weakly bound atoms. Since in this case the mass parameter is given by the reduced mass $\tilde{m} = m_1 m_2 / (m_1 + m_2)$ and not by the generated mass (as for binding of elementary fermions), there are no ambiguities in the extraction of b . One solution has been found with a mass parameter $\tilde{m} = 0.469$ GeV (which corresponds to a system of two hydrogen atoms), slope parameter $b = 2 \cdot 10^6$ fm and $(v/c)^2 = 8.9 \cdot 10^{-31}$, leading to an extremely small binding energy of about $5 \cdot 10^{-36}$ eV. The small binding indicates that this type of binding can play a role only in large macroscopic systems of more than 10^{40} atoms.

The detailed features of density and potentials are very similar to those of the electron in fig. 2 by changing the radial scale by about 16 orders of magnitude. Resulting parameters and extracted radii, momenta and energies are given in table 2. For the scalar state the average momenta of bosons and fermions are $0.22 \cdot 10^{-6}$ GeV, indicating that momentum matching is obtained. Multiplied with (v/c) this yields about $6.7 \cdot 10^{-23}$ GeV for bosons and fermions, in agreement with the boson and fermion energies, as shown in the lower part of table 2. However, for the unstable vector state energy-momentum conservation is not fulfilled for bosons and fermions separately, but still for the sum of boson and fermion contributions; thus, respecting total energy-momentum conservation (19).

Table 2: Solution of an atomic systems bound magnetically, using $\kappa = 1.35$ and $\alpha = 2.14$.

All dimensional quantities in GeV or fm.

	system	$\tilde{m}$	b	(v/c)	$< q_{w_s}^2 >^{1/2}$	$< r_{\psi_s}^2 >^{1/2}$	M_s
	$H - H$	0.469	$2 \cdot 10^6$	$3.0 \cdot 10^{-16}$	$0.22 \pm 0.01 \cdot 10^{-6}$	$1.8 \cdot 10^6$	$4.7 \cdot 10^{-45}$
s	$< q_g^2 >^{1/2} (v/c)$	$< q_f^2 >^{1/2} (v/c)$	$\Sigma < q_{g,f}^2 >^{1/2} (v/c)$		E_g	xM_f	$xM_f - E_g$
0	$6.7 \pm 0.1 \cdot 10^{-23}$	$6.6 \pm 0.2 \cdot 10^{-23}$	$13.5 \pm 0.2 \cdot 10^{-23}$		$-6.8 \cdot 10^{-23}$	$6.7 \cdot 10^{-23}$	$13.5 \cdot 10^{-23}$
1	$6.7 \pm 0.2 \cdot 10^{-23}$	$8.9 \pm 0.5 \cdot 10^{-23}$	$15.6 \pm 0.2 \cdot 10^{-23}$		$-11.0 \cdot 10^{-23}$	$4.3 \cdot 10^{-23}$	$15.3 \cdot 10^{-23}$

This $H - H$ bound state could be related to gravitation. To inspect this possibility, an equivalent first order coupling constant may be defined by relating the radial integral of the potential $V_{3g}^s(r)$ to that of a gravitational potential $V_{gr}(r) = \alpha_{gr} \frac{\hbar c}{r}$. This yields

$$\alpha_{gr} = \frac{\int V_{3g}^s(r) dr}{\int \frac{\hbar c}{r} dr} . \quad (24)$$

The deduced coupling constant $\alpha_{gr} = 5.9 \cdot 10^{-39}$ may be compared to Newton's gravitational constant $G_N = 6.707 \cdot 10^{-39} (\hbar c) GeV^{-2}$, obtained from a gravitation potential of the form $V_{grav}(r) = G_N(m_1 m_2)/r$. Using $m_i = 0.94$ GeV this yields $G_N/(m_1 m_2) = 5.91 \cdot 10^{-39}$, which is in quantitative agreement with the deduced value of α_{gr} .

Following this conjecture, magnetic binding of $\sim 10^{70}$ (hydrogen) atoms leads to large (gravitational) systems with masses and rotation profiles compatible to those observed for galaxies. Other characteristics of gravitation can be understood also, as the deflection of light by solar or galactic systems (gravitational lensing), which is explained by optical deflection on electromagnetic potentials, similar to the scattering of photons from electrons (Compton scattering). As a final point of interest, the present formalism gives rise to a natural solution of Bentley's paradox [9], since the interactions in the present approach (shown e.g. in fig. 2) fall off much faster with distance than Newton's $1/r$ gravitational potential.

6. Summary

Starting from quantum field theory a solution of free particles has been constructed, which are bound by magnetic forces. The structure of this theory is similar to that for hadrons

and atoms, but parity symmetry is broken due to the vector structure of magnetic forces. For elementary fermions this gives rise to bound states, which are of chiral structure, left-handed leptons e , μ and τ and right-handed antileptons $\bar{e}$, $\bar{\mu}$ and $\bar{\tau}$. A similar description may be possible for neutrinos, but for these objects only a tiny binding is possible arising entirely from the elementary fermion spins.

In addition, a magnetically bound system of two hydrogen atoms has been found, which shows an equivalent first order coupling constant in agreement with Newton's gravitational constant G_N . This type of bound state may be the origin of gravitation.

The present bound state formalism has the most simple symmetry structure possible, based on massless elementary bosons and fermions only and requires no external parameters. Further, about 10 boundary conditions could be satisfied by one or two adjustable parameters. This may be taken as strong indication of a fundamental theory, in which all free particles of nature in the hadronic, leptonic, atomic (and possibly gravitational) sector can be understood.

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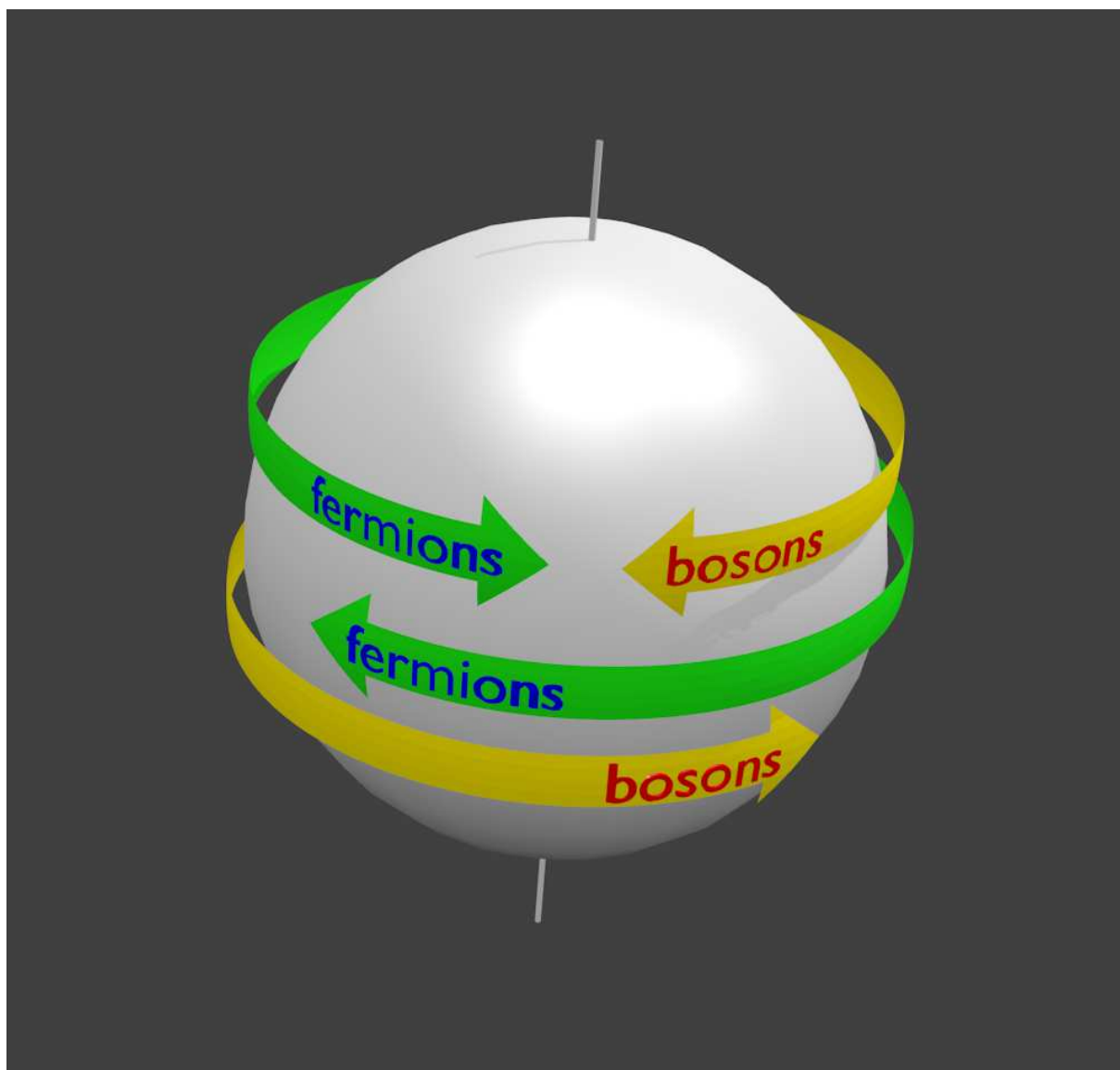


Figure 1: Schematic view of a free particle bound by magnetic forces. The green flashes show the motion of two different fermion components; the rotation of these fermions is counterbalanced by bosons, shown by yellow flashes.

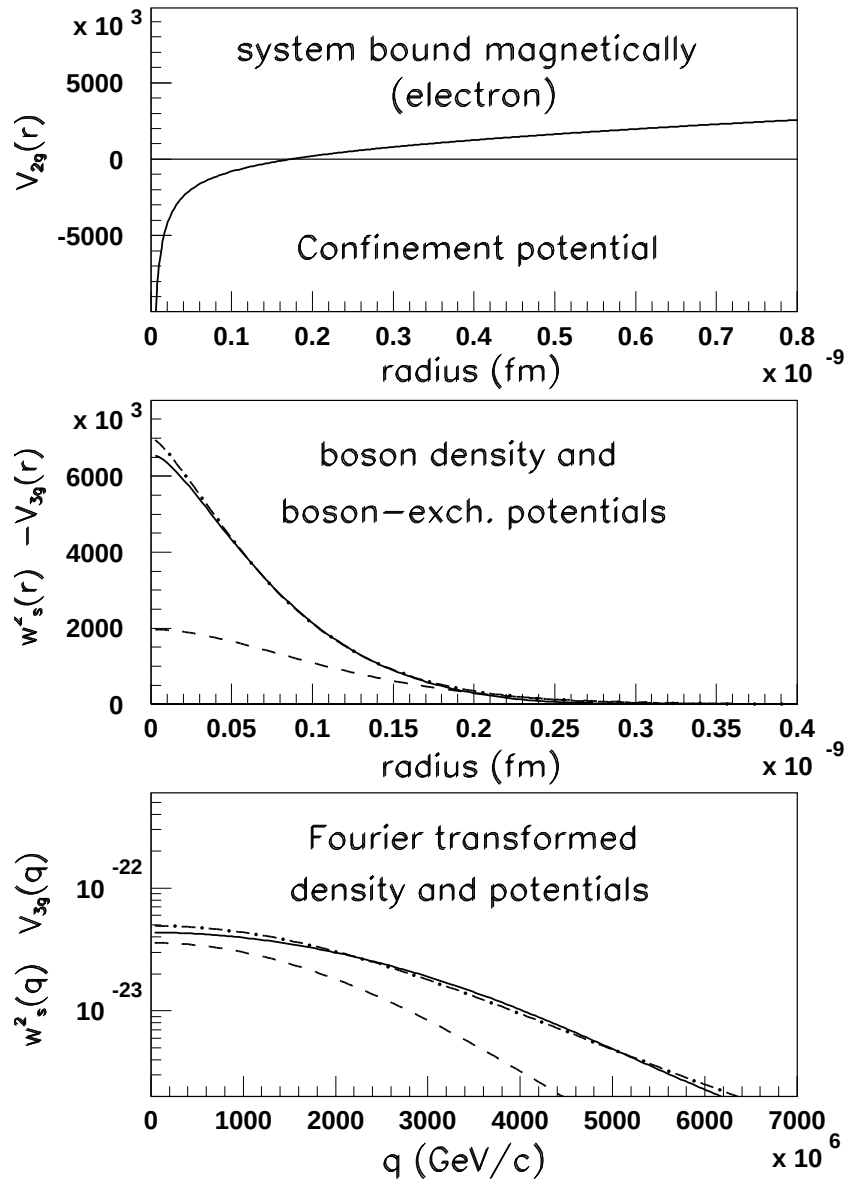


Figure 2: Radial dependence of a system bound magnetically with a root mean square radius $\langle r_{w_s}^2 \rangle^{1/2} = 1.9 \times 10^{-10}$ fm (electron). Upper part: Confinement potential $V_{2g}(r)$. Middle part: Boson density $w_s^2(r)$ (dot-dashed line) and boson-exchange potentials $|V_{3g}^{s,v}(r)|$ given by dashed and solid lines, respectively. Lower part: Fourier transformed density (dot-dashed line) and potentials (dashed and solid lines).

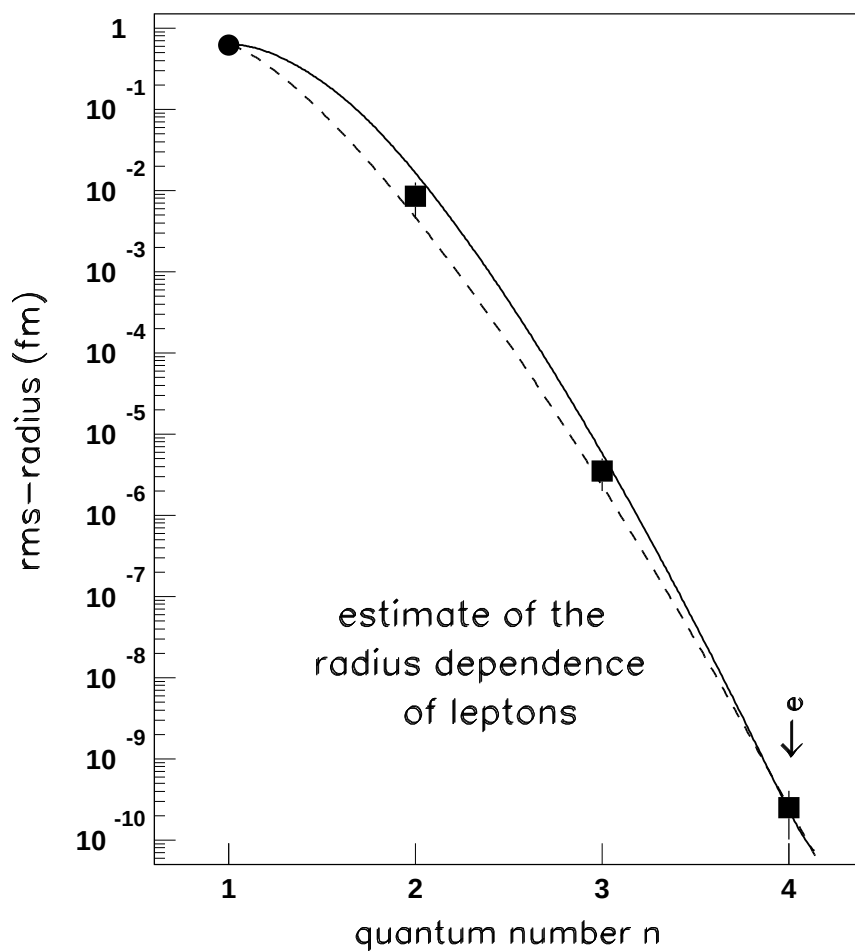


Figure 3: Estimates of fermion root mean square radii of μ and τ leptons, given by solid squares, from a quantum condition demanding that the radii of the different leptons follow the radial density $w_s^2(r)$ (dashed line) or potential $|V_{3g}^s(r)|$ (solid line) of the basic proton system. The rms-radius of the magnetic part of the proton binding potential is given by solid point, the lines are adjusted to give an electron rms-radius of $2 \cdot 10^{-10}$ fm (solid square).