

Joint Optimal Policy for Variable Deteriorating Inventory System of Vendor- Buyer Ordering With Trapezoidal Demand

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Abstract

A joint optimal policy for the vendor and the buyer is analyzed when units in inventory are subject to deterioration and demand is trapezoidal. It is shown numerically that the joint venture reduces the total joint cost significantly when compared with the independent decision of both the players. To entice the buyer to place order of larger size, a permissible credit period is offered by the vendor to the buyer. A negotiation factor is incorporated to share the benefits of cost savings.

Keywords: Vendor-Buyer Joint Decision, Deterioration, CreditPeriod, Trapezoidal Demand

1. Introduction

Silver and Meal(1969), Silver(1979), Xu and Wang (1991), Chung and Ting(1993,1994), Bose et al.(1995), Hariga(1995), Giri and Chaudhari(1997), Lin et al.(2000) etc. discussed optimal ordering policy when demand is linearly changing with

respect to time which is superficial in the market of fashion good, Air seats, Smart phones etc. Mehta and Shah(2003, 2004) assumed the demand to be exponentially time varying which is again unrealistic for newly launched product. Shah et al. (2008) introduced the quadratic demand which is again not observed in the market for indefinite period. In order to have an alternative demand pattern, the trapezoidal demand is considered. This type of demand increases for some time, then gets constant up to some time and afterwards decreases exponentially with time.

Most of the models available in the literature assumed that the buyer is dominant player to make the decision for procurement. This strategy may not be economical for the vendor. An integrated vendor-buyer policy should be analyzed which is beneficial to the players of the supply chain. Clark and Scarf(1960) proposed a mathematical model for vendor-buyer integration. Banerjee(1986) discussed an economic lot-size model when production is finite. Goyal(1988) extended Banerjee's model by relaxing the assumption of the lot-for-lot production. Shah et al.(2008) analyzed joint decision when demand is quadratic.

Deterioration is defined as the decay, spoilage, evaporation and loss of utility of a product from the original one. Fruit and vegetables, cosmetics and medicines, electronic items, blood components, radioactive chemicals, agriculture products are some of the examples of deteriorating commodities. For articles on deteriorating inventory one can refer to Raafat (1991), Shah and Shah (2000) and Goyal and Giri (2001). Yang and Wee (2005) derived a win – win strategy for an integrated system of vendor-buyer when units in inventory are subject to constant rate of deterioration and deterministic constant

demand. Shah et al. (2008) extended above model by incorporating salvage value to the deteriorated units.

In this study, a joint vendor – buyer inventory system is analyzed when demand is trapezoidal and units in inventory are subject to deterioration. A negotiation factor is incorporated to share the savings. The credit period is offered to the buyer to attract the buyer for placing larger order. A numerical example is illustrated to support the proposed model. Sensitivity analysis is carried out to visualize the changes in cost savings.

2. Notations and Assumptions

The proposed study uses following notations and assumptions.

2.1 Notations

A_b	Buyer's ordering cost per order
A_v	Vendor's ordering cost per order
C_b	Buyer's purchase cost per unit
C_v	Vendor's purchase cost per unit
I_b	Inventory carrying charge fraction per unit per annum for the buyer
I_v	Inventory carrying charge fraction per unit per annum for the vendor
θ_b	Deterioration of items in buyer's inventory system
θ_v	Deterioration of items in vendor's inventory system; $0 < \theta_b < \theta_v < 1$
$I_b(t)$	Buyer's inventory level at any instant of time t
$I_v(t)$	Vendor's inventory level at any instant of time t
n	Number of orders during cycle time for the buyer (a decision variable)

- 68 K_b Buyer's total cost per unit time
- 69 K_v Vendor's total cost per unit time
- 70 K_{NJ} Total cost for vendor-buyer inventory System when they take independent
- 71 decision
- 72 K_J Total cost for vendor-buyer inventory System when they take joint decision
- 73 T Vendor's cycle time (a decision variable)
- 74 T_b $(=T/n)$, Buyer's cycle time (a decision variable)
- 75 M Credit period offered by the vendor to the buyer (a decision variable)
- 76 r Continuous discounting rate

77 **2.2 Assumptions**

- 78 • A supply chain of single vendor and single buyer is considered.
- 79 • An inventory system deals with single item.
- 80 • The deterioration rates of items in vendor's and buyer's inventory are different
- 81 and proportional to on hand stock in inventory. There is no repair or replacement
- 82 of deteriorated units during a cycle time.
- 83 • The demand rate is trapezoidal. Its functional form is

$$84 \quad R(t) = \begin{cases} f(t) & ; 0 \leq t \leq u_1 \\ D_0 & ; u_1 \leq t \leq u_2 \\ g(t) & ; u_2 \leq t \leq T \end{cases}$$

85 where $f(t)$ is linear in t ,

86 $D_0 = f(u_1) = g(u_2)$, and $g(t)$ is exponentially decreasing in t . (say)

$$R(t) = \begin{cases} a(1+b_1t) & ; 0 \leq t \leq u_1 \\ a(1+b_1u_1) & ; u_1 \leq t \leq u_2 \\ a(1+b_1u_1)e^{-b_2(-u_2+t)} & ; u_2 \leq t \leq T \end{cases}$$

where a denotes scale demand, $0 < b_1, b_2 < 1$ denotes rates of change of demand. (See Fig. 1)

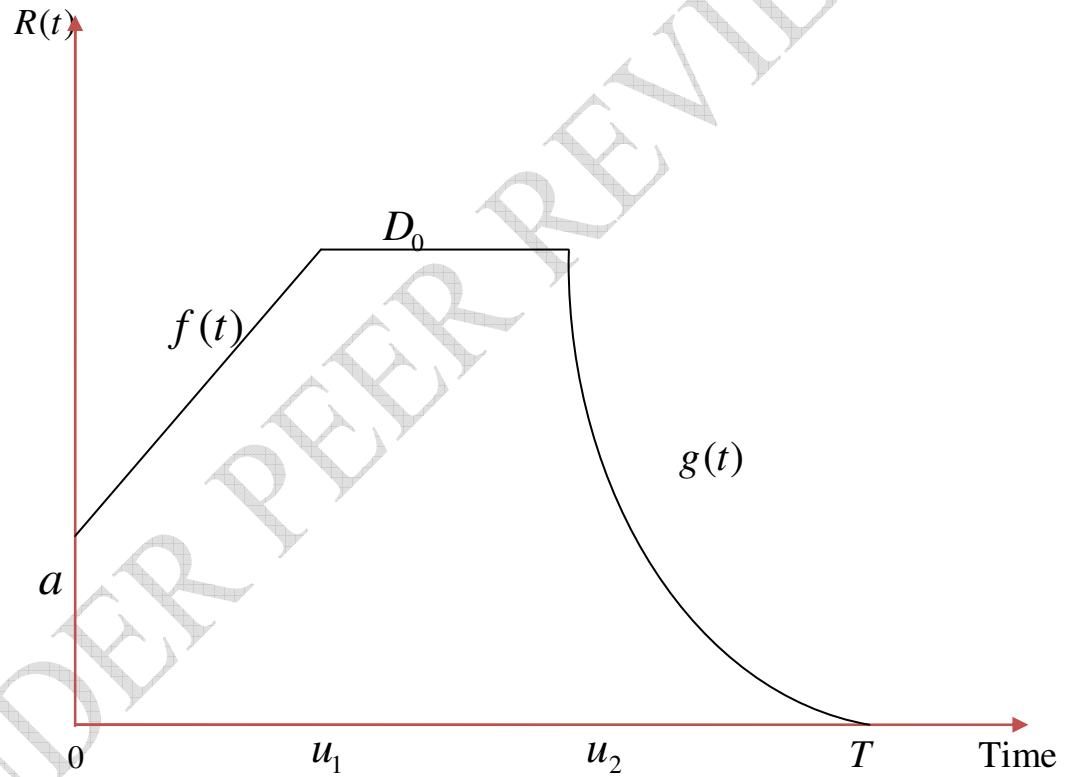


Fig.1 Trapezoidal demand

- The lead time is zero and shortages are not allowed.
- The credit period is offered for settling the accounts due against purchases to attract the buyer to opt a joint decision policy.

3. Mathematical Model

Fig. 2 depicts the time-varying inventory status for the vendor and the buyer.

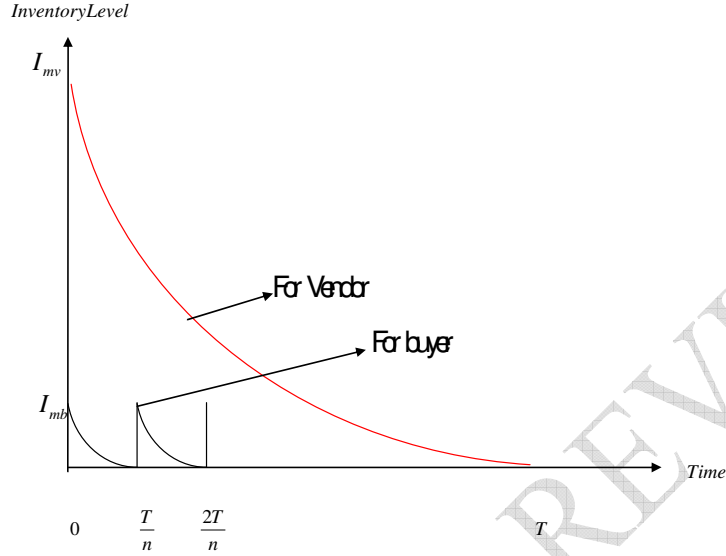


Fig. 2 Vendor –Buyer Inventory Status

The inventory changes due to trapezoidal demand for both vendor and buyer. The rate of change of inventory for both the players is governed by the differential equations:

$$\frac{dI_b(t)}{dt} + \theta_b I_b(t) = -R(t), 0 \leq t \leq T_b \quad (1)$$

$$\frac{dI_v(t)}{dt} + \theta_v I_v(t) = -R(t), 0 \leq t \leq T \quad (2)$$

with the boundary conditions $I_b(T_b) = 0$, $I_v(T) = 0$ and initial conditions $I_b(0) = I_{mb}$,

$I_v(0) = I_{mv}$.

109 The solutions of the differential equations are

110

111

$$I_b(t) = \begin{cases} \left[\frac{a(1+b_1u_1)e^{b_2u_2}}{\theta_b - b_2} \right] \left[e^{-\frac{b_2T}{n} + \frac{\theta_b T}{n} - \theta_b t} - e^{-b_2t} \right] & ; 0 \leq t \leq u_1 \\ \frac{a(1+b_1u_1)}{\theta_b} [-1 + e^{\theta_b u_2 - \theta_b t}] + \\ \left[\frac{a(1+b_1u_1)e^{b_2u_2}}{\theta_b - b_2} \right] \left[e^{-\frac{b_2T}{n} + \frac{\theta_b T}{n} - \theta_b t} - e^{-b_2u_2 + \theta_b u_2 - \theta_b t} \right] & ; u_1 \leq t \leq u_2 \\ a \left[-\frac{1+b_1t}{\theta_b} + \frac{b_1}{\theta_b^2} + \frac{1+b_1u_1}{\theta_b} e^{\theta_b u_1 - \theta_b t} - \frac{b_1}{\theta_b^2} e^{\theta_b u_1 - \theta_b t} \right] + \\ \frac{a(1+b_1u_1)}{\theta_b} [e^{\theta_b u_2 - \theta_b t} - e^{\theta_b u_1 - \theta_b t}] + \\ \left[\frac{a(1+b_1u_1)e^{b_2u_2}}{\theta_b - b_2} \right] \left[e^{-\frac{b_2T}{n} + \frac{\theta_b T}{n} - \theta_b t} - e^{-b_2u_2 + \theta_b u_2 - \theta_b t} \right] & ; u_2 \leq t \leq T_b \end{cases}$$

112

113 (3)

$$I_v(t) = \begin{cases} \left[\frac{a(1+b_1u_1)e^{b_2u_2}}{\theta_v - b_2} \right] [e^{-b_2T + \theta_v T - \theta_v t} - e^{-b_2t}] & ; 0 \leq t \leq u_1 \\ \frac{a(1+b_1u_1)}{\theta_v} [-1 + e^{\theta_v u_2 - \theta_v t}] + \\ \left[\frac{a(1+b_1u_1)e^{b_2u_2}}{\theta_v - b_2} \right] [e^{-b_2T + \theta_v T - \theta_v t} - e^{-b_2u_2 + \theta_v u_2 - \theta_v t}] & ; u_1 \leq t \leq u_2 \\ a \left[-\frac{1+b_1t}{\theta_v} + \frac{b_1}{\theta_v^2} + \frac{1+b_1u_1}{\theta_v} e^{\theta_v u_1 - \theta_v t} - \frac{b_1}{\theta_v^2} e^{\theta_v u_1 - \theta_v t} \right] + \\ \frac{a(1+b_1u_1)}{\theta_v} [e^{\theta_v u_2 - \theta_v t} - e^{\theta_v u_1 - \theta_v t}] + \\ \left[\frac{a(1+b_1u_1)e^{b_2u_2}}{\theta_v - b_2} \right] [e^{-b_2T + \theta_v T - \theta_v t} - e^{-b_2u_2 + \theta_v u_2 - \theta_v t}] & ; u_2 \leq t \leq T \end{cases}$$

114

115

(4)

116 Using $I_b(0) = I_{mb}$, $I_v(0) = I_{mv}$ the maximum procurement quantities for the buyer and the

117 vendor are

$$118 \quad I_{mb} = \left[\frac{a(1+b_1u_1)e^{b_2u_2}}{\theta_b - b_2} \right] \left[e^{\frac{b_2T}{n} + \frac{\theta_b T}{n}} - 1 \right]$$

$$119 \quad I_{mv} = \left[\frac{a(1+b_1u_1)e^{b_2u_2}}{\theta_v - b_2} \right] \left[e^{-b_2T + \theta_v T} - 1 \right]$$

120 respectively.

121 During the cycle time $[0, T]$, the buyer's

- 122 • Purchase Cost ; $PC_b = nC_b I_{mb}$
- 123 • Holding Cost ; $HC_b = nC_b I_b \int_0^{T_b} I_b(t) dt$
- 124 • Ordering Cost ; $OC_b = nA_b$

125 Hence, the buyer's total cost; K_b per unit time is

$$126 \quad K_b = \frac{1}{T} [PC_b + HC_b + OC_b] \quad (5)$$

127 The vendor's inventory is the difference between the vendor-buyer combined
 128 inventory and the buyer's inventory during n-orders. This is known as the joint two-
 129 echelon inventory model. The vendor's

- 130 • Purchase Cost ; $PC_v = nC_v I_{mv}$

131 • Holding Cost ; $HC_v = C_v I_v \left[\int_0^T I_v(t) dt - n \int_0^{T_b} I_b(t) dt \right]$

132 • Ordering Cost ; $OC_v = A_v$

133 Hence, the vendor's total cost; K_v per unit time is

134
$$K_v = \frac{1}{T} [PC_v + HC_v + OC_v] \quad (6)$$

135 The joint total cost K is the sum of K_b and K_v where $T_b = \frac{T}{n}$

136 Thus K is the function of discrete variable n and continuous variable.

137 4. Computational Procedure

138 There are two cases to analyze.

139 **Case 1:** When the vendor and the buyer take decision independently.

140 For given value of n , differentiate K_b with respect to T_b (equivalently, T) and solve

141 $\frac{dK_b}{dT_b} = 0$. This n and T_b minimizes K_v provided

142
$$K_v(n-1) \geq K_v(n) \leq K_v(n+1) \quad (7)$$

143 satisfies.

144 Here, the total cost per unit time with independent decision; K_{NJ} is given by

145
$$K_{NJ} = \min_n [\min_n K_b + K_v] \quad (8)$$

146 **Case 2:** When vendor and buyer make decision jointly.

147 The optimum value of T and n must satisfy the following conditions

148 simultaneously:

149
$$\frac{\partial K}{\partial T} = 0 \text{ and } K(n-1) \geq K(n) \leq K(n+1) \quad (9)$$

150 Thus, the total joint cost is

151
$$K_J = \min_{n,T} [K_b + K_v] \quad (10)$$

152 It is obvious that $K_J \leq K_{NJ}$. Hence, total cost savings Sav_J is defined as

153 $Sav_J = K_{NJ} - K_J$. Now define buyer's cost saving, Sav_b as $Sav_b = \alpha Sav_J$, where $0 \leq \alpha \leq 1$

154 is the negotiation factor. When negotiation factor equals to 0.5, saving gets equally

155 distributed between two players; when it is equal to zero, all saving is in the vendor's

156 pocket and when it is equal to 1, it is in favor of buyer.

157 The present value of unit after a time interval M is e^{-rM} , where r is discounting

158 rate. Solving the following equation

159
$$R(t)C_b(1 - e^{-rM}) = Sav_b \quad (11)$$

160 the buyer's credit period is given by

161
$$M = \frac{1}{r} \ln \left[\frac{C_b R(t)}{C_b R(t) - Sav_b} \right] \quad (12)$$

162 5. Numerical Example and Sensitivity Analysis

163 Consider following inventory parameters values in proper units:

164 $[a \ b_1 \ b_2 \ A_b \ A_v \ C_b \ C_v \ I_b \ I_v \ \theta_b \ \theta_v \ r] = [40000 \ 0.04 \ 0.02 \ 600 \ 3000 \ 10 \ 6$

165 $0.11 \ 0.10 \ .05 \ .08 \ 0.06]$

166 Let $\alpha = 0.5$.

167 The optimal solution is listed in Table 1 for independent and joint decisions.

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169

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Table 1: Optimal Solution for Independent and Joint Decisions

	Case 1	Case 2
	Independent Decision	Joint Decision
n	3	2
T_b	0.146304333	0.1959635
T	0.438913	0.391927
K_b	409194	409546
K_v	254141	253457
K	663335	663003
$PJCR$	-	0.050075188
$M(days)$	-	6.455621012

The buyer's cost and cycle time increases in joint decisions. The vendor gains \$684 and the buyer loses \$352. This hinders the buyer to agree for joint decision. To entice the buyer to joint decision, the vendor offers the buyer a credit period of days with equal sharing of cost savings. This reduces the joint total cost PJCR by 0.050075188 %, where PJCR is defined as $\frac{K_{NJ} - K_J}{K_J} \times 100$.

The convexity of total integrated cost and independent costs are shown in Fig. 3.

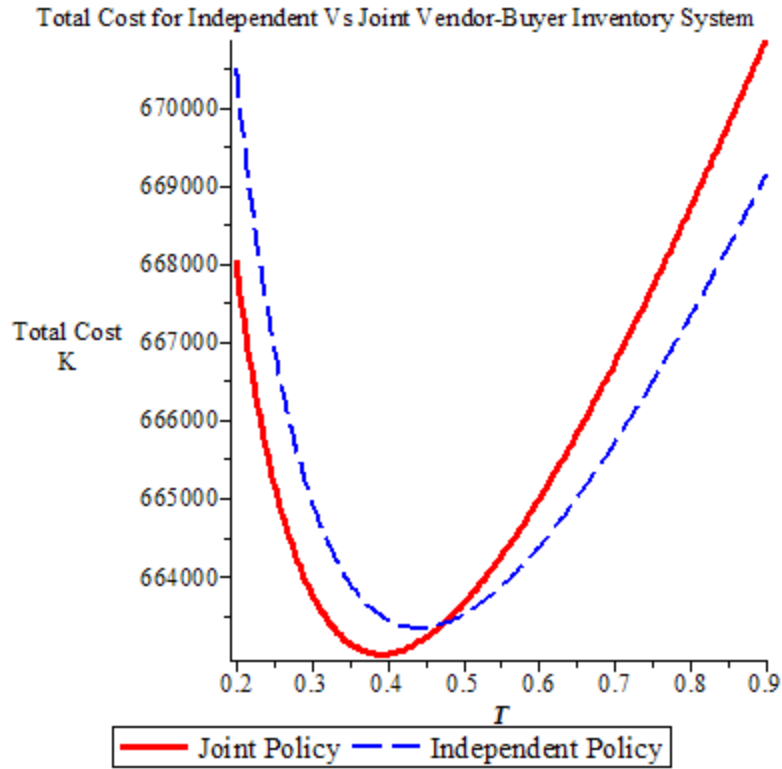


Fig. 3 Total Cost for Independent Vs Joint Vendor-Buyer Inventory System

Table 2: Sensitivity Analysis of Demand Rate

a	24000	32000	40000	48000	56000
K_{NJ}	401803	532724	663335	793726	923950
K_J	401544	532426	663003	793363	923558
$PJCR$	0.064501026	0.055970219	0.050075188	0.045754592	0.042444546
$M(days)$	6.510393077	6.481558237	6.455621012	6.440704274	6.437320861

Observations

- Increase in fixed demand a , decreases percentage of cost reduction and delay period.

Table 3: Sensitivity Analysis of Linear Rate of Change of Demand

b_1	0.024	0.032	0.04	0.048	0.056
K_{NJ}	663060	663198	663335	663473	663611
K_J	662727	662864	663003	663141	663280
$PJCR$	0.050246934	0.05038741	0.050075188	0.050064768	0.04990351
$M(days)$	6.48253545	6.498269843	6.455621012	6.451908917	6.428358053

Observations

- Increase in linear rate of change of demand b_1 , may increase or decrease percentage of cost reduction and delay period.

Table 4: Sensitivity Analysis of Exponential Rate of Change of Demand

b_2	0.012	0.016	0.02	0.024	0.028
K_{NJ}	663653	663497	663335	663168	662996
K_J	663347	663178	663003	662822	662634
$PJCR$	0.046129703	0.048101716	0.050075188	0.052201043	0.05463046
$M(days)$	6.142206701	6.303860301	6.455621012	6.615915098	6.80271129

Observations

- Increase in exponential rate of change of demand b_2 , increases percentage of cost reduction and delay period.

Table 5: Sensitivity Analysis of Buyer's Ordering Cost

A_b	360	480	600	720	840
K_{NJ}	661960	662566	663335	664165	665014
K_J	661741	662382	663003	663607	664195
$PJCR$	0.033094519	0.027778533	0.050075188	0.084085912	0.123307161
$M(days)$	4.522686628	3.683132288	6.455621012	10.55763341	15.10091537

Observations

- Increase in Buyer's Ordering cost A_b , may increase or decreases percentage of cost reduction and delay period.

Table 6: Sensitivity Analysis of Vendor's Ordering Cost

A_v	1800	2400	3000	3600	4200
K_{NJ}	660601	661968	663335	664702	666069
K_J	659684	661413	663003	664482	665873
$PJCR$	0.139005948	0.083911263	0.050075188	0.033108497	0.029435042
$M(days)$	21.10788028	11.65741969	6.455621012	4.001834111	3.362265634

Observations

- Increase in Vendor's Ordering Cost A_v , decreases percentage of cost reduction and delay period.

Table 7: Sensitivity Analysis of Buyer's Purchase Cost

C_b	6	8	10	12	14
K_{NJ}	501511	582345	663335	744382	825445
K_J	500284	581677	663003	744273	825495
$PJCR$	0.245260692	0.11484037	0.050075188	0.014645164	-0.00605697
$M(days)$	35.56472559	15.3972342	6.455621012	1.853298522	-0.76152659

216

217 **Observations**

218

- 219 • Increase in Buyer's purchase cost C_b , decreases percentage of cost reduction and
- 220 delay period significantly.

Table 8: Sensitivity Analysis of Vendor's Purchase Cost

C_v	3.6	4.8	6	7.2	8.4
K_{NJ}	564413	613874	663335	712797	762258
K_J	564569	613816	663003	712139	761231
$PJCR$	-0.0276317	0.009449086	0.050075188	0.092397692	0.134913055
$M(days)$	-2.72030458	1.07110368	6.455621012	13.40843984	21.85033584

221

222 **Observations**

223

- 224 • Increase in Vendor's purchase cost C_v , increases percentage of cost reduction and
- 225 delay period significantly.

226

Table 9: Sensitivity Analysis of Inventory Carrying Charge Fraction of Buyer

I_b	0.066	0.088	0.11	0.132	0.154
K_{NJ}	662148	662704	663335	664000	664679
K_J	661201	662122	663003	663850	664665
$PJCR$	0.143224224	0.087899209	0.050075188	0.022595466	0.002106324
$M(days)$	16.89189119	10.85880018	6.455621012	3.030101841	0.293038529

Observations

- Increase in buyer's inventory Carrying Charge Fraction I_b , decreases percentage of cost reduction and delay period significantly.

Table 10: Sensitivity Analysis of Inventory Carrying Charge Fraction of Vendor

I_v	0.06	0.08	0.1	0.12	0.14
K_{NJ}	661928	662632	663335	664039	664743
K_J	662038	662526	663003	663470	663927
$PJCR$	-0.01661536	0.015999372	0.050075188	0.085761225	0.122905078
$M(days)$	-2.04166732	2.01480826	6.455621012	11.30755438	16.56036679

Observations

- Increase in Vendor's Inventory Carrying Charge Fraction I_v , increases percentage of cost reduction and delay period significantly.

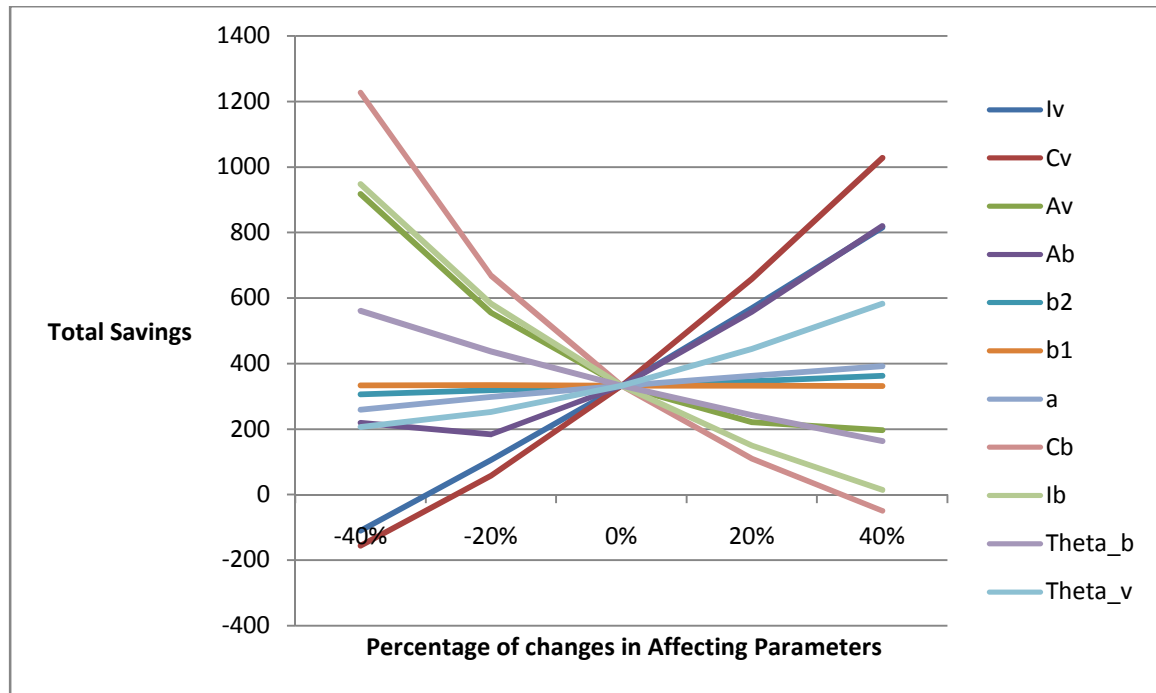


Fig. 4 Total Savings Vs. Percentage of Changes in Affecting Parameters

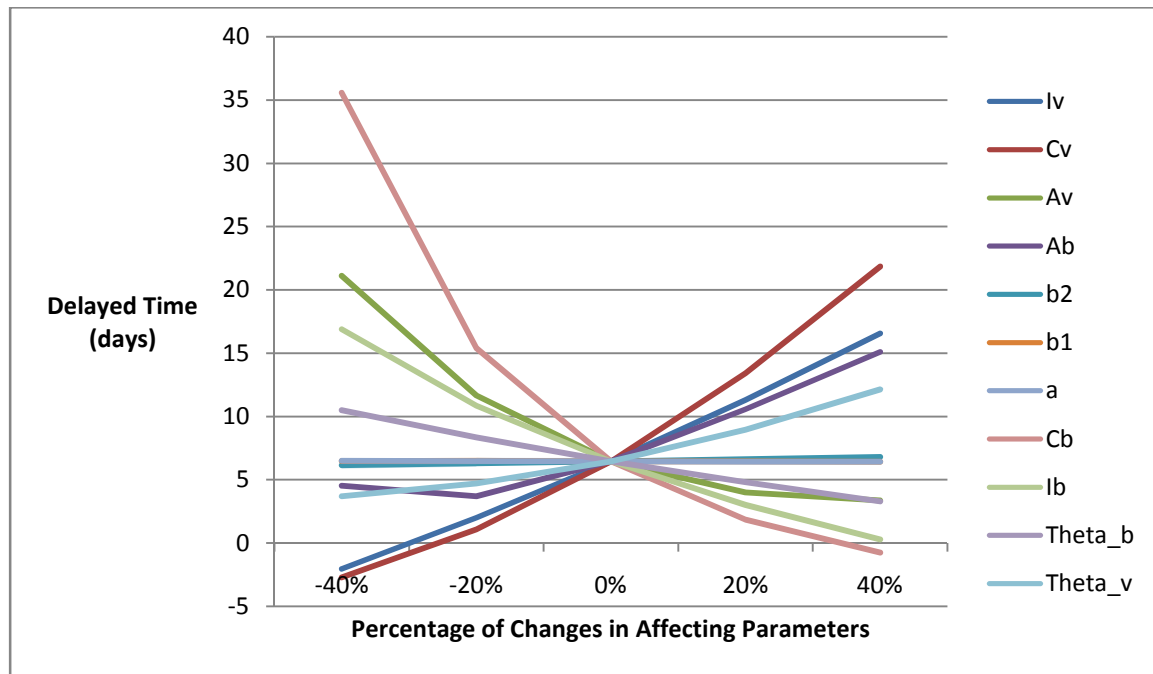


Fig. 5 Delayed Time in Days Vs. Percentage of Changes in Affecting Parameters

6. Conclusions

In this paper, a mathematical model is developed to analyze an optimal ordering policy for a supply chain comprising of vendor-buyer inventory system when demand is trapezoidal. The deterioration rate of units in vendor-buyer inventory system is considered to be different. It is established that the joint decision lowers the total cost of an inventory system, even though the buyer's cost increases significantly. To attract the buyer for a joint decision, a credit period is offered by the vendor to the buyer to settle the account.

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