

A New Structure and Contribution in D -metric Spaces

By

Ali Ahmad Ali Fora*

Yarmouk University

Email address: afora_jo@yahoo.com

Mourad Oqla Massa'deh

Al-Balqa Applied University

Email address: Mourad.oqla@bau.edu.jo

Mohammad Saleh Bataineh

Dept. of Math. Sharjah University

Email address: mbataineh@sharjah.ac.ae

ABSTRACT. In this paper, we define a new topological structure of D -closed, D -continuous and D -fixed point property and discussed of its properties, some result for this subject are also established.

1 Introduction:

The concept of a D -metric space was introduced by Dhage in [2]. A nonempty set X , together with a function $D : X \times X \times X \rightarrow [0, \infty)$ is called a D -metric space, denoted by (X, D) if D satisfies the followings:

- $D_1)$ $D(x, y, z) = 0$ if and only if $x = y = z$ (coincidence),
- $D_2)$ $D(x, y, z) = D(p(x, y, z))$, where p is a permutation of x, y, z (symmetry),
- $D_3)$ $D(x, y, z) \leq D(x, y, a) + D(x, a, z) + D(a, y, z)$ for all $x, y, z, a \in X$ (tetrahedral inequality).

The nonnegative real function D is called a D -metric on X . Dhage [2] claimed that D -metric convergence defines a Hausdorff topology and that the D -metric is (sequentially) continuous in all the three variables. Many authors (see [1]-[11] and [23]-[25]) have taken these claims for granted and used them in proving fixed point theorems in D -metric spaces. For more discussion, we refer the reader to consrilt ([12]-[18] and [21]-[22]). Authors of [20] gave examples to show that in a D -metric space:

- 1) D -metric convergence does not always define a topology.
- 2) Even when D -metric convergence defines a topology, it need not be Hausdorff.
- 3) Even when D -metric convergence defines a metrizable topology, the D -metric need not be continuous even in a single variable.

Definition 1.1 [2, 8]. A sequence $\{x_n\}$ in a D -metric space (X, D) is said to be convergent (or D -convergent) if there exists an element x in X such that $\lim_{n,m} D(x_n, x_m, x) = 0$, i.e. for any $\epsilon > 0$, there exists $j \in \mathbb{N}$ such that $D(x_n, x_m, x) < \epsilon$ for all $n, m \geq j$. In such a case, $\{x_n\}$ is said to converge to x and x is called a limit of $\{x_n\}$. We shall use the notation $\{x_n\} \xrightarrow{D} x$ to denote that $\{x_n\}$ is D -convergent to x .

Definition 1.2 [2, 8]. A sequence $\{x_n\}$ in a D -metric space (X, D) is said to be Cauchy (or D -Cauchy) if, for any $\epsilon > 0$, there exists $j \in \mathbb{N}$ such that $D(x_n, x_m, x_k) < \epsilon$ for all $n, m, k \geq j$.

Definition 1.3 [2, 8]. A D -metric space (X, D) is said to be complete (or D -complete) if every D -Cauchy sequence in X is D -convergent in X .

We shall use the same notation used in [20]. for A^c , namely.

Notation 1.4 [20]. For a subset A of a D -metric space (X, D) , A^c denotes the set $\{x \in X : \text{there exists } x_n \in A \text{ such that } \{x_n\} \xrightarrow{D} x\}$. For any set X , $P(X)$ denotes the power set of X .

S. V. R. Naidu, K.P. R. Rao, and N. Srinivasa Rao have obtained the following nice example.

Example 1.1 [20]. Let $X = A \cup B \cup \{0\}$, where $A = \{2^{-n} : n \in \mathbb{N}\}$ and $B = \{2^n : n \in \mathbb{N}\}$. Then there exists a D -metric on X such that:

- (i) (X, D) is a complete D -metric space in which D -metric convergence does not define a topology.
- (ii) There are convergent sequences in X with infinitely many limits.
- (iii) The operator $\varphi : P(X) \rightarrow P(X)$ defined by $\varphi(A) = A^c$ does not define a closure operator. More precisely, $(B^c)^c \neq B^c$.

2 D -closed sets:

If A is any subset of a D -metric space (X, D) and if $a \in A$, then $\{x_n\} \xrightarrow{D} a$, where $x_n = a$ for $n \in \mathbb{N}$, because $\lim_{n,m} D(x_n, x_m, a) = 0$, Thus $A \subseteq A^c$. Imitating the case of sequentially closed sets we have the following.

Definition 2.1 . A subset E of a D -metric space (X, D) is said to be D -closed provided $E = E^c$ (equivalently $E^c \subseteq E$), i.e. if for any $x_n \in E$ and $p \in X$, if $\{x_n\} \xrightarrow{D} p$, then $p \in E$. The complement of a D -closed set is called D -open. A set in (X, D) will be called D -clopen set if it is D -closed and D -open simultaneously.

The following results are easy to observe.

Proposition 2.2 . Let (X, D) be a D -metric space. If $\{x_n\} \xrightarrow{D} p$ then every D -open set H containing p must contain a tail of $\{x_n\}$.

Proof. Let H be a D -open set in (X, D) containing p . Suppose on the contrary, that H does not contain any tail of $\{x_n\}$. Then there exists a sequence of natural numbers $1 < m_1 < m_2 < \dots$ such that $x_{m_n} \notin H$ for all $n \in \mathbb{N}$. Since $\{x_n\} \xrightarrow{D} p$ therefore $\{x_{m_n}\} \xrightarrow{D} p$. Since $X - H$ is D -closed and $x_{m_n} \in X - H$, therefore $p \in X - H$ which is absurd.

Proposition 2.3 . Every finite set in a D -metric space (X, D) must be D -closed.

Proof. Let A be a finite subset of X and let $x_n \in A, p \in X$ such that $\{x_n\} \xrightarrow{D} p$. Then $\lim_{n,m} D(x_n, x_m, x) = 0$, yields the existence of a natural number j such that $x_n = p$ for all $n \geq j$ (i.e. $\{x_n\}$ has a constant tail $p, p, p, \dots$). Hence $p \in A$.

Proposition 2.4 . Let $D : X \times X \times X \rightarrow [0, \infty)$ be a D -metric on X having a finite range. Then every subset A of X is D -closed.

Proof. Similar to the proof of Proposition 2.3.

Theorem 2.5 The intersection of any collection of D -closed sets in a D -metric space (X, D) is D -closed.

Proof. Let $\mathfrak{F} = \{F_\alpha : \alpha \in \Delta\}$ be a collection of D -closed sets in X and let $x_n \in \bigcap_{\alpha \in \Delta} F_\alpha, p \in X$ such that $\{x_n\} \xrightarrow{D} p$. Since $x_n \in F_\alpha, \{x_n\} \xrightarrow{D} p$ and F_α is a D -closed set, therefore $p \in F_\alpha$ ($\alpha \in \Delta$). Hence $p \in \bigcap_{\alpha \in \Delta} F_\alpha$. Consequently, $\bigcap_{\alpha \in \Delta} F_\alpha$ is D -closed.

Now it is meaningful to have the following definition.

Definition 2.6 . If A is a subset of a D -metric space (X, D) , we define the D -closure of A (denoted by $D - cl(A)$ or $cl_D(A)$) as the intersection of all D -closed sets in X containing A .

It is clear that $D - cl(A)$ is the smallest D -closed set in X containing A . It is also clear that an arbitrary union of D -open sets in a D -metric space (X, D) is still D -open. The fact that the subsets ϕ and X are D -clopen in (X, D) is obvious. Finally, if A is a subset of B then $cl_D(A) \subseteq cl_D(B)$.

Definition 2.7 [20]. A subfamily T^* of X is said to be a supra topology on X if:

- (1) $\phi, X \in T^*$.
- (2) If $F_\alpha \in T^*, \alpha \in \Delta$, then $\bigcup_{\alpha \in \Delta} F_\alpha \in T^*$

(X, T^*) is called a supra topological space. The elements of T^* are called supra open sets in (X, T^*) and complement of a supra open set is called a supra closed set.

Definition 2.8 [20]. The supra closure of a set A is denoted by supra $cl(A)$ and is defined as supra $cl(A) = \bigcap \{B : B \text{ is a supra closed set and } A \subseteq B\}$. The supra interior of a set A is denoted by supra $int(A)$, and defined as supra $int(A) = \bigcup \{B : B \text{ is a supra open set and } B \subseteq A\}$.

Theorem 2.9 . Let (X, D) be a D -metric space and $T_D^* = \{A \subseteq X : A \text{ is } D - \text{open set in } (X, D)\}$. Then we have the following.

- (i) T_D^* is a supra topology on X .
- (ii) Every finite subset of X is supra closed in (X, T_D^*) .
- (iii) For any A subset of X , $cl_D(A) = \text{supra } cl(A)$.

Example 1.1 verifies the following result.

Theorem 2.10 . There exists a D -metric space (X, D) such that:

- (i) There exists $B \subseteq X$ such that $cl_D(cl_D B) \neq cl_D(B)$.
- (ii) There exist subsets M, P of X such that $cl_D(M \cup P) \neq cl_D(M) \cup cl_D(P)$.

Proof. Indeed , if Proposition 2.10 (ii) fails then (X, T_D^*) will be a topological space and this is absurd. As in usual metric space (X, d) , the collection $\mathfrak{S}(d)$ of all open balls is indeed a subbase for the metric topology $T(d)$. Fortunately, $\mathfrak{S}(d)$ is a base for $T(d)$. In a similar way, the collection $\mathfrak{S}^*(D)$ of all D -open sets in a D -metric space (X, D) is a subbase for some topology $T(D)$ on X . Unfortunately, $\mathfrak{S}^*(d)$ need not be a base for some topology on X . So, one starts looking for properties of this topology $T(\mathfrak{S}^*(d))$ generated by $\mathfrak{S}^*(d)$ as a subbase (rather than a base!).

Finally, in this section, we have the following result.

Corollary 2.11 . *Let $D : X \times X \times X \rightarrow [0, \infty)$ be a D -metric on X having a finite range. Then every subset A of X is D -closed, i. e, D generates the discrete topology on X .*

Proof. Immediate consequence of Proposition 2.4.

3 D-continuous functions:

If $(X, \mathfrak{S})$ and (Y, T) are topological spaces such that $(X, \mathfrak{S})$ is first countable at $p \in X$. Then a function $f : (X, \mathfrak{S}) \rightarrow (Y, T)$ is continuous at p if and only if f is sequentially continuous at p (i.e. for any sequence $\{x_n\}$ converging in $(X, \mathfrak{S})$ to p , then $\{f(x_n)\}$ must converge in (Y, T) to $f(p)$).

Imitating this idea in D -metric spaces, we get the following.

Definition 3.1 . *Let $f : (X, D) \rightarrow (Y, \rho)$ be a function between two D -metric spaces. Then f is said to be D_ρ -continuous at $p \in X$ provided that for any sequence $\{x_n\}$ converging in $(X, \mathfrak{S})$ to p , then $\{f(x_n)\}$ must converge in (Y, ρ) to $f(p)$. A function $f : (X, D) \rightarrow (Y, \rho)$ is called D_ρ -continuous if f is D_ρ -continuous at each p in X . In the case $X = Y$ and $D = \rho$, we write D -continuous instead of D_ρ -continuous.*

Definition 3.2 *Let $f : (X, D) \rightarrow (Y, \rho)$ be a function between two D -metric spaces. Then f is said to be D_ρ -weakly continuous at $p \in X$ provided that for any ρ -open set H in Y containing $f(p)$, there exists a D -open set U in X containing p such that $f(U) \subseteq H$. A function $f : (X, D) \rightarrow (Y, \rho)$ is called D_ρ -weakly continuous if f is D_ρ -weakly continuous at each p in X . In the case $X = Y$ and $D = \rho$, we write D -weakly continuous instead of D_ρ -weakly continuous.*

The following result is an analogue to a well-known result in general topology.

Theorem 3.3 . *The followings are equivalent for the function $f : (X, D) \rightarrow (Y, \rho)$ between two D -metric spaces.*

(i) f is D_ρ -weakly continuous.

(ii) For any ρ -open set H in Y , $f^{-1}(H)$ is D -open set in X .

(iii) For any ρ -closed set M in Y , $f^{-1}(M)$ is D -closed set in X .

Proof. (i) $\Rightarrow$ (ii): Let H be a ρ -open set in Y . For each $x \in f^{-1}(H)$, then $f(x) \in H$. Since f is D_ρ -weakly continuous at x , there exists a D -open set U_x such that $x \in U_x$ and $f(U_x) \subseteq H$ (i.e. $U_x \subseteq f^{-1}(H)$). Consequently, $f^{-1}(H)$ is a union of D -open sets in X , and hence is D -open.

(ii) $\Rightarrow$ (iii): Let M be a ρ -closed set in Y . Then $Y - M$ is ρ -open set in Y and hence $f^{-1}(Y - M)$ is a D -open set in X . Consequently, $X - f^{-1}(Y - M)$ is a D -closed set in X , i.e. $f^{-1}(M)$ is a D -closed set in X .

(iii) $\Rightarrow$ (i): To show that f is D_ρ -weakly continuous function, let $p \in X$ and H be any ρ -open set in Y such that $f(p) \in H$. Then $Y - H$ is a ρ -closed set in Y . Hence $f^{-1}(Y - H)$ is a D -closed set in X . Thus $X - f^{-1}(Y - H)$ is a D -open set in X containing p . It is clear that $f(X - f^{-1}(Y - H)) \subseteq H$. Hence f is D_ρ -weakly continuous.

Theorem 3.4 . Let $f : (X, D) \rightarrow (Y, \rho)$ be a function between two D -metric spaces. If f is D_ρ -continuous then f is D_ρ -weakly continuous.

Proof. Let M be any ρ -closed set in Y . To prove that $f^{-1}(M)$ is D -closed in X , let $\{x_n\}$ be any sequence in $f^{-1}(M)$ converging in X to p . Then the sequence $\{f(x_n)\}$ is in M and converging in Y to $f(p)$. The fact that M is ρ -closed set in Y forces $f(p)$ to belong to M . Hence $p \in f^{-1}(M)$.

4 D -fixed point property:

We start this section with the following.

Definition 4.1 (i) A D -metric space (X, D) is said to have the D -fixed point property (abbreviated D -f.p.p.) iff every D -continuous function $f : (X, D) \rightarrow (X, D)$ has a fixed point in X .

(ii) A D -metric space (X, D) is said to have the D -weakly fixed point property (abbreviated D -w.f.p.p.) iff every D -weakly continuous function $f : (X, D) \rightarrow (X, D)$ has a fixed point in X .

The following result is an immediate consequence of Theorem 3.4.

Corollary 4.2 . If a D -metric space has the D -w.f.p.p. then it has the D -f.p.p.

Definition 4.3 . A D -metric space (X, D) is called D -disconnected provided there exists a partition $\{A, B\}$ for X consisting of two D -closed sets in X . (X, D) is called D -connected provided it is not D -disconnected.

Definition 4.4 . Let (X, D) be a D -metric space and A, B be two nonempty subsets of X . Then A, B are called D -separated sets provided $A \cap cl_D(B) = B \cap cl_D(A) = \emptyset$.

The following result is needed for the next theorem.

Lemma 4.5 . In a D -metric space (X, D) , if A, B are two D -separated sets in X such that $A \cup B = X$. Then each of them is a D -clopen set in X .

Proof. Since $B \subseteq cl_D(B)$ and $A \cap cl_D(B) = \emptyset$, therefore $A \cap B = \emptyset$. Now $A \cup B = X$ and $A \cap B = \emptyset$ implies $A = X - B$. The fact that $cl_D(B) \subseteq X - A = B$ implies $B = cl_D(B)$, i.e. B is D -closed. Similarly, A is a D -closed set in X . Consequently, A and B are D -clopen sets in X .

Now the proof of the following result becomes easy to follow.

Theorem 4.6 . The following conditions are equivalent for a D -metric space (X, D) .

- (i) (X, D) is D -disconnected.
- (ii) There exists a D -clopen set A in X such that $\emptyset \neq A \neq X$.
- (iii) X has a partition consisting of two D -open sets.
- (iv) X has a nontrivial D -separation.
- (v) There exists a surjective D_ρ -continuous function $f : (X, D) \rightarrow (\{0, 1\}, \rho)$, where ρ is any D -metric on $\{0, 1\}$.

The following result will be needed later.

Theorem 4.7 . Let (X, D) be a D -metric space and $\{A, B\}$ be a partition of X consisting of D -closed (D -open) sets in X . Let $a \in A$ and $b \in B$ be fixed elements. Then the function $f : (X, D) \rightarrow (X, D)$ defined by: $f(A) = \{b\}$ and $f(B) = \{a\}$, is D -continuous (and hence D -weakly continuous).

Proof. To prove that f is D -continuous, let $\{x_n\}$ be any sequence in X and assume $\{x_n\} \xrightarrow{D} p$. Without loss of generality we may assume $p \in A$. To prove $\{f(x_n)\} \xrightarrow{D} f(p)$ we are going to show that $\lim_{n,m} D(f(x_n), f(x_m), f(p)) = 0$. To prove this, we claim that there exists $j \in \mathbb{N}$ such that for all $n \geq j$, $f(x_n) = b$ (i.e. $x_n \in A$ for n sufficiently large). To prove our claim, suppose not, i.e. for each $j \in \mathbb{N}$ there exists $n_j \in \mathbb{N}$ such that $n_j > j$ and $x_{n_j} \notin A$ (hence $x_{n_j} \in B$). Doing this, we can find an infinite sequence of natural numbers $1 < n_1 < n_2 < \dots$ such that $x_{n_j} \in B$ for all $j \in \mathbb{N}$. Since $\lim_{n,m} D(x_n, x_m, p) = 0$ therefore $\lim_{i,j} D(x_{n_i}, x_{n_j}, p) = 0$. Thus $\{x_{n_j}\} \xrightarrow{D} p$. Now $x_{n_j} \in B$ and B is D -closed in X implies that $p \in B$, this is a contradiction. Now since our claim becomes valid, i.e. a tail of $\{x_n\}$ must be in A . Hence $f(x_n) = b$ for all n large enough. Thus $\lim_{n,m} D(f(x_n), f(x_m), f(p)) = 0$, i.e. $\{f(x_n)\} \xrightarrow{D} f(p)$.

Definition 4.8 . A D -metric space (X, D) is called a D - T_0 -space provided that for any $x, y \in X$, $x \neq y$, there exists a D -open set H such that $H \cap \{x, y\}$ has exactly one element.

The following results are now ready to be proved.

Theorem 4.9 .(i) If (X, D) has the D -f.p.p., then it is D -connected.
(ii) If (X, D) has the D -w.f.p.p., then it is D -connected.

Proof. (i) Let (X, D) be a D -metric space having the D -fixed point property. Suppose on the contrary, that X is D -disconnected. Then X has a partition $\{A, B\}$ consisting of D -closed sets in X . Now pick $a \in A$ and $b \in B$. Define $f : (X, D) \rightarrow (X, D)$ by: $f(A) = \{b\}$ and $f(B) = \{a\}$. Then f is D -continuous according to Theorem 4.7. Notice that f has no fixed point and this contradicts the assumption that (X, D) has the D -w.f.p.p.

(ii) The proof is a direct consequence of (i) together with Proposition 4.2.

Theorem 4.10 .(i) If (X, D) has the D -f.p.p., then it is a D - T_0 -space.
(ii) If (X, D) has the D -w.f.p.p., then it is a D - T_0 -space.

Proof. (i) Let (X, D) be a D -metric space having the D -fixed point property. Suppose on the contrary, that X is not a D - T_0 -space. Then there exist $p, q \in X$, $p \neq q$ such that for any D -open set H in X , either $H \cap \{p, q\} = \emptyset$ or $\{p, q\} \subseteq H$. Define $f : (X, D) \rightarrow (X, D)$ by:

$$f(x) = \begin{cases} p & \text{if } x \neq p \\ q & \text{if } x = p \end{cases}$$

Then f clearly has no fixed point. To prove f is D -continuous, let H be any D -open set in X . Then $f^{-1}(H) = \begin{cases} X & \text{if } p \in H \\ \emptyset & \text{if } p \in X - H \end{cases}$. Notice that for any D -open set H in

X we have: $q \in H$ if and only if $p \in H$. Hence f is D -continuous, which gives us the contradiction.

(ii) The proof is a direct consequence of (i) together with Proposition 4.2.

5 Product of D -metric spaces:

Let us start with the following.

Proposition 5.1 . Let (X_i, D_i) be D -metric spaces ($i = 1, \dots, n$) and let $X = X_1 \times \dots \times X_n$. Define $D : X \times X \times X \rightarrow [0, \infty)$ as follows:

$$D((x_1, \dots, x_n), (y_1, \dots, y_n), (z_1, \dots, z_n)) = \sum_{i=1}^n D_i(x_i, y_i, z_i).$$

Then D is a D -metric on X (we shall denote D by $\sum_{i=1}^n D_i$).

Proof. Straight forward.

Proposition 5.2 . Let (X, D) be a D -metric space and $\emptyset \neq A \subseteq X$. Define $D^* : A \times A \times A \rightarrow [0, \infty)$ as follows: $D^*(x, y, z) = D(x, y, z)$. Then D^* is a D -metric on A (D^* will be denoted by $D^{(A)}$).

Proof. Straight forward.

The following result will be needed later.

Lemma 5.3 . Let (X_i, D_i) be D -metric spaces ($i = 1, \dots, n$) and let $X = X_1 \times \dots \times X_n$. Define $D : X \times X \times X \rightarrow [0, \infty)$ as in Proposition 5.1. Let $z_k = (x_k^{(1)}, x_k^{(2)}, \dots, x_k^{(n)}) \in X$ ($k \in \mathbb{N}$). Then $\{z_k\} \xrightarrow{D} z = (p_1, p_2, \dots, p_n)$ if and only if $\{x_k^j\} \xrightarrow{D_j} p_j$ ($j = 1, 2, \dots, n$).

Proof. Straight forward.

Theorem 5.4 . Let (X_i, D_i) be D -metric spaces ($i = 1, \dots, n$) and let $X = X_1 \times \dots \times X_n$. Define $D : X \times X \times X \rightarrow [0, \infty)$ as in Proposition 5.1. If A_i is a D_i -closed set in (X_i, D_i) for $i = 1, \dots, n$. Then $A_1 \times A_2 \times \dots \times A_n$ is a D -closed set in (X, D) .

Proof. The proof is an immediate consequence of Lemma 5.3.

References

- [1] B. Ahmad, M. Ashraf, and B.E. Rhoades, Fixed point theorems for expansive mappings in D-metric spaces, *Indian J. Pure Appl. Math.* 32(2001), no.10, 1513-1518.
- [2] B.C.Dhage, Generalized metric spaces and mappings with fixed point, *Bull. Calcutta Math. Soc.* 84 (1992), no.4, 329-336.
- [3] B. C. Dhage, On generalized metric spaces and topological structure .II, *Pure Appl. Math. Sci.* 40 (1994), no.1-2, 37-41.
- [4] B. C. Dhage, A common fixed point principle in D-metric spaces, *Bull. Calcutta Math. Soc.* 91 (1999), no.6, 475-480.
- [5] B. C. Dhage, On common fixed points of pairs of coincidentally commuting mappings in D-metric spaces, *Indian J. Pure Appl. Math.* 30(1999), no.4, 395-406.
- [6] B. C. Dhage, Generalized metric spaces and topological structure.I, *An. Stiint. Univ.Al. I. Cuza Iasi. Mat.(N.S.)*46(2000),no.1,3-24.
- [7] B.C.Dhage, Smarti Arya, and Jeong Sheok Ume, A general lemma for fixed-point theorems involving more than two maps in D-metric spaces with applications, *Int.J.Math.Math.Sci.*(2003), no.11,661-672.
- [8] B.C. Dhage, A.M. Pathan, and B. E. Rhoades, A general existence principle for fixed point theorems in D-metric spaces, *Int. J. Math. Math. Sci.* 23 (2000), no.7, 441- 448.
- [9] B.C. Dhage, and B.E. Rhoades, A comparison of two contraction principles, *Math. Sci. Res. Hot-Line* 3 (1999), no. 8, 49-53.
- [10] B.C. Dhage, On common fixed points of quasi-contraction mappings in D-metric spaces, *Indian J. Pure Appl. Math.* 33 (2002), no.5, 677-690.
- [11] B.C. Dhage, Proving fixed point theorems in D-metric spaces via general existence principles, *Indian J. Pure Appl. Math.* 34 (2003), no.4, 609-624.
- [12] PS. Kumari, P. Dinesh, Cyclic contractions and fixed point theorems on various generating spaces, *Fixed Point Theory and Applications.* (2015): 153.
- [13] PS. Kumari, On dislocated quasi metrics, *Journal of Advanced Studies in Topology.* 3(2012), no.2, 66-75.
- [14] PS. Kumari, K. Zoto, and D. Panthi, D - neighborhood system and generalized F-contraction in dislocated metric space, *Springer Plus.* 4(2015), no.1, 1-10.
- [15] PS. Kumari, Ch V. Ramana, and K. Zoto, On quasisymmetric space, *Indian Journal of Science and Technology.* 7(2014), no.10, 1583-1587.

- [16] PS. Kumari, IR. Sarma, and JM. Rao, Metrization theorem for a weaker class of uniformities, *Afrika Matematika*. 27(2016), no. 3-4, 667-672.
- [17] PS. Kumari , S. Muhammad, Some fixed point theorems in generating space of b-quasi-metric family. *Springer Plus*. 5(2016), no.1, 268.
- [18] PS. Kumari, et al, Fixed point theorems and generalizations of dislocated metric spaces, *Indian J. Sci. Technol*. 8 (2015), 154-158.
- [19] A.S.Mashhour, A.A.Allam, F.S.Mahmoud and F. H. Khedr, On supra topological spaces, *Indian J. Pure and Appl. Math*. No.4, 14(1983), 502-510.
- [20] S.V.R. Naidu, K.P.R. Rao, and N. Srinivasa Rao, On the topology of D-metric spaces and generation of D-metric spaces from metric spaces, *Int. J. Math. Math. Sci.* (2004), no.51, 2719-2740.
- [21] IR. Sarma, PS. Kumari, On dislocated metric spaces, *Int. J. Math. Achiev*. 3(2012), no.1, 7-27.
- [22] IR. Sarma , et al, Convergence axioms on dislocated symmetric spaces, *Abstract and Applied Analysis*, Hindawi Publishing Corporation, (2014).
- [23] J. S. Ume, Remarks on non convex minimization theorems and fixed point theorems in complete D-metric spaces, *Indian J. Pure Appl. Math*. 32 (2001), no.1, 25-36.
- [24] J.S.Ume and J.K. Kim, Common fixed point theorems in D-metric spaces with local boundedness, *Indian J. Pure Appl. Math*. 31 (2000), no.7, 865-871.
- [25] T.Veerapandi and K.Chandrasekhara Rao, Fixed points in Dhage metric spaces, *Pure Appl. Math.Sci*. 43 (1996), no.1-2, 9-14.