

A new formulation to calculate effective tension in large pipeline free spans

ABSTRACT

Since the 70's, when offshore oil exploitation advanced, submarine pipeline design has been the subject of many researcher studies. Pipeline free spans are common and frequently assessed. In spite of this, up to this day, there is no simple and specific formula to calculate the effective tension (S_{eff}), which takes into account the pipeline sagging in moderate or large span lengths. Therefore, engineers normally apply finite element analyses, even for preliminary evaluations. On the other hand, there are simple formulae to calculate other parameters such as the deflection, the moment and the natural frequency, which require previous knowledge of the S_{eff} value. In the present contribution, an approximate formula is proposed, in order to solve the beam differential equation for both pinned and fixed end conditions and make it possible to calculate the S_{eff} value using a straightforward and relatively simple equation, which takes into account the pipeline sagging. The values calculated using the proposed method were compared to numerical results obtained for many different cases, varying the span length and the effective tension. The analysis results showed that the proposed equation is highly accurate for practical conditions and, therefore, may become a useful tool for the assessment of the pipeline free spans.

Keywords: Pipeline; free span; effective tension; axial force.

1. INTRODUCTION

The increase of oil demand has driven the development of structures from onshore to offshore [1], but following this same trend the diminishing availability of oil in shallow water reservoirs pushed engineers to design more facilities in deep and, recently, ultra-deep waters. In addition, longer submarine pipelines are required both because of the longer distances to shore as in order to maximize the number of wells covered by the same platform. As a consequence of this, the number of free spans also increases, caused by seabed imperfections and/or crossings. After the installation, scour pits, induced by environmental current turbulence, can also create free spans or expand the existing ones [2]. On the other hand, the unacceptably increased spans require intervention, increasing costs and extending the execution time. Consequently, the assessment of free spans is a very important and frequent procedure in both design and conservation of submarine pipelines, demanding reliable and easy methodologies.

Free spans are evaluated as horizontal beams supported by their ends with applied loads such as temperature, internal and external pressures, self-weight (of the pipe, content, coating, etc.), wave, current and dynamic forces due to vortex induced vibrations (VIV). The three last of these will not be discussed in this work. The temperature increment ΔT acts directly into axial strain $\varepsilon = \Delta T \cdot \alpha_e$, where α_e is the temperature expansion coefficient. Internal and external pressure drives the hoop stress and, consequently, due to the Poisson effect, cause axial stress. In addition, as a second order effect, demonstrated by Palmer and Baldry [3] and after in more details by Sparks [4], internal and external pressure create lateral forces on curved segments, whose behavior can be considered as equivalent to a beam axially loaded by

$$S_{eff} = N_{tr} - p_i A_i + p_e A_e \quad (1)$$

where S_{eff} is the so-called effective tension, N_{tr} is the true steel wall axial force, p_i , p_e , A_i , A_e are the internal and external pressures and cross sections areas, respectively. More discussion can be found in literature about this in [5–7].

For a totally restrained pipe, the code DNV-RP-F105 [8] presents the following formulation:

$$S_{eff} = H_{eff} - \Delta p_i A_i (1 - 2\nu) - A_s E \Delta T \alpha_e \quad (2)$$

where H_{eff} is the effective tension just after the pipe installation (lay tension), Δp_i is the internal pressure increment, ν is the Poisson coefficient, A_s is the steel cross section area and E is the Young modulus. The external pressure p_e is already accounted in H_{eff} .

However, as mentioned before, Equation (2) is valid only for a totally restrained pipe, which is not the condition of free spans, where sagging causes elongation and increases S_{eff} . At the same time, according to the Beam Theory, S_{eff} , together with the self-weight q (N/m) and section properties, defines the sagging. Therefore, in order to calculate the free span S_{eff} a Finite Element Analysis (FEA) is normally performed.

The S_{eff} value is required to calculate the natural frequencies, deflection, true axial force N_{tr} , and bending moment M [8]. With N_{tr} and M , it is possible to determine the axial stresses in the pipe. The natural frequencies are also important for the dynamic VIV evaluation.

Hobbs [9,10] developed a solution based on Beam Theory for the upheaval buckling of submarine pipelines where, as in free spans, the final position and axial force are directly related and unknown. However, due to the complexity of the equations, the calculation of the final configuration based on a given condition of pressure and temperature require numeric computations.

More recently, Vedeld et al [11] developed a semi-analytical solution for natural frequencies with remarkable accuracy and, unlike the formulation in [8], with no limitations of span length and deflection. Nevertheless, it still depends on the value of S_{eff} for the static condition, so he then used an iterative approach to calculate S_{eff} . Sollund et al [12] applied the same solution for very short span lengths, with the span length - outer diameter ratio smaller than 30, where sagging can be neglected and S_{eff} is calculated directly by Equation (2).

The effective tension S_{eff} is a primary result required for several assessments in free spans. However, there are no direct and accurate solutions known in literature for the S_{eff} in spans with a considerable length. This work presents an approximate solution with sufficient accuracy, which engineers can apply in submarine pipeline design, as a reliable and easy tool. Two different conditions will be addressed regarding boundaries: pinned-pinned and fixed-fixed. The soil stiffness at the shoulders will not be considered at this moment.

2. PROPOSED SOLUTION

2.1 Mathematical Problem

According to the Beam Theory, the behavior of a horizontal beam subjected to axial and transversal uniform loads is described by the differential equation

$$EI \frac{d^4 v}{dx^4} - S_{eff} \frac{d^2 v}{dx^2} + q = 0 \quad (3)$$

where I is the area moment of inertia, E is the Young modulus, $v(x)$ is the vertical displacement, x is the longitudinal position along the pipe, S_{eff} is the axial load (positive for tension) and q is the vertical load per length unit, assumed as the submerged self-weight. For the desired boundary conditions, Equation (3) has different analytical solutions, depending if S_{eff} is positive (tension), negative (compression) or null.

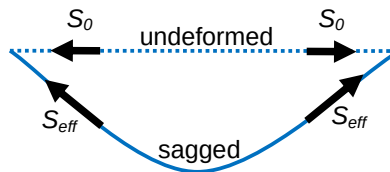


Fig. 1. Effective tension for undeformed and sagged (buckled) positions

In free span segments, the sagging causes an elongation of the pipe and thus S_{eff} increases. Relative to the straight position (undeformed), the post sagging effective tension, assumed constant along span length L , can be calculated by

$$S_{eff} = S_0 + E A_s \varepsilon_e \quad (4)$$

where S_0 is the undeformed effective tension (Fig. 1), calculated by Equation (2), and ε_s is the axial strain due to sagging, derived from

$$\varepsilon_s = 1/L \int_0^L \left[\sqrt{1 + (dv/dx)^2} - 1 \right] dx \quad (5)$$

which can be approximated, for small relative displacements, as:

$$\varepsilon_s \cong 1/2L \int_0^L (dv/dx)^2 dx \quad (6)$$

Theoretically, the system of Equations (3), (4) and (6) could be solved in order to calculate S_{eff} for a given condition of pressure and temperature. However, the solution of Equation (3) depends on the sign of S_{eff} (positive, negative or null) and is not simple, making it unfeasible for a general mathematical solution.

2.2 Solution with an Approximated Displacement Equation

In order to make feasible the derivation of a general solution for the aforementioned problem, an approximated equation for the vertical displacement is used. The expression chosen to take place as the approximate solution of Equation (3) is a fourth degree polynomial, given by Equation (7). The reason for this choice was the good agreement of second and third derivatives with the bending moment and shear force diagrams, respectively, of a beam with a uniform transversal load.

$$\bar{v}(x) = a_4 x^4 + a_3 x^3 + a_2 x^2 + a_1 x + a_0. \quad (7)$$

A relation between the coefficients a_i can be derived applying the proper boundary conditions (BC's) for each configuration, as shown in Table 1.

Table 1. Boundary conditions and derived coefficients

Description	Symbol	Value	
		pinned-pinned	fixed-fixed
Deflection	$v(x=0)$	0	0
	$v(x=L)$	0	0
Declination	$dv/dx(x=0)$	$\tan(\varphi_0)$	0
	$dv/dx(x=L)$	$\tan(-\varphi_0)$	0
Moment	$EI d^2v/dx^2(x=0)$	0	M_0
	$EI d^2v/dx^2(x=L)$	0	M_0
Shear force	$EI d^3v/dx^3(x=0)$	Q_0	Q_0
	$EI d^3v/dx^3(x=L)$	$-Q_0$	$-Q_0$
Polynomial coefficients	a_3	$-2La_4$	$-2La_4$
	a_2	0	L^2a_4
	a_1	L^3a_4	0
	a_0	0	0

Integrating Equation (3), with symmetric BC's (Table 1), will result in

$$EI \frac{d^3v}{dx^3} - S_{eff} \frac{dv}{dx} + q \left(x - \frac{L}{2} \right) = 0 \quad (8)$$

and then repeating the same procedure will become

$$EI \frac{d^2v}{dx^2} - S_{eff} v + \frac{qx^2}{2} (x - L) - M_0 = 0. \quad (9)$$

The left hand of Equations (8) and (9) are differential operators $L(f(x))$ equals to 0 for the exact solution, which means $L(v(x)) = 0$. For the proposed approximate solution, the result will be

$$L(\bar{v}(x)) = R(x) \quad (10)$$

where $R(x)$ is a residual, related to the approximation error. In an attempt to reduce the error to a minimum value, the averaged error along the span length will be forced to be zero, which means

$$\int_0^L R(x) dx = 0 \quad (11)$$

Then Equation (11) is applied related to the differential operator $L(f)$ on Equation (8), for the fixed-fixed condition, or on Equation (9), for the pinned-pinned condition. These different equations were chosen in order to get the best fitting, considering the limitations of the approximate formulation in Equation (7). This procedure will determinate its coefficients as a function of S_{eff} .

$$a_4 = \frac{q}{C_7 S_{eff} L^2 + 24EI} \quad (12)$$

where C_7 is derived depending on the condition (Table 2). The value calculated from the described procedure for the fixed-fixed condition was 0.5, however, tests showed that 0.6 gives a better fitting. The other coefficients (a_3 to a_0) can be directly calculated from a_4 (Table 1).

Table 2. Coefficient values

Coefficient	Value	
	pinned-pinned	fixed-fixed
C_2^*	1	4
C_5^*	1/8	1/12
C_6^*	5/384	1/384
C_7^{**}	2.4	0.6
C_8^{**}	17/70	1/105
* DNV-RP-F105 [8]		
** derived formulation		

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Once the deflection function $v(x)$ is known, by means of its coefficients a_i , it is possible to calculate the axial strain due to the sagging elongation by means of Equation (6):

$$\varepsilon_s = C_8 (a_4)^2 L^6. \quad (13)$$

Combining Equations (4), (12) and (13) will result in the following:

$$A_3 \left(\frac{S_{eff}}{P_{cr}} \right)^3 + A_2 \left(\frac{S_{eff}}{P_{cr}} \right)^2 + A_1 \left(\frac{S_{eff}}{P_{cr}} \right) + A_0 = 0 \quad (14)$$

where $P_{cr} = C_2 \pi^2 EI / L^2$ is the critical buckling load and

$$\begin{aligned} A_3 &= C_2^3 C_7^2 \\ A_2 &= C_2^2 [C_7 (48/\pi^2) - C_2 C_7^2 (S_0/P_{cr})] \\ A_1 &= C_2 [(24/\pi^2)^2 - C_2 C_7 (48/\pi^2)(S_0/P_{cr})] \\ A_0 &= -C_2 (24/\pi^2)^2 (S_0/P_{cr}) - C_2^3 C_8 E A_s (qL)^2 / P_{cr}^3. \end{aligned} \quad (15)$$

In Equation (14), S_{eff} is the only unknown and represents the root of a third degree polynomial. The solution is known in literature as Cardano's formula [13,14]. First, the following parameters need to be defined:

$$\begin{aligned} c_P &= A_1/A_3 - (A_2/A_3)^2/3 \\ c_Q &= A_0/A_3 - A_1 A_2 / (3A_3^2) + 2 [A_2 / (3A_3)]^3 \\ c_D &= (c_Q/2)^2 + (c_P/3)^3 \\ c_R &= \sqrt{(c_Q/2)^2 + |c_D|} \\ c_T &= \arccos(-c_Q/2c_R). \end{aligned} \quad (16)$$

Aiming at the desired root among the three available solutions and assuming $S_{eff}/P_{cr} > -1$, it can be summarized based on previous parameters as

$$\frac{S_{eff}}{P_{cr}} = \begin{cases} (-c_Q/2 + \sqrt{c_D})^{1/3} + (-c_Q/2 - \sqrt{c_D})^{1/3} - A_2/(3A_3) ; & \text{if } c_D \geq 0 \\ 2c_R^{1/3} \cos(c_T/3) - A_2/(3A_3) ; & \text{if } c_D < 0 \end{cases} \quad (17)$$

Then, according to the developed methodology, S_{eff} can be calculated directly from S_0 , which is based on pressure and temperature conditions. The solution basically resorts to solving Equation (17) with its parameters defined in Equations (15) and (16), requiring no further FEA or iterative approach.

In this approximation, S_{eff} was considered constant along the pipe, so the variation due to self-weight was not accounted for, wherefore the calculated value will be that corresponding to the mid-span section.

2.3 Complementary Formulae

Once S_{eff} is known, as mentioned in Section 1, the code DNV-RP-F105 [8] provides simplified formulae to calculate the maximum static moment M and the deflection δ :

$$\begin{aligned} M &= C_5 q L^2 / (1 + S_{eff}/P_{cr}) \\ \delta &= C_6 q L^4 / [EI(1 + S_{eff}/P_{cr})] \end{aligned} \quad (18)$$

where the coefficients C_5 and C_6 are listed in Table 2. In case these formulae do not have enough accuracy, the maximum deflection and momentum can be calculated from the analytical solutions of Equation (3) for a given S_{eff} . For example, for fixed-fixed condition, the analytical solution of maximum moment will be:

$$M = \begin{cases} \frac{q}{2k^2} \left[2 - \frac{kL}{\tanh(kL/2)} \right] & ; \text{ if } S_{eff} > 0 \\ qL^2/12 & ; \text{ if } S_{eff} = 0 \\ \frac{q}{2k^2} \left[\frac{kL \sin(kL)}{1 - \cos(kL)} - 2 \right] & ; \text{ if } S_{eff} < 0 \end{cases} \quad (19)$$

where $k^2 = |S_{eff}| / EI$.

These equations will be used, based on the S_{eff}/P_{cr} value calculated in Equation (17), in order to expand the utility of the proposed formulation. The values obtained will also be compared to numeric results. The natural frequencies will not be evaluated in this work.

The effect of soil flexibility on shoulders could be taken in account by the concept of effective length L_{eff} , presented by Hobbs [15], that is calculated based on soil stiffness, pipe bending stiffness EI and span length L . This approach would bring the proposed solution closer to the real phenomenon.

2.4 Design Sequence

For a design procedure, the effective tension at installation position S_i can be assumed equal to the lay tension H_0 . Then, Equation (14), reformulated to determine S_0 for a given S_{eff} , can be used to obtain the associated undeformed effective tension S_{i0} (Fig. 2a):

$$\left(\frac{S_0}{P_{cr}} \right) = \left(\frac{S_{eff}}{P_{cr}} \right) - \frac{C_8 E A_s (qL)^2}{P_{cr}^3} \left[C_7 \left(\frac{S_{eff}}{P_{cr}} \right) + \frac{24}{C_2 \pi^2} \right]^{-2} \quad (20)$$

The undeformed effective tension for hydrotest (S_{H0}) and operation (S_{O0}) conditions are calculated adding the temperature and internal pressure increment effect according to the Equation (2).

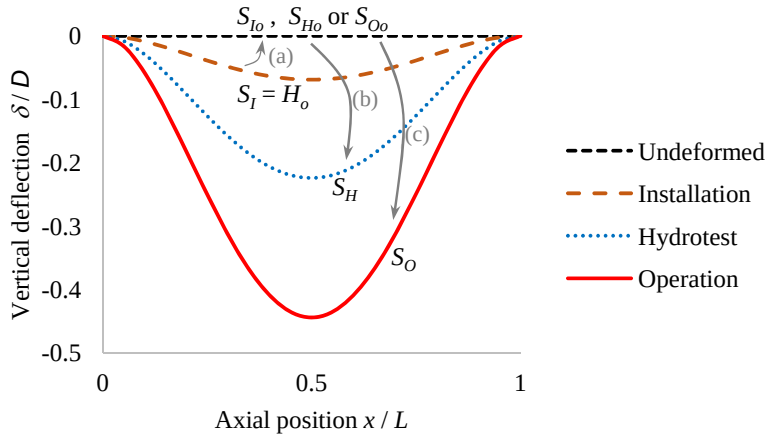


Fig. 2. Example of design sequence for the fixed-fixed condition

Finally, Equation (17) is used to calculate the **corresponding** effective tensions S_H and S_O (Fig. 2b and c).

3. FINITE ELEMENT ANALYSIS

In order to evaluate the accuracy of the proposed solution, a finite element model was implemented in the software ANSYS Mechanical v17, **whose structural matrices are derived based on the principle of virtual work**. The chosen element type was the PIPE289, which is a quadratic three-node beam element based on Timoshenko's beam theory, **that includes first order shear deformation effects and handles large strain and/or rotation nonlinear applications**. Because of the limitation of the first order shear deformation theory, the reliability of this element is limited to pipes with slenderness ratios greater than 30 [16]. In this work, the minimum value of the slenderness ratio calculated was 1350, **remaining inside the applicability range of the referred formulation**. The free span was discretized into elements whose length was set to half de pipe diameter.

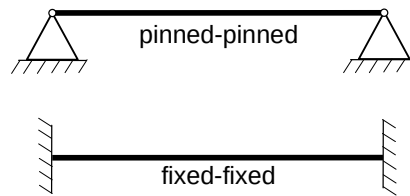


Fig. 3. Boundary condition configurations

Regarding the boundary conditions, for the pinned-pinned configuration, the line end translations were restrained in all three directions and, in order to avoid numerical instability, also in the beam axial rotation. For the fixed-fixed configuration, the line ends were restrained to translate and rotate in all directions.

A distributed vertical load was applied downwards along the pipe, corresponding to the submerged weight. In addition, an internal pressure was defined. The initial effective tension S_0 for each case, calculated by Equation (2), was applied by an imposed axial displacement at one of the ends.

Because of the slenderness of the problem, the simulation was performed considering geometrical nonlinearity, which is calculated based on the Newton-Raphson method. All applied loads, except for the self-weight, were divided into 10 steps, in order to guarantee better convergence and accuracy.

The effective tension result is provided as an output solution from the selected element type. The value considered for the comparison with the proposed solution is that related to the mid-span section. The maximum moment is obtained by the integration of axial stresses at the mid span section for the pinned-pinned configuration and by the moment reaction at the pipe ends for the fixed-fixed configuration. The maximum deflection corresponds to the vertical displacement at the mid-span.

4. RESULTS AND DISCUSSION

4.1 Input Data

The verification cases were generated varying the aimed S_{eff}/P_{cr} values between -0.5 and 1.0. Values lower than -0.5 became too close to the critical buckling load and, therefore, unlikely to occur for normal pipeline design conditions. On the other hand, values greater than 1.0 are too large for spans with acceptable lengths. The length values were considered from $L/D = 40$, increasing this ratio progressively in steps of 20 until the mid-span deflection reached $12D$. This limit was chosen in order to remain within realistic design conditions. A total of 140 cases were analyzed.

The pipe properties used are listed in Table 3.

Table 3. Input data for the verification cases

Description	Symbol	Value	Unit
Outer diameter	D	457.20	mm
Thickness	t	28.575	mm
Young modulus	E	207	GPa
Poisson coefficient	ν	0.3	
Steel density	ρ_s	7,850	kg/m ³
Submerged weight	q	2.576	kN/m
Internal pressure increment	Δp_i	28.75	MPa

The initial effective tension S_0 for each desired S_{eff}/P_{cr} condition is calculated by expliciting S_0/P_{cr} in Equation (14). For the finite element model, Equation (2) is also used to calculate the equivalent displacement that will give the necessary initial tension. Then, after sagging, equilibrium S_{eff} values calculated in FEA are compared with those provided by the proposed solution, described in Section 2.2.

4.2 Comparison between Results from the Proposed and the Numerical Solutions

This section has been prepared to show the accuracy of the equation proposed in this paper comparing its results with those calculated using numerical solutions. These results are obtained varying the relative span lengths L/D and the effective tensions S_{eff}/P_{cr} . Figure 4 presents the difference between S_{eff} values relative to P_{cr} . Considering all the analysed cases, the maximum relative deviation of estimated S_{eff} is -2,1 %, which was obtained for $L/D = 260$ and $S_{eff}/P_{cr} \cong 1.00$ in the fixed-fixed configuration. In the same configuration, for a smaller L/D equal to 140, the relative deviation is smaller than 0.5 % for all values of S_{eff}/P_{cr} . For the pinned-pinned configuration, higher values of S_{eff}/P_{cr} resulted in even smaller deviations, however, slightly larger values occur for the fixed-fixed boundary conditions for large L/D values.

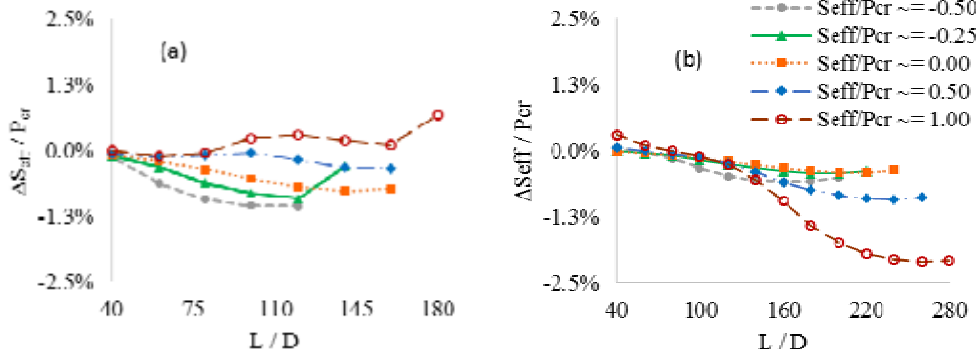


Fig. 4. Difference between effective tension S_{eff} calculated by the proposed formulation and by the numerical analyses relative to critical buckling load P_{cr} for pinned-pinned (a) and fixed-fixed (b) configurations

The true axial force was obtained by Equation (1) based on the calculated S_{eff} value. The maximum relative deviation calculated for the axial force was 0.9 % for $S_{eff}/P_{cr} = -0.50$ (Fig. 5).

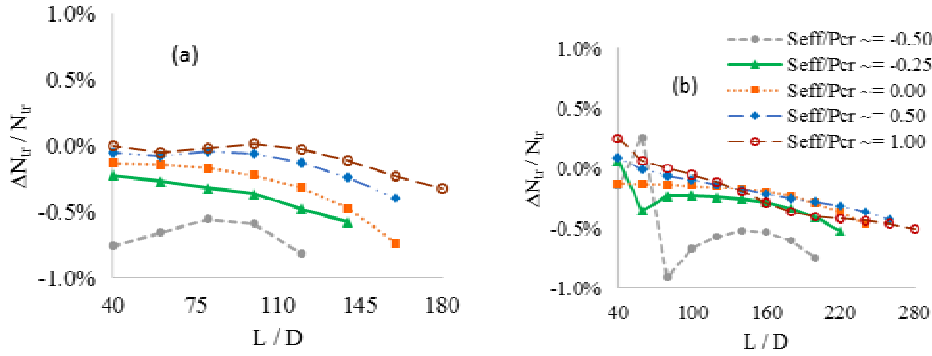


Fig. 5. Relative difference between the true axial force N_{tr} calculated by the proposed formulation and in the numerical analysis for the pinned-pinned (a) and fixed-fixed (b) configurations

As mentioned in Section 2.3, once S_{eff} is known, the maximum deflection (Fig. 6) and moment (Fig. 7) can easily be calculated by the simplified formula given in Equation (18), from DNV-RP-F105 [8]. Regarding the deflections, the larger relative deviation was -3.1 %, calculated for $L/D = 40$ and $S_{eff}/P_{cr} = 1$ (Fig. 6). In this case, the simplified formula calculated a 2.042 mm displacement against 2.109 mm from the FEA, which means a very small discrepancy in absolute value and, therefore, can be disregarded. For all other analyzed cases, this deviation was smaller than -2.3 % (in modulus). For almost all cases in the pinned-pinned configuration, no significant difference was observed for deflection deviation, while in the fixed-fixed cases the discrepancy tends to decrease along with L/D and S_{eff}/P_{cr} combinations.

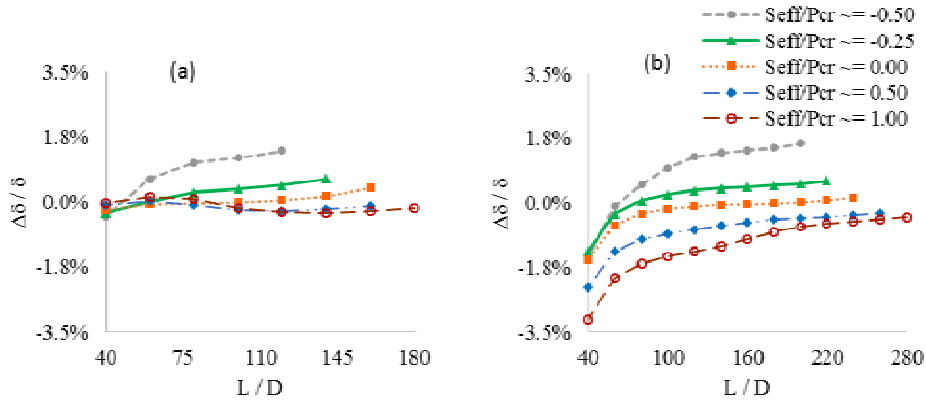


Fig. 6. Relative difference between the deflection δ calculated by the DNV-RP-F105 and the numerical analysis for the pinned-pinned (a) and fixed-fixed (b) configurations

The moment calculated from DNV-RP-F105 [8] for the pinned-pinned condition has presented a deviation from the numerical results smaller than 2,5 % (Fig. 7a). Up this point, the resulting deviations of the simplified formula in Equation (18) are in good agreement with the deviation of the S_{eff} values calculated based the proposed solution, which is smaller than 3 %. This was expected, because those simplified formulae depend on a previously known value of S_{eff} . However, for the fixed-fixed, the calculated divergence is much higher (Fig. 7b). This divergence is nearly constant along analyzed L/D values for each S_{eff}/P_{cr} combination, therefore this formula could be easily adjusted, but it is a third party formula that will not be altered here.

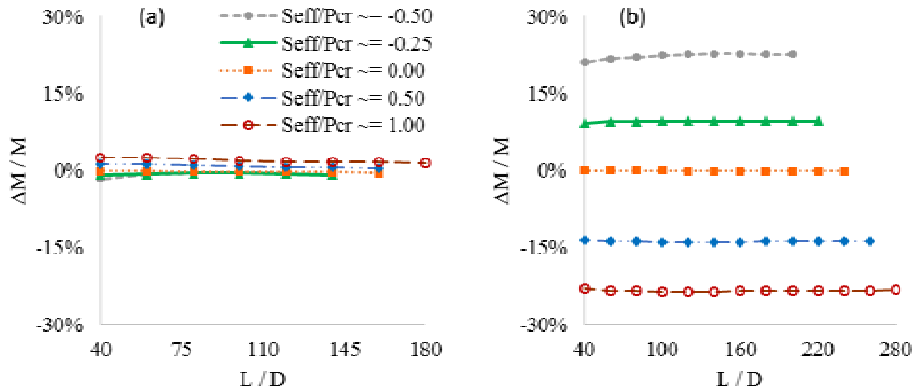


Fig. 7. Relative difference between moment M calculated by the DNV-RP-F105 and by the numerical analysis for pinned-pinned (a) and fixed-fixed (b) configurations

Another way to calculate the moment would be using Equation (19) which was derived here. The results from Equation (19) are plotted in Fig. 8 showing divergence less than 1 % (in modulus) and, therefore, much better than those obtained about with Equation (18).

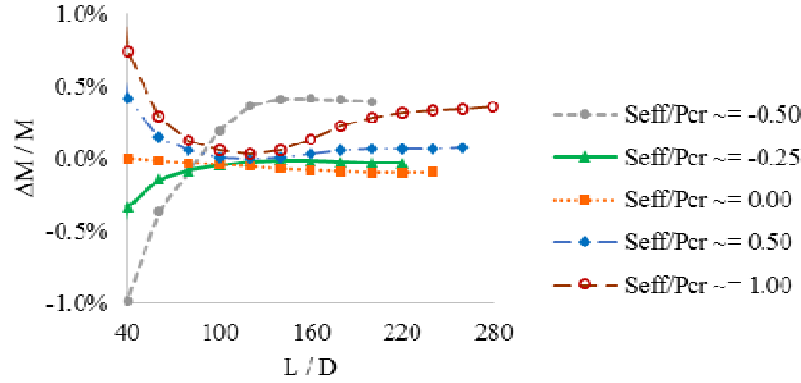


Fig. 8. Relative difference between moment M calculated by analytical solution and by the numerical analysis for fixed-fixed configurations

In Fig. 9 are shown the relative maximum deflection δ/D , which calculation was also based on S_{eff} value from the proposed solution. Since those results are very close to numerical calculation (Fig. 6) there is no need to present those ones too. Maximum values analyzed were $\delta/D = 11.39$ for pinned-pinned and $\delta/D = 11.56$ for fixed-fixed condition.

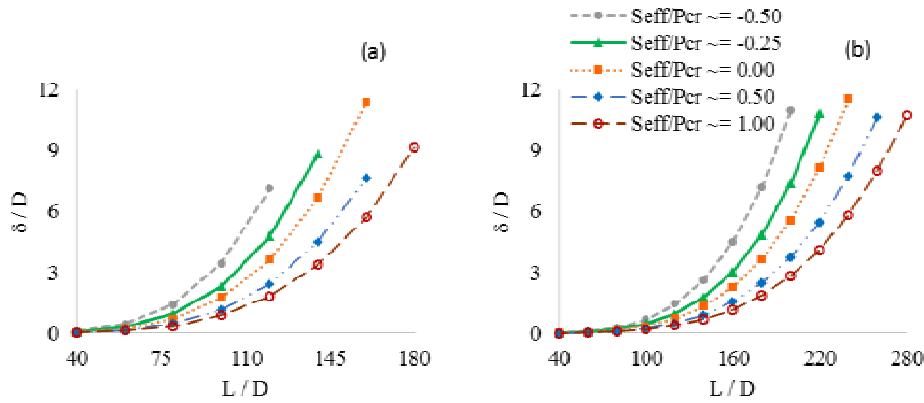


Fig. 9. Maximum deflection δ divided by diameter D calculated by the proposed solution for pinned-pinned (a) and fixed-fixed (b) configurations

4. CONCLUSIONS

Although free spans are common and frequently assessed in submarine pipeline, there is a lack of simple and accurate tools to calculate the effective tension and FEA is usually applied. This value is necessary to calculate many of the important information as true axial force, bending moment, deflection and natural frequency.

The methodology developed in this work is based on an approximated solution and make it possible to derive a straightforward equation to calculate the effective tension S_{eff} for any practical condition of pressure and temperature.

The results from the proposed solution compared to the numerical solution presented high accuracy. The calculated divergence between those solutions were smaller than 2.1 % for relative effective tension (S_{eff}/P_{cr}) and smaller than 0.9 % for the axial force (in modulus).

Based on the S_{eff} values calculated by the proposed solution, the DNV-RP-F105 [8] simplified equations were used to calculate the maximum deflection and moment. Those results also presented great agreement with the numerical solution, except for the moment in fixed-fixed condition. It has been shown, however, that Equation (19), derived here, is an accurate replacement for the Equation (18) from DNV-RP-F105 [8].

Many different cases were calculated, for pinned-pinned and fixed-fixed condition, varying L/D from 40 to 280 (limited for relative deflection $\delta/D < 12$) and S_{eff}/P_{cr} from -0.5 to 1.0. According to those results, the proposed solution appears to be a relatively simple and very accurate tool for free span assessments in any practical condition.

In a future research, the stiffness of the soil on span shoulders could be taken in account in order to develop the same methodology of this work on a more detailed analysis, however, in the meantime, the soil stiffness can be accounted for by the effective length (L_{eff}) [15].

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