

The Coupling Conduction Effects on Natural Convection Flow along a Vertical Flat Plate with Joule Heating and Heat Generation

ABSTRACT

The **out-turn** of heat generation and Joule heating on natural convection flow **through** a vertical flat plate **have** been investigated in the given article. Joule heating and heat conduction due to wall thickness '**b**' are **esteemed** as well in this **analysis**. With **the intent** to **obtain** similarity solutions **to** the problem being **constituted**, the **evolved** equations are made dimensionless **employing appropriate** transformations. The non-dimensional equations are then **modified** into non-linear equations by **bringing into being** a non-similarity transformation. The **out-turn** non-linear **alike** equations **confine** with their **commensurate** boundary conditions **formed** on conduction and convection are solved numerically **applying** the finite difference method **accompanied by** Newton's linearization approximation. The numerical **outcomes** **in terms of** the skin friction coefficient, velocity, temperature, and surface temperature profiles are **shown** both **graphically and in tabular** forms for the different values of the parameters **correlated** with the problem.

Keywords: Natural convection, Joule heating, Heat generation and Prandtl's Number, Magnetohydrodynamics.

1.INTRODUCTION

Free convection flow is often **envisaged** in cooling of nuclear reactors or in the study of the structure of stars and planets. Along with the free convection flow the phenomenon of the boundary layer flow of an electrically conducting fluid to a vertical flat plate in **existence** of Joule-heating, magnetic field and heat generation are also very common, because of their applications in nuclear engineering in **conjunction** with the cooling of reactors. Natural convection heat transfer has **attracted a** considerable attention because of **its widespread** applications in the areas of energy conservations cooling of electrical and electronics components, the design of solar collectors, heat exchangers and **so on**. The

main difficulty **facing the solution to** natural convection problems lies in the determination of the velocity **field which** greatly influences the heat transfer process. El-Amin [1] studied **conjugate** effects of viscous dissipation and Joule heating on MHD forced convection **through** a nonisothermal horizontal cylinder embedded in a **fluid-saturated** porous medium. Gebhart [2] also investigated the **results** of viscous dissipation in a natural flow. Alam *et al.* [3] studied the **consequences** of pressure stress work and viscous dissipation in natural convection flow **towards** a vertical flat plate with heat conduction. Joule heating effects on the coupling of conduction **on MHD** free convection flow from a vertical flat plate have been investigated by Alim *et al.* [4].

Vajravelu *et al.* [5] examined the effects of heat transfer in a viscous fluid on a stretching sheet with viscous dissipation and internal heat generation. Hossain [6] analyzed the viscous and Joule heating sequels on MHD natural convection flow with a variable plate temperature. Hossain *et al.* [7] investigated the effects of free convection flow along a perpendicular wavy surface temperature associated with heat generation/absorption. Miyamoto *et al.* [9] considered the effect of axial heat conduction in a vertical flat plate on free convection heat transfer. Pozzi and Lupo [10] studied the coupling of conduction with laminar natural convection along a flat plate. Molla *et al.* [10] have investigated the effects on MHD free convection flow over a sphere in the availability of heat generation. Effects of conduction variation on MHD free convection flow along a vertical plane plate are considered by Shafiqul Islam *et al.* [13]. Chowdhury, *et al.* [14], performed the MHD free convection flow of viscoelastic fluid past an infinite porous plate. Hossain, M. A. [15] studied the effects of Viscous and Joule

heating effects on MHD free convection flow with variable plate temperature, Hossain, M.A.*et al.* [16] have been investigated the MHD forced and free convection boundary layer flow along a vertical porous plate. The present investigation is mainly concerned with the yielding of Joule heating on natural convection flow along a perpendicular flat plate with heat generation in the entire region from upstream to downstream of a viscous incompressible and electrically conducting fluid. It deals with the upshot of Joule heating parameter Jul , the Prandtl's number Pr and the heat generation parameter Q over the velocity and temperature fields as well as on the skin friction and surface temperature. In the following sections, the detailed derivations of the governing equations for the flow are discussed.

2. GOVERNING EQUATIONS OF THE FLOW

It has been considered that the steady two-dimensional laminar natural convection boundary layer flow of a viscous incompressible electrically conducting fluid along a side of

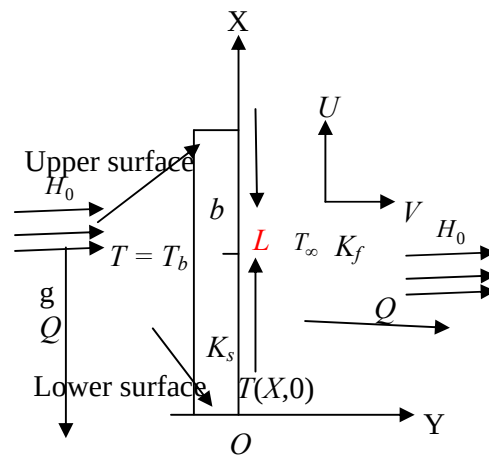


Fig.1: Physical configuration and coordinates system

a vertical flat plate of thickness 'b' insulated on the edges with temperature T_b being maintained on the other side in the presence of a uniformly distributed transverse magnetic field. The flow aspect and the coordinates system are sighted in Fig.1.

With the mathematical statement of the basic conservation acts of mass, momentum, and energy for the steady viscous incompressible and electrically conducting flow, after simplifying we have

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \quad (1)$$

$$U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = \nu \frac{\partial^2 U}{\partial Y^2} + g\beta(T_f - T_\infty) - \frac{\sigma H_0^2 U}{\nu} \quad (2)$$

$$U \frac{\partial T}{\partial X} + V \frac{\partial T}{\partial Y} = \frac{K}{\rho C_p} \frac{\partial^2 T}{\partial Y^2} + \frac{\sigma B_0^2}{\rho C_p} U^2 + \frac{Q_0}{\rho C_p} (T_f - T_\infty) \quad (3)$$

Where, U and V are the velocity components towards the X and Y axis respectively, T is the fluid temperature and T_∞ is the ambient fluid temperature in the boundary layer, g is the gravitational acceleration, K is the thermal conductivity, ρ is the density, C_p is the specific heat constant pressure and ν is the kinematic viscosity. The sum of heat generated or absorbed per unit volume is $Q_0(T - T_\infty)$, Q_0 being either positive or negative constant. The source term describes the heat generation when $Q_0 > 0$ and the heat absorption when $Q_0 < 0$. In the energy equation, heat generation and Joule heating

terms are included. Here for exterior conditions we know, $\partial P / \partial X = \rho_e g$ and $\rho = \rho_e$, P is the pressure, H_0 is the magnetic field strength and σ is the electric conductivity.

The appropriate boundary conditions to be given below

$$U = 0, V = 0, T_f = (X, 0) \text{ at } Y = 0, X > 0, \quad (4)$$

$$U \rightarrow 0, T_f \rightarrow T_\infty \text{ as } Y \rightarrow \infty, X > 0$$

The temperature and the heat flux are considered continuous at the interface for the coupled conditions and at the interface, we must have

$$\frac{k_s}{k_f} \frac{\partial T_{so}}{\partial Y} = \left(\frac{\partial T_f}{\partial Y} \right)_{Y=0} \quad (5)$$

Where k_s and k_f are the thermal conductivity of the solid and the fluid respectively. The temperature T_{so} in the solid as given by A.

Pozzi and M. Lupo is

$$T_{so} = T(X, 0) - \{T_b - T(X, 0)\} \frac{Y}{b} \quad (6)$$

Where $T(X, 0)$ is the unknown temperature at the interface to be determined from the solutions of the equations.

We observe that the equations (1) - (3) together with the boundary conditions (4) - (5) are non-linear partial differential equations. In the following sections, the solution methods of these equations are discussed in details.

3. TRANSFORMATION OF THE GOVERNING EQUATIONS

Equations (1) - (3) may now be non-dimensionalized by using the following

dimensionless dependent and independent variables:

$$x = \frac{X}{L}, y = \frac{Y}{L} d^{1/4}, U = \frac{v}{L} d^{1/2} u, \quad (7)$$

$$V = \frac{v}{L} d^{1/4} v, \theta = \frac{T_f - T_\infty}{T_b - T_\infty},$$

$$L = \frac{v^{2/3}}{g^{1/3}}, d = \beta(T_b - T_\infty)$$

For the problem of natural convection, its parabolic character has no characteristic length; L has been defined in terms of v and g , which are the intrinsic properties of the system. The reference length along the 'y' direction has been modified by a factor $d^{1/4}$ in order to eliminate this quantity from the dimensionless equations and the boundary conditions.

The **magnetohydrodynamic** field in the fluid is governed by the boundary layer equations, which in the non-dimensional form obtained by introducing the dimensionless variables described in (7), may be written as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (8)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + Mu = \frac{\partial^2 u}{\partial y^2} + \theta \quad (9)$$

$$u \frac{\partial \theta}{\partial x} + v \frac{\partial \theta}{\partial y} = \frac{1}{Pr} \frac{\partial^2 \theta}{\partial y^2} + u^2 Jul + Q\theta \quad (10)$$

Where, $Jul = \sigma B_0^2 v d^{1/2} / \rho C_p (T_b - T_\infty)$, the

Joule-heating parameter, $M = \sigma H_0 L^2 / \mu G_r^{1/2}$

is the dimensionless magnetic parameter,

$Pr = \mu C_p / \kappa_f$, the Prandtl's number and

$Q_0 L^2 / \mu C_p d^{1/2} = Q$, where Q is the heat generation parameter.

The corresponding boundary conditions (4) - (6) take the following form:

$$u = v = 0, \theta - 1 = p \left(\frac{\partial \theta}{\partial y} \right) \text{ at } y = 0, \quad (11)$$

$$u \rightarrow 0, \theta \rightarrow 0 \text{ as } y \rightarrow \infty$$

Where p is the conjugate conduction

parameter given by $p = \left(\frac{k_f}{k_s} \right) \left(\frac{y}{L} \right) d^{1/4}$. Here,

the coupling parameter 'p' governs the described problem. The order of magnitude of

'p' depends actually on, b/L , k_f/k_s and $d^{1/4}$

being the order of unity. The term b/L attains

values much greater than one because of L is

small. In case of air, $\left(\frac{k_f}{k_s} \right)$ becomes very

small when the vertical plate is highly conductive i.e. $k_s \gg 1$ and for materials,

$O\left(\frac{k_f}{k_s} \right) = 0.1$ such as glass. Therefore, in

different cases 'p' is different but not always a small number. In the present investigation, we have considered $p = 1$ which is accepted

for b/L of $O\left(\frac{k_f}{k_s} \right)$.

To solve the equations (9) and (10) are

subjected to the boundary conditions (11), the

following transformations are introduced for the flow region starting from upstream to downstream.

$$\begin{aligned}\psi &= x^{4/5}(1+x)^{-1/20} f(x, \eta), \\ \eta &= y x^{-1/5}(1+x)^{-1/20}, \\ \theta &= x^{1/5}(1+x)^{-1/5} h\end{aligned}\quad (12)$$

Here η is the dimensionless similarity variable and ψ is the stream function which satisfies the equation of continuity and $u = \frac{\partial \psi}{\partial y}$, $v = -\frac{\partial \psi}{\partial x}$ and $h(x, \eta)$ is the dimensionless temperature. Therefore, substituting these transformations into equations (9) and (10) and after simplifying, we get the following transformed non-dimensional equations.

$$f''' + \frac{16+15x}{20(1+x)} f f'' - \frac{6+5x}{10(1+x)} f'^2 - Mx^{2/5}(1+x)^{1/10} f' + h = x \left(f' \frac{\partial f'}{\partial x} - f'' \frac{\partial f}{\partial x} \right) \quad (13)$$

$$\begin{aligned}\frac{1}{Pr} h'' + \frac{16+15x}{20(1+x)} f h' - \frac{1}{5(1+x)} f h \\ + Jul x^{7/5}(1+x)^{-1/2} f'^2 + Q x^{1/5}(1+x)^{-1/5} h \\ = x \left(f' \frac{\partial h}{\partial x} - h' \frac{\partial f}{\partial x} \right)\end{aligned}\quad (14)$$

In the above equations, the primes denote differentiation with respect to η .

The boundary conditions (11) after simplifying, we have the following form

$$\begin{aligned}f(x, 0) &= f'(x, 0) = 0, \\ h(x, 0) &= -(1+x)^{1/4} \\ &+ x^{1/5}(1+x)^{1/20} h(x, 0) \\ f'(x, \infty) &= 0, h'(x, \infty) = 0\end{aligned}\quad (15)$$

The solutions of the above equations (13) and (14) together with the boundary conditions (15) enable us to calculate the skin friction τ and the surface temperature θ at the surface in the boundary layer from the following relations:

$$\tau = \mu \left\{ x^{2/5} (1+x)^{-3/20} f''(x, 0) \right\} \quad (16)$$

$$\theta = \left\{ x^{1/5} (1+x)^{-1/5} h(x, 0) \right\} \quad (17)$$

4. METHOD OF SOLUTION

This paper regarding the free convection flow of viscous incompressible fluid towards a uniformly heated vertical flat plate in arrival of Joule heating and heat generation has been investigated using the very efficient implicit finite difference method known as the Keller box scheme developed by Keller [11], which is well documented by Cebeci and Bradshaw [12].

To apply the aforementioned method, Equations (13) and (14) with their boundary conditions (15) are first converted into the following system of first order equations. Here,

x is replaced by ξ so that the coordinates (x, η) are replaced by (ξ, η) . For this purpose, we introduce new dependent variables $u(\xi, \eta)$, $v(\xi, \eta)$, $p(\xi, \eta)$ and $g(\xi, \eta)$ so that the transformed momentum and energy equations can be rewritten as

$$f' = u \quad (18)$$

$$u' = v \quad (19)$$

$$g' = p \quad (20)$$

$$\begin{aligned}v' + P_1 f u' - P_2 u^2 + g - P_3 f' \\ = \xi \left(u \frac{\partial u}{\partial \xi} - v \frac{\partial f}{\partial \xi} \right)\end{aligned}\quad (21)$$

$$\begin{aligned}\frac{1}{Pr} p' + P_1 f p - P_4 u g + P_5 u^2 \\ + P_6 g = \xi \left(u \frac{\partial g}{\partial \xi} - p \frac{\partial f}{\partial \xi} \right)\end{aligned}\quad (22)$$

Where $x = \xi$, $h = g$,

$$P_1 = \frac{16+15\xi}{20(1+\xi)}, P_2 = \frac{6+5\xi}{10(1+\xi)},$$

$$P_3 = Mx^{2/5}(1+x)1/10,$$

$$P_4 = \frac{1}{5(1+\xi)}, P_5 = Jul \xi^{7/5} (1+\xi)^{-1/2}$$

$$\text{and } P_6 = Q \xi^{1/5} (1+\xi)^{-1/5}$$

$$\text{and the boundary conditions (15) are } f(\xi, 0) = 0, u(\xi, 0) = 0, \quad (23)$$

$$g(\xi, 0) = -(1+\xi)^{1/4}$$

$$+\xi^{1/5} (1+\xi)^{1/20} g(\xi, 0)$$

$$u(\xi, \infty) = 0, \quad p(\xi, \infty) = 0$$

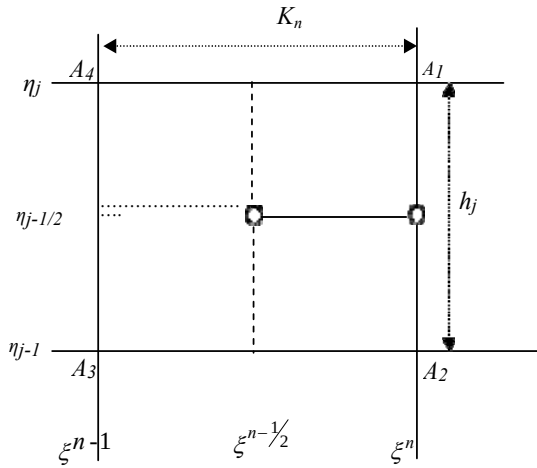


Fig. 2: Net rectangle of difference approximations for the Box scheme.

Here, n and j are just the sequence of numbers on the (ξ, η) plane, having k_n and h_j as the variable mesh widths. Now approximating the quantities f , u , v and p , at the points (ξ^n, η_j) of the net by f_j^n , u_j^n , v_j^n , p_j^n leads to something being termed as net function. It is also employed so that the notation P_j^n for the quantities midways between net points as shown in Fig. 2 and for any net functions as

Now, we consider the net rectangle on the (ξ, η) plane as shown in the Fig. 2 and denote the net points by

$$\xi^0 = 0, \xi^n = \xi^{n-1} + k_n, \quad (24)$$

$$n = 1, 2, 3, \dots, N, \eta_0 = 0,$$

$$\eta_j = \eta_{j-1} + h_j, j = 1, 2, 3, \dots, J$$

$$\xi^{n-1/2} = \frac{1}{2}(\xi^n + \xi^{n-1}) \quad (25)$$

$$\eta_{j-1/2} = \frac{1}{2}(\eta_j + \eta_{j-1/2}) \quad (26)$$

$$g_j^{n-1/2} = \frac{1}{2}(g_j^n + g_j^{n-1}) \quad (28)$$

The finite difference approximation according to box method to the three first-order ordinary differential equations (18)-(20) is written for the

$$\text{midpoint } \left(\xi^n, \eta_{j-1/2} \right)$$

of the segment A_1A_2 as shown in Fig.2.

$$\frac{f_j^n - f_{j-1}^n}{h_j} = u_{j-1/2}^n = \frac{u_{j-1}^n + u_j^n}{2} \quad (29)$$

$$\frac{u_j^n - u_{j-1}^n}{h_j} = v_{j-1/2}^n = \frac{v_j^n + v_{j-1}^n}{2} \quad (30)$$

$$\frac{g_j^n - g_{j-1}^n}{h_j} = p_{j-1/2}^n = \frac{p_j^n + p_{j-1}^n}{2} \quad (31)$$

The finite difference approximation to the first order differential equation (21) and (22) is written for the midpoint $\left(\xi^{n-1/2}, \eta_{j-1/2}\right)$ of the rectangle $A_1 A_2 A_3 A_4$. This procedure yields

$$\frac{1}{2} \left(\frac{v_j^n - v_{j-1}^n}{h_j} \right) + \frac{1}{2} \left(\frac{v_j^{n-1} - v_{j-1}^{n-1}}{h_j} \right) \quad (32)$$

$$+ (P_1 f v)_{j-1/2}^{n-1/2} - (P_2 u^2)_{j-1/2}^{n-1/2} - (P_3 u)_{j-1/2}^{n-1/2} + g$$

$$= \xi_{j-1/2}^{n-1/2} \left(\begin{array}{c} u_{j-1/2}^{n-1/2} \frac{u_{j-1/2}^n - u_{j-1/2}^{n-1}}{k_n} \\ - v_{j-1/2}^{n-1/2} \frac{f_{j-1/2}^n - f_{j-1/2}^{n-1}}{k_n} \end{array} \right)$$

$$\frac{1}{2 \text{Pr}} \left(\frac{p_j^n - p_{j-1}^n}{h_j} \right) + \frac{1}{2 \text{Pr}} \left(\frac{p_j^{n-1} - p_{j-1}^{n-1}}{h_j} \right) \quad (33)$$

$$+ (P_1 f p)_{j-1/2}^{n-1/2} - (P_4 u g)_{j-1/2}^{n-1/2}$$

$$+ (P_5 u^2)_{j-1/2}^{n-1/2} + (P_6 g)_{j-1/2}^{n-1/2}$$

$$= \xi_{j-1/2}^{n-1/2} \left(\begin{array}{c} u_{j-1/2}^{n-1/2} \frac{g_{j-1/2}^n - g_{j-1/2}^{n-1}}{k_n} \\ - p_{j-1/2}^{n-1/2} \frac{f_{j-1/2}^n - f_{j-1/2}^{n-1}}{k_n} \end{array} \right)$$

The above equations are to be linearized using Newton's Quasi-linearization method. Then linear algebraic equations can be rewritten in block matrix that forms a coefficient matrix. The whole methodology which involves mainly reduction to first order pursued by central difference approximations, Newton's Quasi-linearization method and the block Thomas algorithm, is established as the Keller-box method.

5. RESULTS AND DISCUSSION

Here, we have investigated the problem of the steady two dimensional laminar natural convection boundary layer flow of a viscous incompressible electrically conducting fluid along a side of a vertical flat plate of thickness 'b' insulated on the edges with temperature T_b being maintained on the other side in the presence of a uniformly distributed transverse magnetic field and heat generation. Solutions are obtained for the fluid having Prandtl number $Pr = 0.01, 0.10, 0.72, 1.00, 1.74$ and for overall of the Joule heating parameter $Jul = 0.20, 0.40, 0.70, 0.90$ and the heat generation parameter $Q = 0.10, 0.50, 0.80, 1.10$. Also, we consider the controlling magnetic Parameter value only $M = 1.00$, here no variation is not included.

Fig. 3 (a) and 3(b) represent the velocity and the temperature profiles for various values of the Joule heating parameter Jul for particular values of the Prandtl's number $Pr = 0.72$, the magnetic parameter $M = 1.00$ and heat generation parameter $Q = 0.60$. From Fig. 3(a), it is perceived that an enhancement in the Joule heating parameter Jul is associated with a considerable increase in velocity profiles, But close to the surface of the plate, the velocity increases and becomes maximum and then reduces and finally approaches to zero asymptotically. However, Fig. 3(b) exhibits the distribution of the temperature profiles against η for some values of the Joule heating parameter Jul ($=0.90, 0.70, 0.40, 0.20$). It is clearly seen that the temperature distribution increases owing to increasing values of Joule heating parameter Jul and the maximum is adjacent to the plate wall. The local maximum values of temperature profiles are 1.7662, 1.6886, 1.6164, 1.5496 for $Jul =$

0.90, 0.70, 0.40, 0.20 respectively and each of which attains at the surface. Thus the temperature profiles increase 13.98% as Jul increases from 0.20 to 0.90.

Figs. 4(a) and 4(b) exhibit results for velocity and temperature profiles, for various small values of heat generation parameter, $Q = 0.20, 0.40, 0.70, 0.90$ being plotted against η at $Pr = 0.72$, $M = 1.00$ and $Jul = 0.50$. From Fig. 4(a), it is revealed that the velocity profiles $f'(\xi, \eta)$ increases very slightly with the increase in the heat generation parameter, Q . In other words, it indicates that heat generation parameter increases the fluid motion slowly. From Fig. 4(b), it is observed that temperature profiles $\theta(\xi, \eta)$ increase for the growing values of heat generation parameter Q significantly.

Figs. 5(a) and 5(b) exhibit the outcomes for the velocity and temperature profiles, for different small values of Prandtl's number Pr

(=1.74, 1.00, 0.72, 0.10, 0.01) being plotted against η at $Jul=0.60$, $M = 1.00$ and $Q = 0.40$. It is observed from Fig. 5(a) that the velocity profile is affected notably and decreases as the value of Prandtl's number Pr increases. But near the surface of the plate velocity rises significantly and then falls slowly and lastly approaches to zero. Also, it has been evident that the temperature field decreases with increasing values of Prandtl's number Pr in Fig. 5(b).

Numerical values of the velocity gradient and the surface temperature are depicted graphically in Fig. 6 (a) and 6(b) respectively against the axial distance in the interval $[0, 6.4]$ for different values of Joule heating parameter Jul ($= 0.90, 0.70, 0.40, 0.20$). It is seen from Fig. 6(a) that the skin-friction coefficient increases when the Joule heating parameter Jul increases. It is also observed in Fig. 6(b) that the surface temperature distribution increases when Jul increases.

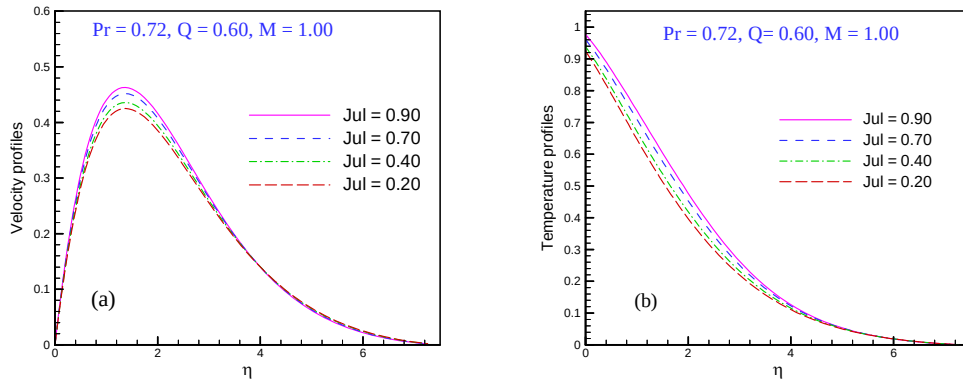


Fig. 3(a) and Fig. 3(b) Variation of dimensionless velocity profiles $f'(\xi, \eta)$ and temperature profiles $\theta(\xi, \eta)$ against dimensionless distance η for different values of Joule heating parameters Jul with $Pr = 0.72$, $M = 1.00$ and $Q = 0.60$.

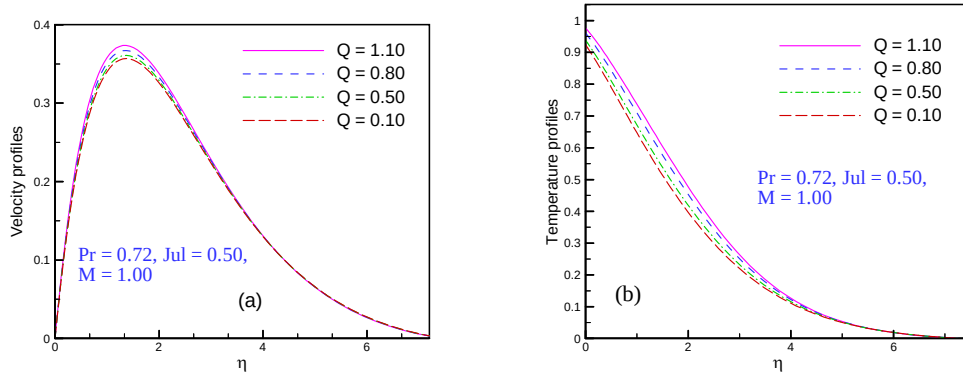


Fig. 4(a) and Fig. 4(b) Variation of dimensionless velocity profiles $f'(\xi, \eta)$ and temperature profiles $\theta(\xi, \eta)$ against dimensionless distance η for distinct values of heat generation parameters Q with $Pr = 0.72$, $M = 1.00$ and $Jul = 0.50$.

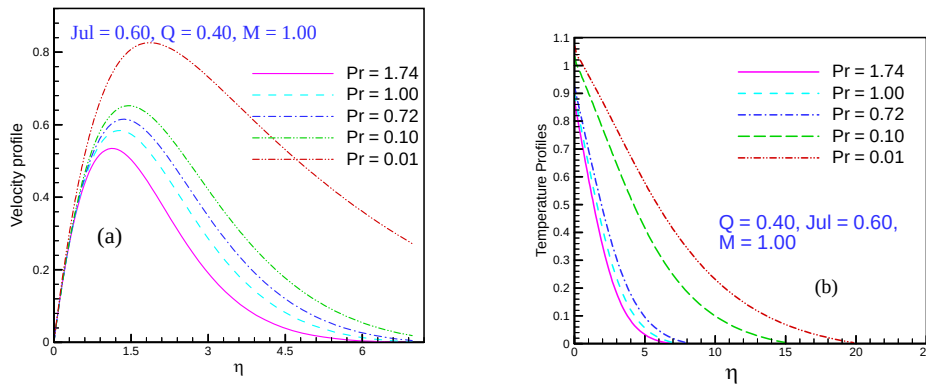


Fig. 5(a) and Fig. 5(b): Variation of dimensionless velocity profiles $f'(\xi, \eta)$ and temperature profiles $\theta(\xi, \eta)$ against dimensionless distance η for different values of Prandtl's number parameters Pr with $Q = 0.40$, $M = 1.00$ and $Jul = 0.60$.

Figs. 7(a) and 7(b) illustrate the variation of skin-friction $f''(\xi, 0)$ and surface temperature distribution $\theta(\xi, 0)$ against ξ for simultaneous values of heat generation parameter Q ($= 1.10, 0.80, 0.50, 0.10$). It is seen from Fig. 7(a) that the skin-friction $f''(\xi, 0)$ increases when the heat generation parameter Q increases. It is also observed from Fig.8 (b) that the surface temperature $\theta(\xi, 0)$ distribution increases as Q increases.

In Fig.8(a), the shear stress coefficient $f''(\xi, 0)$ and Fig.8(b), the surface

temperature $\theta(\xi, 0)$ are presented graphically for different values of the Prandtl's number Pr ($= 0.01, 0.10, 0.72, 1.00, 1.74$) when other values of the controlling parameter $Jul = 0.60$, $M = 1.00$ and $Q = 0.40$. Here we take $Pr = 0.72$ for air, $Pr = 1.00$ for the electrolyte solutions such as salt water and also Pr values of (0.01, 0.10, 0.50) have been used theoretically. Here, it is found that the skin friction and the surface temperature both decrease with increasing values of Prandtl's number.

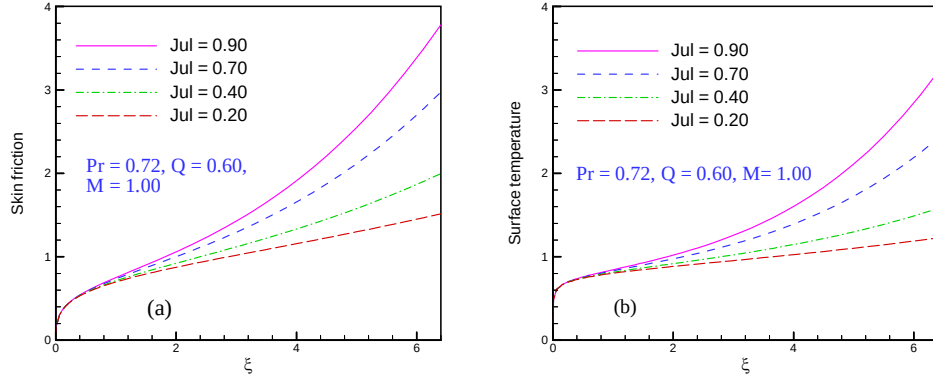


Fig. 6(a) and Fig. 6(b): Variation of skin friction coefficient $f''(\xi, 0)$ and surface temperature $\theta(\xi, 0)$ with dimensionless distance ξ for different values of Joule heating parameters with $Pr = 0.72$, $M = 1.00$ and $Q = 0.60$.

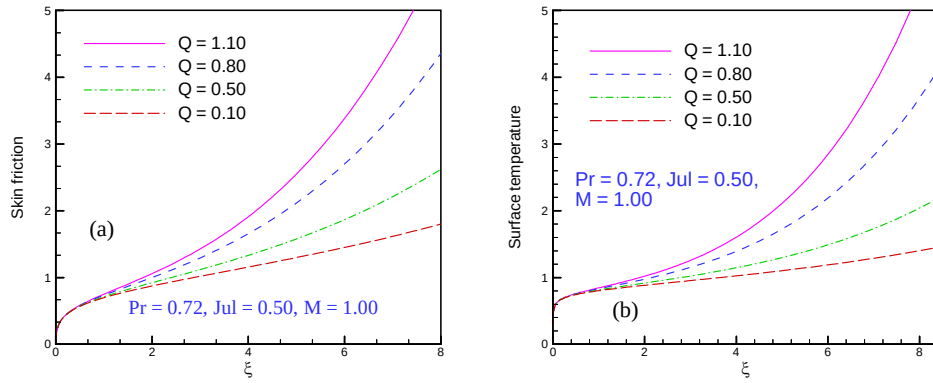


Fig. 7(a) and Fig. 7(b): Variation of skin friction coefficient $f''(\xi, 0)$ and surface temperature $\theta(\xi, 0)$ with dimensionless distance ξ for various values of heat generation parameters Q with $Pr = 0.72$, $M = 1.00$ and $Jul = 0.50$.

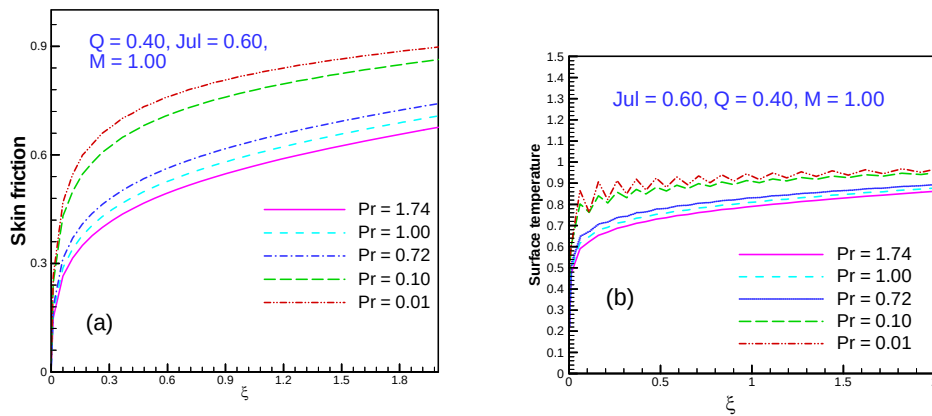


Fig. 8(a) and Fig. 8(b): Variation of skin friction coefficient $f''(\xi, 0)$ and surface temperature $\theta(\xi, 0)$ with dimensionless distance ξ for distinct values of Prandtl's number parameters Q with $Q = 0.40$, $M = 1.00$ and $Jul = 0.60$.

In **Table-1**, skin friction coefficient and surface temperature distribution for various values of heat generation Q when $Jul = 0.50$, $M = 1.00$ and $Pr = 0.72$ have been entered. Here, it is found that the values of skin friction decrease at a different position of ξ for heat generation parameter $Q = 0.10, 0.50, 0.80, 1.10$. The rate of the skin friction increases by 81.68% as the heat generation parameter Q changes from 0.10 to 1.10 at position $\xi = 3.0049$. Moreover, it is examined that the numerical values of the surface temperature distribution increase with rising values of heat generation parameter Q . The rate of increase in surface temperature distribution is 54.26% at position $\xi = 3.0049$ as the heat generation parameter changes from 0.10 to 1.10.

In Table-2 skin friction coefficient and surface temperature distribution for different values of Joule heating parameter Jul when $Q = 0.60$, $M = 1.00$ and $Pr = 0.72$ is being entered. Here, it is found that the values of skin friction decrease at a different position of for Joule heating parameter $Jul = 0.20, 0.40, 0.70, 0.90$. The rate of the skin friction increases by 74.82% as the Joule heating parameter Jul changes from 0.20 to 0.90 at position $\xi = 3.0049$. Moreover, it is observed that the numerical values of the surface temperature distribution increase with growing values of Joule heating parameter Jul . The rate of increase in surface temperature distribution is 56.86% at position $\xi = 3.0049$ as the Joule heating parameter Jul changes from 0.20 to 0.90.

Table -1: Skin friction coefficient and surface temperature distribution for simultaneous values of heat generation parameter Q against ξ with other controlling parameters $Pr = 0.72$, $Jul = 0.60$.

ξ	$Q = 0.10$		$Q = 0.50$		$Q = 0.80$		$Q = 1.10$	
	$f''(\xi, 0)$	$\theta(\xi, 0)$	$f''(\xi, 0)$	$\theta(\xi, 0)$	$f''(\xi, 0)$	$\theta(\xi, 0)$	$f''(\xi, 0)$	$\theta(\xi, 0)$
0.0000	0.0155	0.2052	0.0155	0.2052	0.0155	0.2052	0.0155	0.2052
0.3045	0.5083	0.7352	0.5801	0.8126	0.6663	0.9106	0.7686	1.0338
0.7090	0.6416	0.7907	0.7502	0.8847	0.8851	1.0087	1.0490	1.1701
1.0265	0.7067	0.8152	0.8356	0.9168	0.9982	1.0531	1.1981	1.2332
2.0369	0.8372	0.8614	1.0113	0.9773	1.2373	1.1381	1.5208	1.3559
3.0049	0.9169	0.8885	1.1213	1.0131	1.3913	1.1893	1.7337	1.4311
4.0219	0.9794	0.9095	1.2093	1.0414	1.5169	1.2304	1.9101	1.4924
5.0387	1.0296	0.9263	1.2811	1.0645	1.6211	1.2648	2.0583	1.5443
6.0502	1.0717	0.9405	1.3423	1.0844	1.7111	1.2949	2.1877	1.5903

Table-2: Skin friction coefficient and Surface temperature distribution for different values of Joule heating parameter Jul against ξ with other controlling parameters $Pr = 0.72$, $M = 1.00$ and $Q = 0.60$.

ξ	$Jul = 0.20$		$Jul = 0.40$		$Jul = 0.70$		$Jul = 0.90$	
	$f''(\xi, 0)$	$\theta(\xi, 0)$	$f''(\xi, 0)$	$\theta(\xi, 0)$	$f''(\xi, 0)$	$\theta(\xi, 0)$	$f''(\xi, 0)$	$\theta(\xi, 0)$
0.0000	0.0137	0.1873	0.0137	0.1873	0.0137	0.1873	0.0137	0.1873
0.3045	0.6184	0.8871	0.6356	0.9064	0.6535	0.9270	0.6721	0.9488
0.7090	0.9614	1.1362	0.9990	1.1752	1.0387	1.2170	1.0805	1.2618
1.0265	1.2262	1.3369	1.2819	1.3937	1.3407	1.4549	1.4027	1.5208
2.0369	2.2289	2.1726	2.3586	2.3106	2.4962	2.4605	2.6417	2.6228
3.0049	3.5850	3.4674	3.8199	3.7397	4.0692	4.0360	4.3332	4.3575
4.0219	5.6664	5.7475	6.0662	6.2645	6.4906	6.8279	6.9402	7.4402
5.0387	8.7656	9.6629	9.4141	10.6098	10.1026	11.6428	10.8319	12.7665

6. CONCLUSIONS

The **upshot** of Joule heating parameter *Jul*, the heat generation parameter *Q* and the Prandtl's number *Pr* on MHD free convection boundary layer flow along a **perpendicular** flat plate has been studied introducing a new class of transformations. The transformed non-similar boundary layer governing equations of flow with the boundary conditions based on conduction and convection are numerically solved by using the very efficient implicit finite difference method along with Keller box scheme. From the recent observation, the following conclusions may be drawn:

- The skin friction coefficient, the surface temperature, the velocity and the temperature profiles raise with the increasing values of the Joule heating parameter *Jul*.
- The skin friction coefficient, the surface temperature, the velocity and the temperature profiles are established to increase with the rising values of the heat generation parameter *Q*.
- It has been shown that the skin friction coefficient, the surface temperature distribution, the velocity profile, the temperature profiles decreases over the whole boundary layer with the increase in the Prandtl's number *Pr*.

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