

Complete synchronization of two bidirectionally coupled piecewise linear chaotic systems perturbed by Poisson noise

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ABSTRACT

We establish the sufficient conditions of complete synchronization with probability 1 in the coupled bidirectional piecewise linear chaotic systems with common or different Poisson noises and the positive effects of Poisson noises on the complete synchronization will be found based on the proposed theoretical results. We compare the effects of Poisson noise on the complete synchronization with the case of Gaussian white noise, and then we claim Poisson noise can induce complete synchronization easier than Gaussian white noise in the sense of noise strength. Furthermore, a numerical example is presented to show the excellent agreement between theoretical and numerical analysis.

Keywords: Complete synchronization, Chaotic systems, Poisson noises.

1. INTRODUCTION

The synchronization in chaotic systems plays a key role in many different fields, especially the complete synchronization of bidirectionally coupled piecewise linear chaotic systems with random perturbations [1-9].

Recently, noise-induced complete synchronization was considered since multiplicative and/or additive noises are ubiquitous in natural and synthetic systems [10-12]. One of the remarkable discoveries is that sufficient conditions of complete synchronization between two unidirectionally coupled chaotic systems with Gaussian white noises have been presented in [13, 14]. Later, Xiao and his co-workers analyzed the effects of Gaussian white noises on the complete synchronization of bidirectionally coupled piecewise linear chaotic systems which can describe a number of natural and synthetic systems, such as laser systems, electronic circuits, chemical and biological systems [15-17], and they found that Gaussian white noises had positive effects on the complete synchronization of such systems [4, 6].

Gaussian noise can be used to model different kinds of random fluctuations [18, 19, 28-30], however, it will be not appropriate while there exist large external and/or internal fluctuations in some practical situations and environments [20-26]. Fortunately, Poisson noise can depict random jumps which will converge to Gaussian distribution under general conditions, and it is a natural model for large external and/or internal noise sources such as storage systems,

economic systems, biological systems and so on. Xu and Feng et al. have obtained some good results in biological systems with jumps [25-26]. Meanwhile, the behaviors of the dynamical systems with Poisson noises are very different from the stochastic systems with Gaussian white noise [18,19,31-33]. Therefore, it is important to investigate the dynamical behaviors, just like the complete synchronization phenomenon for bidirectionally coupled piecewise linear chaotic systems with Poisson noises.

Given the limitations of Gaussian white noises and unique nature of the Poisson noises, we in this paper investigate the complete synchronization phenomena for bidirectionally coupled piecewise linear chaotic systems with Poisson noises to find the positive effects of noises, and the advantages of Poisson noises to induce complete synchronization will be considered. Later we will observe from the illustrative examples that numerical results agree well with theoretical analysis.

2. PRELIMINARIES

The following definitions apply:

$C_1(R^k)$ - the space of the functions with continuous first partial derivatives on R^k .

$C_b(R^k)$ - the space of bounded continuous functions on R^k .

$C_b^1(R^k)$ - the subspace of $C_b(R^k)$ constituted by the functions with continuous first partial derivatives.

$C_1^0(R^k \times R_+)$ - the class of functions $V(x, t)$ which have continuous first partial derivatives on $R^k \times R_+$ except possibly at the point $x = 0$.

2.1 The solution of Poisson stochastic differential equations (SDEs)

Consider SDEs of the following form:

$$dX(t) = b(X(t), t)dt + \sigma(X(t-), t)dP(t), X(t_0) = X_0, \quad (1)$$

where

(C1) $X(t)$ is a process with values in R^k ,

$$b(x, t): R^k \times R_+ \rightarrow R^k; \sigma(x, t): R^k \times R_+ \rightarrow R^k \times R^m,$$

The coefficients $b(x, t)$ and $\sigma(x, t)$ are Borel functions and satisfy the usual conditions of the existence and uniqueness theorem:

$\forall x, y \in R^k, t \geq t_0, K_1, K_2$ are constants,

$$|b(x, t) - b(y, t)| + |\sigma(x, t) - \sigma(y, t)| \leq K_1 |x - y|,$$

$$|b(x, t)|^2 + |\sigma(x, t)|^2 \leq K_2 (1 + |x|^2).$$

(C2) $(P(t))_{t \geq 0}$ is a m-dimensional Poisson process with parameter $\lambda = (\lambda_1, \lambda_2, \dots, \lambda_m)^T$ and $P(0) = 0$ with probability 1.

(C3) Random variable X_0 is independent of $(P(t))_{t \geq 0}$ such that $E(X_0^2) < \infty$. Then we have the following conclusions for the solution process of Eq.(1) [32]:

(i) There exists a unique adapted solution process $X(t, X_0, t_0)$ of Eq.(1), which has right continuous sample paths with left limits and $\forall T > t_0$,

$$\sup_{t \in [t_0, T]} E \left[|X(t, X_0, t_0)|^2 \right] < \infty.$$

(ii) Ito's formula

Let v_i^m be a m -dimensional vector with 1 in the i th component and zeros elsewhere and $\{X(t)\}$ is the solution process of Eq.(1), then $\forall V \in C_1(R^k \times R_+)$,

$$V(X(t), t) = V(X(s), s) + \int_s^t \frac{\partial V(X(u), u)}{\partial u} du + \sum_{i=1}^k \int_s^t \frac{\partial V(X(u), u)}{\partial x_i} b_i(X(u), u) du \\ + \sum_{i=1}^m \int_s^t \left[V(X(u-), u) + \sigma(X(u-), u) v_i^m - V(X(u-), u) \right] dP_i(u).$$

(iii) Let $\tilde{A}$ be the extended weak infinitesimal operator of the process

$\{X(t, X_0, t_0)\}$, and $\forall V \in C_b^1(R^k \times R_+)$, we define the operator D :

$$DV(x, t) \equiv \frac{\partial V(x, t)}{\partial t} + \sum_{i=1}^k b_i(x, t) \frac{\partial V(x, t)}{\partial x_i} + \sum_{i=1}^m \lambda_i \left[V(x + \sigma(x, t) v_i^m, t) - V(x, t) \right],$$

then $\tilde{A} = D$ [31-33].

2.2 Some Basic Concepts

Definition 2.1. A function $V(x, t)$ is said to be positive definite (in Lyapunov's sense) in

$R^k \times R_+$, $S_h = \{x \in R^k : |x| < h, h > 0\}$, if

(C4) $V(0, t) = 0, t \in R_+$,

(C5) $V(x, t) \geq W(x), x \in S_h, t \in R_+$, where $W(0) = 0, x \neq 0, W(x) > 0$.

Obviously, it is negative definite if $-V(x, t)$ is positive definite.

Definition 2.2. A function $V(x, t)$ is said to be possessed of an infinitesimal upper limit, if

$$\lim_{x \rightarrow 0} \sup_{t > 0} V(x, t) = 0.$$

Definition 2.3. The trivial solution $X(t) \equiv 0$ of (1) is said to be

(D1) stochastically stable, if for any $t_0 \geq 0, \varepsilon > 0$,

$$\lim_{X_0 \rightarrow 0} P \left\{ \sup_{t \geq t_0} |X(t, X_0, t_0)| > \varepsilon \right\} = 0.$$

(D2) global stochastic asymptotically stable, if it is stochastically stable and also for $t_0 \geq 0, X_0 \in R^k$,

$$P \left\{ \lim_{t \rightarrow \infty} X(t, X_0, t_0) = 0 \right\} = 1.$$

Lemma1. (Global Stochastic Asymptotic Stability Theorem) Suppose that there exists a function $V(x, t)$ which satisfies the following conditions:

105 (C6) $\forall V(x, t) \in C_1^0(R^k \times R_+)$ is a positive definite function with an infinitesimal upper limit,

106 (C7) $DV(x, t), (x, t) \in R^k \times R_+$ is a negative definite function,

107 then the solution to Eq.(1) is global stochastic asymptotically stable [32] .

108

109 3. THE MAIN RESULTS

110

111 In this paper, consider two bidirectionally coupled n -dimensional piecewise linear chaotic
112 systems as follows

$$\begin{aligned} 113 \quad \dot{x}_1 &= f(x_1) + c(x_2 - x_1), \\ 114 \quad \dot{x}_2 &= f(x_2) + c(x_1 - x_2), \end{aligned} \tag{2}$$

115 where c is a positive constant describing the coupling strength between two subsystems,
116 $x_i = (x_{i1}, x_{i2}, \dots, x_{in})^T \in R^n$ ($i=1, 2$) are state vectors, and
117 $f = (f_1, f_2, f_3, \dots, f_n)^T : R^n \rightarrow R^n$ is a piecewise linear vector function describing the
118 dynamics of a single oscillator. Moreover, we also require the piecewise linear
119 function f satisfying the following assumption:

120 **Assumption:** For any $x = (x_1, x_2, \dots, x_n)^T \in R^n, y = (y_1, y_2, \dots, y_n)^T \in R^n$, there exists a
121 positive constant l satisfying

$$122 \quad (x - y)^T [f(x, t) - f(y, t)] \leq l(x - y)^T (x - y). \tag{3}$$

123)

124 The above inequality (3) is usually called the global Lipschitz condition, and l is called the
125 Lipschitz constant. For the continuous smooth chaotic systems, it is difficult to find constant
126 l , such as Lorenz system and Rössler system. While we can find the constant l by
127 inequality proof for some well-known piecewise linear chaotic systems, such as the famous
128 Chua's circuits [15], the Rössler-like system [16], the cellular neural network (CNN) neural
129 model [17], and so on.

130 Eq. (2) represents a classical model of two bidirectionally coupled oscillators, and now we
131 introduce the noisy model as:

$$\begin{aligned} 132 \quad \dot{x}_1 &= f(x_1) + (c + d\xi_1(t))(x_2 - x_1), \\ 133 \quad \dot{x}_2 &= f(x_2) + (c + d\xi_2(t))(x_1 - x_2), \end{aligned} \tag{4}$$

133)

134 where positive constant d denotes the noise strength and ξ_1, ξ_2 are Poisson noises.

135 We in this paper will study system (4) in two different cases:

136 (I) $\xi_1(t) = \xi_2(t)$, i.e. two subsystems are affected by common Poisson noise.

137 (II) $\xi_1(t), \xi_2(t)$, are statistically independent, i.e. two subsystems are affected by different
138 Poisson noises.

139 **3.1 Two bidirectionally coupled piecewise linear chaotic systems perturbed by** 140 **common Poisson noises**

141 In this part, we consider the effect of common noises on the complete synchronization with
142 probability 1. Actually, the two coupled systems (4) are said to achieve complete
143 synchronization if $x_1 = x_2 = m(t)$, $t \rightarrow \infty$. Here $m(t) \in R^n$ is a solution of a single oscillator
144 with $\dot{m} = f(m)$. We usually call $m(t)$ as a synchronization manifold, and here we will use
145 $M(t) = (1/2)(x_1 + x_2)$ instead of synchronization manifold $m(t)$ [27] with

$$146 \quad \dot{M}(t) = (1/2)(f(x_1) + f(x_2)) \square G(x). \quad (5)$$

147)

148 Define the synchronization errors as $e_i(t) = x_i(t) - M(t)$ ($i = 1, 2$), then one gets the error
149 dynamic equations

$$150 \quad \begin{aligned} \dot{e}_1 &= f(x_1) - G(x) + c(e_2 - e_1) + d\xi(t)(e_2 - e_1), \\ \dot{e}_2 &= f(x_2) - G(x) + c(e_1 - e_2) + d\xi(t)(e_1 - e_2), \end{aligned} \quad (6)$$

151)

152 where $e_i(t)$ ($i = 1, 2$) will satisfy:

$$153 \quad e_1 + e_2 = 0. \quad (7)$$

154)

155 The above Eq. (6) can be rewritten as a matrix form

$$156 \quad \dot{E}(t) = F(x) + cC(E) + H(E)\dot{P}(t). \quad (8)$$

157)

158 Here $E(t) = \begin{pmatrix} e_1 \\ e_2 \end{pmatrix}$, $F(x) = \begin{pmatrix} f(x_1) - G(x) \\ f(x_2) - G(x) \end{pmatrix}$, $C(E) = \begin{pmatrix} e_2 - e_1 \\ e_1 - e_2 \end{pmatrix}$, $H(E) = d \begin{pmatrix} e_2 - e_1 \\ e_1 - e_2 \end{pmatrix}$,
159 and $\dot{P}(t) = \xi(t)$.

160 **Theorem 1.** If the coupling strength c , noise strength d , and constants l, λ satisfy

$$l - 2c + \lambda(2d^2 - 2d) < 0, \quad (9)$$

then the coupled system (4) perturbed by common Poisson noises achieves complete synchronization with probability 1.

Proof: The Eq. (8) can be expressed as

$$dE(t) = (F(x) + cC(E))dt + H(E)dP(t). \quad (10)$$

According to **Lemma1**, we choose the positive function

$$V(E) = (1/2)(E^T(t)E(t)), \quad (11)$$

where $E^T(t)$ denotes the transpose of $E(t)$.

Using the infinitesimal operator of SDEs driven by Poisson process to Eq. (11) along with system (10), we have

$$DV(E(t)) = [\partial V(E(t))/\partial E(t)][F(x) + cC(E)] + \lambda[V(E(t) + H(E)) - V(E(t))],$$

where the derivatives of function [4] $V(E(t))$ are $\partial V(E(t))/\partial E(t) = E^T(t)$.

Due to Assumption (3) and Eq. (7), then we obtain

$$\begin{aligned} DV(E(t)) &= e_1^T [f(x_1) - f(m(t))] + e_2^T [f(x_2) - f(m(t))] + c[e_1^T(e_2 - e_1) + e_2^T(e_1 - e_2)] \\ &\quad + (1/2)\lambda \left[\left((E(t) + H(E))^T (E(t) + H(E)) \right) - E^T(t)E(t) \right] \\ &= e_1^T [f(x_1) - f(m(t))] + e_2^T [f(x_2) - f(m(t))] + c[e_1^T(e_2 - e_1) + e_2^T(e_1 - e_2)] \\ &\quad + (1/2)\lambda [de_1^T(e_2 - e_1) + de_2^T(e_1 - e_2) + d(e_2 - e_1)^T e_1 + d(e_1 - e_2)^T e_2 \\ &\quad + d^2(e_2^T - e_1^T)(e_2 - e_1) + d^2(e_1 - e_2)^T(e_1 - e_2)] \\ &= [l - 2c + \lambda(2d^2 - 2d)]E^T(t)E(t). \end{aligned}$$

Now, we employ the condition (9) and the positive definite function $E^T(t)E(t)$ to yield that

$DV(E(t))$ is a negative definite function. Thus, by **Lemma 1**, one can easily conclude the

error equation (8) possesses a global unique solution denoted by $E(t, t_0, E_0)$ for any initial condition. Obviously, $E(t, 0, 0) \equiv 0$ is a trivial solution of error system (8). Then, in terms of **Definition 2.3**, one can easily obtain $\lim_{t \rightarrow \infty} \|E(t)\| = 0$, with probability 1, here, $\|\cdot\|$ stands for Euclidean norm. Namely, the synchronization errors $e_i(t) (i=1, 2)$ converge to zero as $t \rightarrow \infty$ with probability 1.

Consequently, we can conclude the coupled system (4) with common Poisson noises achieves complete synchronization with probability 1.

This completes the proof of **Theorem 1**. □

Remark 1.

Now we wish to present some discussions on the advantages of Poisson noise compared with the case of Gaussian noise based on our proposed results. Xiao and Xu have presented that the two bidirectionally coupled piecewise linear chaotic systems driven by common Gaussian noise were synchronized with probability 1, if the noise strength d satisfies $l - 2c - 2d^2 \leq 0$ ($d \geq \sqrt{(l - 2c)/2}$) in Ref. 4. From their expression, we can see that, with the increase of noise strength, the complete synchronous effect will become much better. However, in our paper we conclude the same systems perturbed by common Poisson noise are synchronized with probability 1, if the noise strength d satisfies $l - 2c + \lambda(2d^2 - 2d) < 0$, namely

$$\left(1 - \sqrt{1 - (l - 2c)/\lambda}\right)/2 < d < \left(1 + \sqrt{1 - (l - 2c)/\lambda}\right)/2 \leq 1.$$

One can find, for fixed l, c , noise strength d in the two coupled systems perturbed by common Poisson noise is bounded. Our result seems rational since noise intensity should be very small in practical problems. Therefore, to some extent, we can claim common Poisson noise is better than common Gaussian case in the sense of comparison of noise strengths to make the systems achieve complete synchronization.

3.2. Two bidirectionally coupled piecewise linear chaotic systems perturbed by different Poisson noises

In this part we will study the effects of different Poisson noises on the occurrence of complete synchronization with probability 1. Similar to common Poisson noise case, we introduce the synchronization errors $e_i(t) (i=1, 2)$, then one can get error equations

$$\begin{aligned} \dot{e}_1 &= f(x_1) - G(x) + c(e_2 - e_1) + (1/2)d(e_2 - e_1)[\xi_1(t) + \xi_2(t)], \\ \dot{e}_2 &= f(x_2) - G(x) + c(e_1 - e_2) + (1/2)d(e_1 - e_2)[\xi_1(t) + \xi_2(t)], \end{aligned} \tag{12}$$

212 The above Eq. (11) can be written as a matrix form

213 $\dot{E}(t) = F(x) + cC(E) + H(E)\dot{P}(t).$ (13
214)

215 Here, the definitions of $E(t)$, $F(t)$ and $C(t)$ are same as in the Sec.3.1,

216 $H(E) = (1/2)d \begin{pmatrix} e_2 - e_1 & e_2 - e_1 \\ e_1 - e_2 & e_1 - e_2 \end{pmatrix}, \dot{P}(t) = \begin{pmatrix} \xi_1(t) \\ \xi_2(t) \end{pmatrix}.$

217 **Theorem 2.** If the coupling strength c , noise strength d , and constants l, λ_1, λ_2 satisfy

218 $l - 2c + [(\lambda_1 + \lambda_2)/2](d^2 - 2d) < 0,$ (14
219)

220 then the coupled system (4) perturbed by different Poisson noises achieve complete
221 synchronization with probability 1.

222 **Proof:** Using the infinitesimal operator of SDEs driven by Poisson process to Eq. (11) along
223 with system (13), one obtains

$$\begin{aligned}
& DV(E(t)) \\
&= \partial V(E(t)) / \partial E(t) [F(x) + cC(E)] + \sum_{i=1}^2 \lambda_i [V(E(t) + H(E)v_i^2) - V(E(t))] \\
&= e_1^T [f(x_1) - f(m(t))] + e_2^T [f(x_2) - f(m(t))] + c [e_1^T (e_2 - e_1) + e_2^T (e_1 - e_2)] \\
&\quad + (1/2) \lambda_1 \left[\left((E(t) + H(E)(1 \ 0)^T)^T (E(t) + H(E)(1 \ 0)^T) \right) - E^T(t)E(t) \right] \\
&\quad + (1/2) \lambda_2 \left[\left((E(t) + H(E)(0 \ 1)^T)^T (E(t) + H(E)(0 \ 1)^T) \right) - E^T(t)E(t) \right] \\
224 \quad &= e_1^T [f(x_1) - f(m(t))] + e_2^T [f(x_2) - f(m(t))] \\
&\quad + c [e_1^T (e_2 - e_1) + e_2^T (e_1 - e_2)] \\
&\quad + (1/2) \lambda_1 \left[\left((d/2)(e_2 - e_1)^T + e_1^T \right) \left((d/2)(e_2 - e_1) + e_1 \right) \right. \\
&\quad \left. + \left((d/2)(e_1 - e_2)^T + e_2^T \right) \left((d/2)(e_1 - e_2) + e_2 \right) - (e_1^T e_1 + e_2^T e_2) \right] \\
&\quad + (d/2) \lambda_2 \left[\left(d/2(e_2 - e_1)^T + e_1^T \right) \left((d/2)(e_2 - e_1) + e_1 \right) \right. \\
&\quad \left. + \left((d/2)(e_1 - e_2)^T + e_2^T \right) \left((d/2)(e_1 - e_2) + e_2 \right) - (e_1^T e_1 + e_2^T e_2) \right] \\
&= [l - 2c + [(\lambda_1 + \lambda_2)/2](d^2 - 2d)] E^T(t)E(t).
\end{aligned}$$

225 Then for the coupling strength c , noise strength d , and constants l, λ_1, λ_2 , if the condition
226 (14) is satisfied, then according to **Lemma1**, the trivial solution of system (13) is global
227 stochastic asymptotically stable, and the synchronization errors $e_i (i=1, 2)$ converge to zero
228 as $t \rightarrow \infty$ with probability 1. So we can see that the presence of different Poisson noises
229 can really induce the complete synchronization.

230 This completes the proof of **Theorem 2**. $\square$

231 **Remark 2.**

232 Similar to the Remark 1, results in different Poisson noises case will be compared with those
233 in Ref. 4. From Ref. 4, they presented the conditions of noise strength as $d \geq \sqrt{l - 2c}$ to
234 achieve complete synchronization, while our conditions of different Poisson noises to
235 achieve complete synchronization can be given as

$$236 \quad \left(1 - 2\sqrt{1 - 2(l - 2c)/(\lambda_1 + \lambda_2)} \right) / 2 < d < \left(1 + 2\sqrt{1 - 2(l - 2c)/(\lambda_1 + \lambda_2)} \right) / 2 \leq 2.$$

237 The above expressions show that, in Ref. 4, noise intensity should take larger values to
238 observe the excellent effects of complete synchronization, and in our paper the small noise
239 strength will make the systems achieve complete synchronization due to the upper bound of

240 noise strength. Therefore, the systems seem much easier to achieve complete
 241 synchronization with different Poisson noises than the case of different Gaussian white
 242 noises with probability 1.

243 In addition, a numerical result will also illustrate our conclusions.

244 4. A NUMERICAL EXAMPLE.

245
 246 In this section, an example is provided and some numerical simulations are performed to
 247 verify the theoretical results. In order to measure the synchronization, we define the
 248 following quantities: $\text{error}(t) = |x_1 - x_2|$, and $x_i = (x_{i1}, x_{i2}, \dots, x_{in})^T, i = 1, 2$ the solution of
 249 Eq. (4).

250 **Example:** In this example we use a three-dimensional neural network [17] to describe the
 251 dynamics of a single oscillator, which can be described as

$$252 \quad \dot{x} = -Dx + Tg(x), \quad (15)$$

253 where $x = (x_1, x_2, x_3)^T \in R^3$ is the state vector, D is an identity matrix,

254 and $g(x) = (g_1(x), g_2(x), g_3(x))^T, g(s) = (|s+1| - |s-1|) / 2$,

$$255 \quad T = \begin{pmatrix} 1.25 & -3.2 & -3.2 \\ -3.2 & 1.1 & -4.4 \\ -3.2 & 4.4 & 1.0 \end{pmatrix}.$$

256 On the basis of the Ref. 23, system (15) has a double-scrolling chaotic attractor (FIG. 1).

257 It is easier to verify that $g(s)$ satisfies $|g(s_1) - g(s_2)| \leq |s_1 - s_2|$. For any $x \in R^3, y \in R^3$,
 258 we have

$$259 \quad \begin{aligned} & (x-y)^T (-Dx + Dy + Tg(x) - Tg(y)) \leq -(x-y)^T D(x-y) + |(x-y)^T T(g(x) - g(y))| \\ & \leq |(x-y)|^T (|T| - D) |x-y| \leq \lambda_m (x-y)^T (x-y), \end{aligned}$$

260 where

$$261 \quad |T| = \begin{pmatrix} 1.25 & 3.2 & 3.2 \\ 3.2 & 1.1 & 4.4 \\ 3.2 & 4.4 & 1.0 \end{pmatrix},$$

262 and $\lambda_m \approx 7.339$ is the maximum eigenvalue of $|T| - D$. So we can acquire the constant
 263 $l \approx 7.339$ in condition (3).

264 The main purpose of this paper is to study the effect of noise on the complete
 265 synchronization. So, in the numerical simulations, we let coupling strength $c=0$, i.e., two
 266 systems just bidirectionally is coupled by multiplicative noise.

267 **Case 1 . A common Poisson noises**

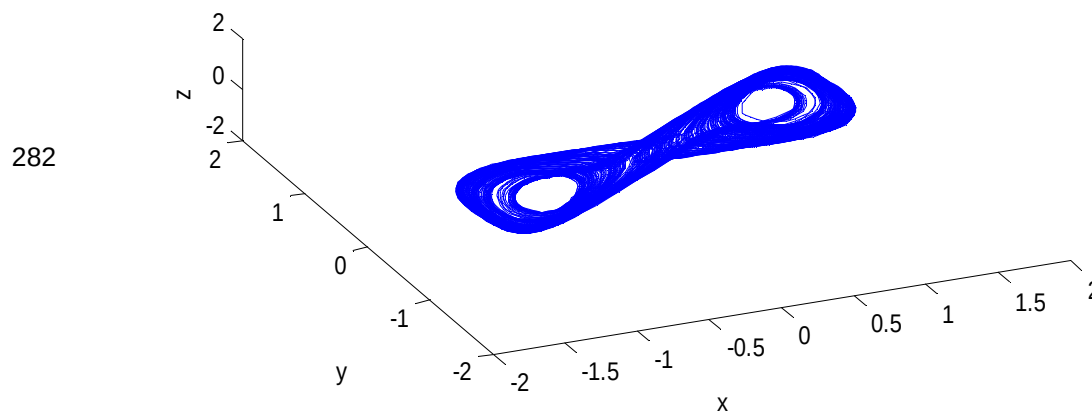
268 By the theoretical results in Sec.3.1, $\lambda = 20$, the two coupled systems are synchronized
 269 with probability 1, with the noise strengths $0.242 < d < 0.758$. Here, we compute the
 270 error(t) with different d to verify the correctness of the theoretical results. The simulation
 271 results are shown in FIG. 2 with $d = 0.4$, FIG. 3 with $d = 0.55$, and FIG. 4 with $d = 0.7$.

272 **Case 2. Two different Poisson noises**

273 According to the theoretical results in Sec.3.2, $\lambda_1 = 10, \lambda_2 = 20$, the two coupled systems
 274 are synchronized with probability 1 with the noise strengths $0.29 < d < 1.71$. Similarly, the
 275 error(t) with different d is computed to verify the correctness of the theoretical results.

276 The simulation results are shown in FIG. 5 with $d = 0.6$, FIG. 6 with $d = 1.0$, and FIG. 7
 277 with $d = 1.7$.

278 In terms of the simulation results, we can see that error(t) converges to zero with time
 279 increase. The individual systems achieve complete synchronization because the coupled
 280 term vanishes. Some chaotic attractors after synchronization achievement are shown in FIG.
 281 8.



283 FIG.1. Chaotic attractors generated by three-dimensional neural network (15) after transient
 284 time $T = 1000$ has been removed.

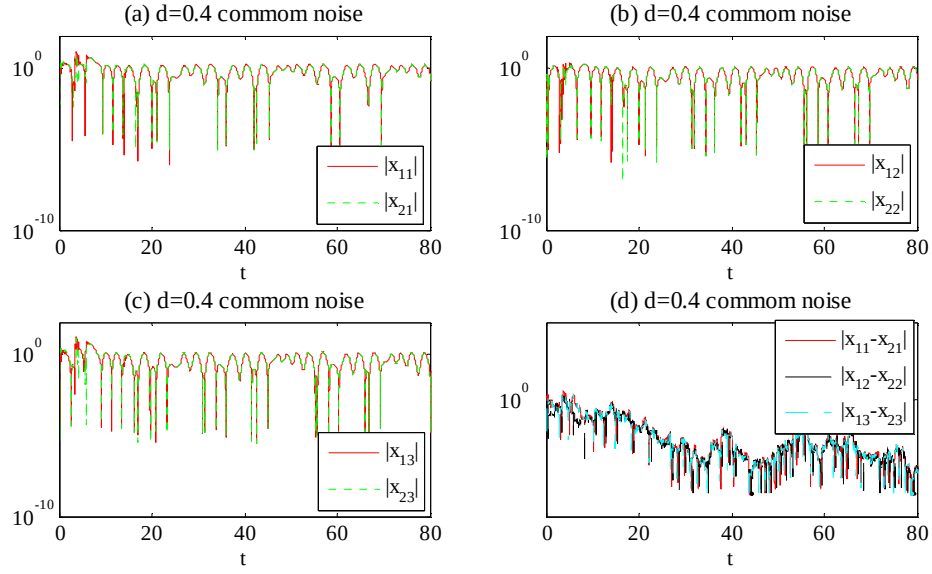


FIG. 2. (a-c) Responses of the systems (15) after synchronization achievement, (d) the temporal evolution of error(t).

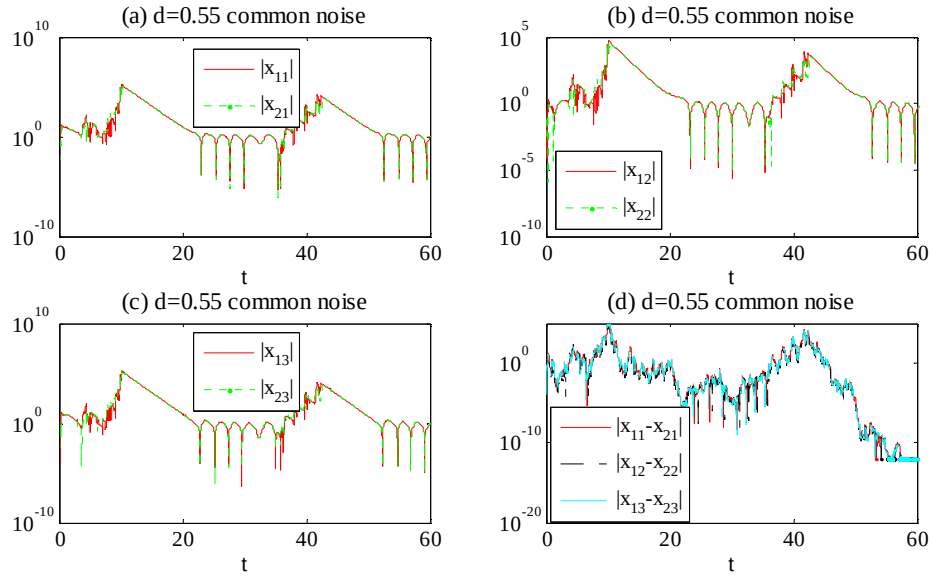
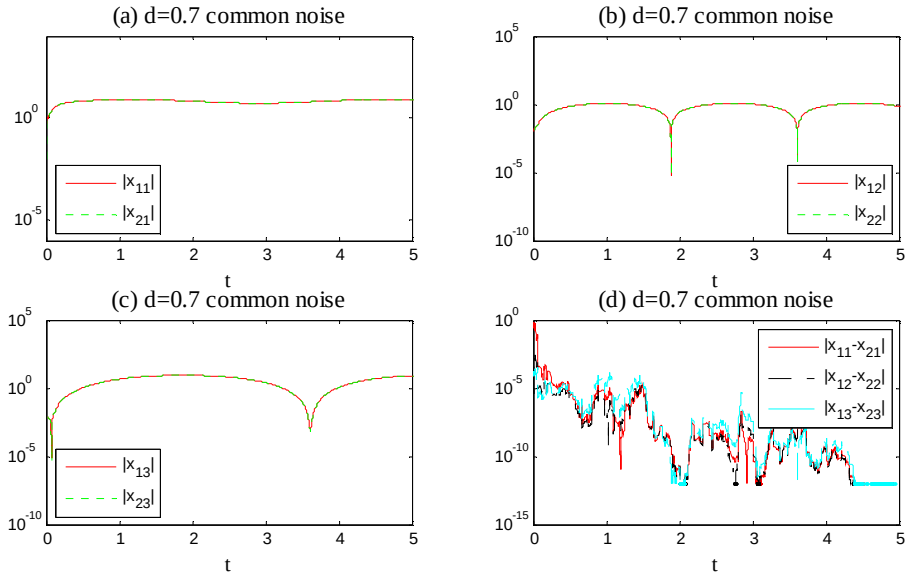


FIG. 3. (a-c) Responses of the systems (15) after synchronization achievement, (d) the temporal evolution of error(t).

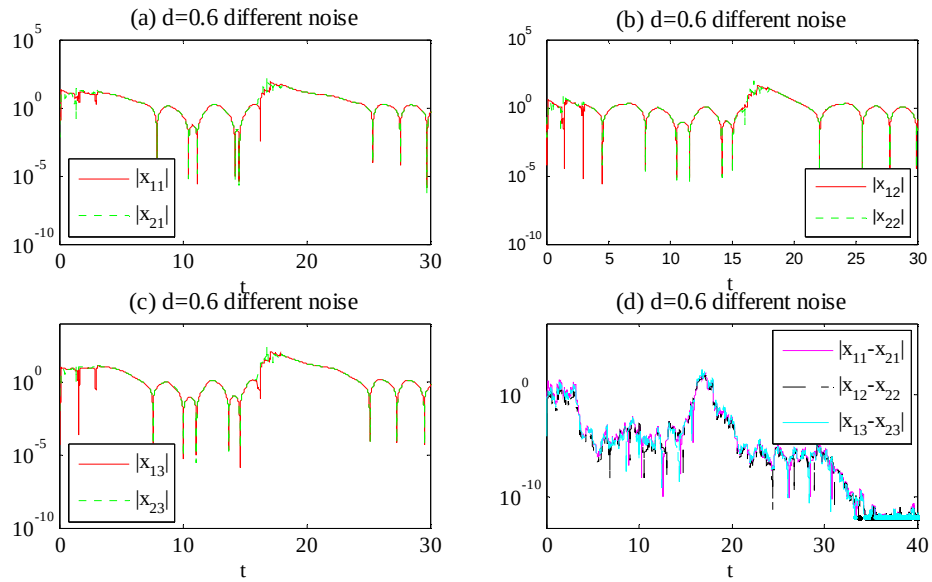
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294

295 FIG. 4. (a-c) Responses of the systems (15) after synchronization achievement, (d) the
296 temporal evolution of error(t).

297



298

299 FIG. 5. (a-c) Responses of the systems (15) after synchronization achievement, (d) the
temporal evolution of error(t).

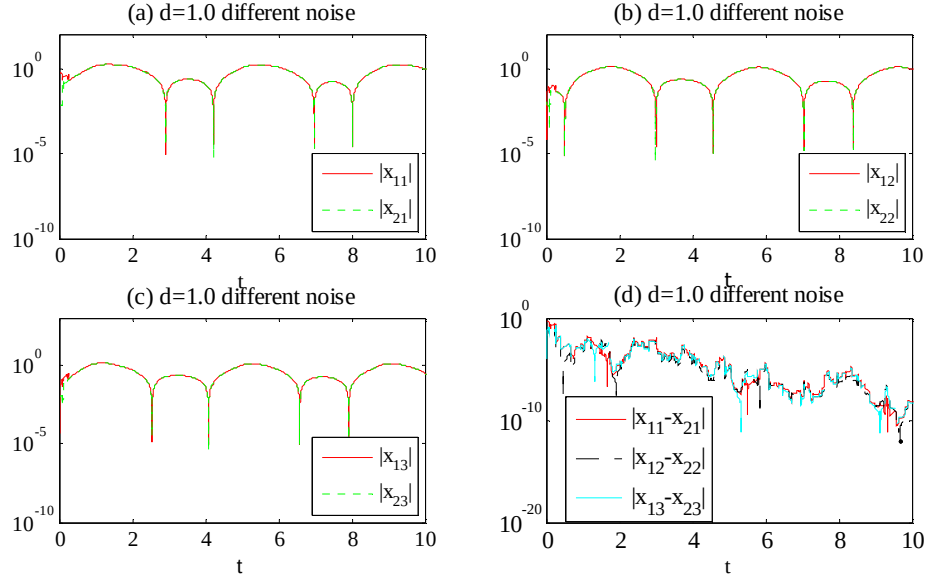


FIG. 6. (a-c) Responses of the systems (15) after synchronization achievement, (d) the temporal evolution of error(t).

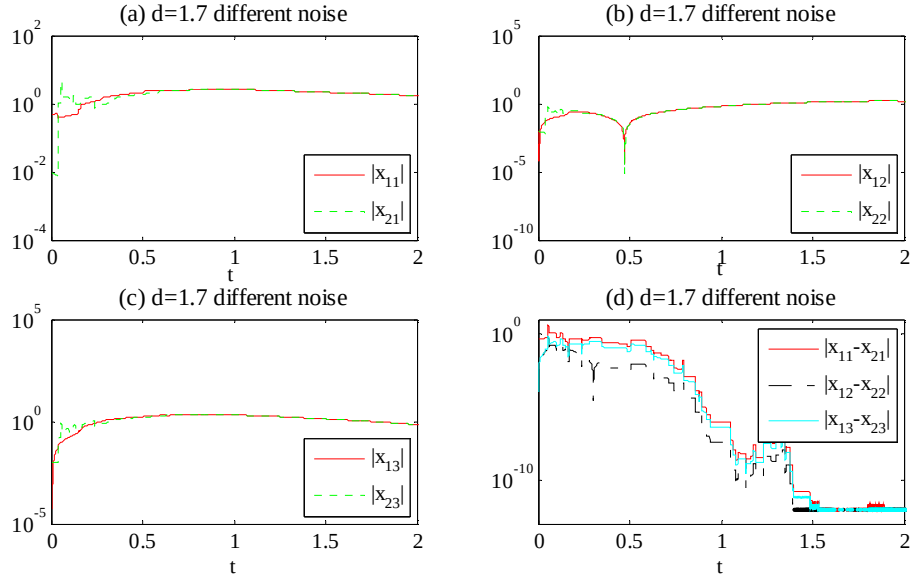


FIG. 7. (a-c) Responses of the systems (15) after synchronization achievement, (d) the temporal evolution of error(t).

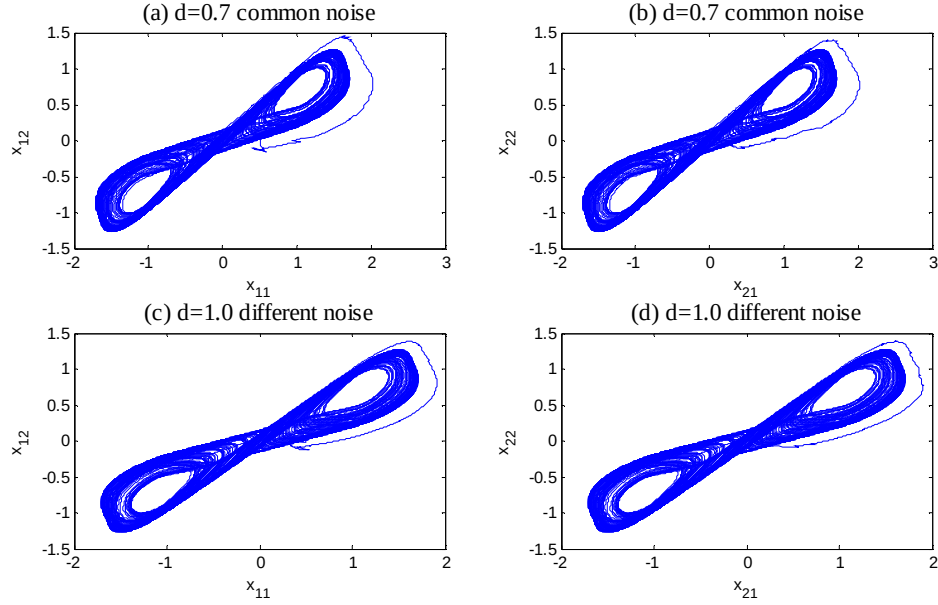


FIG. 8. Chaotic attractors of system (15) after synchronization achievement.

5. CONCLUSIONS

In this paper, complete synchronization of two bidirectionally coupled piecewise linear chaotic systems perturbed by common or different Poisson noises was investigated, and sufficient conditions of the complete synchronization with probability 1 are established. To state the advantages of Poisson noises-induced complete synchronization, the same model in Ref.4 was considered to demonstrate that Poisson noises are easier to achieve complete synchronization than the Gaussian cases via the comparison of noise strengths. We can see that the system (15) perturbed by Gaussian white noise are synchronized with probability 1, if the noise strengths $d > 1.916$ when noise is common, or the noise strengths $d > 2.709$ when noise is different in Ref. 4. It is quite clear the complete synchronous effect will become much better with the increase of noise strength. However, we know that the systems (15) in Poisson noise case are synchronized with probability 1 with the noise strengths $0.242 < d < 0.758$ when noise is common, or the noise strengths $0.29 < d < 1.71$ when noise is different, from the example in this paper. So, the noise strength of the systems perturbed by Poisson noise after complete synchronization achievement is bounded and smaller than the noise strength of the systems perturbed by Gaussian white noise after synchronization achievement. Therefore, through the comparison of two kinds of situations, we can verify that the systems perturbed by Poisson noise are easier to achieve complete synchronization than the case of Gaussian white noise with probability 1 in the sense of comparison of noise strengths.

REFERENCES

- [1] Pecora LM, and Carroll TL. Synchronization in chaotic systems. Phys.Rev.Lett. 1990; 64:821-824.

- [2] Chua L. O, Itoh M., Kocarev L., Eckert K., Chaos synchronization in chua's Circuit. J. Circuits. Syst. Comput. 1993; 3:93.
- [3] Zhou J, Chen Z., Further results on complete synchronization for noise-perturbed chaotic systems. Physics Letters A. 2008; 372:5394-5401.
- [4] Xiao Y., Xu W., Li X., Tang S., The effect of noise on the complete synchronization of two bidirectionally coupled piecewise linear chaotic systems, Chaos. 2009;19: 013131.
- [5] Moukam Kakmeni F. M., Bowong S., Tchawoua C., Nonlinear adaptive synchronization of a class of chaotic systems, Physics Letters A. 352006;5:47-54.
- [6] Xiao Y., Tang S., Xu Y., Theoretical analysis of multiplicative-noise-induced complete synchronization in global coupled dynamical network, Chaos. 2012;22: 013110.
- [7] Zheng S., Zheng L., Impulsive synchronization of bidirectionally coupled chaotic systems, Phys. Scr. 82013;8(3):035004.
- [8] Ott E., Grebogi C., York J. A., Controlling chaos, Phys. Rev. Lett. 1990; 64:1196.
- [9] Theesar S. J. S., Banerjee S., Balasubramaniam P., Adaptive synchronization in noise perturbed chaotic systems, Phys. Scr. 2012;85(6):065010.
- [10] Pikovsky A. S., Comment on "Chaos, Noise, and Synchronization", Phys. Rev. Lett. 1994; 73 : 2931.
- [11] Herzel H., Freund J., Chaos, noise, and synchronization reconsidered, Phys. Rev. E. 1995;52 : 3238.
- [12] Lai C., Zhou C., Synchronization of chaotic maps by symmetric common noise, Europhysics. Lett. 1998;43 : 376.
- [13] Lin W., He Y., Complete synchronization of the noise-perturbed chua's circuits, Chaos. 2005;15: 023705.
- [14] Lin W., Chen G., Using white noise to enhance synchronization of coupled chaotic systems, Chaos. 2006;16 :013134.
- [15] Chua L. O., Wu C. W., Huang A., Zhong G., A universal circuit for studying and generating chaos, I. Routes to chaos, IEEE Trans. Circuits Syst, I: Fundam. Theory Appl. 1993; 40 :732.
- [16] Heisler A., Braun T., Zhang Y., Hu G., Cerdeira H. A., Experimental investigation of partial synchronization in coupled chaotic oscillators, Chaos. 2003;13:185.
- [17] Zou F., Nossek J. A., Bifurcation and chaos in cellular neural networks, Circuits and Systems I: Fundamental Theory and Applications, IEEE Transactions on, 1993; 40:166-173.
- [18] Arnold L., Stochastic differential equations, Theory and Applications, John Wiley and Sons, 1974.
- [19] Friedman A., Stochastic Differential Equations and Applications, Dover Publications, 2006.
- [20] Xu Y., et al., Stochastic bifurcations in a bistable Duffing-Van der Pol oscillator with colored noise, Phys. Rev. E. 2011; 83:056215.
- [21] Tung C. C., Random response of highway bridges to vehicle loads, J. Eng. Mech. 1967;93 : 79-94.
- [22] Xu Y.,Zhang H.Q., Xu W., On stochastic complex beam-beam interaction models with Gaussian colored noise, Physica A. 2007;384:259-272.
- [23] Lin Y. K., Application of non-stationary shot noise in the study of system response to non-stationary excitations, ASME J. Appl. Mech. 1963; 30:555-558.
- [24] Roberts J. B., System response to random impulses, J. Sound.Vib. 1972; 24:23-24.
- [25] Xu Y., Feng J., Li J.J., Zhang H.Q., Stochastic bifurcation for a tumor-immune system with symmetric Lévy noise, Physica A, 2013;392 :4739-4748.
- [26] Xu Y.,Feng J., Li J.J., Zhang H.Q.,Lévy noise induced switch in the gene transcriptional regulatory system, Chaos, 2013; 23:013110.
- [27] Lu W. L., Chen T. P., New approach to synchronization analysis of linearly coupled ordinary differential systems, Physica. D. 2006; 213:214.

- 389 [28] Gihman I. I., Skorokhod A.V., Stochastic differential equations, Springer-Verlag, Berlin:
390 1972.
- 391 [29] Rastovic D., Targeting and synchronization at tokamak with recurrent artificial neural
392 networks, Neural Comput. & Appl.,2012; 21:1065-1069.
- 393 [30] Rastovic D., Fuzzy scaling and stability of tokamaks, J:Fusion Energy., 2009;28:101-
394 106.
- 395 [31] Garcia M. A., Griego R. J., An elementary theory of stochastic differential equation by a
396 Poisson process, Commun. Statist. Stochastic Models. 1994; 10 :335-336.
- 397 [32] Ting Y., The Stability theory of stochastic differential equation driven by a Poisson
398 process, Soochow Journal of Mathematics. 1999; 25:145-165.
- 399 [33] Kushner H. J., Stochastic stability and control, Academic Press, New York: 1967.