

Analytical modeling of the Thermoelectric Effect in Photovoltaic Cells: Combined Solar Photovoltaic and Thermoelectric Generator System (PV+TEG)

Abstract

Aims: Analytical modeling of the combined systems photovoltaic-thermoelectric (PV + TEG). The advantage of these systems is double:

- On the one hand, they allow to cool the photovoltaic cells (PV), which avoids the loss of electrical efficiency observed in the devices,
- On the other hand, recover this lost energy in the form of heat, and transform it into electrical energy thanks to the thermoelectric modules operating in Seebeck mode.

Study design: Laboratory of Radiation Physics LPR, FAST-UAC, 01 BP 526, Cotonou, Benin. Department of Physics (FAST) and Doctoral Formation Materials Science (FDSM), University of Abomey-Calavi, Benin.

Methodology: We considered the temperature distribution in the semiconductor plate of the Thermoelectric Generator System (TEG). We resolved the thermal conductivity equation described by:

$$\frac{\partial T}{\partial t} - a^2 \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) = Q(x, y, z,)$$

Where a^2 is the thermal diffusivity, $Q(x, y, z)$ is the heat flow going from the PV to the TEG module which is dissipated through the latter; using constants variation method. We assumed that the temperature along the y-axis is considered uniform.

Results: The results obtained show, an increasing of the photon generation rate (G) and the photocurrent density (J_{ph}), when the temperature (T) increases.

Conclusion: We noted, an increasing of the fill factor (FF), a reduction of the series resistance (R_s); which improves the quantum efficiency of the combined system (PV+TEG).

Keywords: Strong sunshine, quantum efficiency, thermoelectric module, Seebeck mode, temperature distribution, thermal conductivity.

1. Introduction

35 The major disadvantage of the photovoltaic cells is their low quantum
36 efficiency. Indeed, only a small part of the power of the incident radiation is
37 converted into electricity. Furthermore, unconverted radiations heat the
38 photovoltaic panels, which further reduce their efficiency. Cooling systems are
39 integrated at present in these cells, in order to decrease this temperature and
40 allow a better quantum efficiency of the devices; hence the idea to realize the
41 combined thermoelectric-photovoltaic systems [1].

42 Converting the energy from the Sun directly to electricity in an efficient way
43 is of great interest. Photovoltaic devices (PV) can directly convert parts of the
44 solar spectrum, but a significant part is absorbed as heat. In order to remedy
45 this, a number of combined photovoltaic and heat recovery systems have been
46 proposed recently. The most simple of these convert the heat energy directly to
47 electrical energy using a thermoelectric generator (TEG). The latter utilizes the
48 Seebeck effect to convert heat directly into electrical energy through the
49 movement of charge carriers induced by a temperature span across the TEG [2].
50 A number of such combined systems have been studied recently [3-8].

51 Coupled PV+TEG systems have also been considered solely from a modeling
52 basis [9-17]. Experimental realizations of small combined PV and TEG systems
53 have reported a significant increase in performance compared to that of a PV
54 alone [18-27].

55 Compared with the single PV or TEG energy producing source, the
56 (PV+TEG) hybrid energy source can offer some advantages, namely the better
57 energy security due to the use of multiple energy sources, the higher fuel
58 economy due to the increase of on-board renewable energy, and the higher
59 control flexibility due to the coordination for charging the same pack of
60 batteries. So the Thermoelectric-Photovoltaic hybrid energy source is promising
61 for the application to HVs.

62 The performance of the combined system should be given in terms of both
63 generated electric power and overall system efficiency by highlighting their

64 dependence on environmental conditions, such as temperature and radiation,
65 and on physical properties of the used material.

66 Starting from these considerations the principle of superposition is therefore
67 usable; then the generated electrical power of the overall system will be the sum
68 of the electric powers generated by both modules. Under this assumption the
69 overall efficiency of the system can be calculated as the ratio between the sum
70 of the generated electric powers by each module, and the power of the input
71 system, i.e. the solar radiation available to the PV module.

72 Recently [2] has studied the performance of a combined solar photovoltaic
73 (PV) and thermoelectric generator (TEG) system, using an analytical model for
74 four different types of commercial PVs and a commercial bismuth telluride
75 TEG. The TEG is applied directly on the back of the PV, so that the two devices
76 have the same temperature. The PVs considered are crystalline Si (c-Si),
77 amorphous Si (a-Si), copper indium gallium (di)selenide (CIGS) and cadmium
78 telluride (CdTe) cells. They have shown that, the degradation of PV
79 performance with temperature is shown to dominate the increase in power
80 produced by the TEG, due to the low efficiency of the TEG. For c-Si, CIGS and
81 CdTe PV cells the combined system produces a lower power and has a lower
82 efficiency than the PV alone, whereas for an a-Si cell the total system
83 performance may be slightly increased by the TEG.

84 More recently [28] presents a dynamic theory of the thermoelectric effect that
85 occurs in a uniform unipolar semiconductor under the influence of a time-
86 dependent temperature distribution. To be specific, the temperature distribution
87 is chosen as a plane traveling wave of constant amplitude. The dependence of
88 the thermoelectric current, appearing under these conditions, on the
89 semiconductor properties, as well as on the parameters of the temperature wave,
90 is studied.

91 In this paper, we considered the temperature distribution in the semiconductor
92 plate of the Thermoelectric Generator System (TEG). We resolved the thermal
93 conductivity equation described by:

$$94 \quad \frac{\partial T}{\partial t} - a^2 \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) = Q(x, y, z, t) \quad (1)$$

95 Where a^2 is the thermal diffusivity, $Q(x, y, z, t)$ is the heat flow going from the
96 PV to the TEG module which is dissipated through the latter; using constants
97 variation method. We assumed that the temperature along the y-axis is
98 considered uniform.

99 The paper is organized as follows. In Section 2, the combined PV and TEG
100 system considered is presented. Next, the theoretical approach is established. In
101 Section 3, the solving of the thermal conductivity equation is described. Finally,
102 the conclusion and outlook are listed in Section 4.

103

104 **2. Materials and method**

105 **2.1. The studied system**

106 We consider a combined (PV+TEG) system, where the TEG is mounted directly
107 on the back of the PV. The hot side temperature of the TEG is thus equal to the
108 temperature of the PV, $T_{PV} = T_{TEG.hot}$. The system is shined upon by the Sun,
109 with an arbitrary concentration of the light. The studied setup is illustrated in
110 Fig. 1.

111 We assume that the properties of the PV do not change with solar
112 concentration, but only with temperature. We note that the cold side of the TEG
113 is assumed to be perfectly cooled to the reference temperature 25°C in the
114 remainder of this paper. The system operates at room temperature having as
115 input the solar radiation and as output the total electric power generated by the
116 system. At high solar irradiance the PV module temperature (T_{max}) can reach
117 50-80°C. These values depend on the site, the type of the integration and of the
118 period of year considered.

We assume that the photons provided from the Sun are either converted directly into electrical energy, with fraction η_{PV} , converted into heat, η_T , or not absorbed, η_{non} , such that

$$\eta_{PV} + \eta_T + \eta_{non} = 100\% \quad (2)$$

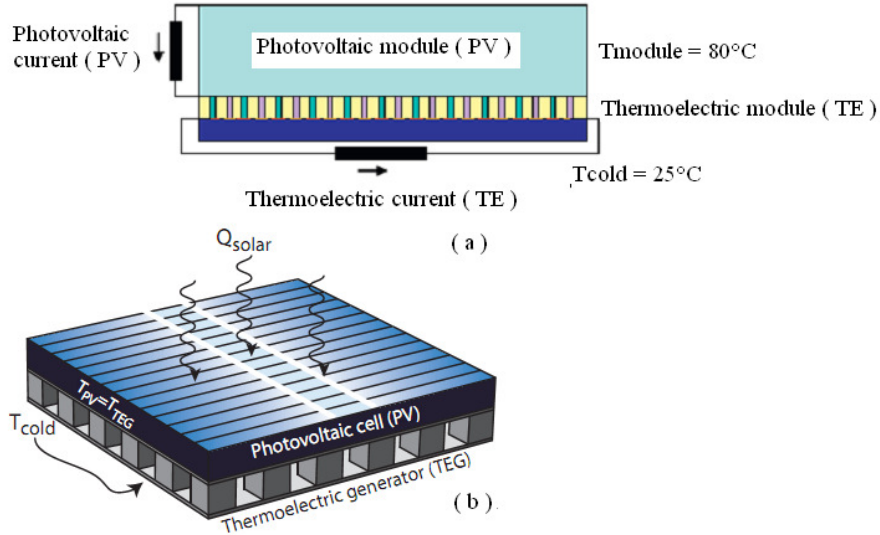


Fig. 1. An illustration of the system considered. a- Equivalent system; current generator, b- the incoming solar radiation hits the PV, which heats up from parts of the radiation. This heat is transferred through the TEG, to the cold side [2]. The fraction of non-absorbed photons is taken to be constant.

$$\eta_{PV} = \eta_{T_{ref}} (1 + \beta(T_{PV,0} - T)) \quad (3)$$

where $\eta_{T_{ref}}$ is the reference temperature, typically 25°C , and β is the temperature coefficient [29-30].

The efficiency of a TEG is generally a nonlinear function of temperature since the thermoelectric material properties vary nonlinearly with temperature. In the case when the material properties do not depend on temperature an analytic expression for the efficiency can be derived. This depends on both the hot and cold side temperature and the temperature span. However, this assumption is not applicable for actual thermoelectric materials [2].

The heat flux available at the hot side of the TEG is assumed to be what remains of the absorbed radiation of the PV power production:

$$Q_{TEG}(T) = Q_{Solar}(1 - \eta_{PV}(T) - \eta_{non}) \quad (4)$$

The total electrical power produced is the sum of the power produced by the PV and by the TEG:

$$P_{tot}(T) = Q_{Solar}\eta_{PV}(T) + Q_{TEG}(T)\eta_{TEG}(T) \quad (5)$$

The maximum total power produced by the combined PV and TEG system can then be obtained. The efficiency of a coupled PV and TEG system are given as.

$$\eta_{PV+TEG}(T) = \eta_{PV}(T) + \eta_{TEG}(T)(1 - \eta_{PV}(T) - \eta_{non}) \quad (6)$$

An amorphous Si (a-Si) solar photovoltaic (PV) and a commercially available bismuth telluride (CdTe) (TEG) is the combined (PV+TEG) system considered in this study.

2. 2. Method: boundary conditions (BCs)

The Boundary conditions (BCs) for equation (1) in the geometry under consideration are as follows. If the upper surface of the sample ($Z = 0$) is illuminated with light of intensity $I = I_0(1 + \cos\phi)$, completely absorbed in an infinity thin near-surface layer of the sample without photocarrier generation, the appropriate (BCs) are as follows:

$$\left. \frac{\partial T}{\partial z} \right|_{Z=0} = -\frac{g_0 I_0}{\kappa} (1 + \cos\phi) \quad (7)$$

where κ : the thermal conductivity of the semiconductor, g_0 : the heating efficiency of the semiconductor by the light.

On the opposite side of the sample ($z = h$), which is the interface with the substrate serving as the thermostat, the BC is written in the form [28].

$$\left. \frac{\partial T}{\partial z} \right|_{Z=h} = -\frac{\chi}{\kappa} (T - T_0) \quad (8)$$

where χ : is the surface thermal conductivity.

Along the x axis, we have:

$$\left. \frac{\partial T}{\partial z} \right|_{Z=0} = \left. \frac{\partial T}{\partial z} \right|_{Z=h} \quad (9)$$

165 These boundary conditions are exploited to solve the equation (1).

166 3. Results and Discussion

167 3.1. Resolution of homogeneous equation

168 The temperature homogeneous distribution in the combined (PV+TEG) system
169 is described by the thermal conductivity equation (10).

$$170 \quad \frac{\partial T}{\partial t} - a^2 \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) = 0 \quad (10)$$

171 The homogeneous solution of equation (10) will be expressed in the form of

$$172 \quad T(x, z, t) = A_1(z) \sin \Phi + A_2(z) \cos \Phi + A_3(z) \quad (11)$$

173 Substituting equation (11) in (equation (1)), one obtains the system of
174 equations for the temperature wave amplitudes $A_1(z) - A_3(z)$.

$$175 \quad A_1''(z) - k^2 A_1(z) + \frac{\omega}{a^2} A_2(z) = 0 \quad (12)$$

$$176 \quad A_2''(z) - k^2 A_2(z) - \frac{\omega}{a^2} A_1(z) = 0 \quad (13)$$

$$177 \quad A_1''(z) = 0 \quad (14)$$

178 with boundary condition (BC), we have:

$$179 \quad A_1'(0) = 0, A_2'(0) = A_3'(0) = -\frac{g_0 I_0}{k} \quad (15)$$

$$180 \quad A_{1,2}'(h) = -\frac{\chi}{k} A_{1,2}(h) \quad (16)$$

$$181 \quad A_3'(h) = -\frac{\chi}{k} (A_3(h) - T_0) \quad (17)$$

182 The general solution to the system of equations (12)-(14) has the following
183 form:

$$184 \quad A_1(z) = \frac{C_1 + iC_3}{2} \cosh \frac{k_1 z}{a} + \frac{C_1 + iC_3}{2} \cosh \frac{k_2 z}{a} + \frac{a(C_2 + iC_4)}{2k_1} \sinh \frac{k_1 z}{a} \\ 185 \quad + \frac{a(C_2 + iC_4)}{2k_2} \sinh \frac{k_2 z}{a} \quad (18)$$

$$186 \quad A_2(z) = \frac{C_3 + iC_1}{2} \cosh \frac{k_1 z}{a} + \frac{C_3 + iC_1}{2} \cosh \frac{k_2 z}{a} + \frac{a(C_4 + iC_2)}{2k_1} \sinh \frac{k_1 z}{a} \\ 187 \quad + \frac{a(C_4 + iC_2)}{2k_2} \sinh \frac{k_2 z}{a}, \quad (19)$$

$$188 \quad A_3(z) = C_5 z + C_6 \quad (20)$$

Where $k_1^2 = a^2 k^2 + i\omega$; $k_2^2 = a^2 k^2 - i\omega$ C_{1-6} are the integration constants.

When $\frac{h}{a}|k_{1,2}| \ll 1$ (For a thermally thin sample), equation (18) and equation (19) become:

$$A_1(z) = C_1 + C_2 z \quad \text{and} \quad A_2(z) = C_3 + C_4 z \quad (21)$$

Substituting equation (20) and equation (21) in equation (15), one obtains:

$$C_2 = 0 \quad \text{and} \quad C_4 = C_5 = -\frac{g_0 I_0}{k} \quad (22)$$

The integration constants C_1 and C_3 are calculated, substituting equation (21) in equation (16), we obtain:

$$C_1 = 0 \quad \text{and} \quad C_3 = \frac{g_0 I_0 (k + \chi h)}{k \chi} \quad (23)$$

C_6 is determined, substituting equation (20) in equation (17). We have:

$$C_6 = T_0 - \frac{g_0 I_0 (k + \chi h)}{k \chi} \quad (24)$$

Once the integration constants have been determined, we have substituted equation (20) and equation (21) in equation (11), and we obtained the following temperature distribution in the thermally thin sample by:

$$T(x, z, t) = T_0 - \frac{g_0 I_0}{k} \left(z - \frac{\kappa + \chi h}{\chi} \right) (1 + \cos \Phi) \quad (25)$$

When the semiconductor is weakly coupled to the substrate, we have ($\chi h \ll \kappa$), and the temperature distribution is given as follows:

$$T(x, z, t) = T_0 + \frac{g_0 I_0}{k} (1 + \cos \Phi) \quad (26)$$

3. 2. Resolution of inhomogeneous equation

The temperature inhomogeneous distribution in the combined (PV+TEG) system is described by the thermal conductivity equation

$$\frac{\partial T}{\partial t} - a^2 \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) = Q(x, y, z, t) \quad (27)$$

where $Q(x, y, z, t) = G_0 e^{-\beta z} + I_0 (1 + \cos \Phi)$:

The general form of the particular solution is given by:

$$Q(x, y, z, t) = A(z) e^{-\beta z} + B(z) \sin \Phi + C(z) \cos \Phi + D(z) \quad (28)$$

215 We have sought the constants $A(z)$, $B(z)$, $C(z)$ and $D(z)$ by the following
216 transformations:

$$217 \quad \frac{\partial Q}{\partial t} = wB(z)\cos\Phi - wc(z)\sin\Phi \quad (29)$$

$$218 \quad \frac{\partial Q}{\partial x} = kB(z)\cos\Phi - kC(z)\sin\Phi \quad (30)$$

$$219 \quad \frac{\partial^2 Q}{\partial x^2} = -k^2B(z)\sin\Phi - k^2C(z)\cos\Phi$$

$$220 \quad \frac{\partial^2 Q}{\partial y^2} = -0 \quad (31)$$

$$221 \quad \frac{\partial Q}{\partial y} = A'(z)e^{-\beta z} - A(z)\beta e^{-\beta z} + B'(z)\sin\Phi - C'(z)\cos\Phi + D'(z) \quad (32)$$

$$222 \quad \frac{\partial^2 Q}{\partial z^2} = A''(z)e^{-\beta z} - A'(z)\beta e^{-\beta z} - A'(z)\beta e^{-\beta z} - A(z)\beta^2 e^{-\beta z} + B''(z)\sin\Phi -$$

$$223 \quad C''(z)\cos\Phi + D''(z) \quad (33)$$

$$224 \quad \frac{\partial^2 Q}{\partial z^2} = (A''(z) - 2A'(z)\beta - A(z)\beta^2)e^{-\beta z} + B''(z)\sin\Phi - C''(z)\cos\Phi + D''(z)$$

$$225 \quad (34)$$

226 By identification, we have:

$$227 \quad A''(z) - 2A'(z)\beta - A(z)\beta^2 = G_0 \quad (35)$$

$$228 \quad B''(z) - k^2B(z) + \frac{w}{a^2}C(z) = I_0 \quad (36)$$

$$229 \quad C''(z) - k^2C(z) - \frac{w}{a^2}B(z) = 0 \quad (37)$$

$$230 \quad D''(z) = -\frac{I_0}{a^2} \quad (38)$$

$$231 \quad D''(z) = -\frac{I_0}{a^2} \quad \text{and} \quad D'(z) = -\frac{I_0}{a^2}z + \delta \quad (39)$$

$$232 \quad D(z) = -\frac{I_0}{2a^2}z^2 + \delta z + \gamma \quad (40)$$

233 We determine the constants δ et γ , using the boundary Conditions (15) and
234 (17):

$$235 \quad D'(z) = -\frac{I_0}{a^2}z + \delta \quad \text{therefore} \quad D'(0) = \delta$$

$$D'(0) = \delta = -\frac{g_0 I_0}{k} \quad (41)$$

We posed:

$$D'(h) = -\frac{\chi}{k} [D(h) - T_0] = D(z) = -\frac{\chi}{k} \left[-\frac{I_0}{2a^2} h^2 - \frac{g_0 I_0}{k} h + \gamma - T_0 \right] \quad (42)$$

$$\text{That to say} \quad D'(h) = -\frac{I_0}{a^2} h - \frac{g_0 I_0}{k} \quad (43)$$

By combining equation (42) and equation (43), we have:

$$-\frac{\chi}{k} \left[-\frac{I_0}{2a^2} h^2 - \frac{g_0 I_0}{k} h + c - T_0 \right] = -\frac{I_0}{a^2} h - \frac{g_0 I_0}{k} \quad (44)$$

$$\text{Then} \quad \gamma = T_0 + \frac{I_0}{2a^2} h^2 + \frac{g_0 I_0}{k} h + \frac{I_0}{\chi a^2} h + \frac{g_0 I_0}{\chi} \quad (45)$$

And we obtain:

$$D(z) = -\frac{I_0}{2a^2} z^2 - \frac{g_0 I_0}{k} z + \left(T_0 + \frac{I_0}{2a^2} h^2 + \frac{g_0 I_0}{k} h + \frac{I_0}{\chi a^2} h + \frac{g_0 I_0}{\chi} \right) \quad (46)$$

The constant B (z) is given by:

$$B(z) = \frac{+iC_3}{2} \cosh \frac{k_1 z}{a} + \frac{C_1 - iC_3}{2} \cosh \frac{k_2 z}{a} + \frac{C_2 + iC_4}{2k_1} \cosh \frac{k_1 z}{a} + \frac{C_2 - iC_4}{2k_2} \cosh \frac{k_2 z}{a} \quad (47)$$

We search now C (z):

We identified C''(z) to I₀ from (equation (37)).

$$C'' = -I_0 \quad (48)$$

$$C'(z) = -I_0 z + \varepsilon \quad (49)$$

$$C(z) = -\frac{I_0}{2} z^2 + \varepsilon z + \vartheta \quad (50)$$

$$\text{where } C'(0) = \varepsilon = -\frac{g_0 I_0}{k}$$

$$\text{as far as } C'(h) = -\frac{\chi}{k} C(h) = -I_0 h - \frac{g_0 I_0}{k} = -\frac{\chi}{k} \left(-\frac{I_0}{2} h^2 + \varepsilon h + \vartheta \right) \quad (51)$$

$$-\frac{I_0}{2} h^2 + -\frac{g_0 I_0}{k} h + \vartheta = I_0 \frac{k}{\chi} h + \frac{g_0 I_0}{\chi} \quad (52)$$

$$\text{and} \quad \vartheta = I_0 \frac{k}{\chi} h + \frac{g_0 I_0}{\chi} + -\frac{I_0}{2} h^2 + \frac{g_0 I_0}{k} h \quad (53)$$

$$C(z) = \frac{-I_0}{2} z^2 - \frac{g_0 I_0}{k} z + I_0 \frac{k}{\chi} h + \frac{g_0 I_0}{\chi} + -\frac{I_0}{2} h^2 + \frac{g_0 I_0}{k} h \quad (54)$$

We search A (z):

$$A''(z) - 2A'(z)\beta - A(z)\beta^2 = G_0 \quad (55)$$

One poses $y'' = A''(z)$; $y' = A'(z)$ et $y = A(z)$

$$\text{we have } y'' - 2\beta y' - \beta^2 y = 0 \quad (56)$$

The characteristic equation of equation (56) is given by:

$$r^2 - 2\beta r - \beta^2 = 0 \quad (57)$$

$$\Delta = 4\beta^2 - 4\beta^2 = 0$$

$$r = \beta$$

The homogeneous solution of equation (56) is given by

$$y_h = (C_1 + C_2 z)e^{\beta z} \quad (58)$$

The particular solution of equation (55) is:

$$y_p = \frac{G_0}{\beta^2} \quad (59)$$

General solution of equation (55) is expressed by:

$$y = y_h + y_p = (C_1 + C_2 z)e^{\beta z} + \frac{G_0}{\beta^2} \quad (60)$$

Identifying y to A(z), we have:

$$A(z) = (C_1 + C_2 z)e^{\beta z} + \frac{G_0}{\beta^2} \quad (61)$$

$$\text{where } e^{\beta z} = \cosh \beta z + \sin \beta z$$

$$\text{Therefore } A(z) = (C_1 + C_2 z) (\cosh \beta z + \sin \beta z) + \frac{G_0}{\beta^2} \quad (62)$$

The general solution of equation (1) becomes:

$$T(x, z, t) = T_0 - \frac{g_0 I_0}{k} \left(z - \frac{\kappa + \chi h}{\chi} \right) (1 + \cos \Phi) + A(z)e^{-\beta z} + B(z) \sin \Phi + C(z) \cos \Phi + D(z) \quad (64)$$

278 For $\chi h \ll \kappa$

$$279 \quad T(x, z, t) = T_0 + \frac{g_0 I_0}{k} (1 + \cos \Phi) + A(z)e^{-\beta z} + B(z)\sin \Phi + C(z)\cos \Phi +$$

$$280 \quad D(z) \quad (65)$$

281 where the constants $A(z)$, $B(z)$, $C(z)$ and $D(z)$ are determined above.

282
283

284 3. 3. Maximum temperature of combined (PV + TEG) system

285 The efficiency as a function of temperature span is given by [21].

$$286 \quad \eta_{TEG} = \alpha \Delta T^2 + \delta \Delta T \quad (66)$$

287 Where $\alpha = -1.21(5) * 10^{-6} K^{-2}$ and $\delta = 4.87(7) * 10^{-4} K^{-1}$ for a cold side
288 temperature of 25°C.

289

290 From equation (66), equation (4) and equation (5), we deduced the temperature
291 at which the total power output is maximum. This is given by [2].

$$292 \quad T_{max} = \frac{1}{4\alpha\beta\eta_{PV,o}} (\pm 2T_C\alpha\beta\eta_{PV,o} \pm 2T_{ref}\alpha\beta\eta_{PV,o} \pm \beta\delta\eta_{PV,o} \pm 2\alpha\eta_{non} \pm$$

$$293 \quad 2\alpha\eta_{PV,o} \pm 2\alpha \pm (4T_C^2\alpha^2\beta^2(\eta_{PV,o})^2 - 8T_C * T_{ref}\alpha^2\beta^2(\eta_{PV,o})^2 +$$

$$294 \quad 4T_{ref}^2\alpha^2\beta^2(\eta_{PV,o})^2 - 4T_C\alpha\beta^2\delta(\eta_{PV,o})^2 + 4T_{ref}\alpha\beta^2\delta(\eta_{PV,o})^2 -$$

$$295 \quad 8T_C\alpha^2\beta\eta_{non}\eta_{PV,o} - 8T_C\alpha^2\beta(\eta_{PV,o})^2 + 8T_{ref}\alpha^2\beta\eta_{non}\eta_{PV,o} +$$

$$296 \quad 8T_{ref}\alpha^2\beta(\eta_{PV,o})^2 + \beta^2\delta^2(\eta_{PV,o})^2 + 8T_C\alpha^2\beta\eta_{PV,o} - 8T_{ref}\alpha^2\beta\eta_{PV,o} +$$

$$297 \quad 8\alpha\beta^2(\eta_{PV,o})^2 + 4\alpha\beta\delta(\eta_{PV,o})^2 + 4\alpha^2(\eta_{non})^2 + 8\alpha^2\eta_{non}\eta_{PV,o} +$$

$$298 \quad 4\alpha^2(\eta_{PV,o})^2 - 4\alpha\beta\delta\eta_{PV,o} - 8\alpha^2\eta_{non} - 8\alpha^2\eta_{PV,o} + 4\alpha^2)^{\frac{1}{2}}) \quad (67)$$

299

300 The total power produced by the (PV +TEG) system can then be found using
301 equation (5) and the solution for the optimum temperature given in the
302 expression above equation (67).

303

304

305

4. Conclusions and Outlook

The performance of a combined solar photovoltaic (PV) and thermoelectric generator (TEG) system is examined, using an analytical model to study the temperature distribution in the semiconductor plate of the Thermoelectric Generator System (TEG). An amorphous Si (a-Si) solar photovoltaic (PV) and a commercially available bismuth telluride (CdTe) (TEG) is the combined (PV+TEG) system considered in this work.

We resolved the thermal conductivity equation, using constants variation method. We assumed that the temperature along the y-axis is considered uniform. The heat flux $Q(x, y, z, t)$ available at the hot side of the TEG is assumed to be what remains of the absorbed radiation of the PV power production. Our results reveal an increasing of the photon generation rate (G) and the photocurrent density (J_{ph}), when the temperature (T) increases. One notes an increasing of the fill factor (FF) and a reduction of the series resistance (R_s); which improves the quantum efficiency of the system (PV+TEG). The efficiency is directly proportional to the produced power, with the proportionally factor being Q_{solar} . The maximum operating temperature of a BiTe TEG is indicated.

Future work can be about:

- 1) - Extensions to the model; 2) - Improvement of the analytical results; 3) - Compare experimental results with those obtained analytically.

References

- [1] Van Sark G J H M. Feasibility of photovoltaic-thermoelectric hybrid moduls. Applied Energy. 2011; 88: 2785-2790.
- [2] Bjørk R, Nielsen KK. The performance of a combined solar photovoltaic (PV) and thermoelectric generator (TEG) system. Solar Energy. 2015; 120: 187-194.

- 334 [3] Sundarraj P, Maity D, Roy SS, Taylor RA. Recent advances in
335 thermoelectric materials and solar thermoelectric generators - a critical review.
336 RSC. Advances. 2014 ; 4: 46860 - 46874.
- 337 [4] Luque A, Marti A. Limiting efficiency of coupled thermal and photovoltaic
338 converters. Solar Energy Materials and Solar Cell. 1999; 58: 147- 165.
- 339 [5] Zhang QJ, Tang XF, Zhai PC, Niino M, Endo C. Recent development in
340 nano and graded thermoelectric materials. Materials Science Forum. 2005; 492:
341 135 - 140.
- 342 [6] Kraemer D, Hu L, Muto A, Chen X, Chen G., Chiesa, M. Photovoltaic-
343 thermoelectric hybrid systems: A general optimization methodology. Applied
344 Physics Letters. 2008; 92: 243503.
- 345 [7] Mizoshiri M, Mikami M, Ozaki, K. Thermal photovoltaic hybrid solar
346 generator using thin-film thermoelectric modules. Japanese Journal of Applied
347 Physics. 2012; 51: 06FL07.
- 348 [8] Ju X, Wang Z, Flamant G, Li P, Zhao W. Numerical analysis and
349 optimization of a spectrum splitting concentration photovoltaic-thermoelectric
350 hybrid system. Solar Energy. 2012; 86: 1941 - 1954.
- 351 [9] Sillman R, Podlowski, L. PV-hybrid and thermoelectric collectors. Solar
352 Energy. 1999; 6: 227-237.
- 353 [10] Messina R, Ben-Abdallah, P. Graphene-based photovoltaic cells for near
354 field thermal energy conversion. Scientific Reports. 2013; 3: 1383.
- 355 [11] Liao T, Lin B, Yang Z. Performance characteristics of a low concentrated
356 photovoltaic-thermoelectric hybrid power generation device. International
357 Journal of Thermal Sciences. 2014; 77: 158 - 164.
- 358 [12] Zhang J, Xuan Y, Yang I. Performance estimation of photovoltaic-
359 thermoelectric hybrid systems. Energy. 2014; 78: 895-903.
- 360 [13] Attivissimo F, Di Nisio A, Lanzolla AML, Paul M. Feasibility of a
361 photovoltaic-thermoelectric generator: performance analysis and simulation

- 362 results. IEEE Transactions on Instrumentation and Measurement. 2015; 64 (5):
363 1158 - 1169.
- 364 [14] Wu YY, Wu SY, Xiao L. Performance analysis of photovoltaic–
365 thermoelectric hybrid system with and without glass cover. Energy Conversion
366 and Management. 2015; 93: 151 - 159.
- 367 [15] Francoeur M, Vaillon R, Pinar Mengüç M. Thermal Impacts on the
368 Performance of Nanoscale-Gap Thermo-photovoltaic Power Generators. IEEE
369 Transactions on Energy Conversion. 2011; 26,: 686-698.
- 370 [16] Lin W, Shih TM, Zheng JC, Zhang Y, Chen J. Performance characteristics
371 of a low concentrated photovoltaic–thermoelectric hybrid power generation
372 device. International Journal of Heat and Mass Transfer. 2014; 74: 121 - 127.
- 373 [17] Xu X, Zhou S, Meyers MM, Sammakia BG, Murray BT. Performance
374 analysis of a combination system of concentrating photovoltaic/thermal
375 collector and thermo-electric generators. Journal of Electronic Packing. 2014;
376 136: 041004.
- 377 [18] Najafi H, Woodbury K A. Modeling and analysis of a combined
378 photovoltaic-thermoelectric power generation system. Journal of Solar
379 Engineering. 2014; 135: 031013 - 8.
- 380 [19] Fisac M, Villasevil F X, Lopez A M. High-efficiency photovoltaic
381 technology including thermoelectric generation. Journal of Power Sources.
382 2014; 252: 264 - 369.
- 383 [20] Wang N, Han L, He H, Park N H, Koumoto K. A novel high-performance
384 photovoltaic-thermoelectric hybrid device. Energy Environmental Science.
385 2011; 4: 3676 - 3679.
- 386 [21] Zhang x, Chau K T, Chan C C. Overview of thermoelectric generation for
387 hybrid vehicles. Journal of Asian Electric Vehicles. 2008; 6 (2): 1119-1124.
- 388 [22] Jiawei Z, Bolin L, Gang C. First-principles calculations of thermal,
389 electrical, and thermoelectric transport properties of semiconductors.
390 Semiconductor Science and Technology. 2016; 32 (4): 043001.

- 391 [23] Hsueh T J, Shieh J M, Yeh Y M. Hybrid Cd-free CIGS solar cell/teg device
392 with ZnO nanowires. *Progress in Photovoltaics*. 2015; 23: 507-512.
- 393 [24] Park K T, Shin S M, Tazebay A S, Um H D, Jung J Y, Jee S W, Oh M W,
394 Park S D, Yoo B, Yu C, Lee J H. Lossless hybridization between photovoltaic
395 and thermoelectric devices. *Scientific Reports*. 2013; 3: 2123.
- 396 [25] Zhang Y, Fang J, He C, Yan H, Wei Z, Li Y. Integrated energy-harvesting
397 system by combining the advantages of polymer solar cells and thermoelectric
398 devices. *Journal of Physical Chemistry C*. 2013; 117: 24685-24691.
- 399 [26] Deng Y, Zhu W, Wang Y, Shi Y. Enhanced performance of solar-driven
400 photovoltaic-thermoelectric hybrid system in an integrated design. *Solar*
401 *Energy*. 2013; 88: 182-191.
- 402 [27] Leonov V, Torfs T, Vullers R, Van Hoof C. Hybrid Thermoelectric-
403 Photovoltaic Generators in Wireless Electroencephalography Diadem and
404 Electrocardiography Shirt. *Journal of Electronic Materials*. 2010; 39 (9): 1674-
405 1680.
- 406 [28] Igor V, *Journal of Applied Physics*. Dynamic thermopower in uniform
407 unipolar semiconductor. 2016; 119 : 095712.
- 408 [29] Drews A, de Keizer A. C, Beyer H G. Monitoring and remote failure
409 detection of grid-connected PV systems based on satellite observations. *Solar*
410 *Energy*. 2007; 81(4): 548-564.
- 411 [30] Skoplaki E, Palyvos J. On the temperature dependence of photovoltaic
412 module electrical performance: A review of efficiency/power correlations. *Solar*
413 *Energy*. 2009; 83: 614-624.