

Fuzzy tangle graph

Abstract:

We will study anew graph , this graph called fuzzy tangle graph, we will study the matrices which represent this graph, and we will discuss the relation between fuzzy tangle graph and dual fuzzy tangle graph. In fuzzy tangle graph the concepts of α -cut tangle graph ,strength of edge are developed.

Key word: Tangle graph, fuzzy tangle graph, and dual fuzzy tangle graph.

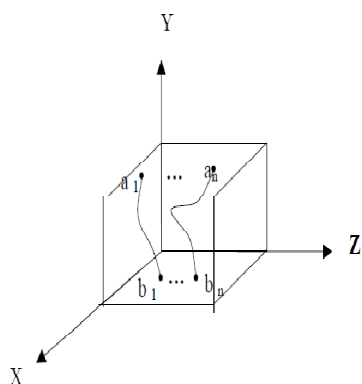
Introduction:

Conway developed tangle theory and invented a system of notation for tabulating knots, nowadays known as Conway notation. Tangle theory can be considered analogous to knot theory except, instead of closed loop we use string whose end are nailed down. Tangles have been shown to be useful in studying DNA topology. It is quite well known that graphs are simply models of relations. A graph is a convenient way of representing information involving relationship between objects. The objects are represented by vertices and relations by edges. When there is vagueness in the description of the objects or in its relationships or in both, it is natural that we need to design a 'Fuzzy Graph Model'. The concept of fuzzy sets and fuzzy relations was introduced by L.A.Zadeh in 1965 [1] and further studied in [2]. It was Rosenfeld [5] who considered fuzzy relations on fuzzy sets and developed the theory of fuzzy graphs in 1975.

Basic concepts:

Definition 1:

Tangle graph: Let D be a unit cube, so $D=\{(x,y,z):0<x,y,z<1\}$ on the top face of cube place n points $a_1, a_2, \dots, a_n$ similarly place on bottom face $b_1, b_2, \dots, b_n$, now join the points $a_1, a_2, \dots, a_n$ with $b_1, b_2, \dots, b_n$ by arcs $d_1, d_2, \dots, d_n$ these arcs are disjoint and each d_i connects some a_j to b_k not connect a_j to a_k or b_j to b_k this called tangle [4].



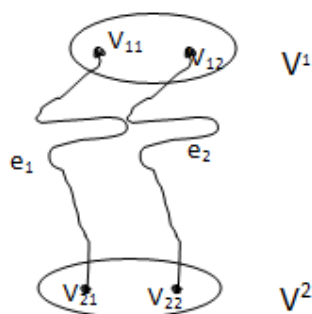


Fig.(1)

Where V^1 and V^2 are outer vertices, v_{11}, v_{12}, v_{21} and v_{22} are inner vertices. e_1 and e_2 are called fibers (edges) of tangle graph.

In fig (1) tangle graph $T(V,E)$ consists of :

$$V = \{ V^1 = \{v_{11}, v_{12}\}, V^2 = \{v_{21}, v_{22}\} \}$$

$$E = \{e_1, e_2\}, e_1 = \{v_{11}, v_{21}\}, e_2 = \{v_{12}, v_{22}\}.$$

Incident matrix:

T	e_1	e_2
V^1	1	1
v_{11}	1_{03}^1	0
v_{12}	0	1_{03}^1
V^2	1	1
v_{21}	1_{03}^1	0
v_{22}	0	1_{03}^1

Where (1) express outer vertices and fiber of tangle graph, (1_{ij}^k) express the fiber of tangle graph analytically, where (k) express the number of fiber, (i) express number of rolls in fiber, and (j) express the number of curves in fiber of tangle graph [5].

Main results:

Dual of tangle graph:

The tangle graph (T^*) is called the dual tangle graph of (T). A tangle graph $T = (V; E_1, E_2, \dots, E_m)$; $V = (v_1, v_2, \dots, v_n)$ { where (V) outer vertex in tangle graph } can be mapped to tangle graph $T^* = (V^*; V_1, V_2, \dots, V_n)$ whose vertices are the points $e_1, e_2, \dots, e_m$, and whose edges are

$V_1, V_2, \dots, V_n$. The incident matrix of dual tangle graph must be the transport matrix of the incident matrix of tangle graph. In fig. (1) We will find T^* :

$$V = \{e_1, e_2\}, V^1 = \{e_1, e_2\}, V^2 = \{e_1, e_2\}, v_{11} = \{e_1\}, v_{12} = \{e_2\}, v_{21} = \{e_1\} \text{ and } v_{22} = \{e_2\}$$

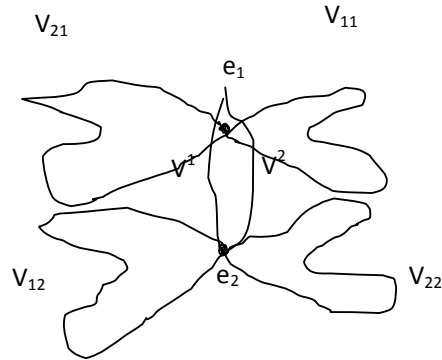


Fig.(2)

T^*	V^1	v_{11}	v_{12}	V^2	v_{21}	v_{22}
e_1	1	1_{03}^1	0	1	1_{03}^1	0
e_2	1	0	1_{03}^1	1	0	1_{03}^1

$$(T)^T = (T^*)$$

Fuzzy tangle graph:

The fuzzy tangle graph is a tangle graph which supply by property , this property is defined by the degree membership of vertex v_i to edge E_j is defined by $\mu_j(v_i)$.

We have two cases of fuzzy tangle graph ,because of in tangle graph consists of inner and outer vertices.

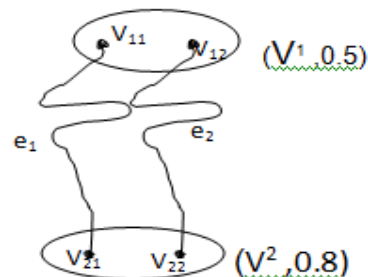
Case 1:

When the outer vertex supplies by property then:

$$E = \{e_1, e_2\}$$

$$V = \{ (V^1, 0.5), v_{11}, v_{12}; (V^2, 0.8), v_{21}, v_{22} \}$$

The incident matrix:



$T^{\sim}$	e_1	e_2
V^1	0.5	0.5
v_{11}	0	0
v_{21}	0	0
V^2	0.8	0.8
v_{21}	0	0
v_{22}	0	0

Fig.3

Dual fuzzy tangle graph:

$V=\{e_1^{\sim}, e_2^{\sim}\}$.

$E^{\sim}=\{V^1=\{(e_1^{\sim}, 0.5), (e_2^{\sim}, 0.5)\}, v_{11}=\{e_1^{\sim}\}, v_{12}=\{e_2^{\sim}\}, V^2=\{(e_1^{\sim}, 0.8), (e_2^{\sim}, 0.8)\}, v_{21}=\{e_1^{\sim}\}$
 $\}$ and $v_{22}=\{e_2^{\sim}\}$.

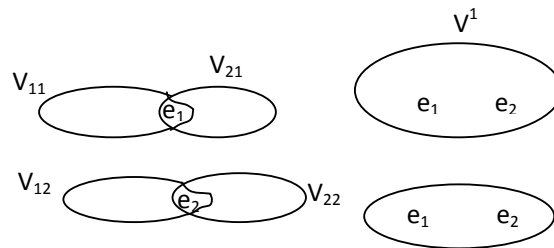


Fig.4

$T^{\sim}$	V^1	v_{11}	v_{21}	V^2	v_{21}	v_{22}
$e_1^{\sim}$	0.5	0	0	0.8	0	0
$e_2^{\sim}$	0.5	0	0	0.8	0	0

$$(T^{\sim})^T = T^{*\sim}$$

Case 2:

When inner vertices supplies by property:

$V^1=\{v_{11}, v_{12}\}$, $V^2=\{v_{21}, v_{22}\}$.

$$\varepsilon = \{ e_1^{\sim}, e_2^{\sim}, e_{12} \}, e_1^{\sim} = \{ (v_{11}, 0.5), (v_{21}, 1) \}, e_2^{\sim} = \{ (v_{12}, 0.8), (v_{22}, 0.5) \} \text{ and } e_{12} = \{ V^1, V^2 \}$$

$T^{\sim}$	$e_1^{\sim}$	$e_2^{\sim}$	e_{12}
V^1	1	1	1
v_{11}	0.5	0	0.5
v_{12}	0	0.8	0.8
V^2	1	1	1
v_{21}	1	0	1
v_{22}	0	0.5	0.5

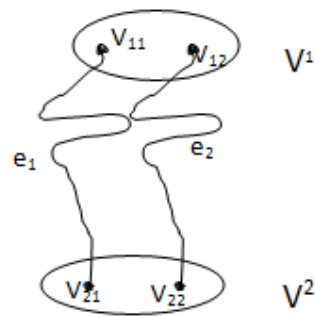


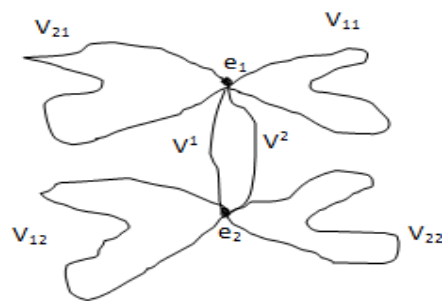
Fig.(5) (fuzzy tangle graph)

Dual fuzzy tangle graph:

$$V = \{ e_1^{\sim}, e_2^{\sim}, e_{12} \}.$$

$$V_{11} = \{ (e_1^{\sim}, 0.5), e_{12} \}, V_{21} = \{ (e_1^{\sim}, 1), e_{12} \}, V_{12} = \{ (e_2^{\sim}, 0.8), e_{12} \}, V_{22} = \{ (e_2^{\sim}, 0.5), e_{12} \} \text{ and } V^1 = V^2 = e_{12}$$

$T^{\sim*}$	V^1	v_{11}	v_{12}	V^2	v_{21}	v_{22}
$e_1^{\sim}$	1	0.5	0	1	1	0
$e_2^{\sim}$	1	0	0.8	1	0	0.5
e_{12}	1	0.5	0.8	1	1	0.5



$T^{\sim T}$ =Fig.6

Strength of edge in fuzzy tangle graph:

We see that some edges contains only vertices having high membership dgree. In figure (5) the edge $e_1^{\sim}$ contains the vertices having member ship at least 0.5. on the other hand $e_2^{\sim}$ as strong as $e_1^{\sim}$.

$T^{\sim}$	$e_1^{\sim}$	$e_2^{\sim}$	e_{12}
V^1	0	0	0
v_{11}	0.5	0	0
v_{12}	0	0.5	0
V^2	0	0	0
v_{21}	0.5	0	0
v_{22}	0	0.5	0

Where V^1, V^2 not supply by any property, then $\mu_j(v^i) = 0$.

α -Cut fuzzy tangle graph:

If we cut a fuzzy tangle graph T at α level , we obtain the α -Cut tangle graph T_α .

1. $T_\alpha = (V_\alpha, E_\alpha)$
2. $V_\alpha = \{ V^i, v_1, v_2, \dots, v_n \}$, where (V^i outer vertex , $v_1, v_2, \dots, v_n$ inner vertices)
3. $E_\alpha = \{ E_{j,\alpha} \mid E_{j,\alpha} \neq \emptyset, j=1,2,\dots,m+1 \}$
4. $E_{j,\alpha} = \{ v_i \mid \mu_j(v_i) \geq \alpha, j=1,2,\dots,m \}$
5. $E_{m+1,\alpha} = \{ v_i \mid \mu_j(v_i) < \alpha \text{ for all } j \}$

In fig.(5) at $\alpha=0.8$ for fuzzy tangle graph ,anew edge (e_3) is added to contain the element v_{11} having less than 0.8 .The tangle graph $T_{0.8}$ is given in the following figure:

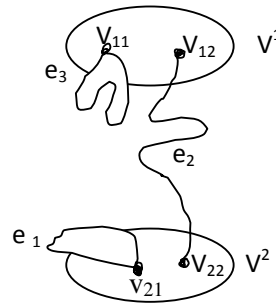


Fig.(7)

The incident matrix:

$T_{0.8}$	e_1	e_2	e_3
V^1	0	1	1
v_{11}	0	0	$(1_{03}^1)'$
v_{12}	0	1_{03}^1	0
V^2	1	1	0
v_{21}	1	0	0
v_{22}	0	1_{03}^1	0

Where $(1_{03}^1)'$ express the new edge.

The dual of α - cut of tangle graph:

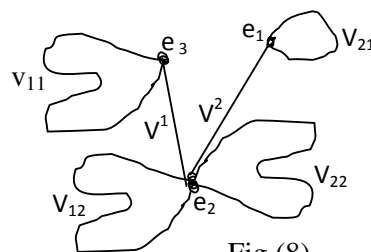


Fig.(8)

The incident matrix:

$T_{0.8}^*$	V^1	v_{11}	v_{12}	V^2	v_{21}	v_{22}
e_1	0	0	0	1	1	0
e_2	1	0	1_{03}^1	1	0	1_{03}^1
e_3	1	$(1_{03}^1)'$	0	0	0	0

$T_{0.8}=T_{0.8}^*$

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