

Genetic Algorithm and Rough Sets Based Hybrid Approach for Economic Environmental Dispatch of Power Systems

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ABSTRACT

In this paper we present a new optimization algorithm for Economic environmental dispatch EED of power systems. The purpose of EED problem is to compute the optimal generation for individual units of the power system by minimizing the fuel cost and emission levels simultaneously, subject to various equality and inequality constraints. The proposed algorithm is population based an evolutionary algorithm which operates in two phases: in the first one, genetic algorithm is implemented as search engine in order to generate approximate true Pareto front. This algorithm based on concept of co-evolution and repair algorithm for handling nonlinear constraints. Also it maintains a finite-sized archive of non-dominated solutions which gets iteratively updated in the presence of new solutions based on the concept of ϵ -dominance. Then, in the second phase, rough sets theory is adopted as local search engine in order to improve the spread of the solutions found so far. Optimization using multiobjective evolutionary algorithms yields not a single optimal solution. However, for practical applications, we need to select one solution which will satisfy the different goals to some extent. TOPSIS method has the ability to identify the best alternative from a finite set of alternatives. The proposed approach is carried out on the standard IEEE 30-bus 6-generator test system. The results demonstrate the capabilities of the proposed approach to generate true and well-distributed Pareto-optimal nondominated solutions of the multiobjective EED problem. Also the comparison with the exiting well-known algorithms demonstrates the superiority of the proposed approach and confirms its potential to solve the multiobjective EED problem.

Keywords: *Economic Environmental Dispatch; Multiobjective Optimization; Genetic Algorithms; Rough Sets; TOPSIS.*

1. Introduction

The generation of electricity from fossil fuel releases several contaminants into the atmosphere. The problem that has attracted much attention is pollution minimization due to the pressing public demand for clean air. Operating at absolute minimum cost can no longer

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26 be the only criterion for dispatching electric power due to increasing concern the
27 environmental consideration. The purpose of Economic Environmental Dispatch (EED)
28 problem is to compute the optimal generation for individual units of the power system by
29 minimizing the fuel cost and emission levels simultaneously, subject to various equality and
30 inequality constraints including the security measures of the power transmission/distribution.

31 To optimize economic environmental dispatch problem, different techniques have been
32 reported in the literature. In past decades, the multiobjective EED problem was converted to
33 a single objective problem by linear combination of different objectives as a weighted sum
34 [1- 4]. The important aspect of this weighted sum method is that a set of Pareto-optimal
35 solutions can be obtained by varying the weights factors. This requires multiple runs as
36 many times as the number of desired Pareto-optimal solutions. Also, this method can be
37 only used to find Pareto-optimal solutions in problems having a convex Pareto-optimal front.
38 In addition, there is no rational basis of determining adequate weights and the objective
39 function so formed may lose significance due to combining noncommensurable objectives.

40 In other research direction [5-7] the multiobjective EED problem was reduced to a single
41 objective optimization problem by treating the emission as a constraint with a permissible
42 limit. This formulation, however, has a severe difficulty in getting the trade-off relations
43 between cost of generation and emission.

44 Goal programming method was also proposed for multiobjective EED problem [8]. In this
45 method, a target or a goal to be achieved for each objective is assigned and the objective
46 function will then try to minimize the distance from the targets to the objectives. The main
47 drawback of this method is that it requires a priori knowledge about the shape of the problem
48 search space.

49 Recently, the direction is to handle both objectives simultaneously as competing
50 objectives instead of simplifying the multiobjective problem to a single objective problem [9-
51 15]. The use and development of heuristics-based multiobjective optimization techniques
52 (Evolutionary Algorithms) have significantly grown. Since they use a population of solutions
53 in their search, multiple Pareto-optimal solutions can, in principle, be found in one single run.
54 Moreover the studies on evolutionary algorithms over the past few years have shown that
55 these models can be efficiently used to eliminate most of the difficulties of classical methods
56 [16-19]. A genetic Algorithms (GA) optimization method is one evolutionary algorithms
57 technique which was successfully applied in real optimization problems.

58 In this paper we present a new multiobjective optimization algorithm to optimize
59 Economic environmental dispatch EED of power systems. The proposed algorithm operates
60 in two phases: in the first one, multiobjective version of genetic algorithm is used as search
61 engine in order to generate approximate true Pareto front. Then in the second phase, rough
62 set theory is adopted as local search engine in order to improve the spread of the solutions
63 found so far. Optimization using multiobjective evolutionary algorithms yields not a single
64 optimal solution. However for practical applications, we need to select one solution which will
65 satisfy the different goals to some extent. TOPSIS method [20,21] has the ability to identify
66 the best alternative from a finite set of alternatives. The proposed algorithm is implemented
67 to the standard IEEE 30-bus 6-generator test system to investigate the effectiveness of the
68 proposed approach and the results are compared to different well-known algorithms reported
69 in literature.

2. Principle of Multiobjective Optimization

A multiobjective Optimization Problem (MOP) can be defined as determining a vector of design variables within a feasible region to minimize a vector of objective functions that usually conflict with each other. Such a problem takes the form:

$$\begin{aligned} \text{Min } F(x) &= (f_1(x), f_2(x), \dots, f_m(x))^T \\ \text{s.t. } x &\in S \\ x &= (x_1, x_2, \dots, x_n)^T \end{aligned} \quad (1)$$

Where $(f_1(x), f_2(x), \dots, f_m(x))$ are the m objectives functions, $(x_1, x_2, \dots, x_n)$ are the n decision variables, and $S \in R^n$ is the solution (parameter) space.

Definition 1. (Pareto optimal solution): x^* is said to be a Pareto optimal solution of MOP if there exists no other feasible x (i.e., $x \in S$) such that, $f_j(x) \leq f_j(x^*)$ for all $j=1, 2, \dots, m$ and $f_j(x) < f_j(x^*)$ for at least one objective function f_j .

Definition 2. [22]. (ϵ -dominance) Let $f: X \rightarrow R^m$ and $a, b \in X$. Then a is said to ϵ -dominate b for some $\epsilon > 0$, denoted as $a \succ_{\epsilon} b$, if and only if for $i \in \{1, \dots, m\}$

$$(1-\epsilon)f_i(a) \leq f_i(b)$$

Definition 3. (ϵ -approximate Pareto set) Let X be a set of decision alternatives and $\epsilon > 0$. Then a set x_{ϵ} is called an ϵ -approximate Pareto set of X , if any vector $a \in x$ is ϵ -dominated by at least one vector $b \in x_{\epsilon}$, i.e

$$\forall a \in x: \exists b \in x_{\epsilon} \text{ such that } b \succ_{\epsilon} a$$

According to definition 2, the ϵ value stands for a relative allowed for the objective values as declared in Figure 1. This is especially important in higher dimensional objective spaces, where the concept of ϵ -dominance can reduce the required number of solutions considerably. Also, makes the algorithms practical by allowing a decision maker to control the resolution of the Pareto set approximation by choosing an appropriate ϵ -value

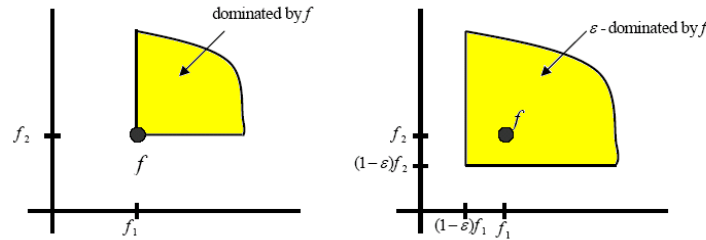


Figure 1. Graphs visualizing the concepts of dominance (left) and ϵ -dominance (right).

3. Economic Environmental Dispatch (EED) of Power Systems

The Economic Environmental Dispatch EED problem is formulated as a nonlinear constrained multiobjective optimization problem which attempts to minimize fuel cost and emission simultaneously which are conflicting with each other, while satisfying various equality and inequality constraints. The problem is formulated as described below.

3.1. Problem Objectives

100 **3.1.1. Minimization of Fuel Cost Objective**

101 The classical economic dispatch problem of finding the optimal combination of power
102 generation, which minimizes the total fuel cost while satisfying the total required demand can
103 be mathematically stated as follows [23]:

$$104 \quad f_1(P_{Gi}) = C_t = \sum_{i=1}^n C_i(P_{Gi}) = \sum_{i=1}^n (a_i + b_i P_{Gi} + c_i P_{Gi}^2) \$ / hr \quad (2)$$

105 Where:

C_t : total fuel cost (\$/hr),

C_i : is fuel cost of generator i

a_i, b_i, c_i : fuel cost coefficients of generator i,

P_{G_i} : power generated (p.u) by generator i,

n: number of generators.

107 **3.1.2. Minimization of Emission objective.**

108 The emission function can be presented as the sum of all types of emission considered,
109 such as NO_x, SO_2 , thermal emission, etc., with suitable pricing or weighting on each
110 pollutant emitted. In the present study, only one type of emission NO_x is taken into account
111 without loss of generality. The amount of NO_x emission is given as a function of generator
112 output, that is, the sum of a quadratic and exponential function:

$$113 \quad f_2(P_{Gi}) = E_{NO_x} = \sum_{i=1}^n [10^{-2} (\alpha_i + \beta_i P_{Gi} + \gamma_i P_{Gi}^2) + \xi_i \exp(\lambda_i P_{Gi})] \text{ ton / hr} \quad (3)$$

114 Where, $\alpha_i, \beta_i, \gamma_i, \xi_i, \lambda_i$: coefficients of the i th generator's NO_x emission characteristic.

115 **3.2 Problem Constraints**

116 **3.2.1 Power balance constraint**

117 The total power generated must supply the total load demand and the transmission losses:

$$118 \quad \sum_{i=1}^n P_{Gi} - P_D - P_{Loss} = 0 \quad (4)$$

119 Where:

120 P_D : total load demand (p.u.), and

P_{loss} : transmission losses (p.u.).

121 The transmission loss is given by [24]:

$$122 \quad P_{Loss} = \sum_{i=1}^n \sum_{j=1}^n [A_{ij} (P_i P_j + Q_i Q_j) + B_{ij} (Q_i P_j - P_i Q_j)] \quad (5)$$

123 Where:

124 $P_i = P_{Gi} - P_{Di}, \quad Q_i = Q_{Gi} - Q_{Di}, \quad A_{ij} = \frac{R_{ij}}{V_i V_j} \cos(\delta_i - \delta_j), \quad B_{ij} = \frac{R_{ij}}{V_i V_j} \sin(\delta_i - \delta_j)$

125 n : number of buses

126 R_{ij} : series resistance connecting buses i and j

127 V_i : voltage magnitude at bus i

128 δ_i : voltage angle at bus i

129 P_i : real power injection at bus i

130 Q_i : reactive power injection at bus i

131 **3.2.2 Power generation constraint**

132 The power generated P_{Gi} by each generator is constrained between its minimum and
133 maximum limits

134 $P_{Gi\min} \leq P_{Gi} \leq P_{Gi\max}, \quad Q_{Gi\min} \leq Q_{Gi} \leq Q_{Gi\max}, \quad V_{i\min} \leq V_i \leq V_{i\max}, \quad i = 1, \dots, n$

135 Where:

136 $P_{Gi\min}$: minimum power generated, and

137 $P_{Gi\max}$: maximum power generated.

138 **3.2.3 Security constraint**

139 A mathematical formulation of the security constrained EED problem would require a very
140 large number of constraints to be considered. However, for typical systems the large
141 proportion of lines has a rather small possibility of becoming overloaded. The EED problem
142 should consider only the small proportion of lines in violation, or near violation of their
143 respective security limits which are identified as the critical lines. We consider only the
144 critical lines that are binding in the optimal solution. The detection of the critical lines is
145 assumed done by the experiences of the DM. An improvement in the security can be
146 obtained by minimizing the following objective function.

147 $S = f(P_{Gi}) = \sum_{j=1}^k (|T_j(P_G)| / T_j^{\max}) \quad (6)$

148 Where, $T_j(P_G)$ is the real power flow $T_j^{\max}$ is the maximum limit of the real power flow of
 149 the j th line and k is the number of monitored lines. The line flow of the j th line is expressed
 150 in terms of the control variables P_{Gs} , by utilizing the generalized generation distribution
 151 factors (GGDF) [24] and is given below.

$$152 \quad T_J(P_G) = \sum_{i=1}^n (D_{ji} P_{Gi}) \quad (7)$$

153 where, D_{ji} is the generalized GGDF for line j , due to generator i

154 For secure operation, the transmission line loading S_ℓ is restricted by its upper limit as

$$155 \quad S_\ell \leq S_{\ell \max}, \ell = 1, \dots, n_\ell$$

156 Where n_ℓ is the number of transmission line.

157 **4. Problem Formulation of Economic Environmental Dispatch (EED)**

158 The multiobjective economic emission load dispatch optimization problem is therefore
 159 formulated as:

$$\begin{aligned} \text{Min } f_1(P_{Gi}) &= C_t = \sum_{i=1}^n (a_i + b_i P_{Gi} + c_i P_{Gi}^2) \text{ \$ / hr} \\ \text{Min } f_2(P_{Gi}) &= E_{NO_x} = \sum_{i=1}^n [10^{-2} (\alpha_i + \beta_i P_{Gi} + \gamma_i P_{Gi}^2) + \xi_i \exp(\lambda_i P_{Gi})] \text{ ton / hr} \\ \text{s.t. } \quad &\sum_{i=1}^n P_{Gi} - P_D - P_{Loss} = 0, \\ 160 \quad &S_\ell \leq S_{\ell \max}, \quad \ell = 1, \dots, n_{Line}, \\ &P_{Gi \min} \leq P_{Gi} \leq P_{Gi \max} \quad i = 1, \dots, n \\ &Q_{Gi \min} \leq Q_{Gi} \leq Q_{Gi \max} \quad i = 1, \dots, n \\ &V_{i \min} \leq V_i \leq V_{i \max} \quad i = 1, \dots, n \end{aligned} \quad (8)$$

161

162 **5. The Proposed Approach**

163 In this section we present a new multiobjective optimization algorithm, the proposed
 164 algorithm operates in two phases: in the first one, multiobjective version of genetic algorithm
 165 is used as search engine in order to generate approximate true Pareto front. Then in the
 166 second phase, rough set theory is adopted as local search engine in order to improve the
 167 spread of the solutions found so far. All equality constraints are replaced by inequality
 168 constraints with additional parameter *tol* to define the precision of the system.
 169

$$f_i(x_1, x_2, \dots, x_n) = 0 \Rightarrow -tol \leq (f_i(x_1, x_2, \dots, x_n)) \leq tol$$

$$\Rightarrow |f_i(x_1, x_2, \dots, x_n)| \leq tol$$

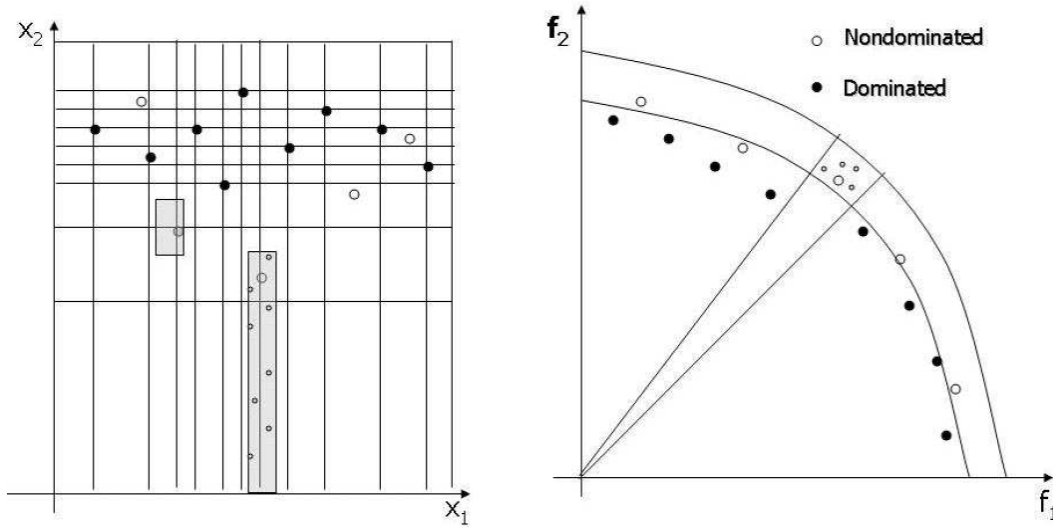
$$\Rightarrow |f_i(x_1, x_2, \dots, x_n)| - tol \leq 0$$

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172 5.1. Use of Rough Sets in Multiobjective Optimization

173 For our proposed approach we will try to investigate the Pareto front using a Rough sets
 174 grid. To do this, we will use an initial approximate of the Pareto front (provided by any
 175 evolutionary algorithm) and will implement a grid in order to get more information about the
 176 front that will let to improve this initial approximation [25].

177 To create this grid, as an input we will have N feasible points divided in two sets: the
 178 nondominated points (NS) and the dominated ones (DS). Using these two sets we want to
 179 create a grid to describe the set NS in order to intensify the search on it. This is, we want to
 180 describe the Pareto front in the decision variable space because then we could easily use
 181 this information to generate more efficient points and then improve this initial Pareto
 182 approximation. **Figure 2** shows how information in objective function space can be
 183 translated into information in decision variable space through the use of a grid. We must
 184 note the importance of the DS sets as in a rough sets method, where the information comes
 185 from the description of the boundary of the two sets NS, DS. Then the more efficient points
 186 provided the better. However, it is also required to provide dominated points, since we need
 187 to estimate the boundary between being dominated and being nondominated. Once the
 188 information is computed we can simply generate more points in the “efficient side”.



189

190 **Figure2. Decision variable space (left) and objective function space (right)[29]**

191 5.2. Structure of an Iterative Multiobjective Search Algorithm

192 The purpose of this section is to informally describe the problem we are dealing with. To this
 193 end, let us first give a template for a large class of iterative search procedures which are
 194 characterized by the generation of a sequence of search points and a finite memory.

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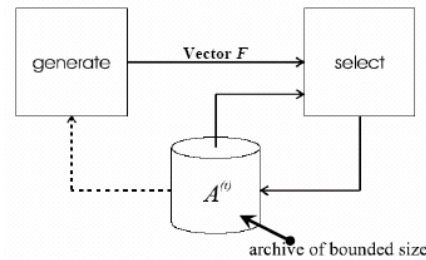
Algorithm 1. Iterative search algorithm

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1.  $t = 0$ 
2.  $A^{(0)} = 0$ 
3. while  $\text{terminate}(A^{(t)}, t) = \text{false}$  do
4.    $t = t + 1$ 
5.    $f^{(t)} = \text{generate}(\cdot)$            {generate new search point}
6.    $A^{(t)} = \text{update}(A^{(t-1)}, f^{(t)})$  {update archive}
7. end while
8. Output :  $A^{(t)}$ 

```

199 An abstract description of a generic iterative search algorithm is given in **Algorithm 1**[22].
 200 The integer t denotes the iteration count, the n -dimensional vector $f^{(t)}$ is the sample
 201 generated at iteration t and the set $A^{(t)}$ will be called the archive at iteration t and should
 202 contain a representative subset of the samples in the objective space
 203 $F = [f_1(x), f_2(x), \dots, f_m(x)]$ generated so far. To simplify the notation, we represent
 204 samples by n -dimensional real vectors f where each coordinate represents one of the
 205 objective values as shown in **Figure 3**.



206

207

Figure 3. Block diagram of Archive/selection strategy

208 The purpose of the function $f^{(t)} = \text{generate}(\cdot)$ is to generate a new solutions in each iteration
 209 t , possibly using the contents of the old archive set $A^{(t-1)}$. The function
 210 $A^{(t)} = \text{update}(A^{(t-1)}, f^{(t)})$ gets the new solutions $\text{generate}(\cdot)$ and the old archive set $A^{(t-1)}$ and
 211 determines the updated one, namely $A^{(t)}$. In general, the purpose of this sample storage is to
 212 gather 'useful' information about the underlying search problem during the run. Its use is
 213 usually two-fold: On the one hand it is used to store the 'best' solutions found so far, on the
 214 other hand the search operator exploits this information to steer the search to promising
 215 regions. This procedure could easily be viewed as an evolutionary algorithm when the
 216 generate operator is associated with variation (recombination and mutation). However, we
 217 would like to point out that all following investigations are equally valid for any kind of

218 iterative process which can be described as **Algorithm 1** and used for approximating the
219 Pareto set of multiobjective optimization problems.

220 **5.3. Constraint Multiobjective Optimization via Genetic Algorithm**

221 In any interesting multiobjective optimization problem, there exist a number of such
222 solutions which are of interest to designers and practitioners. Since no one solution is better
223 than any other solution in the Pareto-optimal set, it is also a goal in a multiobjective
224 optimization to find as many such Pareto-optimal solutions as possible. Unlike most classical
225 search and optimization problems, GAs works with a population of solutions and thus are
226 likely (and unique) candidates for finding multiple Pareto-optimal solutions simultaneously.
227 There are two tasks that are achieved in a multiobjective GA. (1) Convergence to the Pareto-
228 optimal set, and (2) Maintenance of diversity among solutions of the Pareto-optimal set.
229 Here we present a new optimization system, which is based on concept of co-evolution and
230 repair algorithms. Also it is based on the ε -dominance concept. The use of ε -dominance
231 also makes the algorithms practical by allowing a decision maker to control the resolution of
232 the Pareto set approximation by choosing an appropriate ε value.

233 **5.3.1. Initialization Stage**

234 The algorithm uses two separate population, the first population $P^{(t)}$ consists of the
235 individuals which initialized randomly satisfying the search space (The lower and upper
236 bounds), while the second population $R^{(t)}$ consists of reference points which satisfying all
237 constraints (feasible points), However, in order to ensure convergence to the true Pareto-
238 optimal solutions, we concentrated on how elitism could be introduced. So, we propose an
239 “archiving/selection” strategy that guarantees at the same time progress towards the Pareto-
240 optimal set and a covering of the whole range of the non-dominated solutions. The algorithm
241 maintains an externally finite-sized archive $A^{(t)}$ of non-dominated solutions which gets
242 iteratively updated in the presence of new solutions based on the concept of ε -dominance.

243 **5.3.2. Repair Algorithm**

244 The idea of this technique is to separate any feasible individuals in a population from those
245 that are infeasible by repairing infeasible individuals. This approach co-evolves the
246 population of infeasible individuals until they become feasible. Repair process works as
247 follows. Assume, there is a search point $\omega \notin S$ (where S is the feasible space). In such a
248 case the algorithm selects one of the reference points (Better reference point has better
249 chances to be selected), say $r \in S$ and creates random points $\bar{Z}$ from the segment defined
250 between ω, r , but the segment may be extended equally on both sides determined by a user
251 specified parameter $\mu \in [0,1]$. Thus, a new feasible individual is expressed as:

$$252 \quad \left. \begin{aligned} z_1 &= \gamma \cdot \omega + (1 - \gamma) \cdot r \\ z_2 &= (1 - \gamma) \cdot \omega + \gamma \cdot r \end{aligned} \right\}, \gamma = (1 + 2\mu)\delta - \mu, \delta \in [0,1] \quad (9)$$

253 **5.3.3. Evolutionary Algorithm: Phase 1**

254 In the first phase, the proposed algorithm uses two separate population, the first population
255 $P^{(t=0)}$ (where t is the iteration counter) consists of the individuals which initialized randomly
256 satisfying the search space (The lower and upper bounds), while the second population
257 $R^{(0)}$ consists of reference points which satisfying all constraints (feasible points). Also, it

258 stores initially the Pareto-optimal solutions externally in a finite sized archive of non-
 259 dominated solutions $A^{(0)}$. We use cluster algorithm to create the next population P^{t+1} , if
 260 $|P^{(t)}| > |A^{(t)}|$ then new population P^{t+1} consists of all individual from $A^{(t)}$ and the population
 261 $P^{(t)}$ are considered for the clustering procedure to complete P^{t+1} , if $|P^{(t)}| < |A^{(t)}|$ then $|P|$
 262 solutions are picked up at random from $A^{(t)}$ and directly copied to the new population P^{t+1} .

263 Since our goal is to find new nondominated solutions, one simple way to combine multiple
 264 objective functions into a scalar fitness function [26,27] is the following weighted sum
 265 approach

$$266 \quad f(x) = w_1 f_1(x) + \dots + w_i f_i(x) + \dots + w_m f_m(x) = \sum_{j=1}^m w_j f_j(x) \quad (10)$$

267 Where x is a string (i.e., individual), $f(x)$ is a combined fitness function, $f_i(x)$ is the i th
 268 objective function When a pair of strings are selected for a crossover operation, we assign a
 269 random number to each weight as follows:

$$270 \quad w_i = \frac{random_i(.)}{\sum_{j=1}^m random_j(.)}, \quad i = 1, 2, \dots, m \quad (11)$$

271 Calculate the fitness value of each string using the random weights w_i . Select a pair of
 272 strings from the current population according to the following selection probability $\beta(x)$ of a
 273 string x in the population $P^{(t)}$

$$274 \quad \beta(x) = \frac{f(x) - f_{\min}(P^{(t)})}{\sum_{x \in P^{(t)}} \{f(x) - f_{\min}(P^{(t)})\}}, \quad \text{where } f_{\min}(P^{(t)}) = \min\{f(x) \mid x \in P^{(t)}\} \quad (12)$$

275 This step is repeated for selecting $|P|/2$ Pairs of strings from the current populations. For
 276 each selected pair apply crossover operation to generate two new strings, for each strings
 277 generated by crossover operation, apply a mutation operator with a prespecified mutation
 278 probability. The system also includes the survival of some of good individuals without
 279 crossover or mutation. The algorithm maintains a finite-sized archive A^t of non-dominated
 280 solutions which gets iteratively updated in the presence of a new solutions based on the
 281 concept of ε -dominance, such that new solutions are only accepted in the archive if they are
 282 not ε -dominated by any other element in the current archive. The use of ε -dominance also
 283 makes the algorithms practical by allowing a decision maker to control the resolution of the
 284 Pareto set approximation by choosing an appropriate ε value.

285 **5.3.4. Local Search Mechanism Inspired on Rough Sets Theory: Phase 2**

286 Upon termination of phase 1, we start phase2, with initial approximate of the Pareto front
 287 (provided by the proposed algorithm in phase1) which noted as NS. Also all dominated
 288 solutions are marked as DS. It is worth remarking that NS can simply be a list of solutions.

289 From the set NS we choose NNS points previously unselected. If we do not have enough
 290 unselected points, we choose the rest randomly from the set DS. Next, we choose from the

291 set DS, NDS points previously unselected (and in the same way if we do not have enough
 292 unselected points, we complete them in a random fashion) these points will be used to
 293 approximate the boundary between the Pareto front and the rest of the feasible set in
 294 decision variable space. We store these points in the set Items and perform rough sets
 295 iterations:

- 296 1- Range Initialization: for each decision variable i , we compute and sort (from
 297 smallest and highest) the different values it takes in the set Items. Then, for each
 298 decision variable, we have a set of *rang* values and combining all these sets we
 299 have a non-uniform grid in decision variable space.
- 300 2- Compute Atoms: we compute "NNS rectangular atoms" centered in the NNS
 301 efficient points selected. To build a rectangular atom associated to a nondominated
 302 point $x^e \in Items$ we compute the following upper and lower bounds for each decision
 303 variable i :
 - 304 • Lower Bound i : Middle point between x_i^e and the previous value in the set
 305 $rang_i$
 - 306 • Upper Bound i : Middle point between x_i^e and the following value in the set
 307 $rang_i$
 - 308 • If there are no previous or subsequent values in $rang_i$, we consider the
 309 absolute lower or upper bound of variable i . This setting lets the method to
 310 explore close to the feasible set boundaries.
- 311 3- Generate Offspring: inside each atom we randomly generate offspring new points.
 312 Each of these points is sent to the set NS as follows. The idea is that "new solutions
 313 are only accepted in the archive if they are not ϵ -dominated by any other element of
 314 the current archive". If a solution is accepted, all dominated solutions are removed.
 315 **Algorithm 2** shows the operator for ϵ -approximate Pareto set and the pseudo code
 316 of the proposed algorithm is declared in **Algorithm 3**.
 317

Algorithm 2: Operator for ϵ -approximate Pareto set

```

1. INPUT :  $A, x$ 
2. if  $\exists x' \in A$  such that  $x' \succ_{\epsilon} x$  then
3.  $A' = A$ 
4. else
5.  $D = \{x' \in A : x \succ x'\}$ 
6.  $A' = A \cup \{x\} \setminus D$ 
7. end if
8. Output :  $A'$ 

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Algorithm 3: The proposed algorithm(pseudo code of the proposed algorithm)

```

t=0
2. Create  $P^{(0)}, R^{(0)}$ 
3.  $A^{(0)} = \text{nondominated}(P^{(0)})$ 
3. while terminate( $A^{(t)}, t$ ) = false do
4.  $t = t + 1$ 
5.  $P^{(t)} = \text{generate}(A^{(t-1)}, P^{(t-1)})$  {generate new search point}
6.  $A^{(t)} = \text{update}(A^{(t-1)}, P^{(t)})$  {update archive (algorithm 3)}
7. end while
8. Output :  $A^{(t)}$ 
} Phase1

8.  $A^{(t)} \rightarrow \text{NS, Dominted Pointes} \rightarrow \text{DS}$ 
9. Compute "Atom"
10. Comput Upper Boundi, Lower Boundi } Phase2
11. Generate new offspring.
12. Update NS set
13. Output NS

```

326

327 6. Implementation of the proposed approach

328 In order to validate the proposed approach and quantitatively compare its performance with
 329 other MOEAs, we present in this section comparison study applied to the standard IEEE 30-
 330 bus 6-generator test system with two objective. The single-line diagram of this system is
 331 shown in Figure 4 and the detailed data are given in [7,28]. The values of fuel cost and
 332 emission coefficients are given in Table 1. For comparison purposes with the reported
 333 results, the system is considered as losses and the security constraint is released. The
 334 techniques used in this study were developed and implemented using MATLAB
 335 environment. **Table 2 gives Reactive power limit and Voltage limits** and table 3 lists the
 336 parameter setting used in the algorithm for all runs.

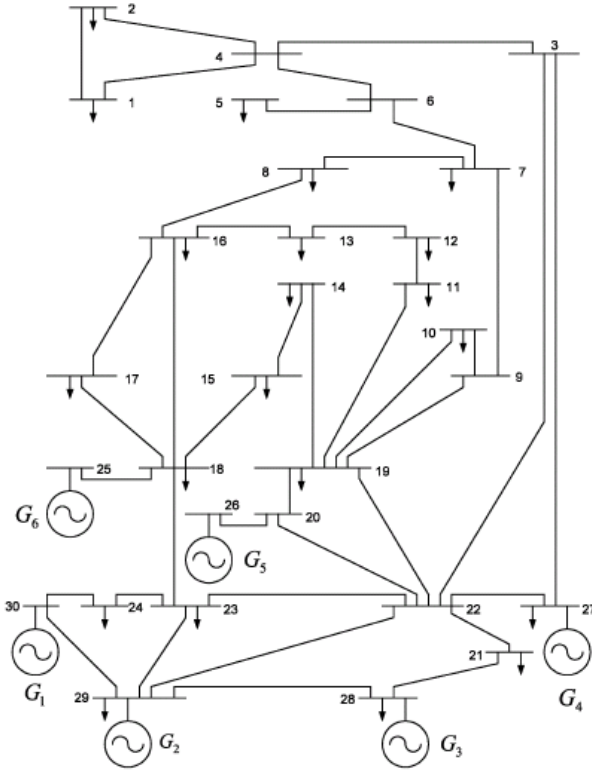


Figure 4: Single line diagram of IEEE 30-bus 6-generator test system

Table 1: Generator fuel cost and emission coefficients

		G1	G2	G3	G4	G5	G6
Cost	a	10	10	20	10	20	10
	b	200	150	180	100	180	150
	c	100	120	40	60	40	100
Emission	α	4.091	2.543	4.258	5.426	4.258	6.131
	β	-5.554	-6.047	-5.094	-3.550	-5.094	-5.555
	γ	6.490	4.638	4.586	3.380	4.586	5.151
	ζ	2.0E-4	5.0E-4	1.0E-6	2.0E-3	1.0E-6	1.0E-5
	λ	2.857	3.333	8.000	2.000	8.000	6.667

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343

Table 2: Reactive power limit and Voltage limits

Bus	Voltage		Reactive power	
	$V_{i \min}$	$V_{i \max}$	$Q_{Gi \min}$	$Q_{Gi \max}$
1	0.9	1.05	-0.2	2.0
2	0.9	1.05	-0.2	2.0
3	0.9	1.05	-0.2	2.0
4	0.9	1.05	-0.2	2.0
5	0.9	1.05	-0.2	2.0
6	0.9	1.05	-0.2	2.0

344 In Table , $Q_{\min}$, $Q_{\max}$, $V_{\min}$ and $V_{\max}$ are in (p.u),

345

346

Table 3: The parameter setting

Population size	200
No. of Generation	200
Crossover probability P_c	0.9
Mutation probability P_m	0.02
Selection operator	Roulette Wheel
Crossover operator	Single point
Mutation operator	Polynomial mutation
Relative tolerance ε	10e-6

347

348 **7. Results and discussions**

349 Figure 5 shows well-distributed Pareto optimal solutions obtained by the proposed algorithm
350 after 200 generations. It is clear from the figure that Pareto-optimal set is well distributed and
351 has satisfactory diversity characteristics compared with Pareto-optimal front of the NPGA[9]
352 and Pareto-optimal front of the SPEA[10].

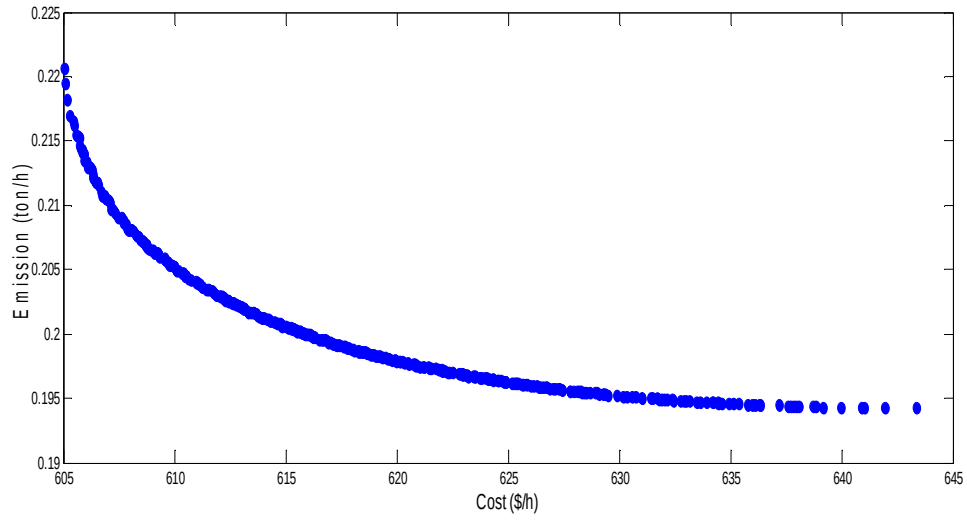


Figure 5: Pareto-optimal front of the proposed approach.

Table 4 and 5 show the best fuel cost and best NO_x emission obtained by proposed algorithm as compared to Nondominated Sorting Genetic Algorithm (NSGA) [8], Niched Pareto Genetic Algorithm (NPGA) [9], Strength Pareto Evolutionary Algorithm (SPEA) [10] and IT-CEMOP [29]. It can be deduced that the proposed algorithm finds comparable minimum fuel cost and comparable minimum NO_x emission to the four evolutionary algorithms.

Table 4: Best total \$/h fuel cost

	NSGA	NPGA	SPEA	IT-CEMOP	Proposed approach
P_{G1}	0.1168	0.1245	0.1086	0.1739	0.928
P_{G2}	0.3165	0.2792	0.3056	0.3578	0.2868
P_{G3}	0.5441	0.6284	0.5818	0.5311	0.5159
P_{G4}	0.9447	1.0264	0.9846	0.9790	0.9896
P_{G5}	0.5498	0.4693	0.5288	0.4429	0.5803
P_{G6}	0.3964	0.39993	0.3584	0.3725	0.3448
Best cost.	608.245	608.147	607.807	606.4533	605.0820
Corresponding Emission	0.21664	0.22364	0.22015	0.2028	0.2206

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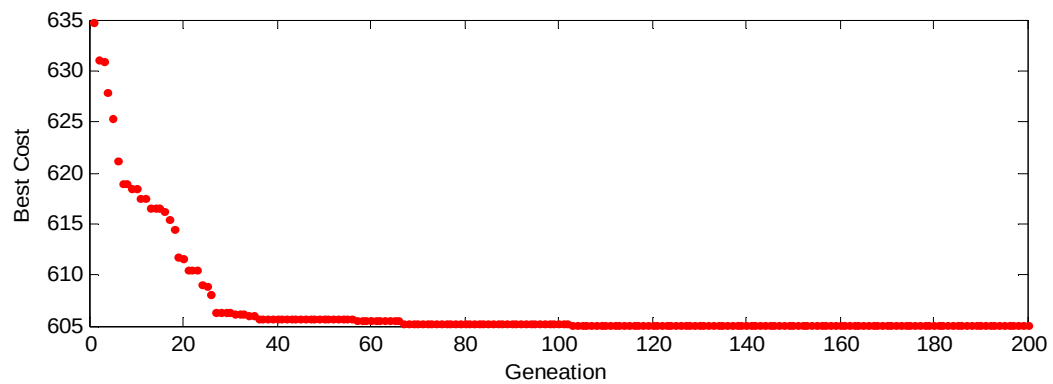
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Table 5: Best total ton/h NO_x emission

	NSGA	NPGA	SPEA	IT-CEMOP	Proposed approach
P_{G1}	0.4113	0.3923	0.4043	0.3885	0.4046
P_{G2}	0.4591	0.4700	0.4525	0.4984	0.4611
P_{G3}	0.5117	0.5565	0.5525	0.5167	0.5289
P_{G4}	0.3724	0.3695	0.4079	0.4502	0.3874
P_{G5}	0.5810	0.5599	0.5468	0.5205	0.5338
P_{G6}	0.5304	0.5163	0.5005	0.5005	0.5014
Best Emission.	0.19432	0.19424	0.19422	0.1882	0.1942
Corresponding cost	647.251	645.984	642.603	642.8976	643.3729

364

365 Convergence of fuel cost and emission objective functions are shown in figure 6 and
 366 figure 7. In the view of our results, the algorithm converges to the optimal solution as in
 367 figure 6 and 7, where the Cost of generating is converges to the optimal value, Also the
 368 Emissions of undesired materials is converges to the optimal value.



369

370

Figure 6 : Convergence of fuel cost objective

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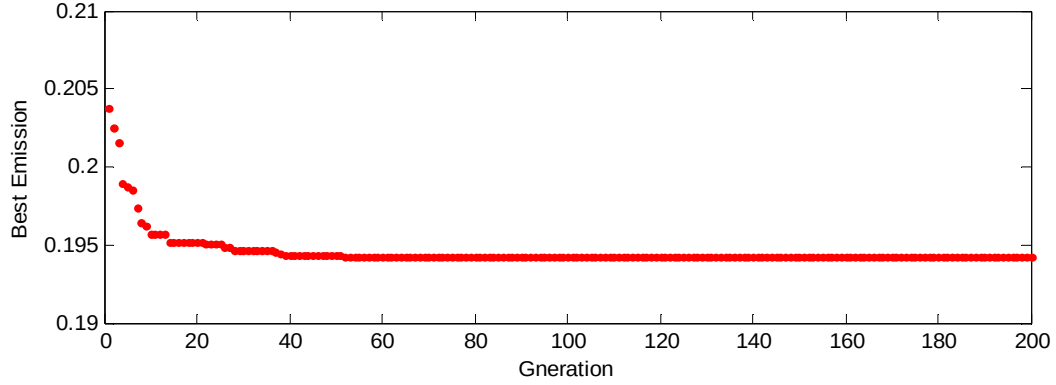


Figure 7: Convergence emission objective

8. Identifying a satisfactory solution

Optimization of the above-formulated objective functions using multiobjective genetic algorithms yields not a single optimal solution, but a set of Pareto optimal solutions, in which one objective cannot be improved without sacrificing other objectives. For practical applications, however, we need to select one solution, which will satisfy the different goals to some extent. Such a solution is called best compromise solution. TOPSIS (Technique for Order Preference by Similarity to Ideal Solution) method [20,21,30] has the ability to identify the best alternative from a finite set of alternatives quickly.

The idea of TOPSIS can be expressed in a series of steps:

(1) Obtain performance data for n alternatives over M criteria x_{ij} ($i=1, \dots, n, j=1, \dots, M$).

(2) Calculate normalized rating (vector normalization is used) r_{ij} .

(3) Develop a set of importance weights W_j , for each of the criteria. The basis for these weights can be anything, but, usually, is ad hoc reflective of relative importance.

$$V_{ij} = w_j \cdot r_{ij} \quad (13)$$

(4) Identify the ideal alternative (extreme performance on each criterion) S^+ .

$$S^+ = \{v_1^+, v_2^+, \dots, v_j^+, \dots, v_m^+\} = \left\{ \left(\max v_{ij} \mid j \in J_1 \right), \left(\min v_{ij} \mid j \in J_2 \right), i = 1, \dots, n \right\} \quad (14)$$

Where J_1 is a set of benefit attributes and J_2 is a set of cost attributes.

(5) Identify the nadir alternative (reverse extreme performance on each criterion) S^- .

$$S^- = \{v_1^-, v_2^-, \dots, v_j^-, \dots, v_m^-\} = \left\{ \left(\min_{ij} v_{ij} \mid j \in J_1 \right), \left(\max_{ij} v_{ij} \mid j \in J_2 \right), i = 1, \dots, n \right\} \quad (15)$$

(6) Develop a distance measure over each criterion to both ideal (D^+) and nadir (D^-).

$$D_i^+ = \sqrt{\sum_j (v_{ij} - v_j^+)^2}, \quad D_i^- = \sqrt{\sum_j (v_{ij} - v_j^-)^2} \quad (16)$$

(7) For each alternative, determine a ratio R equal to the distance to the nadir divided by the sum of the distance to the nadir and the distance to the ideal,

$$R = \frac{D^-}{D^- + D^+} \quad (17)$$

(8) Rank alternative according to ratio R (in Step 7) in descending order.

(9) Recommend the alternative with the maximum ratio

Therefore it can be said that TOPSIS method is attractive since limited subjective input (namely the weight values which reflect the degree of satisfactory of each objective) is needed from the DM to get a satisfactory results from the Pareto set quickly. Also, this method can be classified as interactive approach, where the DM specifies input values according his needs.

Here, we need to select one solution (one operating point), which will satisfy the different goals to some extent. Such a solution is called best compromise solution. The identification of a best compromise solution requires taking into account the preferences expressed by the decision-maker DM, which reflect the degree of satisfactory of each objective. We incorporate relative weights of criterion importance as $\{w_1 = 0.2, w_2 = 0.8\}$, which give relative importance for fuel cost as 0.2 and relative importance for NO_x emission objective as 0.8, the bigger the weighting factor, the more important is the attainment of that objective.

In order to obtain the normalized rating, fuel cost $f_1(\cdot)$, and emission $f_2(\cdot)$ are optimized individually to obtain minimum values of the objectives. The minimum and maximum values of the objectives are given in Table 6 (Minimum values of the objectives are obtained by giving full consideration to one of the objectives and neglecting the others).

For each alternative, determine a ratio R equal to the distance to the nadir divided by the sum of the distance to the nadir and the distance to the ideal as in step 7. Alternatives have been ranked by maximizing the ratio R . It is obvious that all set of solutions are ranked corresponding to the relative weights of criterion importance (degree of satisfactory)

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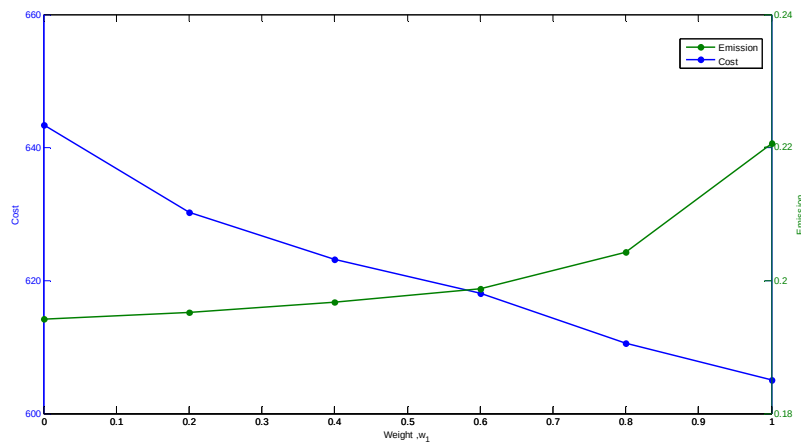
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Table 6: The minimum and maximum values of the objectives

	Max	Min
Fuel cost(\$)	643.3729	605.0802
Emission(ton)	0.2206	0.1942

425 To declare the performance of changing the weights $\{w_1, w_2 \mid w_1 + w_2 = 1\}$ on the best
 426 compromise solution, we plot different values of weight w_1 versus best compromise solution
 427 of $f_1(\cdot)$ (Cost (\$/h)) and versus best compromise solution of $f_2(\cdot)$ (Emission (ton/h)) as in
 428 Figure 8. It is obvious that for each weight (criterion importance), different best compromise
 429 solutions had found proportional to the criterion importance (weighting factor).



430

431 Figure 8: Weight w_1 versus best compromise cost and emission432 **9. Conclusions**

433 The proposed approach presented in this paper was applied to Economic Emission Load
 434 Dispatch (EELD) problem which formulated as multiobjective optimization problem with
 435 competing fuel cost, and emission. The proposed algorithm operates in two Phases: in the
 436 first one, multiobjective version of genetic algorithm is used as search engine in order to
 437 generate approximate true Pareto front. This algorithm based on concept of co-evolution and
 438 repair algorithm. Also it maintains a finite-sized archive of nondominated solutions which
 439 gets iteratively updated in the presence of new solutions based on the concept of ϵ -
 440 dominance. Then in the second phase, rough set theory is adopted as local search engine
 441 in order to improve the spread of the solutions found so far. Our proposed approach keeps
 442 track of all the feasible solutions found during the optimization. The following are the
 443 significant contributions of this paper:

444

- 445 1. The results prove superiority of the proposed approach to those reported in the
 446 literature.
- 447 2. The non-dominated solutions in the obtained Pareto-optimal set are well distributed
 448 and have satisfactory diversity characteristics.

- 449 3. The proposed approach is efficient for solving nonconvex multiobjective optimization
450 where multiple Pareto-optimal solutions can be found in one simulation run.
451 4. The proposed technique has been effectively applied to solve the EED considering
452 two objectives simultaneously, with no limitation in handling more than two
453 objectives.
454 5. TOPSIS method is employed to extract the best compromise solution from the trade-
455 off curve according to the determined weight factor, the bigger the weight factor, the
456 more important is the attainment of that objective.
457

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