

Hydraulic properties of unsaturated soils: A novel procedure

ABSTRACT

Buildings built on shrink-swelling soils may experience serious damage due to volumetric strains. These strains are produced by drying-wetting cycles generated when water content fluctuates in the mass of soil. In order to study this behavior, theoretical models taking into account the soil-water retention curve have been developed. The retention curve is used to indirectly estimate some parameters describing the hydraulic behavior of soils, as for example the hydraulic conductivity and the diffusion coefficient. The precision of indirect methods to determine the hydraulic parameters depends largely on the accuracy of the experimental points of the soil-water retention curve and the equations used to simulate this curve. However, it is possible to skip the second step and simply use the experimental data of the characteristic curve to obtain these parameters. Therefore, a procedure based on an interpolation method with variable increments is proposed. This procedure generates a polynomial equation for the retention curve which simplifies the method and avoids errors caused by the fitting process of the theoretical with the experimental retention curve. In addition, no fitting parameters are required with this procedure. The results obtained with this method are similar to those reported by the traditional indirect methods.

Keywords: unsaturated soils, indirect method, hydraulic conductivity, diffusion coefficient, soil water characteristic curve

1. INTRODUCTION

Shrink-swelling soils can be found anywhere in the world in countries such as Argentina, Australia, Brazil, Canada, Colombia, Cuba, Ecuador, Spain, USA, Mexico, Peru, Russia, South Africa, and Venezuela [1]. Several states in Mexico have reported damages associated with these types of soil. In fact, The American Society of Civil Engineers estimates that 25% of all households in the United States have sustained some damage caused by expansive soils [2]. The US Federal Agency for Emergency Management estimated that the annual loss generated by these soils was around seven billion dollars in 2012. In that sense, the prediction of the hydro-mechanical behavior of shrink-swelling soils is an issue of paramount importance in unsaturated soil mechanics [3,4].

Different indirect methods for predicting the hydraulic parameters of soils through the Soil-Water Retention Curve (SWRC) have been developed. The determination of the hydraulic properties through these indirect methods has shown important advantages over direct methods, since the former reduce significantly the time and the cost of laboratory tests without losing precision [5]. Most indirect methods are based on the use of the SWRC to obtain the hydraulic conductivity (k) and the diffusion coefficient (D). The hydraulic conductivity can be obtained from the saturated permeability and the SWRC of the soil [3,6]. In addition, the diffusion coefficient can be obtained from the hydraulic conductivity and the slope of the SWRC at the considered value of suction.

Among the most popular indirect methods are the equations proposed by van Genuchten[5] based on Mualem's model. These equations simulate the SWRC and the hydraulic conductivity of the soil as a function of matric suction. It is also possible to determine the diffusion coefficient in terms of the volumetric water content (θ) or the degree of saturation of the soil (S_r).

Fredlund and Rahardjo[7] developed a pair of equations based on the proposal by Kunze et al. to estimate the hydraulic conductivity of soils. In this procedure, the full range of water content in the characteristic curve is divided in N intervals of the same magnitude $\Delta\theta$. To this purpose, it is necessary to fit the numerical SWRC with experimental data using a proper equation. This procedure has proven to be adequate for the estimation of the hydraulic properties of soils. However, in some cases, there is poor correlation between the experimental data and the numerical curve, meaning that the parameters

obtained do not represent the properties of the real soil. This is especially true for the case of double structured soils.

Inasmuch as the SWRC is used to predict the hydraulic conductivity and the diffusion coefficient of unsaturated soils, it is of primary importance to simulate this curve as accurately as possible [6,8]. An alternative to the fitting process of the SWRC, is the direct use of the experimental data. In such a case all points of the SWRC are linked together using a polynomial function which is used to determine the hydraulic parameters of the soil. This paper describes the procedure.

2. ANTECEDENTS

2.1 Shrink-Swelling Soils

Expansive soils often contain minerals, such as montmorillonite [2], and are recognized as problematic as they severely compromise the civil structures built on them [9]. Especially lightly loaded structures built on these soils may damage as a result of changes in soil water content [10]. The problems caused by these soils can be attributed to the poor understanding of the volume changes caused by variations in water content [9] in swelling-expansive soils. Expansive soils are capable of adsorbing water in their internal structure, and when water content increases, so does their soil volume. This change in volume can exert sufficient pressure on the structure to cause damage (see Fig.1b). During drying, expansive soils shrink, causing a contraction that can affect building support and give rise to adverse subsidence (see Fig.1a). The wetting-drying process produces shrink-swell cycles that subject structures to repetitive stresses[2].



Fig.1. Building damage: produced by a) shrinkage[11], b) swelling [12].

2.2 Hydraulic Parameters

Water flow in porous media has been of interest since the beginnings of soil mechanics [13]. Water flow induces changes in suction to unsaturated soils [3]. There are seasonal wetting and drying cycles that take place throughout the year. Leaks from water pipes or drainage systems sometimes occur and lead gradually to the saturation of the surrounding soil. These processes produce changes in suction that generate volumetric deformations in the soil. The most severe problems associated with deformations are related to expansive, collapsible, and bad compacted materials. Hence, the description and prediction of water flow through unsaturated soils require an adequate knowledge of their hydraulic conductivity and diffusion coefficient [14,15].

2.2.1 Hydraulic Conductivity (k)

The water flow in saturated soils is commonly described using Darcy's law. This law states that the rate of water flow through a mass of soil is proportional to the gradient of hydraulic head [7]:

$$v_w = -k_w \frac{\partial h_w}{\partial y} \quad (1)$$

where: k_w is the saturated permeability coefficient, $\partial h_w / \partial h$ is the gradient of hydraulic head.

The proportionality term (k_w) in Equation (1) describes the ability of a specific porous medium under specific conditions to transmit a specific fluid [16]. Darcy's law also applies to the flow of water through unsaturated soils, although, in this case, the proportionality term, called hydraulic conductivity, cannot be assumed as constant [7]. This is so because this parameter depends not only on material variables such as the structure of pores (void ratio and porosity) and the properties of the pore fluid (density and viscosity) but also on the relative amount of pore fluid in the system (water content or degree of saturation). The relative amount of fluid in the pores of the system can be described in terms of matric suction $k(\psi)$, degree of saturation $k(S_r)$, or volumetric water content $k(\theta)$ [16]. In that sense, the SWRC can be visualized as the configuration of water-filled pores [7] which is different at wetting and drying. This phenomenon is called hysteresis of the SWRC. When the hydraulic conductivity is plotted in terms of suction, the phenomenon of hysteresis becomes apparent as it is directly linked to the SWRC. However, when this parameter is plotted in terms of volumetric water content, $k(\theta)$, or degree of saturation, $k(S_r)$ hysteresis is not as noticeable [16].

There are two techniques to measure the hydraulic conductivity: the direct and the indirect methods, which can be used in the laboratory or in-situ tests [3]. Among the direct methods the steady state and the instantaneous profile methods are the most commonly used. During a steady-state test the hydraulic head, the matric suction, and the water content of the soil remain constant while the volume of drained water during a certain time is measured. The hydraulic conductivity determined in these conditions is related to the matric suction or water content of the sample. The experiment is repeated for several values of matric suction or water content. Instead, during the instantaneous profile method, a continuous water flow is applied at one end of the sample. Using the SWRC of the soil and psicometers located in different sections of the sample, the hydraulic head (represented by suction) and the change in water content at various points along the specimen can be obtained [7]. Then, using the flow equation it is possible to obtain the hydraulic conductivity as a function of suction. Some permeameters are fabricated in demountable sections with a psicometer placed at the center of each section. This arrangement allows determining directly the water content at each section while the SWRC of the sample is simultaneously obtained. A drawback to these experimental procedures is that they take several months, making them time consuming and expensive [3].

Among the indirect techniques, there are three types of functions: empirical or semi-empirical equations, macroscopic models and statistical models [8]. Empirical or semi-empirical equations can be obtained from the combination of analysis with direct measurements. The resulting equations describe the variation in permeability as a function of suction or volumetric water content [3]. In this category, are the equations proposed by Richards (1931), Wind (1955), Gardner (1958), Richards (1967), Davidson et al. (1969), Cambell (1973), Ahuja (1974), Dane and Klute (1977).

Among macroscopic models, the one proposed by Rahardjo is between the most popular. In this model, water flow is supposed to be laminar [3] and the hydraulic conductivity is written as a function of the effective degree of saturation (S_e), in the form:

$$k_r = S_e^\delta \quad (2)$$

where k_r is the relative hydraulic conductivity, δ is the constant depending on soil type. The effective degree of saturation is defined as

$$S_e = \frac{(S - S_r)}{(S_s - S_r)} \quad (3)$$

where: S is the current degree of saturation of the sample, S_r is the residual degree of saturation and S_s is the maximum degree of saturation reached at wetting. According to Brooks and Corey, this model was made without considering the effect of the size distribution of pores and therefore led to disputes on the value of δ [3].

In statistical models, hydraulic conductivity is determined through the SWRC. Mualem[17], introduced a statistical model based on two assumptions: the first one states that pores of different sizes are randomly distributed in the porous media. The second assumption states that the average flow velocity (u) given by Hagen-Poiseuille's Equation (4), should be incorporated in the model [3]. This parameter writes:

$$\bar{v} = - \left(\frac{r^2 g}{C v} \right) \left(\frac{d\phi}{dx} \right) \quad (4)$$

where: r is the hydraulic radius, $d\phi/dx$ is the hydraulic gradient, g is the gravitational constant, C is a shape factor, and u is the kinematic coefficient of viscosity.

A very popular statistical model is that proposed by Fredlund and Rahardjo[7]. This model was derived from the procedure outlined by Childs and Collis-George [7] using Poiseuille's regime and the bundle of capillary tubes model. This method was improved by Marshall (1958) then modified by Kunze et al. (1968). The hydraulic conductivity of the soil is obtained by dividing in N equal intervals the SWRC of the sample along the volumetric water content axis. The hydraulic conductivity function is then written as:

$$k_i(\theta) = \frac{k_s}{k_{sc}} \frac{T_s^2 \rho_w g \theta_s^p}{2 \mu_w N^2} \sum_{j=i}^m \left\{ \frac{(2j+1-2i)}{(\psi_j)^2} \right\} \quad (5)$$

where: $\Delta\theta$ is the variation in water content for each interval, m represents the last interval corresponding to the lowest volumetric water content on the experimental SWRC (θ_L), i is the interval at which the hydraulic conductivity is being obtained, T_s is the interfacial air-water tensional force, ρ_w is the water density, g is the gravitational acceleration, μ_w is the absolute viscosity of water, θ_s is the saturated volumetric water content, p is a constant considered equal to 1, ψ_j is the matric suction at the j_{th} interval. The term $\sum_{j=1}^m \{(2j+1-2i)(\psi_j)^{-2}\}$ describes the shape of the hydraulic conductivity function, and k_s is the saturated coefficient of permeability measured in laboratory test. Finally, k_{sc} is the theoretical saturated permeability coefficient calculated for $i=1$, in the form:

$$k_{sc} = \frac{T_s^2 \rho_w g \theta_s^p}{2 \mu_w N^2} \sum_{j=1}^m \left\{ \frac{(2j+1-2i)}{(\psi_j)^2} \right\} \quad (6)$$

Notice that by substituting Equation (6) into (5), the hydraulic conductivity (k_i) can be written as:

$$k_i(\theta) = k_s \frac{\sum_{j=i}^m \left\{ \frac{(2j+1-2i)}{(\psi_j)^2} \right\}}{\sum_{j=1}^m \left\{ \frac{(2j+1-2i)}{(\psi_j)^2} \right\}} \quad (7)$$

2.2.2 Diffusion Coefficient (D)

The transient flow of water through soil is a diffusion process controlled by the diffusion coefficient. For an unsaturated soil, this parameter is a function of water content in the form $D(\theta)$. Its value is defined as the ratio of the hydraulic conductivity $k(\theta)$ with the slope $C(\theta)$ of the SWRC at the corresponding water content and divided by the dry density of the soil (ρ_d) [8,16].

$$D_i = \frac{k_i}{C_i \rho_d} \quad (8)$$

The slope of the SWRC represents the variation of water content (θ) per unit of suction change (ψ). This parameter is known as the specific moisture capacity $C(\theta)$ and can be written in the form [16]:

$$C(\theta) = \frac{d\theta}{d\psi} \quad (9)$$

When transient flow experiments are designed to determine directly the diffusion coefficient and the SWRC is available to obtain the specific moisture capacity (C) then, Equation (9) can be used to determine the hydraulic conductivity of the material. Among direct tests, the horizontal infiltration method involves the analysis of the distribution of water content in a horizontal column of soil at different intervals during the gradual increase of water up to 100% of saturation. Given the initial and boundary conditions of the system, the one-dimensional diffusion equation is used to determine the diffusion coefficient. Other methods are disclosed in Lu and Likos[16]. Some disadvantages are that the devices used in these tests are expensive and measurements are slow and difficult to perform. Because of this, researchers have been looking for ways to determine the diffusion coefficient through indirect techniques based on empirical correlations between index properties [18] or the use of the SWRC itself. Among the most commonly used methods are the equation proposed by Aubeny[19] and Li [20], which use the slope $C(\theta)$ of the SWRC, the dry soil density (ρ_d) and the hydraulic conductivity (k). Because these processes reduce the cost and time required for analysis, they are becoming increasingly popular in unsaturated soil mechanics.

Lu and Likos[16] show a procedure derived from the van Genuchten's equation [5], where the SWRC is related to the effective degree of saturation (S_e), in the form:

$$S_e = \left[\frac{1}{1 + (\alpha\psi)^n} \right]^m \quad (10)$$

where: ψ is the suction, α , m , and n are empirical parameters.

Then, the hydraulic conductivity function is written in terms of the effective degree of saturation.

$$k_{rw} = S_e^{1/2} \left[1 - (1 - S_e^{1/m})^m \right]^2 \quad (11)$$

where: k_{rw} is the hydraulic conductivity, S_e is the effective degree of saturation.

Finally, it is possible to calculate the diffusion coefficient using Equation (9).

2.3 Soil-Water Retention Curve

Water flows from larger potential to lower potential zones. The potential energy per unit of water refers to the work required to bring a unit of water from a standard reference state to the considered point. Soil water is subjected to field forces resulting from the attraction of the solid matrix, the presence of solutes, the action of external pressure and the gravitational attraction, among others. The pressure potential is considered positive if the pore water pressure in the soil is larger than the atmospheric pressure, and negative if it is lower. The latter is referred as matric suction (ψ) when taken in absolute value. Matric suction results from capillary forces between solid particles and water[13].

The SWRC is strongly affected by the structure and the pore size distribution (PSD) of the soil. For example, in clays where the pore size may vary from very large to very small, the slope of the curve smoothens [13]. In contrast, in sandy soils, the gradients of the SWRC are steepest because the PSD is more homogeneous.

The SWRC is not unique but it depends on the wetting path. This phenomenon is called hysteresis [14] and is caused by the presence of interconnected pores of different sizes that are randomly distributed. This phenomenon can be reproduced using porous models [21].

The hysteresis of the SWRC is an important phenomenon that must be taken into account during wetting-drying processes [16], since changes in soil moisture depend on hydrological process, such as infiltration, evaporation and evapotranspiration. This process generates wetting-drying cycles which can be reproduced using the primary curves as shown in Fig.2[22].

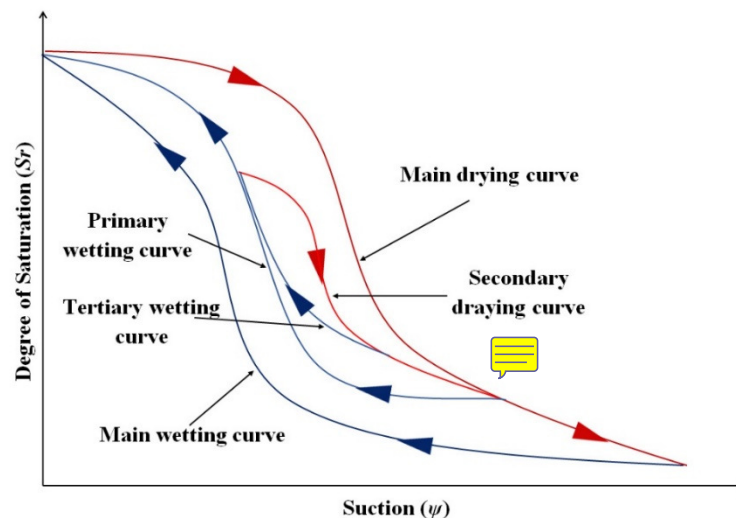


Fig.2. Primary, secondary and tertiary wetting and drying paths of unsaturated soil [22].

The SWRC can be used to determine different parameters that describe the behavior of unsaturated soils [6]. The correct description of these curves is of primary importance when the hydromechanical coupling of unsaturated soils is considered. Hydromechanical coupling refers to the fact that shear strength and volumetric behavior depend not only on the applied stress and suction, but also on the degree of saturation (S_r) of the material [23]. Some methods for the determination of the hydraulic parameters through the fitting of the SWRC with numerical models have been derived. However, in some cases the adjustment of the SWRC cannot be performed accurately, so that the determination of hydraulic parameters is also inaccurate. That is why Fredlund and Rahardjo[7] specify that the reliability of hydraulic parameters depends not only on the equations employed to simulate the SWRC but also on the accuracy to adjust the SWRC.

This study presents an alternative method for determining the hydraulic parameters of unsaturated soils directly through the experimental points of the SWRC. In consequence, the hydraulic parameters are not influence by the accuracy of a fitting process.

3. DESCRIPTION OF THE PROCEDURE

In many cases, the design and construction of buildings on expansive and collapsible soils [9], do not take into account the relationship between hydraulic and mechanical behavior [24,25]. These soils undergo cycles of swelling and shrinkage due to seasonal changes in water content. In that sense, it is important to measure the hydraulic conductivity and diffusion coefficient during wetting-drying cycles in order to assess the influence of hysteresis on these parameters. Therefore, the study of the hydraulic conductivity and diffusion coefficient of deformable unsaturated soils is of great scientific and commercial interest.

At its current state, the model evaluates the hydraulic conductivity (k) and the diffusion coefficient (D) from the primary wetting and drying SWRC. However, the model can be easily extended to simulate wetting-drying cycles.

The proposed procedure develops in three stages and requires the following data: the saturated permeability (k_s), the dry density of the soil (ρ_d), the number of experimental points of the SWRC for each path (n_d and n_w), the saturated water content (θ_s), the residual water content (θ_r), and the number of intervals (M) in which the water content range is divided. M also represents the number of points for which the values of suction, hydraulic conductivity, diffusion coefficient and degree of saturation will be obtained. Fig.3 shows the flowchart of the computer program to determine the hydraulic conductivity and diffusion coefficient of an unsaturated soil through the polynomial adjustment of the SWRC.

During the first stage, the experimental values of matric suction and degree of saturation during drying and wetting (ψ_{di} , Sr_{di} and ψ_{wi} , Sr_{wi} , respectively) are introduced. Then, the SWRC graph is constructed.

With the use of the saturated (θ_s) and residual (θ_r) water contents of each path, it is possible to determine the relative volumetric water content (Θ) in the form:

$$\Theta = \frac{(\theta - \theta_r)}{(\theta_s - \theta_r)} \quad (12)$$

Then, it is possible to plot the SWRC in terms of matric suction or the relative volumetric water content.

These experimental points are stored in a table, where the degree of saturation (Sr), the volumetric water content (θ) and soil suction (ψ) are recorded in different columns. With this table, a polynomial of degree n-1 is determined for each path of the SWRC. Intermediate points are obtained by variable increment interpolation.

In the second stage, the hydraulic conductivity is determined using Equation (5). To this purpose the retention curve can be divided into a number of constant increments (M) of water content using the polynomial form of the SWRC. It is worth mentioning that the accuracy of the method increases with the number of experimental points.

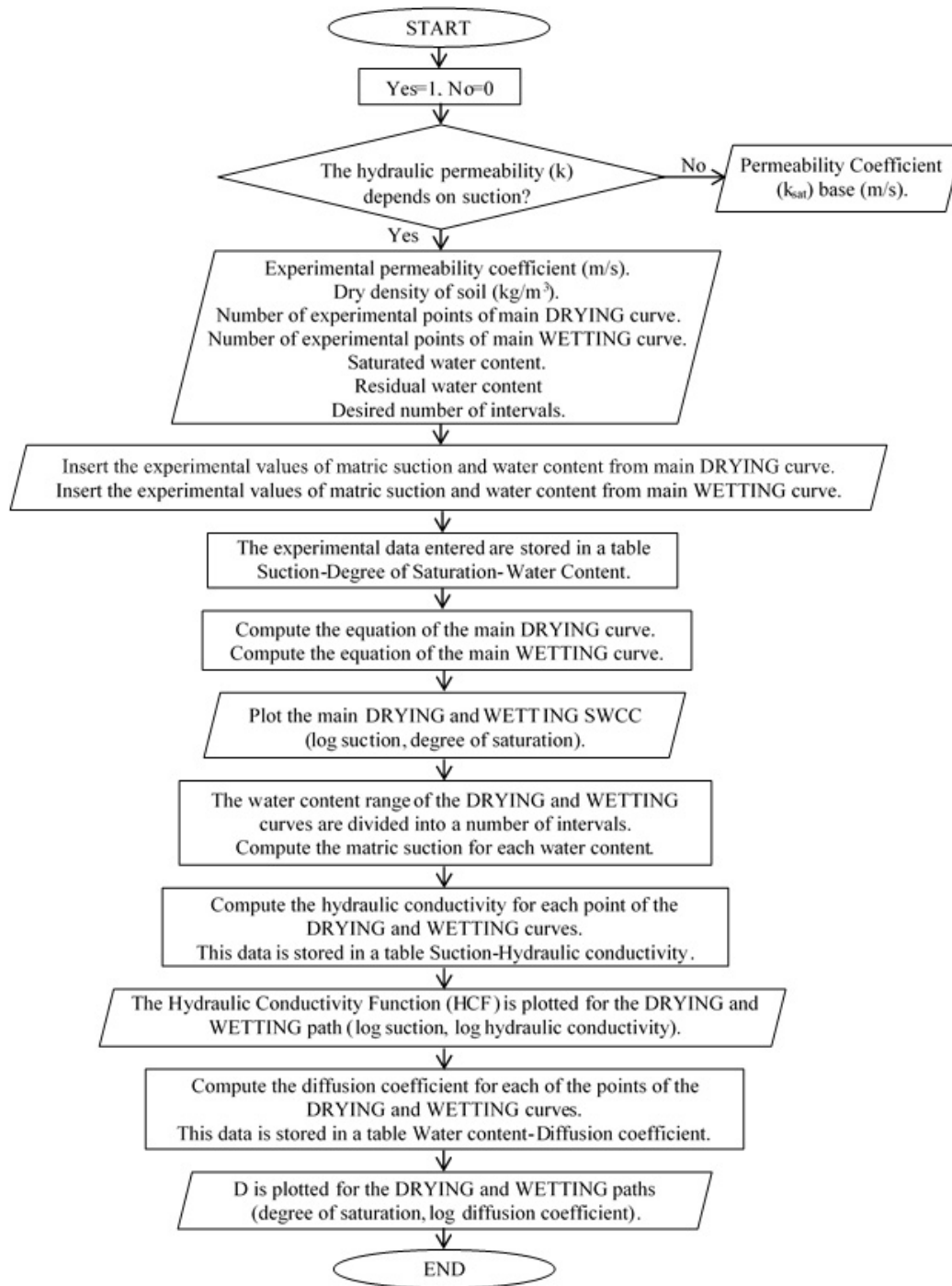


Fig.3. Flow chart for the estimation of the hydraulic parameters of unsaturated soils.

The third and last stage involves the estimation of the diffusion coefficient as proposed by Equation (8). The slope (C) can be determined using the polynomial equation of the SWRC. To this purpose, the slope at the required value of suction is computed (Equation (9)) considering a very small increment of water content (0.001 θ). Thus, for each one of the sections in which the SWRC has been divided there will be an associated water content, a value of suction, a hydraulic conductivity and a coefficient of diffusion.

The process ends with the plot of the degree of saturation or matric suction with the diffusion coefficient (D) for both paths wetting and drying.

4. NUMERICAL COMPARISONS

In order to verify the capabilities of this procedure to determine the hydraulic parameters of the soil through the use of experimental SWRCs, the results reported by some researchers have been used. The results obtained with the proposed method were compared with the experimental data and also with the results obtained from two methods commonly used: the van Genuchten[5] and the Fredlund and Rahardjo [7] methods.

Gonzalez and Adams (1980) reported the SWRCs of a sand using the volumetric pressure plate (see Fig.4). They also reported the hydraulic conductivity obtained with the steady-state method in drying [7]. With the method proposed in this paper, it is possible to determine the hydraulic conductivity for both paths (see Fig.5) and define if the phenomenon of hysteresis has influence on the hydraulic conductivity, as described by Lu and Likos[16]. The results were compared with those obtained from other methods (see Fig.5).

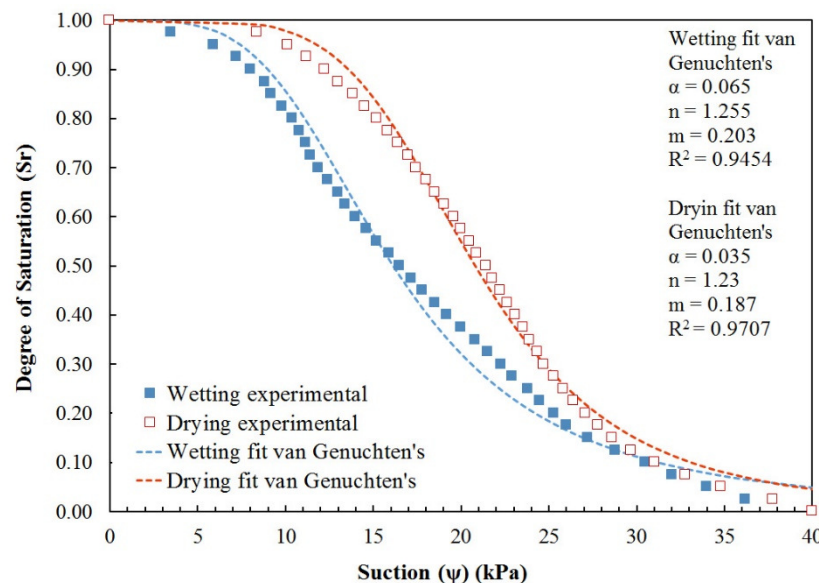


Fig.4. Experimental SWRCs and van Genuchten's fit for a sand reported by Gonzalez and Adams (1980) [7].

Fig.5 shows the comparison of the hydraulic conductivity using different procedures. For the application of the procedure described in this study, the number of intervals $M=40$ corresponds to the number of experimental data. It can be noticed that the results obtained with the method proposed here are closer to the experimental data than other methods (van Genuchten and Fredlund). It is worth mentioning that all the three models show the phenomenon of hysteresis for wetting - drying paths.

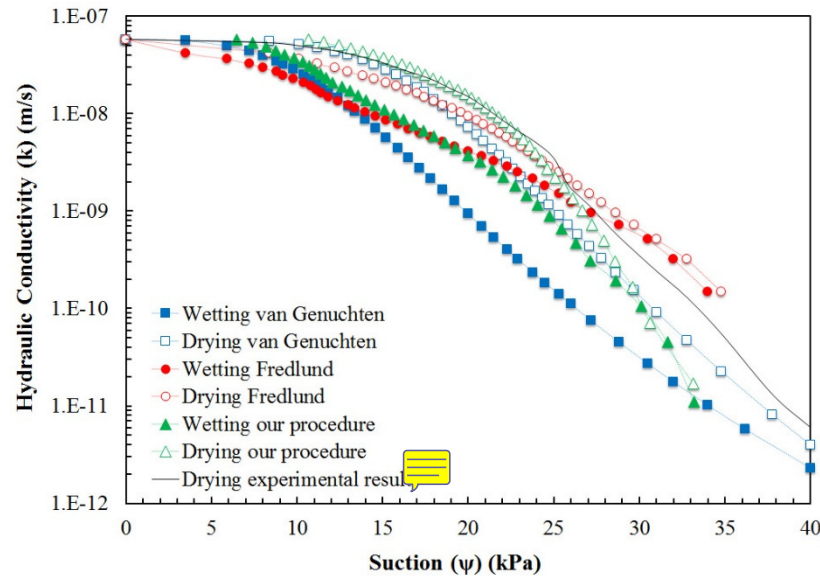


Fig.5. Comparison of the hydraulic conductivity obtained from different methods, using the experimental SWRC reported by Fredlund and Rahardjo[7].

Fig.6 shows the comparison of the diffusion coefficient obtained with the different methods. The results are shown for both paths using the equations proposed by van Genuchten[5]. The result obtained with the van Genuchten's method shows a rather large range of values for the diffusion coefficient (1×10^{-7} and $2.5 \times 10^{-14} \text{ m}^2/\text{s}$) when compared with the range obtained from the other methods. The results obtained with the procedure outlined here are close to those obtained by Fredlund and Li especially for large values of the degree of saturation. While the phenomenon of hysteresis is present in both hydraulic parameters, it is less noticeable in the diffusion coefficient, especially for the Fredlund and Li method and the one proposed here.

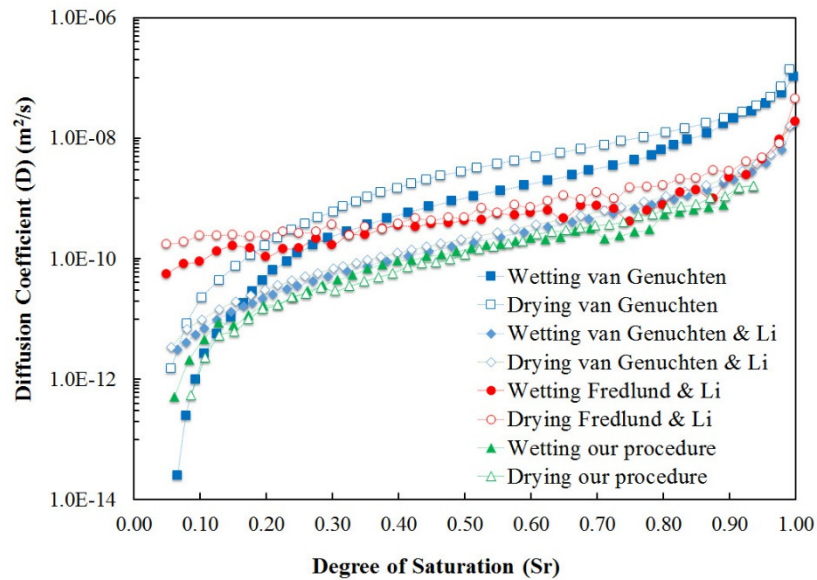


Fig.6. Comparison of diffusion coefficient obtained from different methods, using the experimental SWRC reported by Fredlund and Rahardjo[7].

In order to show the effect of the fitting process of the SWRC on the determination of the hydraulic conductivity, the SWRC of a sandy soil reported by Lu and Likos[16], obtained from the steady state method, is adopted here (see Fig.7).

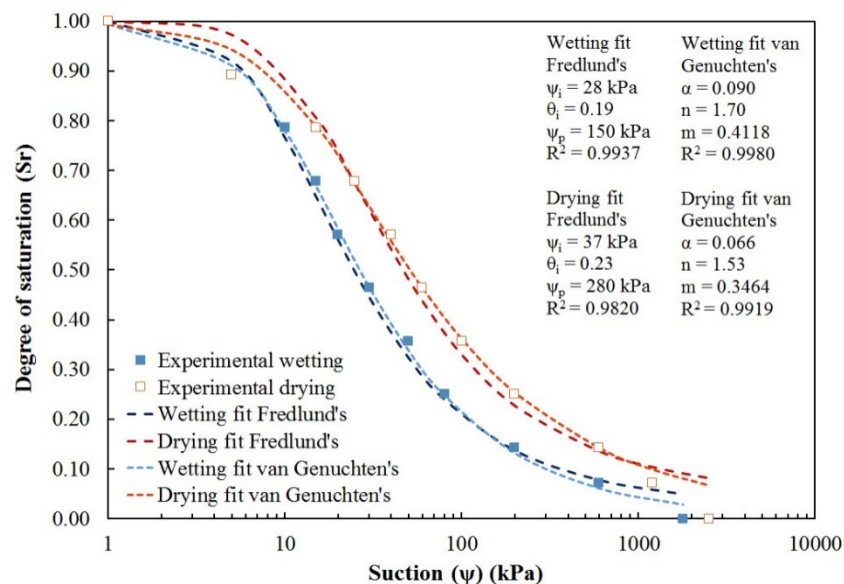


Fig.7. Experimental SWRC of a silty soil reported by Lu and Likos[16].

The SWRCs were fitted using two of the most popular methods in the literature: the Fredlund and Xing [6] method and the van Genuchten's method [5]. Both methods show high correlations however, a better fit

was obtained with van Genuchten's method [5], as can be seen in Fig.7. The hydraulic conductivity was determined using these two methods in addition to the method proposed herein. The results are compared with the experimental points reported by Lu and Likos[16] for both the wetting and the drying paths. In this process, the hydraulic conductivity was determined using 27 sections corresponding to the number of the experimental data. Fig.8 shows that differences between these methods increase when the correlation of the fitted SWRCs decreases. Therefore, low correlations between experimental data and numerical curves results in less precision for the hydraulic parameters.

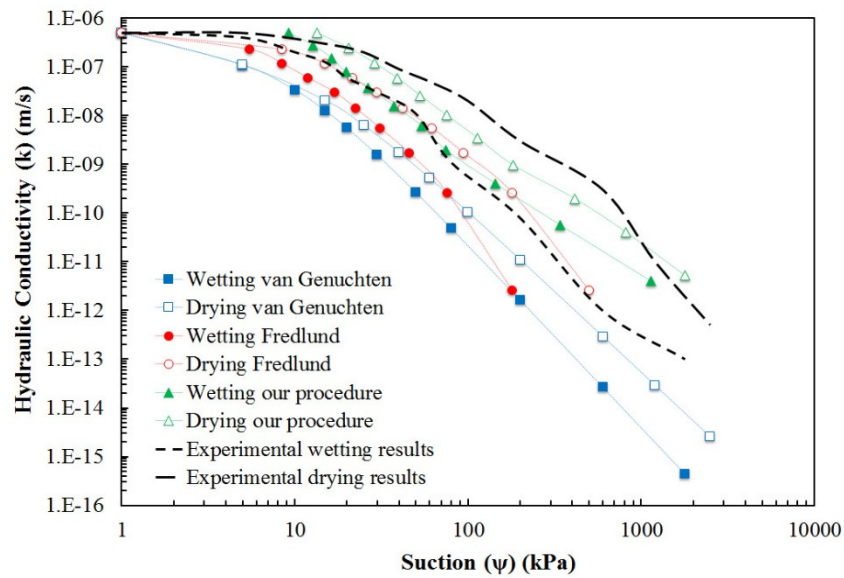


Fig.8. Comparison of hydraulic conductivity obtained from different methods using the SWRC reported by Lu and Likos[16].

It is noteworthy that the results obtained with the method proposed herein are closer to the experimental results than those obtained with the other methods. This shows that our method is an improved tool for predicting the hydraulic parameters through the experimental points of the SWRCs.

The comparison of the diffusion coefficient obtained from different methods is shown in Fig.9. It can be observed that our results are very close to those obtained with the van Genuchten and Li methods. The advantage of our procedure comes from the fact that the polynomial function ensures a 100% correlation with experimental data.

Although the adjustments of the SWRC with experimental data using other methods may have strong correlations ($R^2 > 0.9500$), the hydraulic parameters show larger dispersion with the experimental points when compared with the procedure proposed in this paper. The explanation to these results is provided by Parent et al. [8], who stated that the reliability of hydraulic parameters depends on both the accuracy of the fitting process and the equations used to simulate the SWRC. The procedure presented herein, avoids the use of a predetermined equation as well as the fitting process. Therefore, it is an excellent tool for the determination of the hydraulic parameters of unsaturated soils. The main features of our procedure are: a) It avoids the fitting process of experimental points with a predetermined equation for the SWRC, b) our procedure ensures a 100% correlation between numerical and experimental points, c) this correlation increases the accuracy of the hydraulic parameters obtained for unsaturated soils.

413

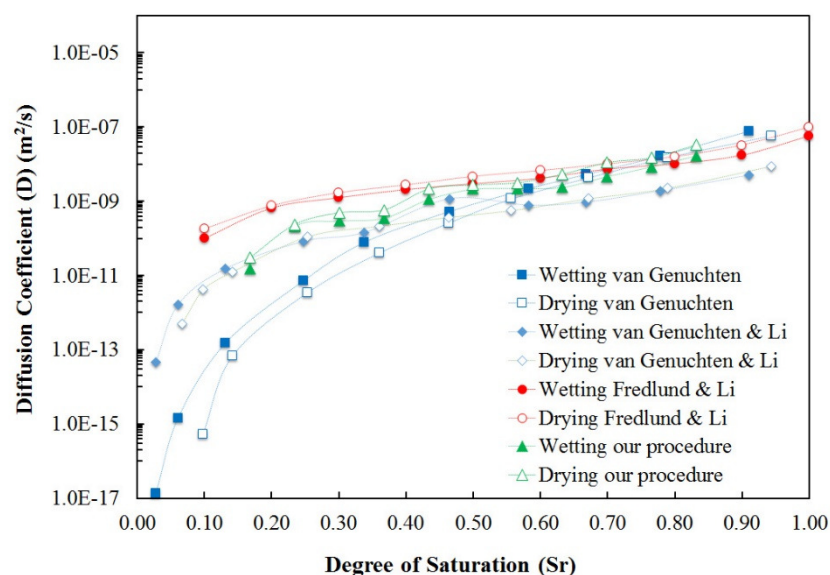


Fig.9. Comparison of diffusion coefficient obtained from different methods using the SWRC reported by Lu and Likos[16].

5. CONCLUSIONS

The procedure described in this paper uses the experimental points of the SWRCs to establish a polynomial equation from which the hydraulic parameters of the soil at wetting and drying can be obtained. This allows the modeling of the hydraulic behavior of soils under wetting-drying cycles. Therefore, it can be helpful for the correct estimation of volumetric changes on shrink-expansive soils. This method shows the following advantages over other existing methods: it ensures a correlation of 100% between experimental and numerical curves. It does not require a predetermined equation for the SWRC. The fitting process between the experimental and the numerical SWRC is avoided. Finally, the comparisons of the hydraulic conductivity obtained with the different methods and the experimental results allow concluding that the method proposed herein provides better results.

REFERENCES

1. López-Lara T, Hernández-Zaragoza JB, Horta J, Rojas E, MinorAG. Physicochemical analysis of unstable clays. Proceedings XVII National Meeting of Soil Mechanics. Mexican Society of Geotechnical Engineering, Cancun, Quintana Roo, Mexico, 2012: 261-268. In Spanish.
2. King K. Expansive soil and expansive clay. Accessed 28 January 2017. Available: <http://geology.com/articles/expansive-soil.shtml>.
3. Sukanta C. Numerical modeling for long term performance of soil-bentonite cut-off wall in unsaturated soil zone. M. Sc. Thesis. Louisiana State University and Agricultural and Mechanical College, 2009.
4. Arroyo H, Rojas E, Pérez-Rea ML, Horta J, Arroyo J. Simulation of the shear strength for unsaturated soils. Comptes Rendus Mécanique. 2013;341:727-742.
5. van Genuchten MT. A closed-form equation for predicting the hydraulic conductivity of unsaturated soils. Soil Science Society of America. 1980;44:892-898.
6. Fredlund DG, Xing A. Equations for the soil-water characteristic curve. Canadian Geotechnical Journal. 1994;31:91-99.

7. Fredlund DG, Rahardjo H. Soil mechanics for unsaturated soils, New York: Wiley-Interscience, 1993:124-135.
8. Parent SE, Abdolazadeh AM, Nuth M, Cabral AR. Hydraulic conductivity and water retention curve of highly compressible materials from a mechanistic approach through phenomenological models, Developments in hydraulic conductivity research, InTech, Chapter 3, 2011.
9. Puppala JP, Manossuthikij T, Chittoori CS. Swell shrinkage characterization of unsaturated expansive clays from Texas. Engineering Geology. 2013;164:187-194.
10. Jahangir E, Deck O, Masroui F. An analytical model of soil-structure interaction with swelling soils during droughts. Computers and Geotechnics. 2013;54:16-32.
11. Basement systems. Foundation settlement & structural damage. Accessed 2 February 2017. Available: <http://www.basementsystems.ca/foundation-repair/wall-cracks-repair/settlement.html>.
12. Home general contractor. Fixing a home's foundation. Accessed 2 February 2017. Available: <http://homegeneralcontractor.com/fixing-a-homes-foundation/>.
13. Pliego M. Capillary penetration dynamics in complex structures. Ph.D. Thesis, Universidad Autónoma de Querétaro, 2012.
14. Meza V. Partially saturated soils, from research to university lecture. Earth Sciences Bulletin. 2012; 31: 23-28.
15. Galaviz-González R, Pérez-Rea ML, Arroyo H. Prediction models of water retention curves for partially saturated soils. Proceedings 9th International Congress of Engineering. Autonomous University of Querétaro. Santiago de Querétaro, Mexico. 2013. Spanish.
16. Lu N, Likos WJ. Unsaturated soil mechanics, New Jersey: John Wiley & Sons. Inc., 2004.
17. Mualem Y. Hydraulic conductivity of unsaturated soils: Prediction and formulas. Invited review, A. Klute, Ed. Chapter 31, Monograph No. 9 of the ASA: Methods of Soil Analysis, Part 1, 1986.
18. Thakur AB. Determination of diffusion coefficient through laboratory tests and analytically validating it using empirical relations for unsaturated soils. M. Sc. Thesis. Texas A&M University, 2005.
19. Aubeny C, Lytton RL, Tang D. Simplified analysis of unsteady moisture flow through unsaturated soils. 82nd Annual Meeting of Transportation Research Board, 2003.
20. Li J. Two dimensional simulation of a stiffened slab on expansive soil subject to a leaking underground water pipe. Fourth International Conference on Unsaturated Soils, 2006:2098-2109.
21. Rojas E, Chávez O. Volumetric behavior of unsaturated soils. Canadian Geotechnical Journal. 2013;50:209-222.
22. Han-Chen H, Yih-Chi T, Chen-Wuing L, Chu-Hui C. A novel hysteresis model in unsaturated soil. Hydrological Processes. 2005;19:1653-1665.
23. Horta J, Rojas E, Pérez-Rea ML, López-Lara T, Zaragoza JB. A random solid-porous model to simulate the retention curves of soils. Int. J. Numer. Anal. Methods Geomech. 2013;37:932-944.
24. Likos WJ. Pore-scale model for water retention in unsaturated sand. Proceedings of the 6th International Conference on Micromechanics of Granular media. American Institute of Physics. 2009. English.
25. Zhou C. Modelling of suction effect on fabric yielding and kinematic hardening of reconstituted soils. In: Unsaturated Soils – Theoretical & Numerical Advances in Unsaturated Soil Mechanics. 2009:629-634.