

Numerical study of temperature dependent viscosity and thermal conductivity on a natural convection flow over a sphere in presence of magneto hydrodynamics

ABSTRACT

The objective of this research is to investigate the numerical study of temperature dependent viscosity and thermal conductivity on natural convection flow of an electrically conducting fluid over an isothermal sphere in presence of magneto hydrodynamic. Viscosity is considered variation with temperature and also thermal conductivity is taken as a linear function of temperature. The governing equations are transformed into dimensionless non-similar equations by using set of suitable transformations and solved numerically by finite difference method along with Newton's linearization approximation. The computational findings for dimensionless velocity profiles, temperature profiles, local skin friction coefficient and local heat transfer coefficient are displayed in graphically and also in tabular forms for different vales of the Prandtl's number Pr , dependent viscosity parameter ε , dependent thermal conductivity parameter γ and magnetic parameter M .

Keywords: Natural convection, magneto hydrodynamic, temperature dependent viscosity and dependent thermal conductivity.

1. INTRODUCTION

Free convection heat transfer has gained considerable attention because of its numerous applications in the areas of energy conservation cooling of electrical and electronics components, lubrication technologies, drying technologies, food processing, float glass production, design of solar collectors, heat transfer in solar ponds and heat exchangers etc. Many practical heat transfer applications involve with the conversion of some forms of mechanical, electrical, nuclear or chemical energy to thermal energy. A vast number of research articles have been published considering

different fluid flow systems. Alam, M. Md. *et al.* [1] established the viscous dissipation effect on MHD natural convection flow over a sphere in presence of heat generation. Aktar, S. *et al.* [2] analyzed the viscous dissipation effects on natural convection flow alone a sphere with heat generation. Miraj, *et al.* [3] studied the effects of pressure work and radiation on natural convection flow around a sphere with heat generation. Molla, M. M. *et al.* [4] have been investigated the effects in natural convection flow from an isothermal sphere with temperature dependent thermal conductivity. Safiqul Islam, A. K. M. *et al.* [5]

performed the effects of temperature dependent thermal conductivity on MHD free convection flow along a vertical flat plate with heat generation and joule heating. Nasrin, R. *et al.* [6] studied the combined effects of viscous dissipation and temperature dependent thermal conductivity on magneto hydrodynamic (MHD) free convection flow with conduction and Joule heating along a vertical flat plate. Effect of variable viscosity and thermal conductivity of micro polar fluid in a porous channel in presence of magnetic field is considered by Gitima Patoway [7]. Nasrin, R. *et al.* [8] performed the effects on MHD free convection flow along a vertical flat plate with thermal conductivity and viscosity depending on temperature. The effects of temperature dependent viscosity and viscous dissipation on MHD convection flow from an isothermal horizontal circular cylinder in the presence of stress work and heat generation is analyzed by Nabil. *et al.* [9]. safiquil Islam, K. M. *et al.* [10] established the effects of temperature dependent thermal conductivity on natural convection flow along a vertical flat plate with heat generation. The effect of temperature dependent viscosity on MHD natural convection flow from an isothermal sphere is studied by Molla M. M. *et al.* [11]. Heat generation effects on MHD natural convection flow along a vertical wavy surface with variable thermal conductivity is considered by Alim M.

2. GOVERNING EQUATIONS OF THE FLOW

The steady two-dimensional natural convection boundary layer flow of an incompressible viscous and electrically conducting fluid over a sphere of the radius a

M. *et al.* [12]. Md.Raihanul Haque *et al.* [13] have been investigated the effects of viscous dissipation on natural convection flow over a sphere with temperature dependent thermal conductivity. Cebeci, T., & Brashaw, P, [14] analyzed the physical and computational aspects of convective heat transfer. Keller H. B. [15] made numerical methods in boundary layer theory. Molla, M. M. *et al.* [16] performed the natural convection flow along a vertical wavy surface with uniform surface temperature in presence of heat generation absorption. Free convection boundary layer on an isothermal sphere in a micro polar fluid is considered by Nazar, *et al.* [17]. Mriaj, *et al.* [18] established the Joule heating effect on magneto hydrodynamic natural convection flow along a vertical wavy surface with viscosity dependent on temperature. Parveen, *et al.* [19] studied the conjugate effects of radiation and joule heating on magneto hydrodynamic free convection flow along a sphere with heat generation. It is observed that the combined effect of temperature dependent viscosity and thermal conductivity in presence on MHD natural convection flow over a sphere has received a little attention. So, in this article, we are interested to investigate the effects of temperature dependent viscosity and thermal conductivity on MHD natural convection flow over a sphere.

has been considered. In this analysis T_w is assumed as the constant temperature at the surface of the sphere where $T_w > T_\infty$ and T_∞ being the ambient temperature of the fluid and also T is the temperature of the fluid in the boundary layer.

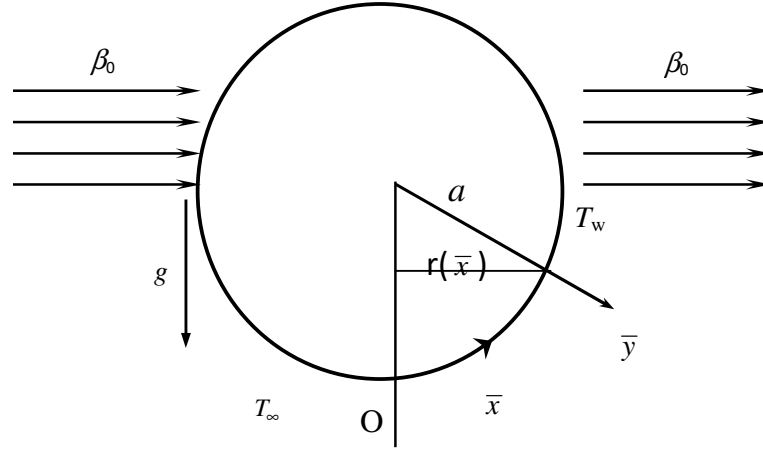


Fig. 1: Physical model and coordinate system

The conservation equations for the flow characterized with steady, laminar and two dimensional boundary layers, the continuity, momentum and energy equations can be written as:

$$\frac{\partial}{\partial \bar{x}}(r \bar{u}) + \frac{\partial}{\partial \bar{y}}(r \bar{v}) = 0 \quad (1)$$

$$\begin{aligned} \bar{u} \frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v} \frac{\partial \bar{u}}{\partial \bar{y}} &= \frac{1}{\rho} \frac{\delta}{\delta \bar{y}} \left(\mu \frac{\delta \bar{u}}{\delta \bar{y}} \right) \\ + g \beta (T - T_{\infty}) \sin \left(\frac{\bar{x}}{a} \right) &- \frac{\sigma_0 \beta_0^2}{\rho} \bar{u} \end{aligned} \quad (2)$$

$$\begin{aligned} \bar{u} \frac{\partial T}{\partial \bar{x}} + \bar{v} \frac{\partial T}{\partial \bar{y}} &= \\ \frac{1}{\rho C_p} \left(\frac{\partial k_f}{\partial \bar{y}} \frac{\partial T}{\partial \bar{y}} + k \frac{\partial^2 T}{\partial \bar{y}^2} \right) \end{aligned} \quad (3)$$

The boundary conditions for the governing equations are

$$\left. \begin{aligned} \bar{u} = \bar{v} = 0, \quad T = T_w \quad \text{on} \quad \bar{y} = 0 \\ \bar{u} \rightarrow 0, T \rightarrow T_{\infty} \quad \text{at} \quad \bar{y} \rightarrow \infty \end{aligned} \right\} \quad (4)$$

$$\begin{aligned} r(\bar{x}) &= a \sin(\bar{x}), \quad r = r(\bar{x}), \\ r &= a \sin \bar{x} \end{aligned} \quad (5)$$

We have, r is the radial distance from the symmetrical axis to the surface of the sphere, $\bar{u}$ and $\bar{v}$ are velocity components along the $\bar{x}$ and $\bar{y}$ respectively, $k(T)$ is the thermal conductivity of the fluid depending on the fluid temperature T , σ_0 is the electric conduction and β_0 is the magnetic field strength. Here we will consider $\mu = \frac{\mu_{\infty}}{1 + \alpha(T - T_{\infty})}$ is the temperature dependent viscosity where $\alpha = \frac{1}{\mu_f} \left(\frac{\delta \mu}{\delta T} \right)_f$. We also consider the form of the temperature dependent thermal

conductivity which is proposed by Charraudeau (1975), as follows

$$k = k_{\infty} \left(1 + \gamma^* (T - T_{\infty}) \right), \text{ where } k_{\infty} \text{ is}$$

the thermal conductivity of the ambient

$$\text{fluid and } \gamma^* = \frac{1}{k_f} \left(\frac{\partial k}{\partial T} \right)_f = \frac{\gamma}{(T_w - T_{\infty})};$$

$$\gamma = \frac{1}{k_f} \left(\frac{\partial k}{\partial T} \right) (T_w - T_{\infty}), \text{ where } \gamma^* \text{ is a}$$

constant.

3. TRANSFORMATION OF THE GOVERNING EQUATIONS:

The above equations (1)-(3) are made dimensionless by using the following substitutions:

$$\begin{aligned} \xi &= \frac{\bar{x}}{a}, \eta = Gr^{1/4} \frac{\bar{y}}{a}, \\ u &= \frac{\bar{u}}{u_0} = \frac{a}{\nu} Gr^{1/2} \bar{u}, \\ v &= \frac{a}{\nu} Gr^{1/4} \bar{v}, \theta = \frac{T - T_{\infty}}{T_w - T_{\infty}} \end{aligned} \quad (6)$$

The equations (1) to (3) can be converted into dimensionless with the help of equation (6) which is modified the following forms:

$$\frac{\delta}{\delta \xi} (ru) + \frac{\delta}{\delta \eta} (rv) = 0 \quad (7)$$

$$u \frac{\partial u}{\partial \xi} + v \frac{\partial u}{\partial \eta} = \frac{-\varepsilon}{(1 + \varepsilon \theta)^2} \frac{\partial u}{\partial \eta} \frac{\partial \theta}{\partial \eta} \quad (8)$$

$$\begin{aligned} &+ \frac{1}{1 + \varepsilon \theta} \frac{\partial^2 u}{\partial \eta^2} + \theta \sin \xi - Mu \\ &u \frac{\partial \theta}{\partial \xi} + v \frac{\partial \theta}{\partial \eta} = \end{aligned} \quad (9)$$

$$\frac{1}{Pr} (1 + \gamma \theta) \frac{\partial^2 \theta}{\partial \eta^2} + \frac{1}{Pr} \gamma \left(\frac{\partial \theta}{\partial \eta} \right)^2$$

With the boundary conditions

$$\left. \begin{aligned} u &= v = 0, \quad \theta = 1 \\ \text{at } \xi &= 0, \text{ for any } \eta \\ u &= v = 0, \quad \theta = 1 \\ \text{at } \eta &= 0, \quad \xi > 0 \\ u &\rightarrow 0, \quad \theta \rightarrow 0 \\ \text{as } \eta &\rightarrow \infty, \quad \xi > 0 \end{aligned} \right\} \quad (10)$$

Here, $\mu = \frac{\mu_{\infty}}{1 + \alpha(T - T_{\infty})}$ is the variation

viscosity, $\alpha = \frac{1}{\mu_f} \left(\frac{\delta \mu}{\delta T} \right)_f$ and

$\varepsilon = \frac{1}{\mu_f} \left(\frac{\delta \mu}{\delta T} \right)_f (T - T_{\infty})$ is the temperature

dependent viscosity parameter,

$M = \frac{\sigma_0 \beta_0^2 a^2}{\mu Gr^{1/2}}$ is the magneto hydrodynamics

parameter.

To solve equations (8) and (9) subject to the boundary conditions (10), we introduce the following variables u and v where $\psi = \xi r(\xi) f(\xi, \eta)$ and $\psi(\xi, \eta)$ is a non-dimensional stream function which is related to the velocity components in the usual way as

$$u = \frac{1}{r} \frac{\partial \psi}{\partial \eta} \text{ and } v = -\frac{1}{r} \frac{\partial \psi}{\partial \xi} \quad (11)$$

We may proceed to transform the conservation of momentum and energy

equations (8) and (9) into the new co-ordinates. To facilitate the transformation, it is useful to have the velocity components explicitly expressed in terms of the new variables. Therefore we obtained

$$\begin{aligned} u &= \frac{1}{r} \frac{\partial \psi}{\partial \eta} \quad \text{and} \\ v &= -\frac{1}{r} \frac{\partial \psi}{\partial \xi} = -f - \xi \frac{\delta f}{\delta \xi} - \frac{\xi f}{r} \frac{\delta r}{\delta \xi} \\ u &= \xi \frac{\delta f}{\delta \eta}, \frac{\delta u}{\delta \xi} = \frac{\delta f}{\delta \eta} + \xi \frac{\delta^2 f}{\delta \eta \delta \xi} \\ \frac{\delta u}{\delta \eta} &= \xi \frac{\delta^2 f}{\delta \eta^2}, \frac{\delta^2 u}{\delta \eta^2} = \xi \frac{\delta^3 f}{\delta \eta^3} \end{aligned} \quad (12)$$

The momentum equation (8) reduces the above substitution

$$\begin{aligned} \frac{1}{1+\varepsilon\theta} \frac{\delta^3 f}{\delta \eta^3} + \left(1 + \frac{\xi}{\sin \xi} \cos \xi\right) f \frac{\delta^2 f}{\delta \eta^2} \\ - \left(\frac{\delta f}{\delta \eta}\right)^2 - \frac{\varepsilon}{(1+\varepsilon\theta)^2} \frac{\delta \theta}{\delta \eta} \frac{\delta^2 f}{\delta \eta^2} \\ + \frac{\theta \sin \xi}{\xi} - M \frac{\delta f}{\delta \eta} = \\ \xi \left(\frac{\delta f}{\delta \eta} \frac{\delta^2 f}{\delta \eta \delta \xi} - \frac{\delta f}{\delta \xi} \frac{\delta^2 f}{\delta \eta^2} \right) \end{aligned} \quad (13)$$

And also the energy equation (9) reduces

$$\begin{aligned} \frac{1}{pr} (1+\gamma\theta) \frac{\delta^2 \theta}{\delta \eta^2} + \frac{1}{pr} \gamma \left(\frac{\delta \theta}{\delta \eta} \right)^2 \\ + \left(1 + \frac{\xi}{\sin \xi} \cos \xi\right) f \frac{\delta \theta}{\delta \eta} \\ = \xi \left(\frac{\delta f}{\delta \eta} \frac{\delta \theta}{\delta \xi} - \frac{\delta f}{\delta \xi} \frac{\delta \theta}{\delta \eta} \right) \end{aligned} \quad (14)$$

At the lower stagnation point of the sphere *i.e.*

$\xi \approx 0$, equations (13) and (14) reduced to the following ordinary differential equations:

$$\begin{aligned} \frac{1}{1+\varepsilon\theta} f''' + 2f f'' - (f')^2 \\ - \frac{\varepsilon}{(1+\varepsilon\theta)^2} \theta' f'' + \theta - M f' = 0 \end{aligned} \quad (15)$$

$$\frac{1}{Pr} (1+\gamma\theta) \theta'' + \frac{1}{Pr} \gamma (\theta')^2 + 2f \theta' = 0 \quad (16)$$

The boundary conditions as mentioned in equations (10) then take the following form

$$\begin{cases} f(0) = f'(0) = 0, \theta(0) = 1 \\ f' \rightarrow 0, \theta \rightarrow 0 \text{ as } \eta \rightarrow \infty \end{cases} \quad (17)$$

Where primes denote the differentiation of the function with respect to η . In practical applications, the physical quantities of principle interest are the wall shearing stress

τ_w and the rate of heat transfer, which lead to compute the coefficient of skin-friction Cf_ξ and the Nusselt number Nu_ξ respectively. These can be written in non-dimensional form as

$$\begin{aligned} Nu_\xi &= \frac{aGr^{-1/4}}{k_f (T_w - T_\infty)} q_w \\ \text{and } Cf_\xi &= \frac{Gr^{-3/4} a^2}{\mu v} \tau_w \end{aligned} \quad (18)$$

Where $\tau_w = \mu \left(\frac{\partial U}{\partial Y} \right)_{Y=0}$ and

$q_w = -k_f \left(\frac{\partial T}{\partial Y} \right)_{Y=0}$. Using the new variables

(6) and (11), we have the simplified form of the local heat transfer and the local skin- friction coefficient as

$$\theta'(\xi, 0) = Nu_\xi = - \left(\frac{\partial \theta}{\partial \eta} \right)_{\eta=0} \quad (19)$$

$$\text{and } f''(\xi, 0) = Cf_\xi = \xi \left(\frac{\partial^2 f}{\partial \eta^2} \right)_{\eta=0}$$

4. METHOD OF SOLUTION

There are different types of numerical techniques available depending on the types of problem to be solved. Finite difference method is one of the important methods both for programming and rapid convergence. The transformed boundary layer equations will be solved numerically with the help of implicit finite method together with the Keller-box scheme [9]. To begin with, the partial differential equations (13) and (14) are first converted into a system of first differential equations. Then these equations are expressed in finite difference forms by approximating the functions and their derivatives in terms of the center difference. Denoting the mesh points in the ξ and η -plane by ξ_i and η_j where $i = 1, 2, 3, \dots, M$ and $j = 1, 2, 3, \dots, N$, central difference approximations are made

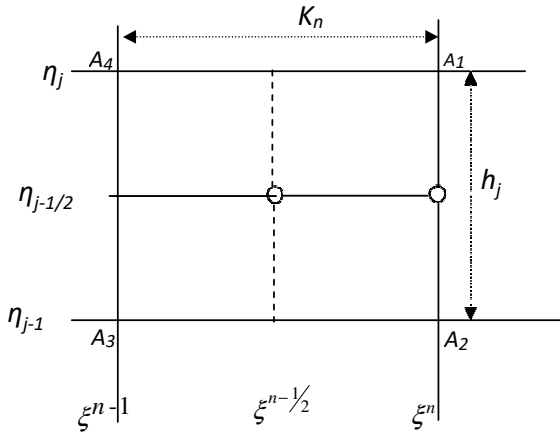


Fig. 2: Net rectangle of difference approximations for the Box scheme.

which shown in **Fig.2**, such that those equations involving ξ explicitly are centered at $\left(\xi^{n-1/2}, \eta_{j-1/2}\right)$ and the remainder at $\left(\xi^n, \eta_{j-1/2}\right)$ where $\eta_{j-1/2} = \frac{1}{2}(\eta_j + \eta_{j-1/2})$ etc. The above central difference approximations reduces the system of first differential equations for to a set of non-linear difference equations for the unknown at ξ_i in terms of their values at ξ_{i-1} . The resulting set of non-linear difference equations are solved by using the Newton's quasi-linearization method. The Jacobian matrix has a block-tridiagonal structure and the difference equations are solved using a block-matrix version of the Thomas algorithm. The details of the computational procedure have been discussed further by in the book by Cebeci and Bradshaw [8]. The equations (13) and (14) finally can be written in the following forms

$$\begin{aligned}
& \frac{1}{2} \left(P_1 \frac{v_j^n - v_{j-1}^n}{h_j} \right) + \frac{1}{2} \left(P_1 \frac{v_j^{n-1} - v_{j-1}^{n-1}}{h_j} \right) \\
& + (P_2 f v)_{j-\frac{1}{2}}^{n-\frac{1}{2}} - (u^2)_{j-\frac{1}{2}}^{n-\frac{1}{2}} - (P_3 g v)_{j-\frac{1}{2}}^{n-\frac{1}{2}} \\
& + (P_4 g)_{j-\frac{1}{2}}^{n-\frac{1}{2}} - (P_5 u)_{j-\frac{1}{2}}^{n-\frac{1}{2}} = \\
& \xi_{j-\frac{1}{2}}^{n-\frac{1}{2}} \left(\begin{aligned} & u_{j-\frac{1}{2}}^{n-\frac{1}{2}} \frac{u_{j-\frac{1}{2}}^n - u_{j-\frac{1}{2}}^{n-1}}{k_n} \\ & - v_{j-\frac{1}{2}}^{n-\frac{1}{2}} \frac{f_{j-\frac{1}{2}}^n - f_{j-\frac{1}{2}}^{n-1}}{k_n} \end{aligned} \right)
\end{aligned} \tag{20}$$

$$\begin{aligned}
& \frac{1}{2Pr} \left(P_6 \frac{p_j^n - p_{j-1}^n}{h_j} \right) + \frac{1}{2Pr} \left(P_6 \frac{p_j^{n-1} - p_{j-1}^{n-1}}{h_j} \right) \\
& + \left(\frac{P_7 p^2}{Pr} \right)_{j-\frac{1}{2}}^{n-\frac{1}{2}} + (P_2 f p)_{j-\frac{1}{2}}^{n-\frac{1}{2}} \\
& = \xi_{j-\frac{1}{2}}^{n-\frac{1}{2}} \left(\begin{aligned} & u_{j-\frac{1}{2}}^{n-\frac{1}{2}} \frac{g_{j-\frac{1}{2}}^n - g_{j-\frac{1}{2}}^{n-1}}{k_n} \\ & - p_{j-\frac{1}{2}}^{n-\frac{1}{2}} \frac{f_{j-\frac{1}{2}}^n - f_{j-\frac{1}{2}}^{n-1}}{k_n} \end{aligned} \right)
\end{aligned} \tag{21}$$

$$\text{Where, } P_1 = \left(1 + \frac{\xi \cos \xi}{\sin \xi} \right), \quad P_2 = -\frac{\varepsilon}{(1 + \varepsilon \theta)^2},$$

$$P_3 = \frac{\sin \xi}{\xi}, \quad P_4 = M, \quad P_5 = 1 + \gamma \theta \text{ and } P_6 = \gamma.$$

5. RESULTS AND DISCUSSION

In this article, results of the numerical studied of temperature dependent variable viscosity and thermal conductivity on MHD natural convection flow over a sphere has been investigated. Solutions are obtained for the fluid having Prandtl's number Pr ($= 1.00, 1.74, 2.00, 3.00$), temperature dependent viscosity parameter ε ($= 0.30, 0.50, 0.80, 1.00, 1.20, 1.50$), thermal conductivity parameter γ ($= 0.30, 0.50, 0.80, 1.00$) against η at any position of ξ and for a wide range of values of

magnetic parameter M ($= 1.50, 1.20, 0.90, 0.70, 0.30$). Also the results for the skin friction coefficient and the rate of heat transfer have been obtained for fluids having Prandtl's number Pr , the magnetic parameter or Hartmann number M , temperature dependent viscosity parameter ε and thermal conductivity parameter γ at a different position of ξ .

From figure 3(a) it is observed that velocity distribution increases as the values of

dependent thermal conductivity parameter γ ($=0.30, 0.50, 0.80, 1.00$) increase in the region $\eta \in [0, 9]$ but near the surface of the sphere, velocity increases significantly and then decreases slowly and finally approaches to zero. The maximum values of the velocity are recorded to be 0.43056, 0.44332, 0.47589 and 0.53463 for $\gamma = 0.30, 0.50, 0.80$ and 1.00 respectively which occur at $\eta = 1.17520$ for first, second and third maximum values and at $\eta = 1.23788$ for last maximum values. Here we see that the velocity increases by 24.17 % as γ increases from 0.30 to 1.00. From figure 3(b), it is seen that when the values of dependent thermal conductivity parameter γ increases in the region $\eta \in [0, 9]$, the temperature distribution also increases. But near the surface of the sphere temperature profiles are maximum and then decreases away from the surface and finally take asymptotic value.

From figure 4(a) it is observed that velocity distribution increases as the values of temperature dependent viscosity parameter $\mathcal{E}$ ($= 0.30, 0.50, 0.80, 1.00, 1.20, 1.50$) increase in the region $\eta \in [0, 9]$ but near the surface of the sphere, velocity increases significantly and then decreases slowly and finally approaches to zero. From figure 4(b), it is seen that when the values of temperature dependent viscosity parameter $\mathcal{E}$ increase in the region $\eta \in [0, 9]$, the temperature distribution decreases. It is observed that the maximum temperature has been found at the surface on the sphere.

The effects of magnetic parameter or Hartmann number M for $Pr = 0.72$, $\mathcal{E} = 0.80$ and $\gamma = 0.50$ on the velocity profiles and

temperature profiles are shown in figures 5(a) to 5(b). Figure 5(a) and figure 5(b) represent respectively the effects of the magnetic parameter M on the velocity and temperature profiles. From these figures, it is seen that the velocity profiles decrease with the increasing values of M and the temperature profiles increase with the increasing values of M .

Figure 6(a) and figure 6(b) indicate the effects of the Prandtl's number Pr with $M = 1.20$, $\mathcal{E} = 0.70$ and $\gamma = 0.90$ on the velocity profiles and the temperature profiles. From figure 6(a) it is observed that the increasing values of Prandtl's number Pr lead to decrease the velocity profiles. The maximum values of the velocity are recorded to be 0.50815, 0.44815, 0.38592 and 0.36155 for $Pr = 1.00, 1.74, 2.00$ and 3.00 respectively which occur at $\eta = 1.23788$ for first, second maximum value and $\eta = 0.99806$ for third and fourth maximum values. Here it is depicted that the velocity decreases by 31.54 % as Pr increases from 1.00 to 3.00. Again from figure 6(b) it is observed that the temperature profiles decrease with the increasing values of Prandtl's number Pr . But near the surface of the sphere temperature profiles are maximum and then decreases away from the surface and finally take asymptotic value.

The values of the skin friction coefficient Cf_ξ and Nusselt number Nu_ξ for different values dependent thermal conductivity parameter γ while $Pr = 0.72$, $M = 1.00$ and $\mathcal{E} = 1.60$ are recorded which is shown in Table-2. Here it is found that the values of the skin friction coefficient Cf_ξ increase at a different position of ξ for dependent thermal conductivity parameter $\gamma = 0.30, 0.50, 0.80, 1.00$. The rate

of the local skin friction coefficient Cf_ξ increases by 11.74% as the dependent thermal conductivity parameter γ changes from 0.30 to 1.00 and $\xi = 1.30900$ which is shown in Fig.7(a). Furthermore, it is seen that the numerical values of the Nusselt number Nu_ξ increase for increasing values of dependent thermal conductivity parameter γ . The rate of increase of the local Nusselt number Nu_ξ is 194.02% at position $\xi = 1.30900$ as the dependent thermal conductivity parameter γ changes from 0.30 to 1.00 which is shown in Fig, 7(b).

Figure 8(a) and figure 8(b), represents the effects for different values of dependent viscosity parameter $\mathcal{E}$ for the magnetic parameter $M = 1.00$, dependent thermal conductivity parameter $\gamma = 0.70$ and Prandtl's number $Pr = 0.72$ on the reduced skin friction coefficient Cf_ξ and the rate of heat transfer Nu_ξ . The skin friction coefficient Cf_ξ increases with the increase of dependent viscosity parameter $\mathcal{E}$, also heat transfer coefficient Nu_ξ increase with the increasing values of dependent viscosity parameter $\mathcal{E}$.

The effects of magnetic parameter or Hartmann number M for $Pr = 0.72$, $\mathcal{E} = 0.80$ and $\gamma = 0.90$ on the skin friction coefficient Cf_ξ and the heat transfer coefficient Nu_ξ are shown in figure 9(a) to figure 9(b). From figure 9(a) and figure 9(b) it is evident that the increasing values of magnetic parameter M leads to decrease both the skin friction coefficient Cf_ξ and the heat transfer co-efficient Nu_ξ .

In figure 10(a), the shear stress coefficient Cf_ξ and figure 10(b), the Nusselt number Nu_ξ are shown graphically for different values of the Prandtl's number Pr ($=1.00, 1.74, 2.00, 3.00$) when other values of the controlling parameter $M=1.20$, $\mathcal{E}=0.70$ and $\gamma=0.90$. Four values 1.00, 1.74, 2.00, 3.00, are taken for Pr , 1.00 corresponds to electrolyte solutions such as salt water, $Pr = 1.74, 2.00$ and 3.00 have been used theoretically. Here, it is found that the skin friction coefficient and the heat transfer coefficient both decrease for increasing values of Prandtl's number.

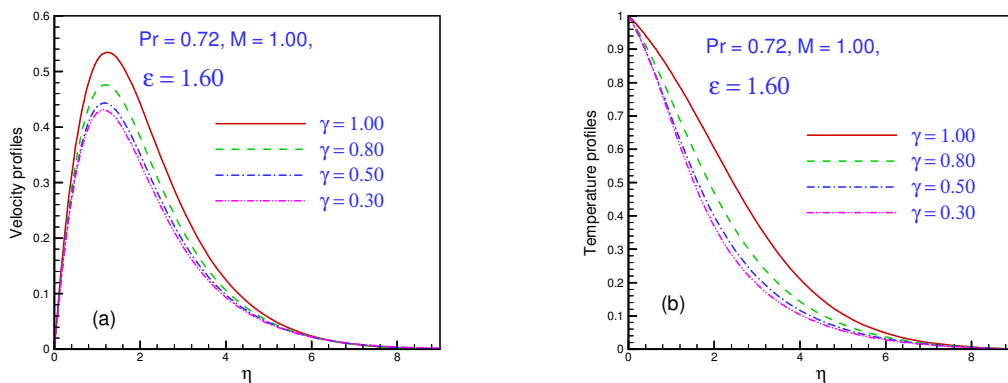


Fig.3(a) and Fig.3(b): Variation of dimensionless velocity profiles $f'(\eta, \xi)$ and temperature profiles

$\theta(\eta, \xi)$ against dimensionless distance η for different values of dependent thermal conductivity parameter γ with $Pr = 0.72$, $\varepsilon = 1.60$ and $M = 1.00$.

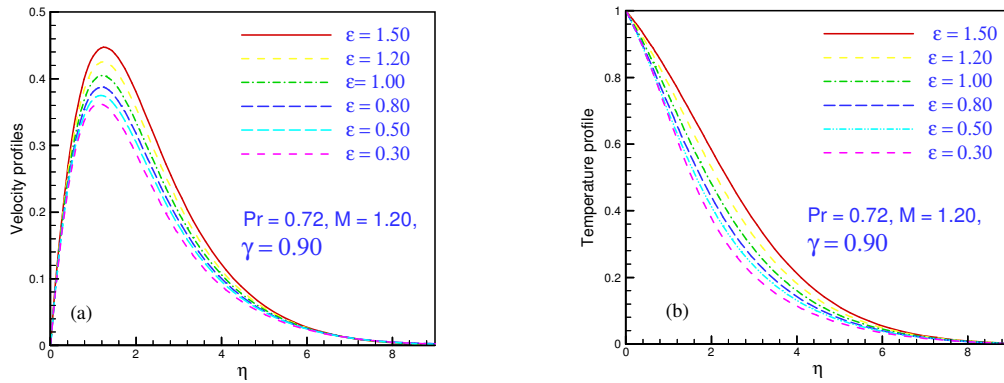


Fig.4 (a) and Fig.4 (b): Variation of dimensionless velocity profiles $f'(\eta, \xi)$ and temperature profiles $\theta(\eta, \xi)$ against dimensionless distance η for different values of variation temperature dependent viscosity parameter ε with $Pr = 0.72$, $\gamma = 0.90$ and $M = 1.20$.

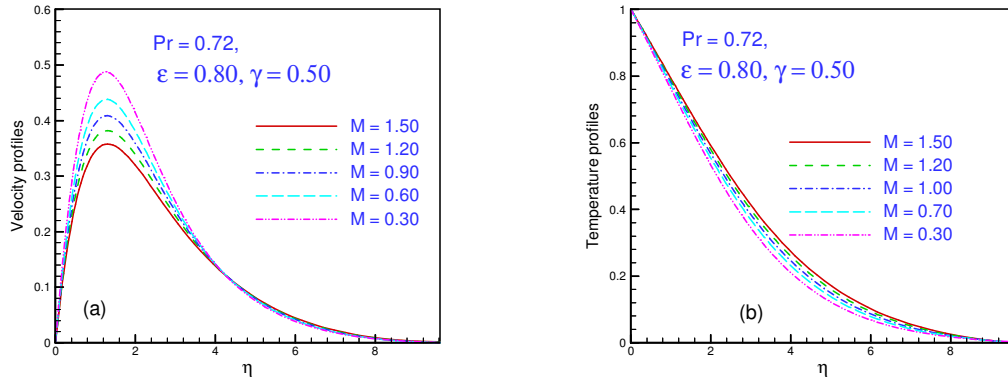


Fig.5 (a) and Fig.5 (b): Variation of dimensionless velocity profiles $f'(\eta, \xi)$ and temperature profiles $\theta(\eta, \xi)$ against dimensionless distance η for different values of magneto hydrodynamics parameter M with $Pr = 0.72$, $\gamma = 0.50$ and $\varepsilon = 0.80$.

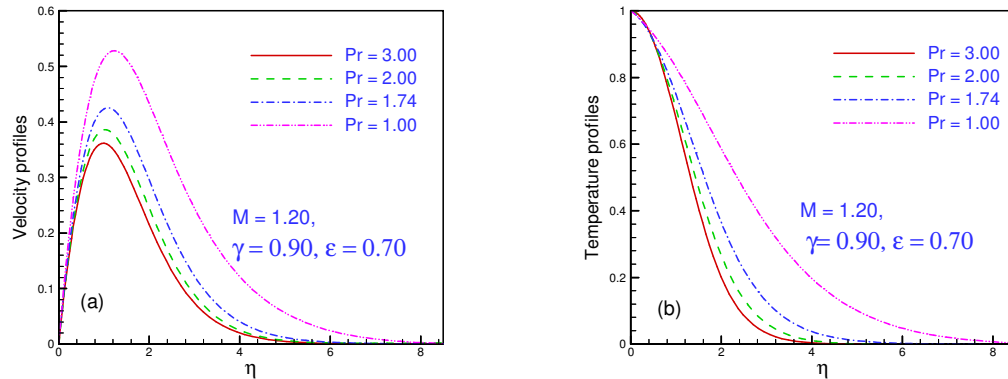


Fig.6 (a) and Fig.6 (b): Variation of dimensionless velocity profiles $f'(\eta, \xi)$ and temperature profiles $\theta(\eta, \xi)$ against dimensionless distance η for different values of prandtl's number Pr with $M = 1.20$, $\gamma = 0.90$ and $\varepsilon = 0.70$.

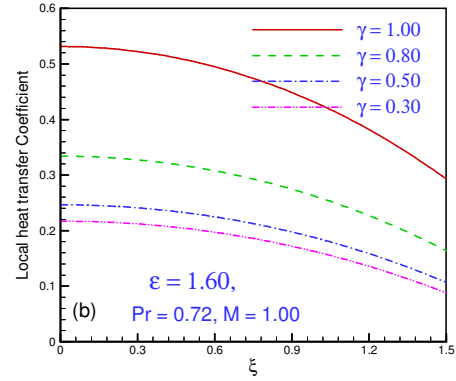
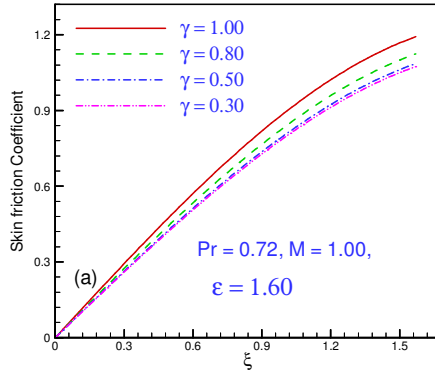


Fig.7 (a) and Fig.7 (b): Variation of dimensionless skin friction coefficient Cf_ξ and the Nusselt number, Nu_ξ against dimensionless distance ξ for different values of dependent thermal conductivity parameter γ with $Pr = 0.72$, $\varepsilon = 1.60$ and $M = 1.00$.

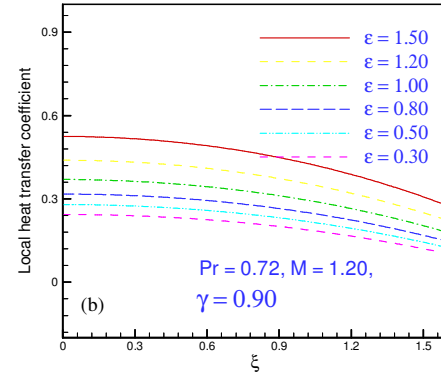
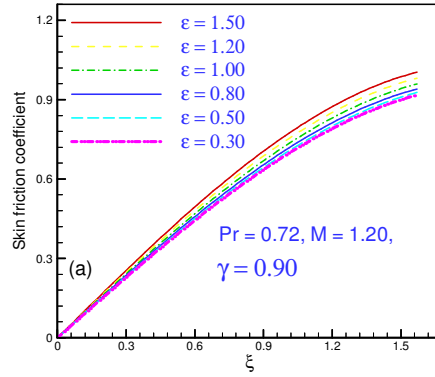


Fig.8 (a) and Fig.8 (b): Variation of dimensionless skin friction coefficient Cf_ξ and the Nusselt number, Nu_ξ against dimensionless distance ξ for different values of temperature dependent viscosity parameter ε with $Pr = 0.72$, $\gamma = 0.90$ and $M = 1.20$.

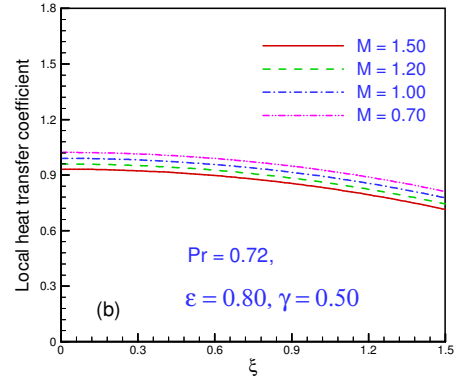
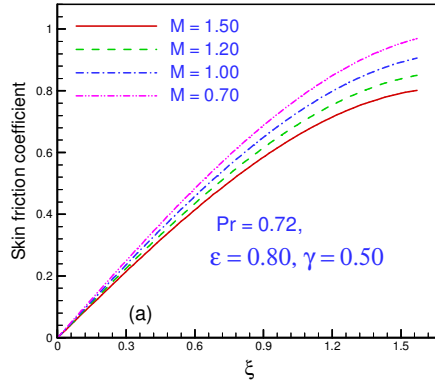


Fig.9 (a) and Fig.9 (b): Variation of dimensionless skin friction coefficient Cf_ξ and the Nusselt number, Nu_ξ against dimensionless distance ξ for different values of magneto hydrodynamics parameter M with $Pr = 0.72$, $\gamma = 0.50$ and $\varepsilon = 0.80$.

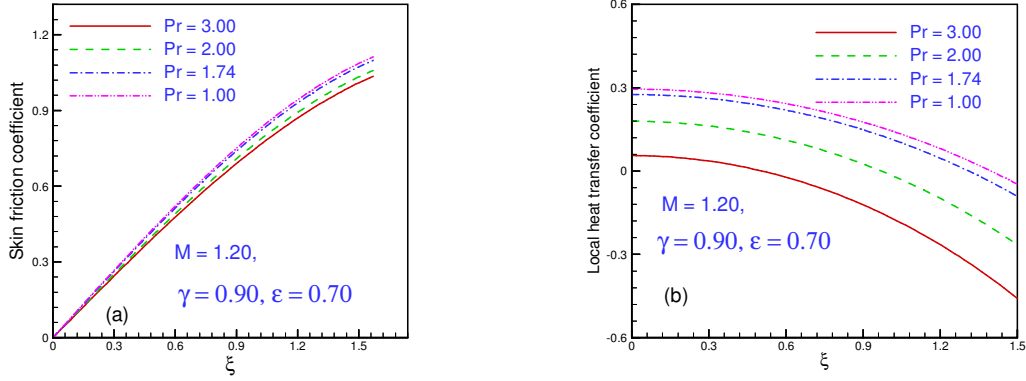


Fig.10 (a) and Fig.10 (b): Variation of dimensionless skin friction coefficient Cf_ξ and the Nusselt number, Nu_ξ against dimensionless distance ξ for different values of prandtl's number Pr with $M = 1.20$, $\gamma = 0.90$ and $\varepsilon = 0.70$.

In order to verify the accuracy of the present work, the values of non-dimensional heat transfer parameter Nu_ξ for $\gamma = \varepsilon = M = 0.00$ having Prandtl's number $Pr = 0.70, 7.00$ at different position of ξ (in degree) are compared with those reported by Nazar *et al.* [17] and Molla *et al.* [16] as presented in **Table-1**. The results are found to be in excellent agreement.

Table-1: Comparisons of the present numerical results of Nu_ξ for the Prandtl's numbers $Pr = 0.70, 7.00$ without effect of the dependent thermal conductivity parameter, magneto hydrodynamics parameter and variation viscosity parameter with those obtained by Molla *et al.* [16] and Nazar *et al.* [17]

ξ in degree	$Pr = 0.70$			$Pr = 7.00$		
	Nazar <i>et al</i> [17]	Molla <i>et al</i> [16]	Present	Nazar <i>et al</i> [17]	Molla <i>et al.</i> [16]	Present
0	0.4576	0.4576	0.4495	0.9595	0.9582	0.9547
10	0.4565	0.4564	0.4481	0.9572	0.9558	0.9495
20	0.4533	0.4532	0.4484	0.9506	0.9492	0.9434
30	0.4480	0.4479	0.4421	0.9397	0.9383	0.9296
40	0.4405	0.4404	0.4352	0.9239	0.9231	0.9154
50	0.4308	0.4307	0.4268	0.9045	0.9034	0.8968
60	0.4189	0.4188	0.4136	0.8801	0.8791	0.8642
70	0.4046	0.4045	0.3982	0.8510	0.8501	0.8395
80	0.3879	0.3877	0.3769	0.8168	0.8161	0.8133
90	0.3684	0.3683	0.3607	0.7774	0.7768	0.7707

Table-2: Skin friction coefficient and rate of heat transfer against ξ for the different values of dependent thermal conductivity parameter γ with other controlling parameters $Pr = 0.72, M = 1.00$

and $\mathcal{E} = 1.60$.

	$\gamma = 0.30$		$\gamma = 0.50$		$\gamma = 0.80$		$\gamma = 1.00$	
ξ	Cf_ξ	Nu_ξ	Cf_ξ	Nu_ξ	Cf_ξ	Nu_ξ	Cf_ξ	Nu_ξ
0.00000	0.00000	0.21711	0.00000	0.24642	0.00000	0.33411	0.00000	0.53153
0.10472	0.09113	0.21648	0.09232	0.24571	0.09621	0.33323	0.10309	0.53031
0.20944	0.18170	0.21462	0.18411	0.24370	0.19187	0.33080	0.20551	0.52692
0.50615	0.43068	0.20286	0.43630	0.23097	0.45449	0.31531	0.48640	0.50536
0.80285	0.65890	0.18119	0.66732	0.20757	0.69451	0.28677	0.74220	0.46551
1.01229	0.80129	0.15965	0.81128	0.18429	0.84349	0.25841	0.90011	0.42587
1.20428	0.91443	0.13511	0.92550	0.15781	0.96118	0.22615	1.02388	0.38058
1.30900	0.96808	0.11972	0.97958	0.14119	1.01657	0.20586	1.08164	0.35210
1.50098	1.04979	0.08760	1.06173	0.10655	1.10004	0.16353	1.16757	0.29252
1.57078	1.07372	0.07464	1.08571	0.09256	1.12412	0.14640	1.19188	0.26839

In **Table - 2** are given the tabular values of the skin friction coefficient Cf_ξ and the Nusselt number Nu_ξ for different values of dependent thermal conductivity γ while $Pr = 0.72$, $\mathcal{E} = 1.60$ and $M = 1.00$. Here it is found that the values of local skin friction coefficient Cf_ξ increase at different position of ξ for dependent thermal conductivity $\gamma = 0.30, 0.50, 0.80, 1.00$. The rate of skin friction coefficient Cf_ξ is increased by 9.882% as the

dependent thermal conductivity γ varies from 0.30 to 1.00 and $\xi = 1.01229$. Furthermore, it is seen that the numerical values of the Nusselt number Nu_ξ increase for increasing values of dependent thermal conductivity γ . The rate of increase the Nusselt number is 26.622% at position $\xi = 1.01229$ as the dependent thermal conductivity γ varies from 0.30 to 1.00.

6. Conclusions

From the present problem the following conclusions may be drawn:

- Significant effects of magnetic parameter M , variation viscosity parameter $\mathcal{E}$ and dependent thermal conductivity parameter γ on velocity and temperature profiles as well as on skin

friction coefficient Cf_ξ and the rate of heat transfer Nu_ξ have been found in this investigation.

- An increase in the values of magnetic parameter or Hartmann number M lead to decrease both the skin friction coefficient Cf_ξ and the rate of heat

transfer Nu_{ξ} . The velocity profiles also decrease but the temperature profiles increase.

- All the velocity profiles, temperature profiles, the skin friction coefficient Cf_{ξ} and the rate of heat transfer Nu_{ξ}

Increase significantly when the values of dependent thermal conductivity γ increase while $Pr = 0.72$, $M = 1.00$ and $\mathcal{E} = 1.60$.

- As temperature dependent viscosity parameter $\mathcal{E}$ increases, both the velocity and the temperature profiles also increase significantly. Also same results for skin friction coefficients Cf_{ξ} and heat transfer coefficients Nu_{ξ} .
- Increasing values of Prandtl's number Pr lead to decrease the velocity profiles, the temperature profiles, the skin friction coefficient Nu_{ξ} and also the rate of heat transfer Nu_{ξ} .

7. References:

- [1] Alam, M. M, Alim, M. A, Chowdhury, M. M. K. Viscous Dissipation Effect on MHD Natural Convection Flow over a Sphere in the Presence of Heat Generation. Nonlinear Analysis, Modelling and control. 2007; 12(4): 447-459.
- [2] Aktar, S, Mahmuda Binte Mostofa Ruma., Parveen, N. Viscous Dissipation Effects on Natural Convection Flow along a Sphere with Heat Generation. Global Journal of Science Frontier Research. 2010; 10(1).
- [3] Miraj, M, Alim, M.A., Shahidul Alam, Karim, M.R Conjugate effects of radiation and viscous dissipation on natural convection flow over a sphere with pressure work. International Journal of Science and Technology. 2012; 1(3)
- [4] Molla, M. M., Rahman, A, Rahman, L. T. Natural convection flow from an isothermal sphere with temperature dependent thermal conductivity. J. Archit. Marine Eng. 2005; 2: 53–64.
- [5] Safiqul Islam, A. K. M, Alim, M. A, Sarker, ATM. M. R(2010): "Effects of temperature dependent thermal conductivity on MHD free convection flow along a vertical flat plate with heat generation and joule heating", The international conference on marine Technology, 11-12 December 2010, BUET, Dhaka, Bangladesh.
- [6] Nasrin, R. and Alim, M. A., (2009): "Combined effects of viscous dissipation and temperature dependent thermal conductivity on magnetohydrodynamic (MHD) free convection flow with conduction and joule heating along a vertical flat plate", Journal of Naval Architecture and Marine Engineering, JNAME; 2009; 6(1):30-40.
- [7] Gitima Patoway (2012): "Effect of variable viscosity and thermal conductivity of micropolar fluid in a porous channel in presence of magnetic field". International Journal for basic Social Science (IJBSS) ISBN:/2319-2968. 2012;1(3):69-77.

- [8] Nasrin, R. and Alim, M. A., "MHD free convection flow along a vertical flat plate with thermal conductivity and viscosity depending on temperature", *Journal of Naval Architecture and Marine Engineering*, 2009.
- [9] Nabil T.M.Eldabe, Elsayed M.A.Elbashbeshy, I.K.Youssef, Ahmed M.Sedki(December-2014):"The effects of temperature dependent viscosity and viscous dissipation on MHD convection flow from an isothermal horizontal circular cylinder in the presence of stress work and heat generation", *Journal of European Scientific*, ISSN:1857-7881; 10(36).
- [10] K.M.safiqul Islam, M.A.Alim, M.M.A. Sarker, A.F.M. Khodadad Khan December-(2012), "Effects of temperature dependent thermal conductivity on natural convection flow along a vertical flat plate with heat generation", *Journal of Naval Architecture and Marine Engineering*.
- [11] M.M.Molla, S.C.Saha, M. A.Hossain (2012) : "The effect of temperature Dependent viscosity on MHD natural convection flow from an isothermal sphere", *Journal of Applied fluid mechanics*; 5(2):25-31.
- [12] Md.Abdul Alim, Md. Rezaul Karim, Md. Miraj akand (March 2012): "Heat Generation effects on MHD natural convection flow along a vertical wavy surface with variable thermal conductivity", *Journal of Computational Mathematics*; 2:42-50.
- [13] Md.Raihanul Haque, M.M.Ali2, Alam M.M., M.A.Alim(2014) : "Effects of viscous dissipation on natural convection flow over a sphere with temperature dependent thermal conductivity", *Journal of Computer and Mathematical sciences*; 5:1- 12.
- [14] Cebeci, T., & Brashaw, P, (1984): "Physical and Computational Aspects of Convective Heat Transfer", spring, New York.
- [15] Keller H.B., (1978): "Numerical methods in boundary layer theory", *Annual Rev. Fluid Mech.*; 10:417-443.
- [16] Molla, M.M., Hossain, M.A. and Yao, L.S., (2004): "Natural convection flow along a vertical wavy surface with uniform surface temperature in presence of heat generation absorption", *Int. J. Thermal Science*, 43: 157-163.
- [17] Nazar, R., Amin, N., Grosan, T. and Pop, I., (2002a): "Free convection boundary layer on an isothermal sphere in a micro polar fluid", *Int. Comm.Heat Mass Transfer*; 129(3):377-386.
- [18] Ali, M. Md., Alim, A. Md., and Andallah, S. L., (2011): " Conjugate effects of radiation and joule heating on magneto hydrodynamic free convection flow along a sphere with heat generation", *American journal of computational mathematics*; 01:18-25.
- [19] Parveen, N., and Alim, A. Md., (2011): "Joule heating effect on magneto hydrodynamic natural convection flow along a vertical wavy surface with viscosity dependent on temperature", *International Journal of Engineering & Technology*; 3(1):1-11.