



SDI Review Form 1.6

Journal Name:	<u>British Journal of Mathematics & Computer Science</u>
Manuscript Number:	Ms_BJMCS_34336
Title of the Manuscript:	Convergence Analysis of a Non-overlapping DDM for Optimal Absorbing Boundary Control Problems Governed by Wave Equations
Type of the Article	

General guideline for Peer Review process:

This journal's peer review policy states that **NO** manuscript should be rejected only on the basis of '**lack of Novelty**', provided the manuscript is scientifically robust and technically sound.

To know the complete guideline for Peer Review process, reviewers are requested to visit this link:

(<http://www.sciencedomain.org/page.php?id=sdi-general-editorial-policy#Peer-Review-Guideline>)



SDI Review Form 1.6

PART 1: Review Comments

	Reviewer's comment	Author's comment (if agreed with reviewer, correct the manuscript and highlight that part in the manuscript. It is mandatory that authors should write his/her feedback here)
Compulsory REVISION comments	This paper discusses a non-overlapping domain decomposition method (DDM) for solving the optimal boundary control problems governed by wave equations with absorbing boundary condition. An integral mean method is applied to present an explicit flux calculation on the inter-domain. Hence, the local problems are communicated on the interfaces between sub-domains. The full parallel and discrete schemes are established for solving these local problems, and the stability of the schemes is proven. In addition, a priori error estimates for state, costate and control variables are derived. The convergence results are well-discussed. However, the presentation of the paper shall be further improved. Some comments are given below: 1. When mention the reference in the text, the word "reference(s)" shall not be used. It is suggested to mention the citation without the word "reference(s)". 2. The correct way to mention the citation in the text is (author(s), year), not using [author(s), (year)]. 3. PDE is the short form for partial differential equations. The full name of PDE shall be used at the first time appearing in the text. 4. When mention the citation in the text, it is suggested not using "from (author(s), year) to (author(s), year)". It is suggested to use	1. We delete the word "reference(s)" in the revised manuscript. 2. The citation "[author(s),(year)]" was produced by using the Latex Template of this Journal, which was downloaded from the website of this journal. We can't change them. 3. We use the full name of PDE at the first time. Please see it at Line 4, page 2. 4. We change the citations "from... to..." by citing the references out. Please see the revised manuscript, such as Line 2, Line -14, page 2; Line -3, page 3. 5. On page 3, below Eq. (2.3), we have defined d_t as a bounded operator. Please see it. 6. The last line of Eq.(4.28) is a notational expression, which marks the terms of the above terms as I_i.



SDI Review Form 1.6

	“(author(s), year) and (author(s), year)” 5. The sentence “We note that $d_t p$ is not the time derivative dp/dt of p in the sense of distributions.” does not give the actual meaning of the term $d_t p$. What does this term $d_t p$ stand for? 6. In Eq. (4.28), the last line is not clear, look like not related to the previous lines of the equation. 7. In the proof of Lemma 4.7, it is suggested not using Step 1 and Step 2. Normally, “Step” is used for calculation procedure, not using in the proof. Perhaps, use Part 1 and Part 2. 8. In Eq. (2.1), (2.4), (3.5), (4.4), the integrals are not in proper form. In fact, the integral form is $\int_{\Omega} \cdot dt$, but in the equations mentioned, the integral form $\int_{\Omega} \cdot$ is used 9. See the sentence before Eq. (3.2), “... $0 < H < \min\{\text{diam}(\Omega_1), \text{diam}(\Omega_2)\}$, ...”, what does <i>diam</i> stand for? 10. Start from Eq. (3.5), the bracket $\langle \cdot, \cdot \rangle$ is used. This bracket $\langle \cdot, \cdot \rangle$ means for the inner product and it is suggested to mention the meaning of this bracket $\langle \cdot, \cdot \rangle$ in the text after Eq. (3.5). 11. The brackets $\langle \cdot, \cdot \rangle$ and $(\cdot, \cdot)$ can be used for the inner product. So, please use either one for consistency in the text. See Eq. (3.9). 12. There are some terms in over bar form, that is, $\bar{}$, are these terms represent conjugate term? It is suggested to declare the meaning of this over bar in the text. 13. Some terms are put in this bracket $[\]$, any	7. We use the word “Part ” to take place of “Step”. 8. In this manuscript, for simplicity to write and without confusions, the integral was used by omitting the integral element dx, dt, ds and $dxdt$, since the integrals were used on the domains with different dimension. For example, $\int_{\Omega} u$ means $\int_{\Omega} u dx$, $\int_{\Gamma_N \times (0,T)} u$ means $\int_0^T \int_{\Gamma_N} u ds dt$. These kind of notations are normally adopted in the research papers in the field of Optimal Control Problem. 9. Here, $\text{diam}(\Omega)$ stands for the diameter of the domain Ω. We change it as $\text{diameter}(\Omega)$ 10. We explain the meaning of the bracket in the sentences below Eq. (3.6), on Page 6. 11. See the above explanation. 12. The over bar is to calculate the integral mean value of a function, which is defined as Eq.(3.2), page 5. 13. The bracket $[v]$ is to define the jump value of the function v on the boundary Γ. We had defined this by Eq.(3.1), page 4
--	--	--



SDI Review Form 1.6

	meaning for this bracket [] in the content? See Eq. (3.5), (3.6), (3.7), (3.12), (3.18), (3.21), (3.22), (3.24), (3.27) and some equations in Section 4. 14. Some references are not cited in the text. Please check.	14. See the above explanation 4. We cite all references out, instead of using "references from.....to".
<u>Minor</u> REVISION comments	1. Do not use the notation Γ in the text when mention boundary in the abstract, in the second last paragraph in Section 1, and in the conclusion. 2. When use "such as" to give some examples, "and so on" shall not be used. 3. The short form of domain decomposed method, which is DDM, is introduced at the first time it appears in the text, then the term DDM shall be used for the following text. 4. In Eq, (2.2), (3.6) and (3.7), the word "on" and "in" shall not put in the <i>italic</i> form. 5. Normally, the numbering of the equations is labelled from (1), (2), (3), ... and so on, not labelling according to the section number, but thesis is in this way. 6. The notations in Figures 1 and 2 are too tiny, not clear. For convenience, those are highlighted in the yellow color shall be revised properly.	1. We delete the notation Γ in the abstract and the conclusion. 2. We delete "and so on". 3. We use the term DDM for the later appearance of "domain decomposition method" . 4. We change the form of "on" and "in" in Eq. (2.2), (3.6) and (3.7). 5. Since we used the Late Template of this journal, the numbering of the equations is just labelled as (1.1), (2.1),.... and so on. We can't change the numbering form by ourselves. Maybe, the editor of the journal will change them. 6. We amplify the size of Figure1 and 2. The notations in figures are clear now. According the reviewer's comments, we have revised the highlighted contents properly. Thanks much for the reviewer's valuable comments and suggestions
<u>Optional/General</u> comments		