

A comparison of Two Novel Methods for Handling a Certain Class of Problems of Digital Circuit Design

Abstract

With the advent of digital computers, several prominent problems of digital circuit design emerged. A particular class of these problems, (called Type-2 problems) can be divided into two subclasses depending on whether an honest translator is possible or a sneaky translator is warranted. The case of an honest translator is simply an inverse problem of logic in which knowledge of the vectorial function $Z(X)$ is utilized to produce its inverse vectorial function $X(Z)$. Though an old method of solving type-2 problems was known almost half a century ago, two modern methods are now possible, namely the method of Boolean-equation solving and the method of input-domain constraining. The emergence of these two methods are due to several recent developments including new techniques for (a) suppressing unwanted variables, (b) listing all particular solutions of a 'big' Boolean equation, and (c) utilizing don't-care representations. This paper exposes and illustrates these two novel methods, with a stress on comparing them together and demonstrating their superiority to (as well as agreement with) the old method. This purpose is achieved by way of three typical classical examples. The Boolean-equation-solving method seems to involve certain unwarranted steps that might be possibly skipped. However, its utility can be extended beyond type-2 problems.

Key words: 'Big' Boolean algebras, Digital circuit design, Honest and sneaky translators, Type-2 problems.

1. Introduction

A classical problem of digital circuit design can be described by the general layout of Fig. 1. Various variants or special cases of this layout appeared on early scholarly work on the subject, including the pioneering contributions of Ledley [1, 2], Bell [3], and Brown [4]. Figure 1 shows a combinational logic circuit C that is comprised of two subnetworks A and B , while the final output can be viewed as output $s(Y, Z)$ or $(s(X, Y, Z))$ of network B or output $t(X, Y)$ of network C . If the shaded path in Fig. 1 can be omitted, network B is called an 'honest translator' [4], while if it is needed (or used), this network is called a 'sneaky translator' [4]. Based on Fig. 1, three

40 ‘elementary problems of digital circuit design’ are proposed [1-5],
41 which are

42 **Type-1 problem:** Given $z(X)$ and $s(X, Y, Z)$, find $t(X, Y)$.

43 **Type-2 problem:** Given $z(X)$ and $t(X, Y)$ find $s(Y, Z)$ if an honest
44 translator is possible; otherwise find $s(X, Y, Z)$ for a sneaky translator.

45 **Type-3 problem:** Given $s(Y, Z)$ or $s(X, Y, Z)$ as well as $t(X, Y)$, find
46 $z(X)$.

47 The type-1 problem is solved trivially by direct substitution and does
48 not warrant further attention. The type-2 problem is handled by
49 Ledley [1, 2], Bell [3], Brown [4], and Rushdi [5], and will be revised
50 further herein. The type-3 problem is treated by Ledley [1, 2] and
51 much more recently (*via* modern techniques) by Rushdi and Ahmad
52 [6].

53

54 We now address the probably intriguing question: Why should we
55 bother revisit a problem that was really hot as back as half a century
56 ago. The answer is that new computational methods associated with
57 useful conceptual insight have recently been made possible through a
58 variety of breakthrough developments made in the past few years.
59 These include:

60

- 61 1. The development of a method to suppress certain variables Y
62 in a parent equation $h(X, Y, Z) = 1$ so as to replace it by a
63 simpler equation $g(X, Z) = 1$ that can be used to solve for Z in
64 terms of X producing the set of solutions $Z(X, Y)$ of the parent
65 equation $h(X, Y, Z)$ that are independent of Y [7].
- 66 2. The emergence of a novel method to list all the particular
67 solutions of a ‘big’ Boolean equation in a very compact space,
68 thereby allowing easy selection of a specific particular solution
69 that enjoys certain desirable features [6, 8, 9].
- 70 3. The convenience and power of rewriting interval based
71 conditions of the form $g_l \leq g \leq g_u$ as a don’t-care based one
72 $g = g_l \vee d(g_u)$ [11-14].

Utilizing the aforementioned developments, this paper introduces and compares two novel techniques for handling type-2 problems of digital circuit design. These are:

1. A technique using solutions of ‘big’ Boolean equations. If an honest translator is possible, we are able to handle the inverse problem of logic by using $\mathbf{Z}(\mathbf{X})$ to obtain $\mathbf{X}(\mathbf{Z})$ and hence produce $t(\mathbf{X}, \mathbf{Y}) = s(\mathbf{Z}(\mathbf{X}), \mathbf{Y})$. Otherwise, we produce a sneaky translator $t(\mathbf{X}, \mathbf{Z}, \mathbf{Y})$. The essence of this technique is quite similar to that used for type-3 problems in [6].
2. A technique that enhances the earlier one of input-domain constraining developed by Brown [4] by augmenting it by the don’t-care terminology by Reusch [9], Rushdi [5, 11-13], and Rushdi and Albarakati [14]. This enhancement allows easy minimization as well as the possibility of handling larger problems *via* the Variable-Entered Karnaugh Map (VEKM) rather than the Conventional Karnaugh Map (CKM) [5, 11-14].

Details and characteristics of the two novel techniques are clarified herein *via* three detailed examples. Example 1 and 2 were earlier handled by Bell [3], while Example 3 was treated by Ledley [2] and Brown [4] *via* the conventional old method. As we proceed, we will demonstrate advantages of our two competing novel methods compared to the conventional method. These advantages include conceptual clarity, high speed, aggregation of tasks, and better control on outcomes. Such advantages do not pertain only to pedagogical issues but might also be of significant benefit to practical digital design. This introductory section is followed by Sections 2-4 which discuss the aforementioned three design examples. Section 5 concludes the paper.

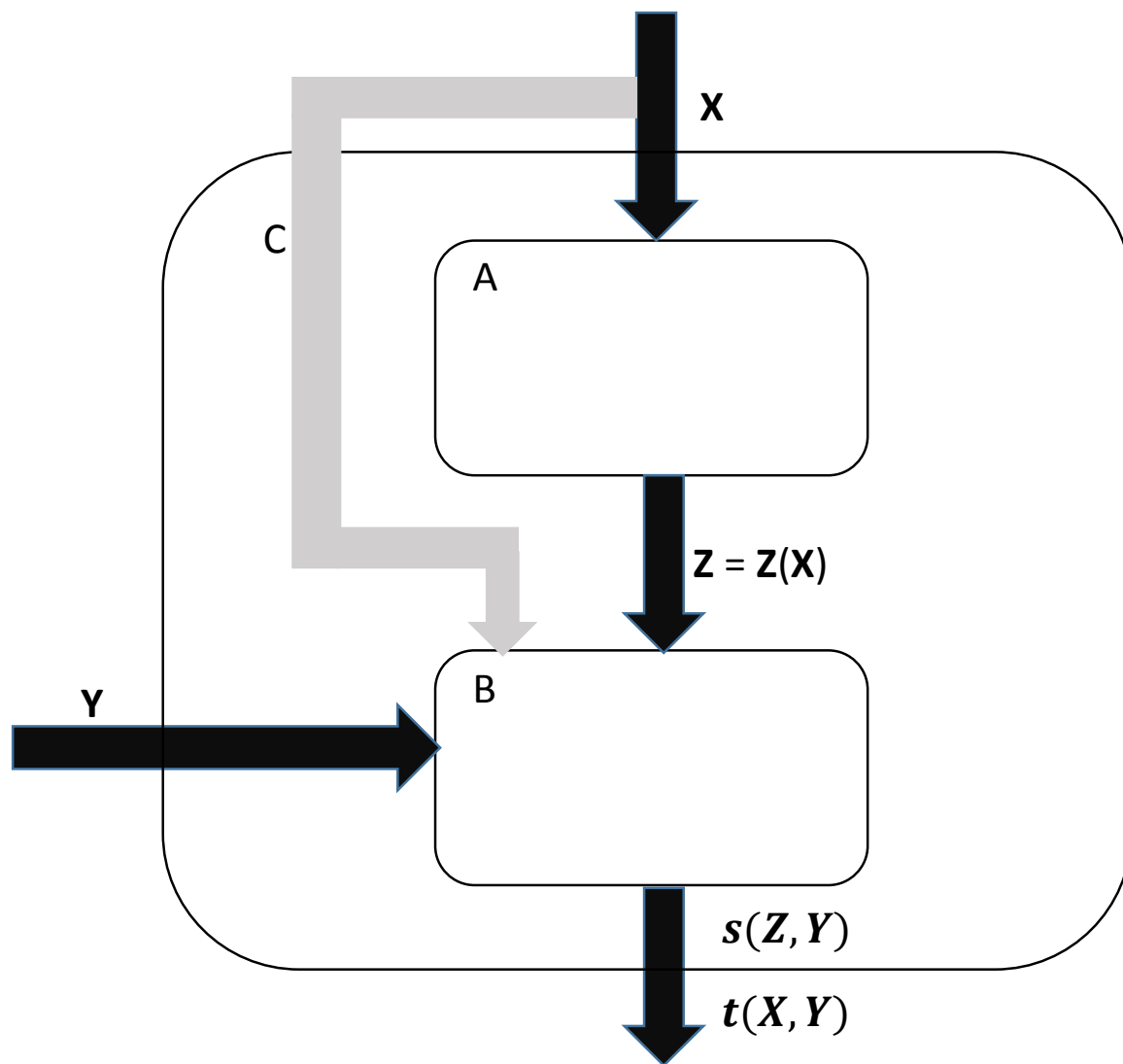


Fig. 1. Layout of a general problem of digital circuit design. If the sneaky (lightly shaded) path is needed or used, the function $s(Z, Y)$ is replaced by $s(X, Z, Y)$.

2. A Small Example Circuit

This example was earlier handled by Bell [3] by the long and old conventional method, and it admits the existence of an honest translator. There are two inputs for subnetwork B (outputs for subnetwork A)

$$Z_1 = X_1 \vee \bar{X}_2, \quad (1)$$

$$Z_2 = X_1 \bar{X}_2, \quad (2)$$

and a single desirable output for the overall network C

$$t(\mathbf{X}) = \bar{X}_1 \vee X_2, \quad (3)$$

and there are no side inputs $\mathbf{Y}$ for subnetwork B. In our first novel method we obtain $s(\mathbf{Z})$ by first solving an inverse problem of logic (*i.e.*, using $\mathbf{Z}(\mathbf{X})$ to derive $\mathbf{X}(\mathbf{Z})$) and then obtaining $s(\mathbf{Z})$ *via*

$$s(\mathbf{Z}) = t(\mathbf{X}(\mathbf{Z})). \quad (4)$$

The information given by $\mathbf{Z}(\mathbf{X})$ can be formulated as a single Boolean equation of the form

$$g(\mathbf{X}, \mathbf{Z}) = (Z_1 \odot (X_1 \vee \bar{X}_2)) (Z_2 \odot (X_1 \bar{X}_2)) = 1. \quad (5)$$

The two-valued Boolean function $g(\mathbf{X}, \mathbf{Z}): B_2^4 \rightarrow B_2$ is now viewed as a ‘big’ Boolean function $g(\mathbf{X}) = B_{16}^2 \rightarrow B_{16}$, where B_{16} is the free Boolean algebra $FB(Z_1, Z_2)$, so that the roles of Z_1 and Z_2 is switched from that of independent variables to that of generators of B_{16} [14-25]. The natural map for $g(\mathbf{X})$ is shown in Fig. 2. Its entries are some of the 4 atoms of $B_{16} = FB(Z_1, Z_2)$ which are expressed in terms of the generators Z_1 and Z_2 . Since these generators look like “variables” (in fact, they were originally variables), the map is called a Variable-Entered-Karnaugh map (VEKM). We use the technique developed in [15-18] to construct the auxiliary function $G(\mathbf{X}, \mathbf{p})$ to be used in deriving the parametric solution of $g(\mathbf{X}) = 1$. These solutions are expressed in terms of a single arbitrary parameter $p \in B_2$ as [6, 15, 16, 20]

$$X_1 = Z_1 Z_2 \vee Z_1 \bar{Z}_2 p \vee d(\bar{Z}_1 Z_2), \quad (6a)$$

$$X_2 = \bar{Z}_1 \bar{Z}_2 \vee Z_1 \bar{Z}_2 p \vee d(\bar{Z}_1 Z_2), \quad (6b)$$

$$\bar{X}_1 = \bar{Z}_1\bar{Z}_2 \vee Z_1\bar{Z}_2\bar{p} \vee d(\bar{Z}_1Z_2), \quad (6c)$$

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156 Substitution of (6b) and (6c) in (3) and (4) results in the following
157 expression

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$$s(\mathbf{Z}) = \bar{Z}_1\bar{Z}_2 \vee Z_1\bar{Z}_2\bar{p} \vee \bar{Z}_1\bar{Z}_2 \vee Z_1\bar{Z}_2p \vee d(\bar{Z}_1Z_2) \\ = \bar{Z}_2 \vee d(\bar{Z}_1), \quad (7)$$

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162 in which the parameter p disappears. However, depending on whether
163 we nullify or assert the don't-care term in (7), the following two
164 solutions are obtained for $s(\mathbf{Z})$

165

$$s(\mathbf{Z}) = \bar{Z}_2, \quad (8a)$$

167

168 or

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$$s(\mathbf{Z}) = \bar{Z}_2 \vee \bar{Z}_1. \quad (8b)$$

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173 in agreement with the solution obtained in Bell [3].

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		X_1
	$Z_1\bar{Z}_2$	Z_1Z_2
	$\bar{Z}_1\bar{Z}_2$	$Z_1\bar{Z}_2$
		X_2

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$$g(\mathbf{X})$$

Fig. 2. The natural map for $g(\mathbf{X})$ in Equation (5). Each map entry is a particular minterm or atom over (Z_1, Z_2) . The polarity of Z_1 is positive over $(X_1 \vee \bar{X}_2)$, while that of Z_2 is positive over $X_1\bar{X}_2$.

	X_1	
	$\begin{array}{c} Z_1 \bar{Z}_2(\bar{p}) \\ \vee d(\bar{Z}_1 Z_2) \end{array}$	$\begin{array}{c} Z_1 Z_2(1) \\ \vee d(\bar{Z}_1 Z_2) \end{array}$
	$\begin{array}{c} \bar{Z}_1 \bar{Z}_2(1) \\ \vee d(\bar{Z}_1 Z_2) \end{array}$	$\begin{array}{c} Z_1 \bar{Z}_2(p) \\ \vee d(\bar{Z}_1 Z_2) \end{array}$
	X_2	

$$G(\mathbf{X}, \mathbf{p})$$

Fig. 3. The auxiliary function used to derive the parametric solutions of $g(\mathbf{X}) = 1$ in Section 2.

Now, we employ our second novel method adapted from the input-domain constraining method of Brown [4]. Our basic information $\mathbf{Z}(\mathbf{X})$ can be cast in the equational form $h(\mathbf{X}, \mathbf{Z}) = 0$ where $h(\mathbf{X}, \mathbf{Z}) = \bar{g}(\mathbf{X}, \mathbf{Z})$ is represented by the natural map for $h(\mathbf{X})$: $B_{16}^2 \rightarrow B_{16}$ in Fig. 4. According to Brown [4], our current problem can be described by

$$\{h = 0\} \Rightarrow \{s = t\}, \quad (9)$$

which has the interval solution

$$\bar{h}t \leq s \leq h \vee t. \quad (10)$$

We now rewrite the double-inequality (10) in the don't-care notation [11-14]

$$\begin{aligned} s &= \bar{h}t \vee d(h \vee t) \\ &= \bar{h}t \vee d(h) \\ &= gt \vee d(\bar{g}). \end{aligned} \tag{11}$$

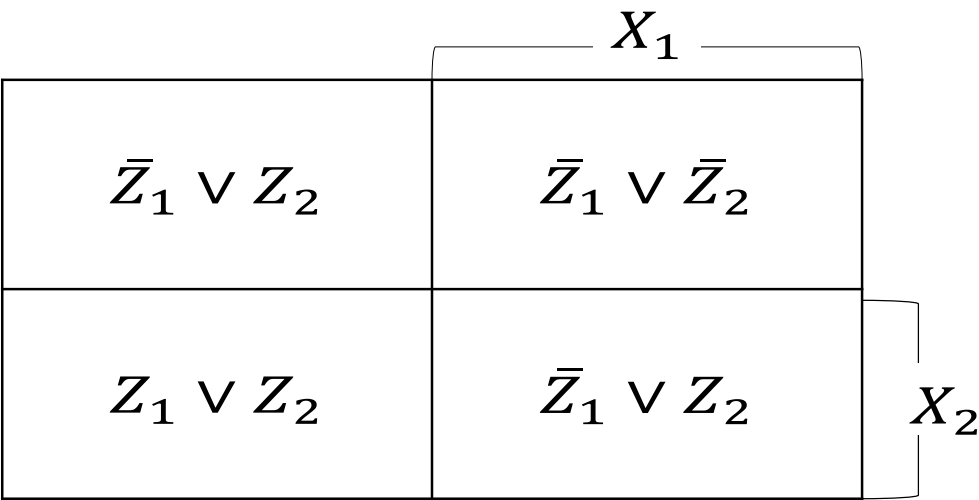
and use the maps in Figs. 2, 4, and 5 for $\bar{h} = g, h$, and t to construct the natural map for s in Fig. 6 according to (11). This map is simplified by noting that (11) indicates that $s = g \vee d(\bar{g}) = g \vee d(1)$ when $t = 1$, and that $s = d(\bar{g})$ when $t = 0$. Figure 6 can immediately be used to produce an expression of s that is a function of both $\mathbf{X}$ and $\mathbf{Z}$, *i.e.*, a sneaky translator, of the form

$$\begin{aligned} s(\mathbf{X}, \mathbf{Z}) &= (\bar{X}_1\bar{X}_2 \vee X_1X_2)(Z_1\bar{Z}_2 \vee d(1)) \vee \bar{X}_1X_2(\bar{Z}_1\bar{Z}_2 \vee d(1)) \vee \\ &\quad X_1\bar{X}_2d(\bar{Z}_1 \vee \bar{Z}_2). \end{aligned} \tag{12}$$

However, we note that none of the four atoms of $FB(Z_1, Z_2)$ has a mixed assertion in the cells of the map of Fig. 6. In fact, each of atoms $Z_1\bar{Z}_2$ and $\bar{Z}_1\bar{Z}_2$ is only asserted positively in this map (in cells $\bar{X}_1\bar{X}_2$ and X_1X_2 for the former atom and cell $\bar{X}_1X_2$ for the latter one), while the atom Z_1Z_2 is only asserted negatively (in cell $X_1\bar{X}_2$). By contrast, the atom $Z_1\bar{Z}_2$ is neither asserted positively or negatively in the map. The lack of any mixed assertion is equivalent to the condition demanded by Brown [4] for an honest translator *i.e.*, one for which s is independent of $\mathbf{X}$ [5]. According to the VEKM “enlargement” rule [11, 26, 27], the VEKM in Fig. 6 might be read to yield any of the two solutions in (8). Alternatively, we note that, for an honest translator s is a disjunction of all positively asserted atoms and possibly any of the totally don't-care atoms [5], *i.e.*, s is given by

$$\begin{aligned} s &= Z_1\bar{Z}_2 \vee \bar{Z}_1\bar{Z}_2 \vee d(Z_1Z_2) \\ &= \bar{Z}_2 \vee d(Z_1), \end{aligned} \tag{13}$$

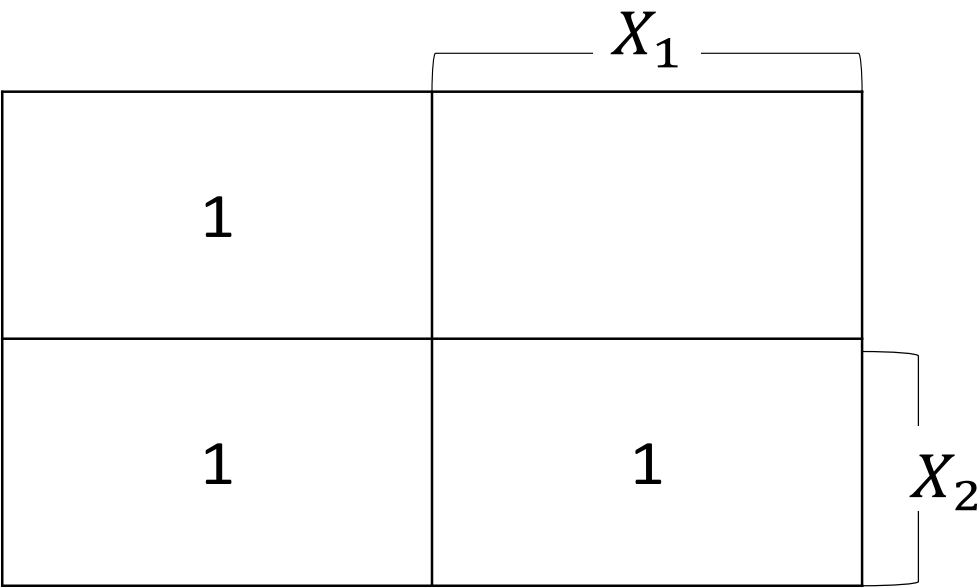
241 is agreement with (8).
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$$h(X, Z) = 0$$

247
248 Fig. 4. The natural map for $h(X, Z)$ needed in the second method
249 in Sec. 2.
250



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252
253

$$t(X) = \bar{X}_1 \vee X_2$$

Fig. 5. Conventional Karnaugh map for $t(\mathbf{X})$ expressed by (3) in Sec. 2.

	X_1	
	$Z_1\bar{Z}_2 \vee d(1)$	$d(\bar{Z}_1 \vee \bar{Z}_2)$
	$\bar{Z}_1\bar{Z}_2 \vee d(1)$	$Z_1\bar{Z}_2 \vee d(1)$
	X_2	

$$s(\mathbf{X}, \mathbf{Z}) = \bar{h}t \vee d(h)$$

Fig. 6. Simplified natural map for $s(\mathbf{X}, \mathbf{Z})$ in Sec. 2.

3. An Asymmetric Example Circuit

This example was used by Bell [3] to demonstrate the power of his method and show how it is applied when the number of elements of $\mathbf{Z}$ differs from that of $\mathbf{X}$. In the current example, these two numbers are 5 and 4, respectively, while they were 2 and 2 in the example of Section 2. Again, there is no side inputs $\mathbf{Y}$, and $\mathbf{Z}(\mathbf{X})$ is given by

$$Z_1(\mathbf{X}) = X_1, \quad (14a)$$

$$Z_2(\mathbf{X}) = X_1\bar{X}_4 \vee X_3X_4, \quad (14b)$$

$$Z_3(\mathbf{X}) = \bar{X}_1(\bar{X}_2X_4 \vee X_2X_3) \vee X_1(X_2\bar{X}_4 \vee \bar{X}_2\bar{X}_3), \quad (14c)$$

$$Z_4(\mathbf{X}) = X_1X_3 \vee X_4(X_1 \vee \bar{X}_2), \quad (14d)$$

$$Z_5(\mathbf{X}) = \bar{X}_4(X_1X_3 \vee X_2\bar{X}_3) \vee \bar{X}_1X_2X_3X_4, \quad (14e)$$

and the required single output for the overall network C is

$$t(\mathbf{X}) = \bar{X}_1 \bar{X}_2 \vee \bar{X}_3 (\bar{X}_1 \vee \bar{X}_2 \bar{X}_4 \vee X_2 X_4). \quad (15)$$

The information supplied by $\mathbf{Z}(\mathbf{X})$ in (14) is equivalent to a single Boolean equation of the form

$$g(\mathbf{X}, \mathbf{Z}) = \bigwedge_{i=1}^5 (Z_i \odot Z_i(\mathbf{X})) = 1. \quad (16)$$

The function $g(\mathbf{X}, \mathbf{Z}): B_2^9 \rightarrow B_2$ is viewed as $g(\mathbf{X}): B^4 \rightarrow B$, where $B = FB(Z_1, Z_2, Z_3, Z_4, Z_5)$ has 5 generators, $2^5 = 32$ atoms, and $2^{32} = 4,294,967,296$ elements. The natural map for $g(\mathbf{X})$ is shown in Fig. 7. An entry in the cell $\mathbf{X}$ of this map is an atom of B of the form $Z_1^{Z_1(\mathbf{X})} Z_2^{Z_2(\mathbf{X})} Z_3^{Z_3(\mathbf{X})} Z_4^{Z_4(\mathbf{X})} Z_5^{Z_5(\mathbf{X})}$. Since $Z_1(\mathbf{X}) = X_1$, the literal $Z_1^{Z_1(\mathbf{X})} = Z_1^{X_1}$ is Z_1 when $X_1 = 1$ and $\bar{Z}_1$ when $X_1 = 0$. For convenience, the map demonstrates the construction of $Z_3^{Z_3(\mathbf{X})}$ (as an example) by showing the loops of $Z_3(\mathbf{X})$ in (14c), so that $Z_3(\mathbf{X})$ appears as Z_3 inside these loops and as $\bar{Z}_3$ outside them. Out of the 32 atoms of B , the atom $Z_1 Z_2 \bar{Z}_3 Z_4 \bar{Z}_5$ has two appearances in Fig. 7, which are each appended by one of the orthonormal tags p_1 and $\bar{p}_1$ in Fig. 8, for the auxiliary function, while the atom $\bar{Z}_1 \bar{Z}_2 \bar{Z}_3 \bar{Z}_4 \bar{Z}_5$ has three appearances in Fig. 7, which are each ANDed with one of the orthonormal tags, p_2 , $\bar{p}_2 p_3$ and $\bar{p}_2 \bar{p}_3$ in Fig. 8, where each of p_1 , p_2 and $p_3 \in B_2$ [6, 16]. There are 11 other atoms with a single appearance in Fig. 7, which are each tagged by (1) in Fig. 8, while the remaining 19 atoms do not appear at all in Fig. 7, and must be nullified as a consistency condition and further entered don't-care in the cells of Fig. 8. There are $2^3 = 8$ particular solutions for $\mathbf{X}$. Figure 9 shows solutions for the $\mathbf{X}$ variables, their complements, and their products, culminating in a map for $s = t$ where t is given by (15). Note that the three parameters p_1 , p_2 and p_3 disappeared from the final map for s , which makes s parametrically unique (albeit incompletely specified). The map for s in Fig. 9(l) is particularly covered by the two essential prime-implicant dotted loops $\bar{Z}_1 Z_4$ and $\bar{Z}_2 \bar{Z}_3$ (shown solid). To complete the coverage of s , either of the

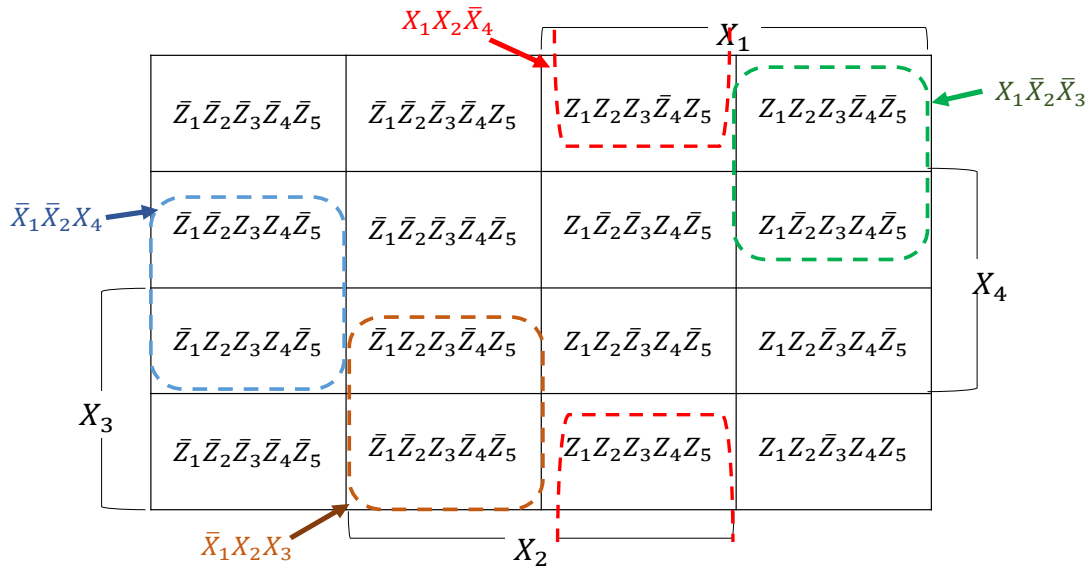
dotted non-essential loops $Z_1\bar{Z}_4\bar{Z}_5$ or $Z_2\bar{Z}_4\bar{Z}_5$ is needed. Hence, there are two minimal solutions of s .

$$s = \bar{Z}_1Z_4 \vee \bar{Z}_2\bar{Z}_3 \vee Z_1\bar{Z}_4\bar{Z}_5, \quad (17a)$$

$$s = \bar{Z}_1Z_4 \vee \bar{Z}_2\bar{Z}_3 \vee Z_2\bar{Z}_4\bar{Z}_5, \quad (17b)$$

in agreement with the results of Bell [3].

Now, we consider the second method for this example. Figure 10 represents the map for $t(\mathbf{X})$ in (15), while Fig. 11 utilizes Fig. 7 (and its complement) together with Fig. 10 to produce a map for $t(\mathbf{X}, \mathbf{Z})$, which can be used to find a sneaky translator. However, we note that an honest translator is possible since none of the 32 atoms of $FB(Z_1, Z_2, Z_3, Z_4, Z_5)$ has a mixed assertion in Fig. 11. In fact, there are 6 atoms that are asserted positively, 7 atoms that are asserted negatively, and 19 atoms asserted neither way. Hence s can be made an honest translator as a disjunction of the 6 positively asserted atoms and, possibly, any of the totally don't-care (non-asserted) atoms. This precisely expresses s by the map in Fig. 9(l), thereby recovering our earlier results.



$g(\mathbf{X})$

Fig. 7. The natural map for $g(\mathbf{X})$ in (16). Loops for $Z_3(\mathbf{X})$ are shown, inside which Z_3 appears and outside which $\bar{Z}_3$ appears.

X_1				
$\bar{Z}_1\bar{Z}_2\bar{Z}_3\bar{Z}_4\bar{Z}_5\bar{p}_2\bar{p}_3 \vee$ $d(\bar{Z}_1\bar{Z}_2\bar{Z}_3 \vee \bar{Z}_2\bar{Z}_3\bar{Z}_4 \vee$ $Z_1\bar{Z}_2\bar{Z}_5 \vee Z_1\bar{Z}_2\bar{Z}_4 \vee$ $\bar{Z}_1Z_2\bar{Z}_4\bar{Z}_5 \vee \bar{Z}_1\bar{Z}_2\bar{Z}_3Z_4 \vee$ $\bar{Z}_1Z_2Z_3Z_4 \vee Z_1Z_2Z_3Z_4\bar{Z}_5)$	$\bar{Z}_1\bar{Z}_2\bar{Z}_3\bar{Z}_4Z_5 \vee$ $d(\bar{Z}_1\bar{Z}_2\bar{Z}_3 \vee \bar{Z}_2\bar{Z}_3\bar{Z}_4 \vee$ $Z_1\bar{Z}_2\bar{Z}_5 \vee Z_1\bar{Z}_2\bar{Z}_4 \vee$ $\bar{Z}_1Z_2\bar{Z}_4\bar{Z}_5 \vee \bar{Z}_1\bar{Z}_2\bar{Z}_3Z_4 \vee$ $\bar{Z}_1Z_2Z_3Z_4 \vee Z_1Z_2Z_3Z_4\bar{Z}_5)$	$Z_1Z_2Z_3\bar{Z}_4Z_5 \vee$ $d(\bar{Z}_1\bar{Z}_2\bar{Z}_3 \vee \bar{Z}_2\bar{Z}_3\bar{Z}_4 \vee$ $Z_1\bar{Z}_2\bar{Z}_5 \vee Z_1\bar{Z}_2\bar{Z}_4 \vee$ $\bar{Z}_1Z_2\bar{Z}_4\bar{Z}_5 \vee \bar{Z}_1\bar{Z}_2\bar{Z}_3Z_4 \vee$ $\bar{Z}_1Z_2Z_3Z_4 \vee Z_1Z_2Z_3Z_4\bar{Z}_5)$	$Z_1Z_2Z_3Z_4\bar{Z}_5 \vee$ $d(\bar{Z}_1\bar{Z}_2\bar{Z}_3 \vee \bar{Z}_2\bar{Z}_3\bar{Z}_4 \vee$ $Z_1\bar{Z}_2\bar{Z}_5 \vee Z_1\bar{Z}_2\bar{Z}_4 \vee$ $\bar{Z}_1Z_2\bar{Z}_4\bar{Z}_5 \vee \bar{Z}_1\bar{Z}_2\bar{Z}_3Z_4 \vee$ $\bar{Z}_1Z_2Z_3Z_4 \vee Z_1Z_2Z_3Z_4\bar{Z}_5)$	
X_3	$\bar{Z}_1\bar{Z}_2Z_3Z_4\bar{Z}_5 \vee$ $d(\bar{Z}_1\bar{Z}_2\bar{Z}_3 \vee \bar{Z}_2\bar{Z}_3\bar{Z}_4 \vee$ $Z_1\bar{Z}_2\bar{Z}_5 \vee Z_1\bar{Z}_2\bar{Z}_4 \vee$ $\bar{Z}_1Z_2\bar{Z}_4\bar{Z}_5 \vee \bar{Z}_1\bar{Z}_2\bar{Z}_3Z_4 \vee$ $\bar{Z}_1\bar{Z}_2Z_3Z_4 \vee Z_1Z_2Z_3Z_4\bar{Z}_5)$	$\bar{Z}_1\bar{Z}_2\bar{Z}_3\bar{Z}_4\bar{Z}_5 \vee$ $d(\bar{Z}_1\bar{Z}_2\bar{Z}_3 \vee \bar{Z}_2\bar{Z}_3\bar{Z}_4 \vee$ $Z_1\bar{Z}_2\bar{Z}_5 \vee Z_1\bar{Z}_2\bar{Z}_4 \vee$ $\bar{Z}_1Z_2\bar{Z}_4\bar{Z}_5 \vee \bar{Z}_1\bar{Z}_2\bar{Z}_3Z_4 \vee$ $\bar{Z}_1\bar{Z}_2Z_3Z_4 \vee Z_1Z_2Z_3Z_4\bar{Z}_5)$	$Z_1\bar{Z}_2\bar{Z}_3Z_4\bar{Z}_5 \vee$ $d(\bar{Z}_1\bar{Z}_2\bar{Z}_3 \vee \bar{Z}_2\bar{Z}_3\bar{Z}_4 \vee$ $Z_1\bar{Z}_2\bar{Z}_5 \vee Z_1\bar{Z}_2\bar{Z}_4 \vee$ $\bar{Z}_1Z_2\bar{Z}_4\bar{Z}_5 \vee \bar{Z}_1\bar{Z}_2\bar{Z}_3Z_4 \vee$ $\bar{Z}_1\bar{Z}_2Z_3Z_4 \vee Z_1Z_2Z_3Z_4\bar{Z}_5)$	X_4
	$\bar{Z}_1Z_2Z_3Z_4\bar{Z}_5 \vee$ $d(\bar{Z}_1\bar{Z}_2\bar{Z}_3 \vee \bar{Z}_2\bar{Z}_3\bar{Z}_4 \vee$ $Z_1\bar{Z}_2\bar{Z}_5 \vee Z_1\bar{Z}_2\bar{Z}_4 \vee$ $\bar{Z}_1Z_2\bar{Z}_4\bar{Z}_5 \vee \bar{Z}_1\bar{Z}_2\bar{Z}_3Z_4 \vee$ $\bar{Z}_1\bar{Z}_2Z_3Z_4 \vee Z_1Z_2Z_3Z_4\bar{Z}_5)$	$\bar{Z}_1Z_2Z_3Z_4\bar{Z}_5 \vee$ $d(\bar{Z}_1\bar{Z}_2\bar{Z}_3 \vee \bar{Z}_2\bar{Z}_3\bar{Z}_4 \vee$ $Z_1\bar{Z}_2\bar{Z}_5 \vee Z_1\bar{Z}_2\bar{Z}_4 \vee$ $\bar{Z}_1Z_2\bar{Z}_4\bar{Z}_5 \vee \bar{Z}_1\bar{Z}_2\bar{Z}_3Z_4 \vee$ $\bar{Z}_1\bar{Z}_2Z_3Z_4 \vee Z_1Z_2Z_3Z_4\bar{Z}_5)$	$Z_1Z_2\bar{Z}_3Z_4\bar{Z}_5\bar{p}_1 \vee$ $d(\bar{Z}_1\bar{Z}_2\bar{Z}_3 \vee \bar{Z}_2\bar{Z}_3\bar{Z}_4 \vee$ $Z_1\bar{Z}_2\bar{Z}_5 \vee Z_1\bar{Z}_2\bar{Z}_4 \vee$ $\bar{Z}_1Z_2\bar{Z}_4\bar{Z}_5 \vee \bar{Z}_1\bar{Z}_2\bar{Z}_3Z_4 \vee$ $\bar{Z}_1\bar{Z}_2Z_3Z_4 \vee Z_1Z_2Z_3Z_4\bar{Z}_5)$	
	$\bar{Z}_1\bar{Z}_2\bar{Z}_3\bar{Z}_4\bar{Z}_5\bar{p}_2\bar{p}_3 \vee$ $d(\bar{Z}_1\bar{Z}_2\bar{Z}_3 \vee \bar{Z}_2\bar{Z}_3\bar{Z}_4 \vee$ $Z_1\bar{Z}_2\bar{Z}_5 \vee Z_1\bar{Z}_2\bar{Z}_4 \vee$ $\bar{Z}_1Z_2\bar{Z}_4\bar{Z}_5 \vee \bar{Z}_1\bar{Z}_2\bar{Z}_3Z_4 \vee$ $\bar{Z}_1Z_2Z_3Z_4 \vee Z_1Z_2Z_3Z_4\bar{Z}_5)$	$\bar{Z}_1\bar{Z}_2\bar{Z}_3\bar{Z}_4\bar{Z}_5 \vee$ $d(\bar{Z}_1\bar{Z}_2\bar{Z}_3 \vee \bar{Z}_2\bar{Z}_3\bar{Z}_4 \vee$ $Z_1\bar{Z}_2\bar{Z}_5 \vee Z_1\bar{Z}_2\bar{Z}_4 \vee$ $\bar{Z}_1Z_2\bar{Z}_4\bar{Z}_5 \vee \bar{Z}_1\bar{Z}_2\bar{Z}_3Z_4 \vee$ $\bar{Z}_1Z_2Z_3Z_4 \vee Z_1Z_2Z_3Z_4\bar{Z}_5)$	$Z_1Z_2Z_3Z_4\bar{Z}_5 \vee$ $d(\bar{Z}_1\bar{Z}_2\bar{Z}_3 \vee \bar{Z}_2\bar{Z}_3\bar{Z}_4 \vee$ $Z_1\bar{Z}_2\bar{Z}_5 \vee Z_1\bar{Z}_2\bar{Z}_4 \vee$ $\bar{Z}_1Z_2\bar{Z}_4\bar{Z}_5 \vee \bar{Z}_1\bar{Z}_2\bar{Z}_3Z_4 \vee$ $\bar{Z}_1Z_2Z_3Z_4 \vee Z_1Z_2Z_3Z_4\bar{Z}_5)$	
X_2				

$$G(\mathbf{X}, p)$$

Fig. 8. The auxiliary function used to derive the parametric solutions of $g(\mathbf{X}) = 1$ in Section 3.

z_1								
1	1	d	d	d		d	d	
z_5	1	d	1	d	d		d	d
	d	d	d	d			d	d
	d	1	1	d		d		
	z_3		z_2			z_3		
z_4								

(a) $\bar{X}_1$

								z_1			
z_5	p_2	1	d	d	d		d	d			
	1	d	1	d	d	1	d	d			
	d	d	d	d		1	d	d			
	d			d	p_1	d		1			
								z_3			
								z_2			
								z_3			
								z_4			

(b) X_2

z_1								
z_5	$\bar{p}_2\bar{p}_3$ $\vee \bar{p}_2p_3$		d	d	d	1	d	d
		d		d	d		d	d
	d	d	d	d	1		d	d
	d	1	1	d	$\bar{p}_1$	d	1	
z_3		z_2			z_3			
z_4								

(c) $\bar{X}_2$

z_1								
z_5	$\bar{p}_2\bar{p}_3$ $\vee p_2$		d	d	d	1	d	d
	1	d		d	d	1	d	d
	d	d	d	d			d	d
	d		1	d		d	1	1
z_2		z_3			z_4			

(d) $\bar{X}_3$

z_1							
z_5	p_2		d	d	d		d
		d	1	d	d		d
	d	d	d	d			d
	d	1	1	d	1	d	1
z_3		z_2			z_3		
z_4							

(e) X_4

z_1								
z_5	$\bar{p}_2\bar{p}_3$ $\vee \bar{p}_2p_3$	1	d	d	d	1	d	d
	1	d		d	d	1	d	d
	d	d	d	d	1	1	d	d
	d			d		d		
z_3		z_2			z_3			z_4

(f) $\bar{X}_4$

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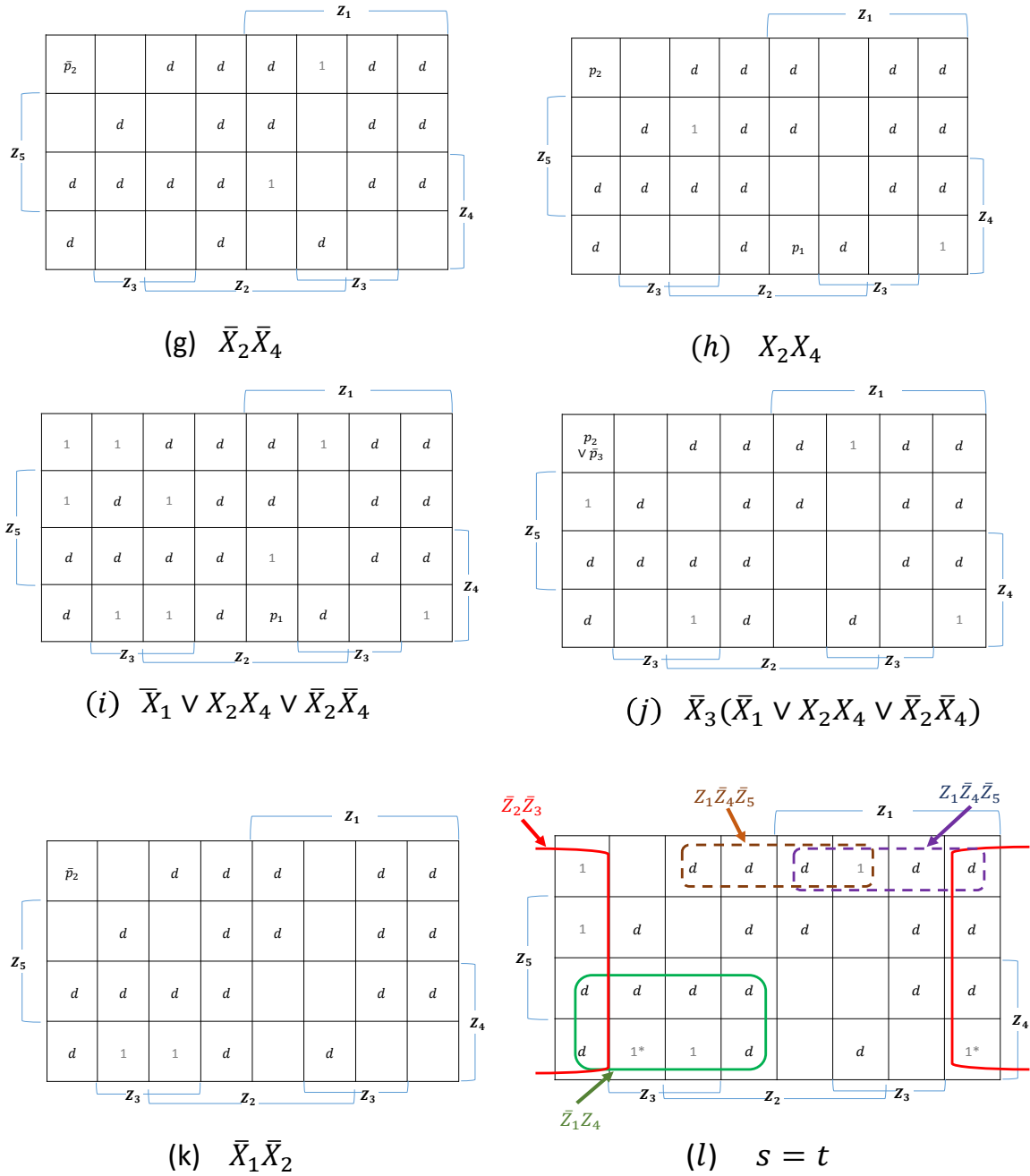


Fig. 9. Solutions for the pertinent X variables, their complements and products in Section 3, culminating in a map for $s = t$ as given by (15).

X_3		X_1		X_4
		1	1	
		1	1	
		1	1	
		X_2		

$$t(X)$$

Fig. 10. Conventional Karnaugh map for $t(X)$ expressed by (15) in Sec. 3.

X_3		X_1		X_4
		$\bar{Z}_1\bar{Z}_2\bar{Z}_3\bar{Z}_4\bar{Z}_5 \vee d(1)$	$\bar{Z}_1\bar{Z}_2\bar{Z}_3\bar{Z}_4Z_5 \vee d(1)$	
		$\bar{Z}_1\bar{Z}_2Z_3Z_4\bar{Z}_5 \vee d(1)$	$\bar{Z}_1\bar{Z}_2\bar{Z}_3Z_4\bar{Z}_5 \vee d(1)$	
		$\bar{Z}_1Z_2Z_3Z_4\bar{Z}_5 \vee d(1)$	$d(Z_1 \vee \bar{Z}_2 \vee \bar{Z}_3 \vee Z_4 \vee \bar{Z}_5)$	
		X_2		

$$s(X, Z) = tg \vee d(\bar{g})$$

Fig. 11. Natural map for $s(X, Z)$ in Sec. 3.

4. A Circuit Example with Side Input

This example was solved earlier by Ledley [2] and Brown [4] and differs from the earlier two examples, since it has side inputs $\mathbf{Y}$. The example is specified by:

$$t(\mathbf{X}, \mathbf{Y}) = X_1 X_2 \vee Y_1 \bar{X}_2 X_3 \vee Y_2 (X_1 \bar{X}_3 \vee X_2), \quad (18)$$

$$\mathbf{Z}(\mathbf{X}) = \begin{bmatrix} X_1 \bar{X}_2 \vee \bar{X}_1 X_3 \\ \bar{X}_1 X_2 \vee X_1 \bar{X}_3 \\ X_1 X_2 \vee \bar{X}_2 X_3 \end{bmatrix}. \quad (19)$$

We rewrite (19) in equational form as

$$g(\mathbf{X}, \mathbf{Z}) = (Z_1 \odot (X_1 \bar{X}_2 \vee \bar{X}_1 X_3))(Z_2 \odot (\bar{X}_1 X_2 \vee X_1 \bar{X}_3))(Z_3 \odot (X_1 X_2 \vee \bar{X}_2 X_3)) = 1, \quad (20)$$

and proceed to solve it for $\mathbf{X}$ in terms of $\mathbf{Z}$. In Figs (12a)-(12c), we draw maps of the factors in (19), and then AND them cell-wise to draw the natural map of g as a function of $\mathbf{X}$ over the big Boolean algebra $FB(Z_1, Z_2, Z_3)$. Figure 12(d) shows that four atoms of $FB(Z_1, Z_2, Z_3)$, namely $\bar{Z}_1 \bar{Z}_2 \bar{Z}_3$, $\bar{Z}_1 Z_2 \bar{Z}_3$, $\bar{Z}_1 Z_2 Z_3$, and $\bar{Z}_1 \bar{Z}_2 Z_3$ appear once each, and two atoms, *viz.*, $Z_1 \bar{Z}_2 Z_3$, and $Z_1 Z_2 \bar{Z}_3$ appear twice each, while the remaining two atoms ($Z_1 Z_2 Z_3$, and $Z_1 \bar{Z}_2 \bar{Z}_3$) never appear. This means that the consistency condition is

$$Z_1 Z_2 Z_3 \vee Z_1 \bar{Z}_2 \bar{Z}_3 = 0, \quad (21)$$

and the number of particular solutions subject to that condition is $(1)^4 * (2)^2 = 4$. The auxiliary function $G(\mathbf{X}, \mathbf{p})$ is shown in Fig. 13 where we used a parameter p_1 to create orthonormal tags $\{p_1, \bar{p}_1\}$ for the atom $Z_1 Z_2 \bar{Z}_3$, and an independent parameter p_2 to create orthonormal tags $\{p_2, \bar{p}_2\}$ for the atom $Z_1 \bar{Z}_2 Z_3$. From Fig. 13, we obtain the following expressions for the $\mathbf{X}$ variables and some of their complements

$$X_1 = \bar{Z}_1 Z_3 \vee Z_1 Z_2 \bar{Z}_3 \bar{p}_1 \vee Z_1 \bar{Z}_2 Z_3 p_2 \vee d(Z_1 Z_2 Z_3 \vee Z_1 \bar{Z}_2 \bar{Z}_3),$$

(22a)

$$X_2 = \bar{Z}_1 Z_3 \vee \bar{Z}_1 Z_2 \vee Z_1 Z_2 \bar{Z}_3 p_1 \vee d(Z_1 Z_2 Z_3 \vee Z_1 \bar{Z}_2 \bar{Z}_3),$$

(22b)

$$X_3 = \bar{Z}_2 Z_3 \vee Z_1 Z_2 \bar{Z}_3 p_1 \vee d(Z_1 Z_2 Z_3 \vee Z_1 \bar{Z}_2 \bar{Z}_3),$$

(22c)

$$\bar{X}_2 = \bar{Z}_1 \bar{Z}_2 \bar{Z}_3 \vee Z_1 Z_2 \bar{Z}_3 \bar{p}_1 \vee Z_1 \bar{Z}_2 Z_3 \vee d(Z_1 Z_2 Z_3 \vee Z_1 \bar{Z}_2 \bar{Z}_3),$$

(22d)

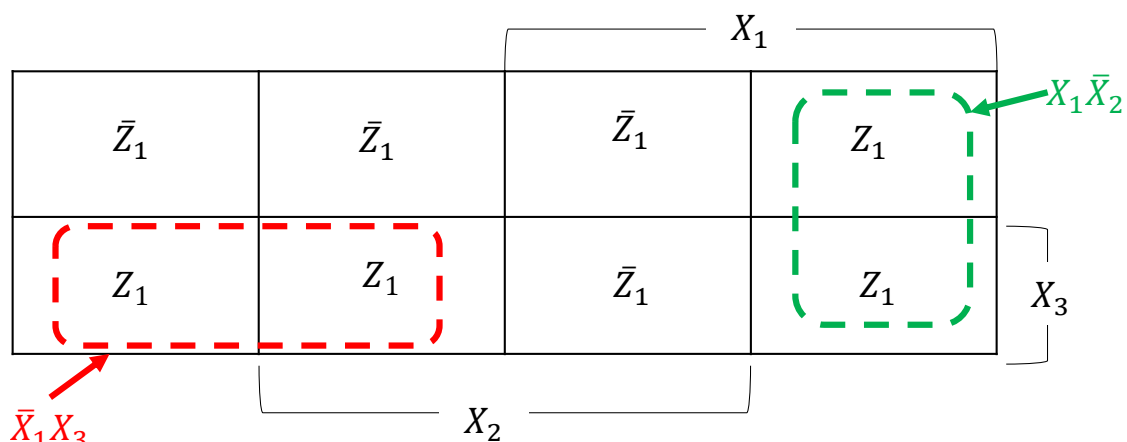
$$\bar{X}_3 = \bar{Z}_1 \bar{Z}_3 \vee \bar{Z}_1 Z_2 \vee Z_1 Z_2 \bar{Z}_3 \bar{p}_1 \vee d(Z_1 Z_2 Z_3 \vee Z_1 \bar{Z}_2 \bar{Z}_3).$$

(22e)

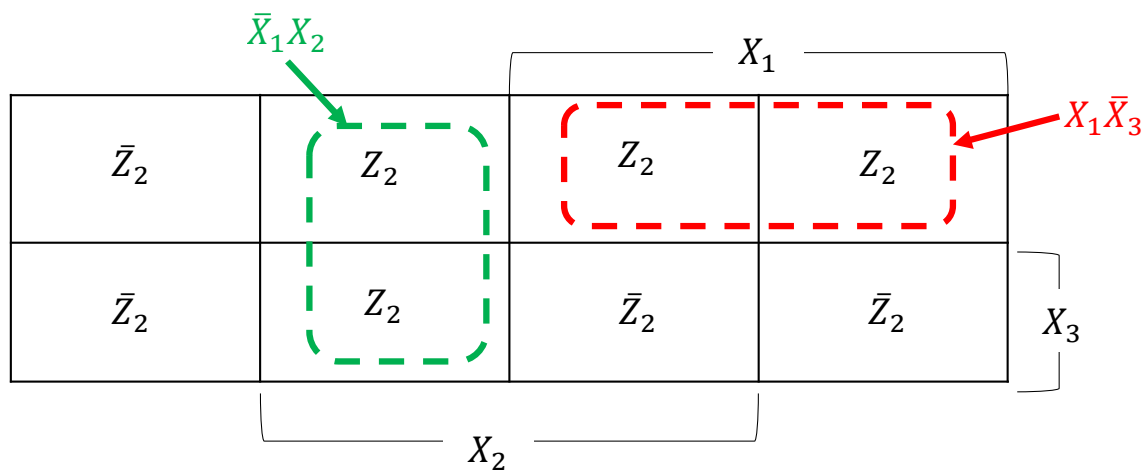
Finally, $t(\mathbf{X}, \mathbf{Y})$ in (18) is replaced by $s(\mathbf{Z}, \mathbf{Y})$, using intelligent multiplication [28-34]

$$s(\mathbf{Z}, \mathbf{Y}) = \bar{Z}_1 Z_3 \vee Y_1 Z_1 \bar{Z}_2 Z_3 \vee Y_2 (\bar{Z}_1 Z_2 \vee Z_2 \bar{Z}_3) \vee d(Z_1 Z_2 Z_3 \vee Z_1 \bar{Z}_2 \bar{Z}_3).$$

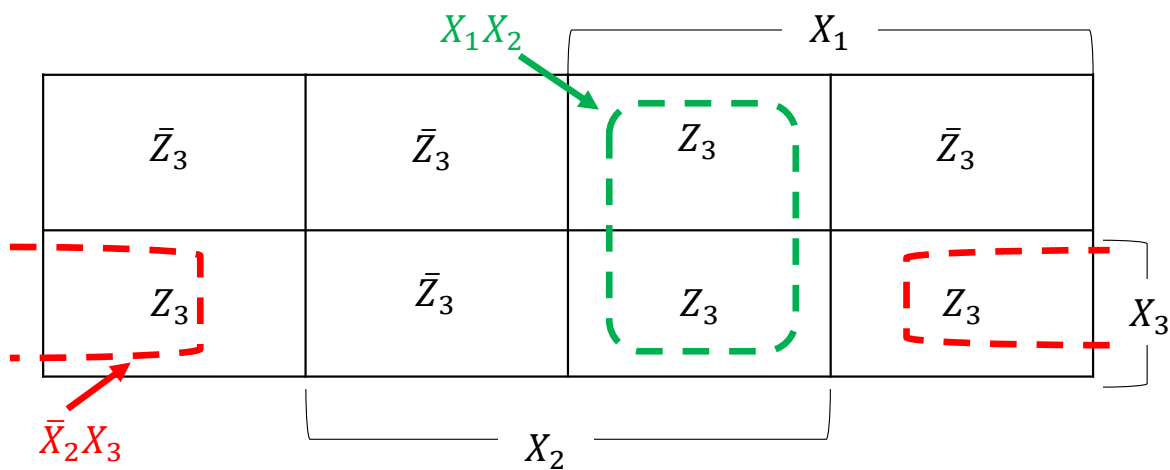
(23)



$$(a) \quad Z_1 \odot Z_1(\mathbf{X}) = Z_1 \odot (X_1\bar{X}_2 \vee \bar{X}_1X_3)$$



$$(b) \quad Z_2 \odot Z_2(\mathbf{X}) = Z_2 \odot (\bar{X}_1X_2 \vee X_1\bar{X}_3)$$



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$$(c) \quad Z_3 \odot Z_3(\mathbf{X}) = Z_3 \odot (X_1 X_2 \vee \bar{X}_2 X_3)$$

460

X_1			
$\bar{Z}_1 \bar{Z}_2 \bar{Z}_3$	$\bar{Z}_1 Z_2 \bar{Z}_3$	$\bar{Z}_1 Z_2 Z_3$	$Z_1 Z_2 \bar{Z}_3$
$Z_1 \bar{Z}_2 Z_3$	$Z_1 Z_2 \bar{Z}_3$	$\bar{Z}_1 \bar{Z}_2 Z_3$	$Z_1 \bar{Z}_2 Z_3$
X_2			
X_3			

461

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463

$$(d) \quad g(\mathbf{X})$$

464

Fig. 12. Gradual derivation of the natural map $g(\mathbf{X})$ of in Sec. 4.

465

466

X_1			
$\bar{Z}_1 \bar{Z}_2 \bar{Z}_3(1) \vee d(Z_1 Z_2 Z_3 \vee Z_1 \bar{Z}_2 \bar{Z}_3)$	$\bar{Z}_1 Z_2 \bar{Z}_3(1) \vee d(Z_1 Z_2 Z_3 \vee Z_1 \bar{Z}_2 \bar{Z}_3)$	$\bar{Z}_1 Z_2 Z_3(1) \vee d(Z_1 Z_2 Z_3 \vee Z_1 \bar{Z}_2 \bar{Z}_3)$	$Z_1 Z_2 \bar{Z}_3(\bar{p}_1) \vee d(Z_1 Z_2 Z_3 \vee Z_1 \bar{Z}_2 \bar{Z}_3)$
$Z_1 \bar{Z}_2 Z_3(\bar{p}_2) \vee d(Z_1 Z_2 Z_3 \vee Z_1 \bar{Z}_2 \bar{Z}_3)$	$Z_1 Z_2 \bar{Z}_3(p_1) \vee d(Z_1 Z_2 Z_3 \vee Z_1 \bar{Z}_2 \bar{Z}_3)$	$\bar{Z}_1 \bar{Z}_2 Z_3(1) \vee d(Z_1 Z_2 Z_3 \vee Z_1 \bar{Z}_2 \bar{Z}_3)$	$Z_1 \bar{Z}_2 Z_3(p_2) \vee d(Z_1 Z_2 Z_3 \vee Z_1 \bar{Z}_2 \bar{Z}_3)$
X_2			
X_3			

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468

$$G(\mathbf{X}, \mathbf{p})$$

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Fig. 13. The auxiliary function needed in Sec. 4.

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The function $s(\mathbf{Z}, \mathbf{Y})$ is represented by the VEKM of Fig. 14, which can be read to yield the minimal expression (obtained earlier by Brown [4])

$$s(\mathbf{Z}, \mathbf{Y}) = \bar{Z}_1 Z_3 \vee Y_1 Z_3 \vee Y_2 Z_2. \quad (24)$$

Figure 14 can also be used to verify the non-minimal solution obtained by Ledley [2]

$$s(\mathbf{Z}, \mathbf{Y}) = \bar{Z}_1 Z_3 \vee Y_1 Z_1 \bar{Z}_2 \vee Y_2 Z_2 \bar{Z}_3. \quad (25)$$

Now, we consider the second method for this example. Figure 15 represents a VEKM for $t(\mathbf{X}, \mathbf{Y})$ in (18). When one utilizes Fig. 12(d) (and its complement) to produce a representation of $s = tg \vee d(\bar{g})$, Fig. 16 is simply obtained.

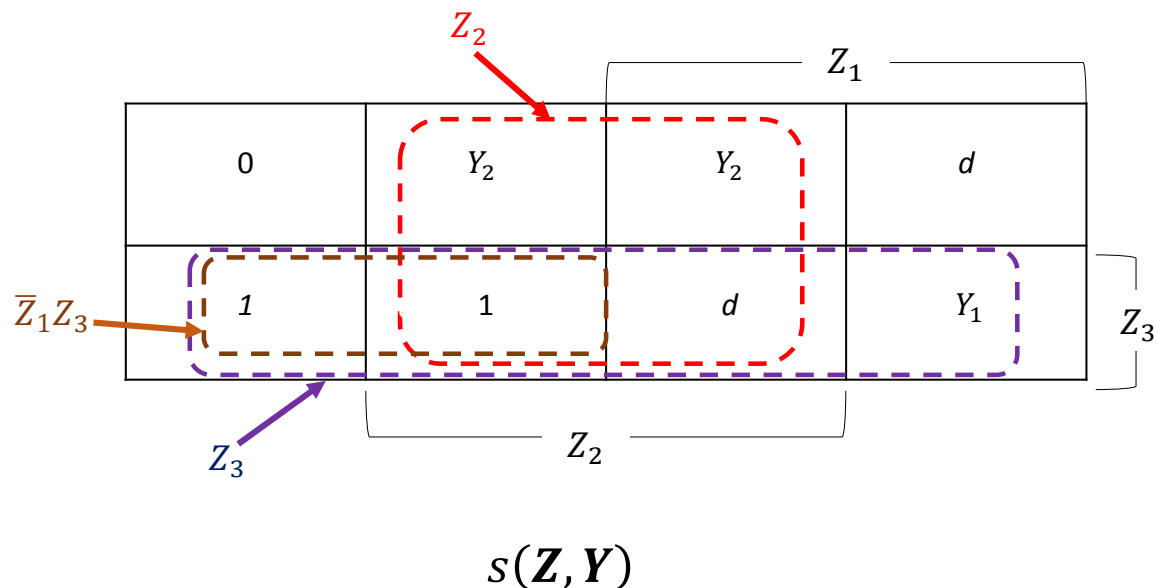


Fig. 14. A VEKM representation for $s(\mathbf{Z}, \mathbf{Y})$ in Sec. 4.

		X_1		
0	Y_2	1	Y_2	
Y_1	Y_2	1	Y_1	X_3
	X_2			

$$t(X, Y)$$

Fig. 15. A VEKM representing $t(X, Y)$ in (18) of Sec. 4.

		X_1		
$d(Z_1 \vee Z_2 \vee Z_3)$	$Y_2 \bar{Z}_1 Z_2 \bar{Z}_3 \vee d(Z_1 \vee \bar{Z}_2 \vee Z_3)$	$\bar{Z}_1 Z_2 Z_3 \vee d(1)$	$Y_2 Z_1 Z_2 \bar{Z}_3 \vee d(\bar{Z}_1 \vee \bar{Z}_2 \vee Z_3)$	
$Y_1 Z_1 \bar{Z}_2 Z_3 \vee d(\bar{Z}_1 \vee Z_2 \vee \bar{Z}_3)$	$Y_2 Z_1 Z_2 \bar{Z}_3 \vee d(\bar{Z}_1 \vee \bar{Z}_2 \vee Z_3)$	$\bar{Z}_1 \bar{Z}_2 Z_3 \vee d(1)$	$Y_1 Z_1 \bar{Z}_2 Z_3 \vee d(Z_1 \vee \bar{Z}_2 \vee Z_3)$	X_3
	X_2			

$$s(X, Y, Z) = tg \vee d(\bar{g})$$

Fig. 16. A VEKM representing $s(X, Y, Z)$ in Sec. 4.

with $s(X, Y, Z)$ viewed as $s(X): B^3 \rightarrow B$, where $B = FB(Z_1, Z_2, Z_3, Y_1, Y_2)$ has 32 atoms. Again, none of these atoms (which can be viewed as the cell products of the 5-variable Karnaugh map in Fig. 17 has a mixed assertion. In fact, we are forced to enter a

single entry of 1 in 14 cells of the map in Fig. 17, to enter a unique 0 entry in 10 cells of that map, and to leave 8 cells intact (presumably don't-cares). This means that $s(\mathbf{X}, \mathbf{Y}, \mathbf{Z})$ can be represented by the map in Fig. 17 as $s(\mathbf{Z}, \mathbf{Y})$ independently of $\mathbf{X}$, and it has the minimal coverage depicted by the three loops in Fig. 17 which corresponds to the expression (24).

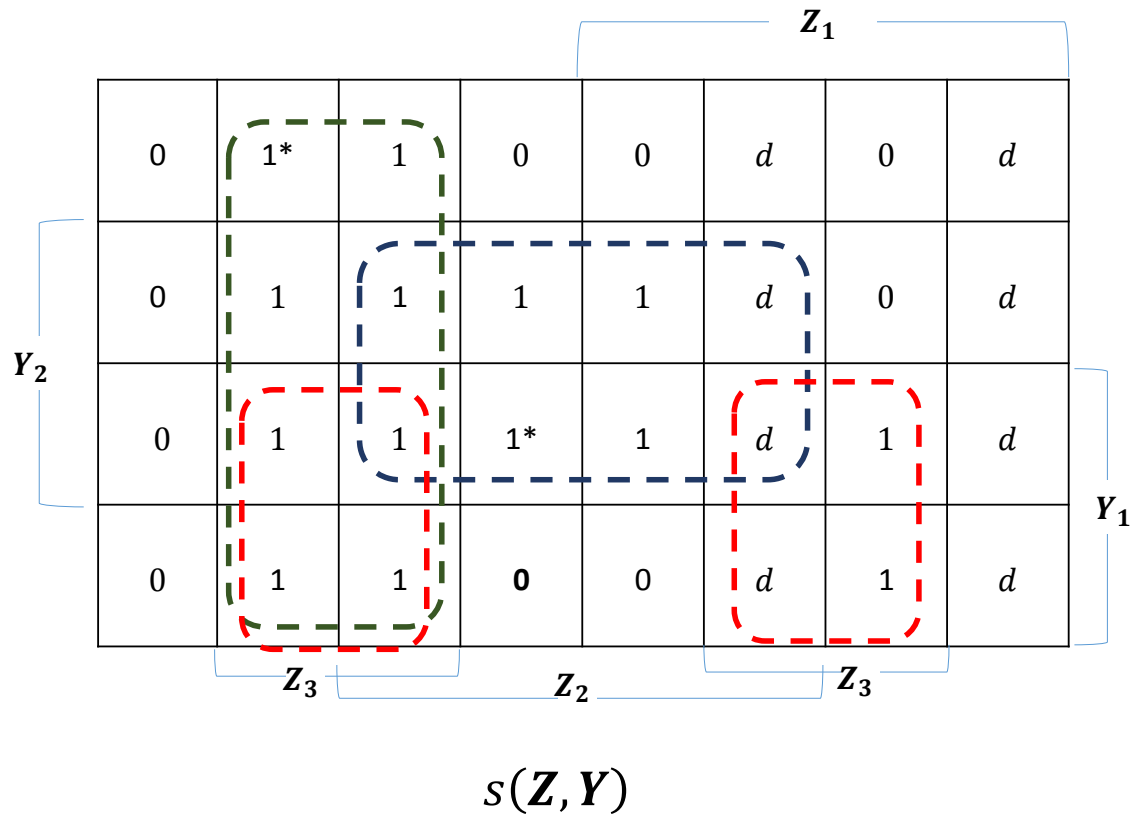


Fig. 17. A 5-variable Karnaugh map for admitting a representation of $s(\mathbf{X}, \mathbf{Y}, \mathbf{Z})$ that is independent of $\mathbf{X}$.

5. Conclusions

This paper is a part of our consolidated effort to modernize techniques for handling fundamental problems of digital circuit design. We devote this paper to handling what is called Type-2 problems, with our sister publication [6] dedicated to Type-3 problems. While our

treatment of Type-3 problems in [6] relied solely on equation-solving in big Boolean algebras, we considered herein a similar treatment for Type-2 problems, but we exposed it in comparison with another method [5] that adapts the input-domain constraining technique *via* utilization of variable-entered Karnaugh map (VEKMs) together with modern don't-care notation. Our method used in [6] as well as our two methods employed herein outperform the conventional method both in speed and clarity. Our main concern herein is to decide which of our two novel competing methods is preferable. The answer seems to be a matter of personal taste convenience, and relative experience. There is definitely some similarity, interaction, and conceptual relationship between the two methods. We note that the Boolean-equation-solving method produces parametric solutions that lead to several parametric solutions expressed in terms of a few parameters. However, these parameters disappeared later in the final stage of problem solving. Therefore, the Boolean-equation-solving method seems to involve certain unwarranted steps that might be possibly skipped. However, its utility is not restricted to type-2 problems, since it handles type-3 problems as well. In all the examples worked out herein, the two methods obtained exactly the same answers. These answers agreed with (or occasionally were more compact than) answers obtained conventionally.

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