

The impulse interactive cuts of an entropy functional measure on trajectories of Markov diffusion process, integrating in information path functional, and applications

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Abstract

The introduced entropy integral measure on random trajectories (EF) is defined by the process additive functional with functions drift and diffusion reducing this functional on trajectories to a regular integral functional. This functional evaluates the cutting process impulses imposed through an impulse control.

The impulse control cuts the process EF on elementary information impulses $\downarrow\uparrow$ which the introduced information path functional (IPF) integrates in a bit composing an information process.

Compared to Shannon entropy measure of a random state, cutting the process on separated states decreases quantity of process information by the amount, concealed in correlation connections between these states, which hold a hidden process information infinite.

The impulse elementary interactive cutting action provides both reduction of the observing process entropy and discrete unit of the cutting entropy memorized as an information Bit.

That defines information as memorized entropy cutting in random observations which process interactions. Cutting maximum of minimal impulse information and transferring minimal entropy between impulses implement the paper maxmin-minimax principle of converting process entropy to information.

Each cutoff sequentially and automatically converts entropy to information, naturally encoding information Bit from random process, which connects the Bits sequences in the IPF and the EF predicts next cut.

The impulse curvature, enclosing the entropy of interacting impulses, enables converting energy of an external (natural) process to bit of the interacting process which memorizes the bit by delivering the Landauer energy.

Finding the conditions creating a natural bit memorized in interacting curved impulse and the conditions generating a code of the interacting information process which assembles the natural bits.

The natural encoding satisfied Landauer's principle and compensates for the cost of Maxwell's Demon.

The energy of a specific interaction limits the universal code length.

It is shown that each IPF dimensional cut measures Feller kernel information.

The n -dimensional process cutoff generates a finite information measure, integrated in the IPF whose information approaches the EF measure at $n \rightarrow \infty$, restricting maximal information of the Markov diffusion process.

Estimation extracting information confirms non-additivity of the EF measured fractions.

The multiple applications validate the results.

Introduction

Conventional information science generally considers an information *process*, but traditionally uses the probability measure for random *states* and Shannon's entropy measure as the uncertainty *function* of the states (Shannon [1], Jaynes [2], Kolmogorov [3], others).

Such a measure evaluates information for a sequence of each process' states and does not measure *inner connections* between these states *through all process*.

The problem is to find an *integral* information measure, evaluating the inner connection and dependencies of the *random* process' states to measure the process' total information. Such measure should integrate information enclosed in both continuous and discrete processes.

Using definition of conditional information entropy and applying it to transitional probability density' measure, defined along the Markov diffusion process, we introduce the process' integral information measure on the process' trajectories.

Such an entropy functional (EF) measure has an analogy with Feynman's path functional [4], which Kac [5] has expanded on trajectories of Brownian processes.

The difference is transformation of the transition probability's density through the process' multiplicative and additive functionals whose mathematical expectation of a random trajectory defines the EF functional measure.

To estimate inner connections and dependencies of the diffusion process' states with EF, we use a jump-wise control action, cutting off the diffusion process through its de-correlation.

The cutting actions, while measuring the EF along the process trajectory, implement multiple sequence of the transitive transformations, which instantaneously convert probability of Markovian process to probability of Brownian process and vice versa.

Theory of additive functional is developed by Dynkin [6,7], Girsanov [8], Ventsel [9], Blumenthal, Gettoor [10], Ito, Ikeda, Watanabe [11,12], Prokhorov, Rozanov [13], Gikhman, Skorochod [14], other authors.

The information measures of a random process were introduced by Fisher [15], Kulback, Leibler [16], Rényi [17], Stratonovich [18] and some others.

Fisher's logarithmic measure calculates the covariance matrices associated with maximum-likelihood estimates in statistics. Rényi's entropy is a generalization of Shannon entropy by introducing probability of a discrete random variable with various parameter of the probability's power. Kullback-Leibler's divergence between the probabilities of a process' states distributions is measured by a relative Shannon's entropy. Stratonovich specified Radon-Nikodym's density for a probability density measure, applying it to Shannon's entropy of a random process.

Controllable Markov processes with the optimal stopping and estimation methods are studied by Dynkin [19], Gihman, Skorochod [20], Krylov [21], Harrison *et al* [22], Handson [23], Bensoussan, Lions [24], Bronstein *et al* [25], and many others.

Feller [26, 27] introduced a diffusion operator of “kernel”, which absorbs the diffusion motion on its limited time interval, covering Markov property.

McKean and Tanaka [28] connected the Feller’s kernel with additive functionals of Brownian path and related these functionals to both an instant of the path’ passage boundary time, limited the kernel, and to the kernel finite energy.

Fukushima *et al* [29] estimated an energy, covering the kernel, and the minimal boundary time.

Borodin and Gorin [30] studied a correlation function of a discrete time Markov process with the transitional probability measure of Feller’s kernel that links the kernel’s measure to the correlations.

The cutting off kernel information measure is still *unspecified*. The paper cutoff impulse control “intervenes” in the diffusion process during the minimal boundary instant time of a kernel’s jumping mechanism [29].

This leads to extraction of information hidden in a kernel, which is evaluated through the additive functional at such cut off.

A deterministic “intervention” in a random process, considered by Yushkevich [31], Dempster and Ye [32], others, studied mostly a diffusion process with a drift function.

Using the jump transition probability under the impulse control, which is described through the additive functional, defined by both drift and diffusion, allows us studying a more general case.

The paper shows that cutting off the entropy integral information measure on the separated states’ measures decreases the quantity of process information by the amount, which was concealed in the correlation connections between the separating states. This integral functional measure accumulates more process information than the sum of the information measures of its separated states and integrates information covered in both continuous process and discrete impulses cutting information of the process.

Study discrete impulses shows how each generates information and transmit it between the impulses.

Each impulse intervention interacts with the process in different forms: the cutoff-reaction, observation, measurement, which brings both irreversibility and a memory of cutting entropy.

While each cutoff reveals information hidden in the process before the cutoff, the paper information path functional (IPF) integrates all cutoffs impulse information.

The n -dimensional process cutoffs generates a finite information measure, integrated by information path functional (IPF) whose total information approaches the EF *measure* at $n \rightarrow \infty$, restricting maximal information of the Markov diffusion process.

In studied impulse delta-function cutoff step-down cut generates maximal information while the step-up action delivers minimal information from impulse cut to next impulse step-down action.

Extracting maximum of minimal impulse information and transferring minimal entropy between impulses implement maxmin-minimax principle of converting process entropy to information.

Each cutoff consequentially and automatically converts entropy to information, holding information Bit of random process, which connects the Bits sequences in the IPF.

Each EF dimensional cut measures Feller kernel information.

Estimation extracting information confirms non-additivity of the EF measured process fractions.

The paper organizes in 8 Sections:

Section 1 introduces the EF, defined on Markov diffusion process through the process' additive functional of transitive probabilities, which determine functions drift and diffusion of Ito's stochastic differential equation. The relational EF probability measure, conditional to Feller kernel, evaluates optimal transition to the kernel, minimizing the EF which originates math problem solving in the paper.

Section 2 evaluates information of the process' cutoff action by applying Dirac's impulse control, estimates the controls' extracting information, which includes the Feller kernel, evaluates its maximum; along with extracting maximum of minimal impulse information and transferring minimal entropy between impulses- as maxmin-minimax principle of converting process entropy to information.

Section 3 evaluates information of the cutting process' regular EF integral by the impulse control.

Section 4 properties of the EF cutoff functional, confirms non-additivity of the EF measured process' fractions, defines information path functional (IPF) with its properties.

Section 5 applies the theoretical results to impulse natural encoding.

Section 6 includes estimation information hidden by interstates' connections.

Section 7 demonstrates implementation of the theory in practical systems.

Section 8 analyzes the path functional in multi-dimensional observing process and discusses the results.

(Numbering formulas in each part's section starts with (1..) following (2..) in next section of the part, and so on. At citation of these formulas in other part, we use the number of the part as the first number (1.1...), (2.1..), (3.1.), (2.2..), etc.)

1. Entropy functional on the trajectories

Let us have Markov diffusion process $\tilde{x}_t$ with transition probabilities $P(s, \tilde{x}, t, B)$, $\mathcal{O}$ -algebra $\Psi(s, t)$ created by events $\{\tilde{x}(\tau) \in B\}$ at $s \leq \tau \leq t$, and conditional probability distributions $P_{s,x}$ on $\Psi(s, t)$.

A family of the real or complex random values $\varphi_s^t = \varphi_s^t(\omega)$ depending on $s \leq t$ defines an *additive functional* of process $\tilde{x}_t = \tilde{x}(t)$ [27, 28], if each $\varphi_s^t = \varphi_s^t(\omega)$ is measured regarding the related $\mathcal{O}$ -algebra $\Psi(s, t)$ at any $s \leq \tau \leq t$ with probability 1 at $\varphi_s^t = \varphi_s^\tau + \varphi_\tau^t$, and $E_{s,x}[\exp(-\varphi_s^t(\omega))] < \infty$, where $E_{s,x}[\cdot]$ are the related mathematical expectations.

Transformation of probability measures [7, p.48]:

$$\tilde{P}_{s,x}(d\omega) = p(\omega)P_{s,x}(d\omega), \quad (1.1)$$

on trajectories of Markov process $(\tilde{x}_t, P_{s,x})$ holds distributions $\tilde{P}_{s,x} = \tilde{P}_{s,x}(A)$ on extensive $\mathcal{O}$ -algebra $\Psi(s, \infty)$ with density measure:

$$p(\omega) = \frac{\tilde{P}_{s,x}(d\omega)}{P_{s,x}(d\omega)} = \exp\{-\varphi_s^t(\omega)\}, \quad (1.2)$$

and transitional probabilities of transformed diffusion process ζ_t with the additive functional:

$$\tilde{P}(s, \zeta, t, B) = \int_{\tilde{x}(t) \in B} \exp\{-\varphi_s^t(\omega)\} P_{s,x}(d\omega) \quad (1.3)$$

Applying the definition of conditional entropy [17] to mathematical expectation of logarithmic probability *functional density measure* (1.2) for process $\tilde{x}_t$ regarding process ζ_t we have

$$S(\tilde{x}_t / \zeta_t) = E\{-\ln[p(\omega)]\} = \int_{\tilde{x}(t) \in B} -\ln[p(\omega)] \tilde{P}_{s,x}(d\omega), \quad (1.4)$$

where $E = E_{x,s,\tilde{x}_t}$ is conditional mathematical expectation, taken along the process trajectories $\tilde{x}_t$ at a varied $(\tilde{x}, s)$ (by analogy with M.Kac [5, 194-218]).

From (1.2) and (1.3) we get conditional entropy functional expressed via the additive functional on the trajectories of considered diffusion processes:

$$S[\tilde{x}_t / \zeta_t] = E[\varphi_s^t(\omega)], \quad (1.5)$$

which is entropy measure of a distance between distributions $\tilde{P}_{s,x}, P_{s,x}$.

Minimum of this functional, depending on the additive functional measures closeness above distributions:

$$\min_{\varphi_s^t} S[\tilde{x}_t / \zeta_t] = S^0. \quad (1.5a)$$

Let diffusion process $\tilde{x}_t$ be a solution of stochastic Ito n -dimensional differential Eqs [12]:

$$d\tilde{x}_t = a(t, \tilde{x}_t)dt + \sigma(t, \tilde{x}_t)d\xi_t, \tilde{x}_s = \eta, t \in [s, T] = \Delta, s \in [0, T] \subset R_+^1, \quad (1.6)$$

with standard limitations[19] on drift function $a = a(t, x)$, function of diffusion $\sigma = \sigma(t, x)$, and Wiener process $\xi_t = \xi(t, \omega)$, defined on a probability space of elementary random events $\omega \in \Omega$ for variables located in R^n .

The process additive functional, according Girsanov's Theorem [8], satisfies the form [13,p.355-360], [14,Theorem11] with its upper limit T :

$$\varphi_s^T = 1/2 \int_s^T a(t, \tilde{x})^T (2b(t, \tilde{x}))^{-1} a(t, \tilde{x}_t) dt + \int_s^T \sigma(t, \tilde{x})^{-1} a(t, \tilde{x}) d\xi(t), 2b(t, \tilde{x}) = \sigma(t, \tilde{x})\sigma^T(t, \tilde{x}) > 0. \quad (1.7)$$

Functional (1.7) describes transformation of the Markov processes' random time traversing the various sections of a trajectory.

Let the transformed process be

$$\varsigma_t = \int_s^t \sigma(v, \xi_v) d\xi_v \quad (1.8)$$

having the same diffusion as the initial process, but the zero drift.

Measures $\tilde{P}_{s,x} = \tilde{P}_{s,x}(A)$, defined for diffusion process ς_t (1.8) (with transitional probability (1.2)) holds dispersion $b(t, \tilde{x})$ of $\tilde{x}_t$.

Process ς_t is a transformed version of process $\tilde{x}_t$ whose transition probability satisfies (1.1), and transformed probability $\tilde{P}_{s,x}$ for this process evaluates the Feller kernel measure [27, 29].

Since transformed process ς_t (1.8) has the same diffusion matrix but zero drift, the right part of additive functional in (1.5) satisfies

$$E\left[\int_s^T (\sigma(t, \tilde{x})^{-1} a(t, \tilde{x}) d\xi(t))\right] = 0. \quad (1.9)$$

We come to entropy functional expressed via parameters of the Ito stochastic equation in form:

$$S[\tilde{x}_t / \varsigma_t] = 1/2 E\left[\int_s^T a(t, \tilde{x})^T (2b(t, \tilde{x}))^{-1} a(t, \tilde{x}) dt\right]. \quad (1.10)$$

Formulas (1.2-1.5), (1.6), (1.7), (1.8) and (1.10) are in [33] with related citations and references.

The entropy functional in forms (1.4, 1.5, 1.10) is an *information indicator* of distinction between the probability measures of processes $\tilde{x}_t$ and ς_t ; it measures a *quantity of information* of process $\tilde{x}_t$ regarding process ς_t . For the process' equivalent measures, this quantity is zero, and it is positive for the process' equivalent measures.

Since mathematical expectation on the *process' trajectories* (1.5) is conditional to probability measure of Feller kernel, it is invariant at Markovian transformations defined through Radon-Nikodym's probability density measure (1.3) where both $P_{s,x}$ and $\tilde{P}_{s,x}$ are defined.

Thus, integral (1.10) is the entropy measure of Markov process $\tilde{x}_t$ while being conditional to the kernel probability measure. Entropy measure (1.4) is conditional to any transformed Markov diffusion process, not necessary satisfying (1.10).

Measuring conditional entropy (1.4), (1.10) relatively to diffusion process ς_t , which models standard perturbations in controllable systems, is practically usable.

Comments. Differential entropy defined on set of random variable X with probability density $p(v)$ as a Shannon form of conditional entropy:

$$H_v = E_x[-\ln p(v)] = -\int_X p(v) \ln p(v) dv, \quad (1.11)$$

is not equal to (1.4), and is not invariant under change of variables and the process.

Therefore, it cannot be an entropy measure for an arbitrary continuous process, which is unlimited by special requirements [35, pp.224-238].

Extension of Shannon's discrete entropy to continuous probability distributions requires preserving its invariant measure on any measured process. For these reasons, Shannon entropy (type H_v) with its density $p(v)$ is not founded for continuous process and is not sufficient and applicable for considered random process. •

The problem consists of evaluation information, that carries entropy functional (EF) at transforming above distributions while moving probability measure of processes $\tilde{\chi}_t$ to probability measure of ς_t .

The transformation or movement involves an action on the EF through changing the drift function of it additive functional. Measuring EF along trajectory of process $\tilde{\chi}_t$ requires a multiple sequence of such transformations, which instantaneously convert probability of Markovian process to probability of Brownian process and vice versa:

$$P_{s,x} \rightarrow \tilde{P}_{s,x}, \tilde{P}_{s,x} \rightarrow P_{s,x}. \quad (1.12)$$

Such nonstop transformations bring instant conditional probability's Brownian measure to the EF, implementing direct and correct measuring the random process entropy through this Radon-Nikodym's probability density measure.

Formulas (1.4), (1.10) directly measure the EF by the functional of the process trajectories' function drift and diffusion and connect it with conditional-density probabilities taken along the random trajectories.

The instant action on the functional drifts requires finding function which immediately executes that.

The requirement satisfies Dirac's delta-function and its discrete form represented through Heaviside's step-up and step-down functions cutting the process on small intervals.

Concurrent cutting each process dimension will instantaneously implement these transformations for n -dimensional Markov process under the manifold cutoff impulses.

Cutting the process by applying impulse control extracts concealed hidden information, which evaluates the EF process cutoff fractions.

2. The evaluation of the cutting process EF fractions by an impulse control

Let us define control on the space $KC(\Delta, U)$ of a piece-wise continuous step functions u_t at $t \in \Delta$:

$$u_- \stackrel{\text{def}}{=} \lim_{t \rightarrow \tau_k - 0} u(t, \tilde{\chi}_{\tau_k}), u_+ \stackrel{\text{def}}{=} \lim_{t \rightarrow \tau_k + 0} u(t, \tilde{\chi}_{\tau_k}), \quad (2.1)$$

which are differentiable on the set

$$\Delta^o = \Delta \setminus \{\tau_k\}_{k=1}^m, k=1, \dots, m, \quad (2.1a)$$

and applied on diffusion process $\tilde{\chi}_t$ from moment τ_{k-o} to τ_k , and then from moment τ_k to τ_{k+o} , implementing the process' transformations $\tilde{\chi}_t(\tau_{k-o}) \rightarrow \varsigma_t(\tau_k) \rightarrow \tilde{\chi}_t(\tau_{k+o})$; n -dimensional process holds m such transformations.

At a vicinity of moment τ_k , between the jump of control u_- and the jump of control u_+ , we consider a control *impulse*

$$\delta u_{\pm}(\tau_k) = u_-(\tau_{k-o}) + u_+(\tau_{k+o}). \quad (2.2)$$

The following statement evaluates the EF information contributions at such transformations.

Proposition 1.

Entropy functional (1.10) at the switching moments $t = \tau_k$ of control (2.2) takes values

$$\Delta S[\tilde{\chi}_t(\delta u_{\pm}(\tau_k))] = 1/2, \quad (2.3)$$

and at locality of $t = \tau_k$: at $\tau_{k-o} \rightarrow \tau_k$ and $\tau_k \rightarrow \tau_{k+o}$, produced by each of the impulse control's step functions in (2.1), is estimated by

$$\Delta S[\tilde{\chi}_t(u_-(\tau_k))] = 1/4, u_- = u_-(\tau_k), \tau_{k-o} \rightarrow \tau_k \quad (2.3a)$$

and

$$\Delta S[\tilde{x}_t(u_+(\tau_k))]=1/4, \quad u_+=u_+(\tau_k), \quad \tau_k \rightarrow \tau_{k+o}. \quad (2.3b)$$

Proof. The jump of the control function u_- in (2.1) from moment τ_{k-o} to τ_k , acting on the diffusion process, might cut off this process after moment $\tau_{k-o} \rightarrow \tau_k$. The cut off diffusion process has the same drift vector and the diffusion matrix as the initial diffusion process.

The additive functional for this cut off has the form [13]:

$$\varphi_s^{t-} = \begin{cases} 0, & t \leq \tau_{k-o}; \\ \infty, & t > \tau_k \end{cases}, \quad (2.4)$$

The jump of the control function u_+ (2.1) from τ_k to τ_{k+o} might cut off the diffusion process *after* moment $\tau_k \rightarrow \tau_{k+o}$ with the related additive functional

$$\varphi_s^{t+} = \begin{cases} \infty, & t > \tau_k \\ 0, & t \leq \tau_{k+o} \end{cases}. \quad (2.5)$$

For the control impulse (2.2), the additive functional at a vicinity of $t = \tau_k$ acquires the form of an *impulse function*

$$\varphi_s^{t-} + \varphi_s^{t+} = \delta\varphi_s^{\mp}, \quad (2.6)$$

which summarizes (2.3) and (2.4).

Entropy functional (1.1.10) following from (2.4-2.5) takes values

$$\Delta S[\tilde{x}_t(u_-(t \leq \tau_{k-o}; t > \tau_k))] = E[\varphi_s^{t-}] = \begin{cases} 0, & t \leq \tau_{k-o} \\ \infty, & t > \tau_k \end{cases}, \quad (2.7a)$$

$$\Delta S[\tilde{x}_t(u_+(t > \tau_k; t \leq \tau_{k+o}))] = E[\varphi_s^{t+}] = \begin{cases} \infty, & t > \tau_k \\ 0, & t \leq \tau_{k+o} \end{cases}, \quad (2.7b)$$

changing from 0 to ∞ and back from ∞ to 0, acquiring *absolute maximum* at $t > \tau_k$, between τ_{k-o} and τ_{k+o} .

The multiplicative functional [11], related to (2.4-2.5), are:

$$p_s^{t-} = \begin{cases} 0, & t \leq \tau_{k-o} \\ 1, & t > \tau_k \end{cases}, \quad p_s^{t+} = \begin{cases} 1, & t > \tau_k \\ 0, & t \leq \tau_{k+o} \end{cases}. \quad (2.8)$$

Control impulse (2.2) provides an impulse probability density in form of multiplicative functional

$$\delta p_s^{\mp} = p_s^{t-} p_s^{t+}, \quad (2.9)$$

where $\delta p_s^{\mp}$ holds $\delta[\tau_k]$ -function, which determines probabilities $\tilde{P}_{s,x}(d\omega) = 0$ at $t \leq \tau_{k-o}, t \leq \tau_{k+o}$ and $\tilde{P}_{s,x}(d\omega) = P_{s,x}(d\omega)$ at $t > \tau_k$.

For the cutoff diffusion process, transitional probability (at $t \leq \tau_{k-o}$ and $t \leq \tau_{k+o}$) turns to zero, and the states $\tilde{x}(\tau_k - o), \tilde{x}(\tau_k + o)$ become independent, while their mutual time correlations *are dissolved*:

$$r_{\tau_k - o, \tau_k + o} = E[\tilde{x}(\tau_k - o), \tilde{x}(\tau_k + o)] \rightarrow 0. \quad (2.10)$$

Entropy increment $\Delta S[\tilde{x}_t(\delta u_{\pm}(\tau_k))]$ of additive functional $\delta\varphi_s^{\mp}$ (2.5), produced within or at a border of control impulse (2.2), defines equality

$$E[\varphi_s^{t-} + \varphi_s^{t+}] = E[\delta\varphi_s^\mp] = \int_{\tau_{k-o}}^{\tau_{k+o}} \delta\varphi_s^\mp(\omega) P_\delta(d\omega), \quad (2.11)$$

where $P_\delta(d\omega)$ is a probability evaluation of the impulse $\delta\varphi_s^\mp$.

Integral of symmetric δ -function $\delta\varphi_s^\mp$ between the above time intervals on the border is

$$E[\delta\varphi_s^\mp] = 1/2 P_\delta(\tau_k) \text{ at } \tau_k = \tau_{k-o}, \text{ or } \tau_k = \tau_{k+o}. \quad (2.12)$$

The impulse, produced by deterministic controls (2.2) for each process dimension, is random with probability at τ_k -locality

$$P_{\delta c}(\tau_k) = 1, k = 1, \dots, m. \quad (2.13)$$

This probability holds a jump-diffusion transition probability in (2.12) (according to [19]), which is conserved during the jump.

From (2.11)-(2.13) follows estimation of the entropy functional's increment under impulse control (2.2) applying at $t = \tau_k$ in form

$$\Delta S[\tilde{x}_t(\delta u_\pm(\tau_k))] = E[\delta\varphi_s^\mp] = 1/2, \quad (2.14)$$

which proves (2.3), while delta impulse $\delta\varphi_s^\mp \rightarrow \infty$ brings absolute maximum to (2.14) within each k cutoff impulse.

Symmetrical entropy contributions (2.6) at a vicinity of $t = \tau_k$:

$$E[\varphi_s^{t-}] = \Delta S[\tilde{x}_t(u_-(t \leq \tau_{k-o}; t > \tau_k))] \text{ and } E[\varphi_s^{t+}] = \Delta S[\tilde{x}_t(u_+(t > \tau_k; t \leq \tau_{k+o}))] \quad (2.15)$$

estimate relations

$$\Delta S[\tilde{x}_t(u_-(t \leq \tau_{k-o}; t > \tau_k))] = 1/4, u_- = u_-(\tau_k), \tau_k \rightarrow \tau_{k-o}, \quad (2.16a)$$

$$\Delta S[\tilde{x}_t(u_+(t > \tau_k; t \leq \tau_{k+o}))] = 1/4, u_+ = u_+(\tau_k), \tau_k \rightarrow \tau_{k+o}, \quad (2.16b)$$

which proves (2.3a,b).

Entropy functional (1.1.10), defined through Radon-Nikodym's probability density measure (1.1.3), holds all properties of the considered cutoff controllable process, where both $P_{s,x}$ and $\tilde{P}_{s,x}$ are defined.

Thus, cutting correlations (2.10) extracts entropy of hidden process information which directly measures each δ -cutoff:

$$\Delta I_k[\tilde{x}_t(\delta u(\tau_k))] = \Delta S[\tilde{x}_t(\delta u_\pm(\tau_k))] = 1/2. \quad (2.17)$$

Known information measures do not provide such measuring.

According the definition of entropy functional (1.1.5), it is measured in natural $\ln$ where each its Nat equals $\log_2 e \cong 1.44 \text{bits}$; therefore, it does not using Shannon entropy measure. •

Corollaries

From the Proposition it follows that:

(a)-Stepwise control function $u_- = u_-(\tau_k)$, implementing transformation $\tilde{x}_t(\tau_{k-o}) \rightarrow \varsigma_t(\tau_k)$, converts the EF from its minimum at $t \leq \tau_{k-o}$ (2.16a) to maximum at $\tau_{k-o} \rightarrow \tau_k$ (2.17);

(b)-Stepwise control function $u_+ = u_+(\tau_k)$, implementing transformation $\varsigma_t(\tau_k) \rightarrow \tilde{x}_t(\tau_{k+o})$, converts the EF from its maximum at $\tau_{k-o} \rightarrow \tau_k$ (2.17) to minimum at $\tau_k \rightarrow \tau_{k+o}$ (2.16b);

(c)-Impulse control function $\delta u_{\tau_k}^\pm$, implementing transformations $\tilde{x}_t(\tau_{k-o}) \rightarrow \varsigma_t(\tau_k) \rightarrow \tilde{x}_t(\tau_{k+o})$, switches the EF from its minimum to maximum and back from maximum to minimum, while the maximum of the entropy functional at a vicinity of $t = \tau_k$ allows the impulse control to deliver *maximal amount* of information (2.17) from these transformations;

(d)-Dissolving the correlation between the process' cutoff points (2.10) cuts *functional connections* at these discrete points, which border the Feller kernel measure [27, 29];

(e)-The relation of that measure to additive functional [29] in form (1.1.7) allows evaluating the *kernel's information* by the EF (1.5). The jump action (2.2) on Markov process, associated with “killing its drift”, selects the Feller measure of the kernel [12,29,35], while the cutoff EF *provides information measure* of the Feller kernel (2.17);

(g)-Stepwise control $u_- = u_-(\tau_k)$, transferring the EF from $\tau_{k-o} \rightarrow \tau_k$, maximizes by moment τ_k the minimal information increment (brought at $t \rightarrow \tau_{k-o}$), implementing condition

$$\max_{\tau_k} \min_{\tau_{k-o}} \Delta I_k[\tilde{x}_t(\delta u(\tau_k))]; \quad (2.17a)$$

(f)-Stepwise control $u_+ = u_+(\tau_k)$, transferring the EF from $\tau_k \rightarrow \tau_{k+o}$, kills the additive functional at stopping moment τ_{k+o} minimizing the maximal information increment by the end of this transformation, implementing condition

$$\min_{\tau_{k+o}} \max_{\tau_k} \Delta I_k[\tilde{x}_t(\delta u(\tau_k))]. \quad (2.17b)$$

Such transformation associates with killing the Markovian process at the rate of increment of related additive functional $d\varphi_s^{ti} / \varphi_s^{ti}$ for each single dimension i [11]. Control $u_+ = u_+(\tau_k)$ transfers the rate of killed Markov process to a process with probability (2.13), conserved during the jump, and starts a certain (non-random) process with the eigenvalue of diffusion operator [39], creating the process that balances the killing at the same rate [40].

The step-wise controls, acting on the multi-dimensional diffusion process dimensions, sequentially stops and starts the process, evaluating its multiple functional information.

The dissolved element of the correlation matrix at these moments provides independence of the cutting off fractions, leading to orthogonality of their correlation matrix •.

3. The evaluation of the cutting process regular entropy integral functional by an impulse control

3.1. The regular entropy integral functional. Example

The EF integrant in (1.1.10) is *partially observable* via measuring only covariation function on the process' trajectories.

For a single dimensional Eq. (1.1.6) with drift function $a = c\tilde{x}(t)$ at given nonrandom function $c = c(t)$ and diffusion $\sigma = \sigma(t)$ entropy functional (1.1.10) acquires form

$$S[\tilde{x}_t / \varsigma_t] = 1/2 \int_s^T E[c^2(t)\tilde{x}^2(t)\sigma^{-2}(t)]dt, \quad (1.1)$$

from which, at $\sigma(t)$ and nonrandom function $c(t)$, we get

$$S[\tilde{x}_t / \varsigma_t] = 1/2 \int_s^T [c^2(t)\sigma^{-2}(t)E_{s,x}[x^2(t)]]dt = 1/2 \int_s^T c^2[2b(t)]^{-1}r_s dt, \quad (1.2)$$

where for the diffusion process, the following relations hold true:

$$2b(t) = \sigma(t)^2 = dr/dt = \dot{r}_t, E_{s,x}[x^2(t)] = r_s. \quad (1.3)$$

This allows *identifying* the entropy functional on observed Markov process $\tilde{X}_t = \tilde{X}(t)$ by measuring above correlation functions, applying positive function $u(t) = c^2(t)$ and *representing* functional (1.1.10) through a regular integral of non-random functions

$$A(s, t) = [2b(t)]^{-1} r_s = \dot{r}_t^{-1} r_s \quad (1.4a) \quad \text{and} \quad u(t) = c^2(t) \quad (1.4b)$$

in form

$$S[\tilde{X}_t / \zeta_t] = 1/2 \int_s^T u(t) A(t, s) dt, \quad (1.4)$$

The n -dimensional functional integrant (1.4) follows directly from related n -dimensional covariations (1.3), dispersion matrix, and applying n -dimensional function $u(t)$. At given nonrandom function $u(t)$, (1.4) is regular integral which measures the entropy functional of Markov process at the probability transformation (1.1.2) with additive functional (1.1.7), where the integrant averages the additive functional.

3.2. Application of variation condition (1.1.5a)

Proposition 2.1.

Integral (1.4), satisfying variation condition (1.1.5a) at linear function $c^2(t) = u(t) = c^2 t$, has form

$$S[\tilde{X}_t / \zeta_t] = 1/2 \int_s^T u(t) o(t) dt, \quad (2.1)$$

where the extreme of function (1.4a) holds minimum

$$A(t, s_k^{+o}) = o(s) b_k(s_k^{+o}) / b_k(t) = o(t), \quad (2.2)$$

which decreases with growing time $t = s_k^{+o} + o(t)$ at $t \rightarrow T$ and fixed both $b_k(s_k^{+o})$ and

$$o(s) = A(s, s). \quad (2.3)$$

Since satisfaction of this variation condition includes transitive transformation of a current distribution to that of Feller kernel, $b_k(t)$ is transition dispersion at this transformation, which is growing with the time of the transformation.

Proof. Applying Euler's equation to variation condition (1.1.5a) of this integral, at simple linear function $c^2(t) = u(t) = c^2 t$ and fixed c^2 on starting moment s_k^{+o} , we get

$$\partial A(t, s_k^{+o}) / \partial t = \dot{A}(t, s_k^{+o}) t + A(t, s_k^{+o}) = 0, \quad (2.4)$$

and

$$t \partial A(t, s_k^{+o}) / \partial t + A(t, s_k^{+o}) = 0, \partial A(t, s_k^{+o}) / A(t, s_k^{+o}) + \partial t / t = 0, t \neq 0, A(t, s_k^{+o}) \neq 0, \partial t / t \neq 0. \quad (2.4a)$$

From these it follows

$$\ln A(t, s_k^{+o}) + \ln Ct = \ln[A(t, s_k^{+o}) Ct] = 0, A(t, s_k^{+o}) Ct = 1,$$

$$A(t, s_k^{+o})^{-1} = Ct, \quad (2.5a)$$

$$A(s_k^{+o}, t = s_k^{+o})^{-1} = C s_k^{+o}, C = A(s_k^{+o}, s_k^{+o})^{-1} / s \quad (2.5b)$$

On a current time interval $\Delta_t = t - s_k^{+o}$, relations (2.5a,b) determine functions

$$A(s_k^{+o}, s_k^{+o}) / A(t, s_k^{+o}) = t / s_k^{+o}. \quad (2.6)$$

Relations (1.3) allow representations

$$A(s_k^{+o}, s_k^{+o}) = r_k(s_k^{+o}) / 2b_k(s_k^{+o}) , \quad (2.7a)$$

$$A(s_k^{+o}, t) = r_k(s_k^{+o}) / 2b_k(t), \quad (2.7b)$$

where correlation function

$$r(s) = \int_s^{s+o(s)} 2b(t)dt = 2b(s)o(s) \quad (2.8)$$

leads to

$$A(s, s) = [2b(s)]^{-1} 2b(s)o(s) = o(s). \quad (2.9)$$

Substitution (2.9) and (2.7a,b) in (2.6) brings

$$t = s_k^{+o} b_k(t) / b_k(s_k^{+o}). \quad (2.10)$$

This shows that with growing time $t = s_k^{+o} + o(t)$ dispersion $b_k(t)$ has tendency to grow.

Substituting (2.10) and (2.9) in (2.5a) determines function

$$A(t, s_k^{+o}) = o(s) b_k(s_k^{+o}) / b_k(t) = o(t), \quad \Delta_t = o(t) \quad (2.11)$$

which decreases with growing time $t \rightarrow T$.

The extreme holds minimum since second derivative of function (2.5a) is positive. •

Comments 2.1. On interval Δ_t , moment $t = t_k$ in equation (2.10) in form

$$t = s_k^{+o} \dot{r}(t) / \dot{r}(s_k), \quad (2.12)$$

$$\text{at } b(t) = 1 / 2\dot{r}(t), b(s_k) = 1 / 2\dot{r}(s_k), \quad (2.12a)$$

can be identified by controlling correlation:

$$t_k \cong s_k r_k(t_k) / r_k(s_k) . \quad \bullet \quad (2.12b)$$

Thus, EF (2.10) integrates time of correlation which function $u(t) = u_t$ cuts.

3.3. Impulse action on the entropy integral

Let define $u(t) = u_t$ on space $KC(\Delta, U)$ of a piece-wise continuous step-functions $u_t(u_-^t, u_+^t)$ at $t \in \Delta$,

applying to integrant of (2.1) the difference of step-down $u_-^t = u_-(t) / \delta_o$ and step-up $u_+^t = u_+(t + \delta_o) / \delta_o$

$$\text{functions at fixed interval } \delta_o : \quad u_t^{\delta_o} = [u_-(t) - u_+(t + \delta_o)] / \delta_o = u_t^{\delta_o} \quad (3.1)$$

which forms Dirac's delta-function at

$$\lim_{\delta_o \rightarrow 0} = \delta u_t . \quad (3.2)$$

Proposition 3.1.

Entropy integral (2.1) under impulse control (3.2) takes the following information values:

(a)-at a switching impulse middle locality $t = \tau_k$:

$$S[\tilde{x}_t / \zeta_t]_{t=\tau_k} = 1 / 2 \text{ Nats}, \quad (3.3)$$

(b)- at switching impulse left locality $t = \tau_k^{-o}$:

$$S[\tilde{x}_t / \varsigma_t]_{t=\tau_k^{-o}} = 1/4Nats \quad (3.3a)$$

(c)-and at switching impulse right locality $t = \tau_k^{+o}$:

$$S[\tilde{x}_t / \varsigma_t]_{t=\tau_k^{+o}} = 1/4Nats. \quad (3.3b)$$

Proof. Applying delta-function $c^2(t, \tau_k) = \delta u_t(t - \tau_k)$ to integral

$$\Delta S[\tilde{x}_t / \varsigma_t]_{t=\tau_k^{-o}}^{t=\tau_k^{+o}} = 1/2 \int_{\tau_k^{-}}^{\tau_k^{+}} \delta u_t(t - \tau_k) o(t) dt, \quad \tau_k^{-o} < \tau_k < \tau_k^{+o} \quad (3.4)$$

determines functions [36,p.678-681]

$$\Delta S[\tilde{x}_t / \varsigma_t]_{t=\tau_k^{-o}}^{t=\tau_k^{+o}} = \begin{cases} 0, t < \tau_k^{-o} \\ 1/4o(\tau_k^{-o}), t = \tau_k^{-o} \\ 1/4o(\tau_k^{+o}), t = \tau_k^{+o} \\ 1/2o(\tau_k), t = \tau_k, \tau_k^{-o} < \tau_k < \tau_k^{+o} \end{cases} \quad (3.5)$$

Or such cutoff brings amount of entropy integral $\Delta S[\tilde{x}_t / \varsigma_t]_{t=\tau_k} = 1/2o(\tau_k) = 1/2Nats$, while on borders of

interval $o(\tau_k)$ the integral amounts are $\Delta S[\tilde{x}_t / \varsigma_t]_{t=\tau_k^{-o}} = 1/4o(\tau_k^{-o})Nats$ and $\Delta S[\tilde{x}_t / \varsigma_t]_{t=\tau_k^{+o}} = 1/4o(\tau_k^{+o})Nats$

accordingly. • The results concur with Sec.1.1.2.

Sum of the instances for the cutting interval:

$$\sum_{t=\tau_k^{-o}}^{t=\tau_k^{+o}} \Delta S[\tilde{x}_t / \varsigma_t] = 1/4o(\tau_k^{-o}) + 1/2o(\tau_k) + 1/4o(\tau_k^{+o}) = o_k \quad (3.6)$$

evaluates constant *Nat* fraction of the cutoff EF on this interval, which the interval encloses.

The invariant cutting fractions follow from variation condition (1.1.5a) imposed on (1.4), which leads to (2.10) and (3.4).

4. Properties of the cutoff functional

Let us consider a sum of increments (3.2.6) under impulse control $\delta u(\tau_k)$, cutting the process x_t at moments $\tau_k, k=1,2,...,m$ along the process' trajectory on intervals

$$s > \tau_1 > t_1 > \tau_2 > t_2, ..., t_{k-1} > \tau_k > t_k, ..., t_{m-1} > \tau_m > t_m = T, \quad (4.1)$$

where borders of these interval t_{k-1}, t_k belong to the EF fractions before the cuts:

$$\Delta S_k[\tilde{x}_t(t \rightarrow t_k)], \Delta S_{k+1}[\tilde{x}_t(t \rightarrow t_{k+1})]. \quad (4.2)$$

Inside the border, at $t_{k-1} \leq \tau_{k-o}, \tau_k, \tau_{k+o} \leq t_k$, arise cutting information contributions $\Delta I_k[\tilde{x}_t(\delta u(\tau_k))]$.

1. Applying additive principle for the process' information, measured at the moments of dissolving correlation, we get summary information

$$I_{m0} = \sum_{k=1}^m \Delta I_k[\tilde{x}_t(\delta u(\tau_k))] = \Delta I_1[\tilde{x}_t(\delta u(\tau_1))] + \Delta I_2[\tilde{x}_t(\delta u(\tau_2))] + ..., + \Delta I_m[\tilde{x}_t(\delta u(\tau_m))] \quad (4.3)$$

where impulses $\delta u(\tau_k)$ implement transitional transformations (1.1.2-1.1.3), initiating the Feller kernels along the process and extracting total kernel information for n -dimensional process with m cuts off.

2. Sum $S_{mo} \big|_s^T$ of the additive fractions of EF on the finite time intervals $s, t_1; t_1 + o_1, t_2; \dots, t_{k-1} + o_{k-1}, t_k; \dots, t_{m-1} + o_{m-1}, t_m = T$; at

$$S_{mo} \big|_s^T = \Delta S_{10}[\tilde{x}_t / \varsigma_t] \big|_s^{t_1} + \Delta S_{20}[\tilde{x}_t / \varsigma_t] \big|_{t_1+o_1}^{t_2} + \dots + \Delta S_{mo}[\tilde{x}_t / \varsigma_t] \big|_{t_{m-1}+o_m}^{t_m} \quad (4.4)$$

approaches I_{mo} (3.3) at $o_k \rightarrow \tau_k, k=1, 2, \dots, m$:

$$\lim_{o_k \rightarrow \tau_k} S_{mo} \big|_s^T = I_{mo}, \quad (4.5)$$

while at any $o(\tau_k) > \tau_k$,

$$I_{mo} < S_{mo}. \quad (4.5a)$$

3. Sum (4.4) is less than $S[\tilde{x}_t / \varsigma_t]_s^T$, which is defined by the additive functional (1.1.7).

As a result, additive principle for a process' information, measured by the EF, is *violated*:

$$S_m \big|_s^T < S[\tilde{x}_t / \varsigma_t]_s^T. \quad (4.6)$$

4. *Information path functional* (IPF) defines distributed actions multi-dimensional delta-function on entropy functional (1.1.10) through the additive functional for all dimensions:

$$I_{pm} = \delta_m \{S[\tilde{x}_t / \varsigma_t]\} = 1/2E \left\{ \int_s^T \delta_m [a(t, \tilde{x})^T (2b(t, \tilde{x}))^{-1} a(t, \tilde{x}) dt] \right\} \quad (4.7)$$

which determines sum (4.2) of information measures $\Delta I_k[\tilde{x}_t(\delta u(\tau_k))]$ along the path on cutting process intervals (3.3.1), (3.3.6). In a limit it leads to

$$I_p = \lim_{m \rightarrow \infty} \sum_{k=1}^m \Delta I_k[\tilde{x}_t(\delta u(\tau_k))]. \quad (4.7a)$$

Formal definition (4.7) allows the IPF representation by Furies integral [36] leading to frequency analysis with Furies series.

5. IPF is the sum of *extracted* information which approaches theoretical measure (1.1.10):

$$I_p = \lim_{m \rightarrow \infty} I_{mo} \big|_s^T = \lim_{m \rightarrow \infty} S_{mo} \big|_s^T \rightarrow S[\tilde{x}_t / \varsigma_t]_s^T, \quad (4.8)$$

if all finite time intervals $t_1 - s = o_1, t_2 - t_1 = o_2, \dots, t_{k-1} - t_k = o_k, \dots, t_m - t_{m-1} = o_m$, at $t_m = T$ satisfy condition

$$(T - s) = \lim_{m \rightarrow \infty} \sum_{t=s, m}^{t=T} o_m(t). \quad (4.8a)$$

Since according to (3.3.6, 4.6) each cutting interval encloses invariant information measure, the limited I_p limits the initially undefined (in (1.1.7)) upper time T of EF integral.

It also brings direct connection I_p and $T - s$.

Therefore, at infinite sequence of the time intervals, this sequence has limit

$$\lim_{m \rightarrow \infty} o(t_m) \rightarrow 0, \quad (4.8b)$$

and sum of such sequence is finite [36, p.130, 4.8-1].)

Realization (4.7), (4.8), (4.8a, b) requires applying the impulse controls at each instant $(\tilde{x}, s), (\tilde{x}, s + o(s)), \dots$ along the process trajectories of conditional math expectation (1.1.10).

However for any *finite* number m of these instants, the *integral* process information, composed from the information, measured for the process' fractions, is not complete.

The I_p properties:

1. The IPF measures information of the cutting process's interstate connections hidden by the states correlations, which are not covered by traditional Shannon information measure.

2. Since each cutting $\Delta S_k[\tilde{x}_t(\delta u(\tau_k))] = \Delta I_k[\tilde{x}_t(\delta u(\tau_k))]$ maximizes the cutting information, I_p measures a total (integral) maximal information on the path.

The cutting control provides equal maxmin-minimax information contributions

$$\max_{\tau_k} \min_{\tau_{k-o}} \Delta I_k[\tilde{x}_t(\delta u(\tau_k))] = \min_{\tau_{k+1}} \max_{\tau_k} \Delta I_k[\tilde{x}_t(\delta u(\tau_k))] \quad (4.9)$$

on each path $t_{k-1} \rightarrow (\tau_{k-o} \rightarrow \tau_k \rightarrow \tau_{k+o}) \rightarrow t_k$ from cutting t_{k-1} to following cut t_k (*Coroll.a-c*).

3. If each k -cutoff “kills” its process dimension at moment τ_{k+o} , then $m=n$, and (4.7-4.8a,b) require infinite process dimensions.

4. At $m=n \rightarrow \infty$, $o_k = t_k - t_{k-1} \rightarrow \tau_k$, the process time

$$(T-s) = \lim_{n \rightarrow \infty} \sum_{t=s, m}^{t=T} \tau_k(t) \quad (4.10)$$

approaches the summary of the discrete time intervals cutting all correlations.

5. Sequential cuts transform the IPF information contributions from each maximum through minimum to next maximal information contributions

$$\max_{\tau_k} \Delta I_k[\tilde{x}_t(\delta u(\tau_k))] \rightarrow \min_{\tau_{k+o}} \Delta I_k[\tilde{x}_t(\delta u(\tau_k))] \rightarrow \max_{\tau_{k+1}} \Delta I_k[\tilde{x}_t(\delta u(\tau_{k+1}))], \quad (4.11)$$

where each next maximum decreases at the cutoff moments

$$\max_{\tau_{k+1}} \Delta I_k[\tilde{x}_t(\delta u(\tau_{k+1}))] < \max_{\tau_k} \Delta I_k[\tilde{x}_t(\delta u(\tau_k))]. \quad (4.12)$$

Each Dirac delta-function preserves its cutting information (2. 3.6).

The information contribution by finale interval o_m at its inner ending moment τ_{m+o} , according to (2.2.9),(4.8a), satisfies

$$\min_{\tau_{m+o}} \Delta I_m[\tilde{x}_t(\delta u(\tau_m))] \rightarrow 0, \quad (4.13)$$

which limits sum (4.3) at $m=n \rightarrow \infty$.

6. Since EF functional $S[\tilde{x}_t / \zeta_t]_s^T$ limits growth of $S_{mo} |_s^T$ in (4.8), it limits the IPF in (4.5a),(4.13), hence, the IPF approaches the EF functional during time (4.10) at unlimited increase of the process dimensions.

7. Because upper time T in both the EF integral and IPF functional is limited by (4.8a) and (4.10), the entropy integral converges in the path functional, and both of them are restricted at the unlimited dimension number.

8. For any of these limitations, EF measure, taken along the process trajectories for time $(T-s)$, limits maximum of total process information, while IPF extracts maximum of the process hidden information during the same time and brings more information than Shannon traditional information measure for multiple states of the process.

9. Information density

$$I_{ko_k} = I_k[\tilde{x}_t(\delta u(\tau_k))] / \tau_k \quad (4.14)$$

grows with absolute maximum at

$$I_{ko_n} \rightarrow \infty. \quad (4.15)$$

The time transition to each following Nat decreases (satisfying the variation condition) since that Nat integrates all preceding Nats concentrating the integral information in the final IPF Nat, which Feller kernel absorbs.

The IPF formally defines the distributed actions of multi-dimensional delta-function on the EF via the multi-dimensional additive functional (1.1.7), which leads to analytical solution and representation by Furies series.

Final a finite integral information approaches that generated by the last impulse during the final finite $\tau_{k=n}$, which is Kronicker impulse-discrete analog of Dirac delta-function taking values 0 and 1.

Conditional probability satisfies Kolmogorov's 1-0 [37] law for function $f(x)|\xi$ of ξ, x infinite sequence of independent random variables:

$$P_s(f(x)|\xi) = \begin{cases} 1, & f(x)|\xi \geq 0 \\ 0, & f(x)|\xi < 0 \end{cases} \quad (4.16)$$

This probability measure has applied for the impulse probing of an observable random process, which holds opposite Yes-No probabilities – as the unit of probability impulse step-function [41].

Comments. Number M of equal probable possibilities determines Hartley's quantity of information $H = \ln M$ measured on Nats, which for the impulse $M = 2$ holds $H = \ln 2 \text{Nat}$. The impulse information measured in bits is $I = 1/\ln 2 \ln M = \lg M \text{bit} = 1 \text{bit}$. The correlation cutting by the impulse brings information 0.75Nat from which $\delta S_u \cong 0.0568 \text{Nat}$ delivers the impulse controls. •

Thus, the IPF integral information evaluates maximal speed of enclosing information (4.14) in that finite impulse time interval, which is the impulse cutting time delivering the correlation's hidden information $\ln 2$.

All integrated information enfolds the Feller kernel whose time and energy evaluate results [29].

Minimal physical time interval limits the light time interval $\delta t_r \cong 1.33 \times 10^{-15} \text{sec}$ defined by the light wavelength $\delta l_m \cong 4 \times 10^{-7} \text{m}$. That allows us estimate maximal information density (4.14):

$$I_{ko_k} \cong \ln 2 / 1.33 \times 10^{-15} \cong 5.2116 \times 10^{+15} \text{Nat} / \text{s} . \quad (4.17)$$

Variety of the impulse $\downarrow \uparrow$ physical interactions unites the considered impulse information model, which EF-IPF integrate (with more details in [41]).

5. Natural encoding information of the cutting impulses

The question is how actually a natural process creates its natural Bit? At what conditions?

Can such multiple bits generate natural code, for example DNA triplet code?

As it's shown, the impulse cut of information hidden in correlation encodes the impulse Nat unit, which can be memorized during that impulse time interval in the ending impulse stopping state. Such impulse encoding merges its memory with the time of encoding information, which minimizes that time.

The encoded Bit within time interval has maximal theoretical admissible density (4.17) concentrating the IPF in Feller kernel during the minimal time interval and space interval.

The definition information as phenomenon of interaction whose cutting impulse enables originate information and encodes it in the interacting physical states leads to potential artificial generation of both natural origin and its encoding during an observation. Such observable process is a path from prior uncertainty to its resolution–conversion to posterior information and automatic encoding in arising an information

Observer. This process starts with growing memory of cutting process correlation which after each cut binds the impulse code sequence and memorizes both the code time followed by its length; being a source of logical complexity [3]. The cutting off random process brings information Bit that, transferring to the nearest impulse, keeps persistence continuation of the impulse sequence and the Bits. Such persistent Bit sequentially and automatically converts entropy to information, holding the cutoff information of random process, which connects the Bits sequences in the IPF. The EF presents a potential informational path functional of the Markov process until the applied impulse, carrying the cutoff contributions, transforms it to the IPF.

Markov random process is a source of each information contribution, whose entropy increment of cutting random states delivers information, hidden between these states. The IPF formally defines the distributed actions of multi-dimensional delta-function on the EF via the multi-dimensional additive functional (1.1.7), which leads to analytical solution and representation by Furies series.

Impulse information, as the resolution of the EF uncertainty through the IPF, progressively develops the encoding information program memorizing in current real time. Encoding each Bit extracts its hidden position by the cut, which erases the Bit of information at cost of energy.

According to Landauer principle [42], any logically irreversible manipulation with information, such as encoding leads to erasure the information in a dissipative irreversible process.

Erasure of information Bit requires spending entropy $S = k_B \ln 2$ (k_B is Boltzman constant).

Maxwell demon [43] associates the entropy of Landauer's principle with the acquisition of information Bit equal to the entropy inevitable cost for the erasure. Erasure of the Bit requires at least work $W = k_B \theta \ln 2$ at absolute temperature θ , which theoretically *limits the finite energy recourses or the time of performing such operation*.

Bennett [44] found that any computation (encoding) can be performed using only reversible steps, which in principle requires no dissipation and no power spending. However, specific reversible computer needs to reproduce the map of inputs to outputs, erasing everything else that requires the energy cost.

Information cannot be copied with perfect accuracy according to no-cloning principle [45]; observing the copy disturbs the original states of the system, creating the errors requiring erasure.

Bell [46, 47] shows that information can be encoded in nonlocal correlations between the different parts of a physical system, which have interacted and then separated.

In Shor algorithm [48], the properties of the correlations between the “input register” and “output register” of a computer functions require huge memory to store. This nonlocal information is hard to decode, the time of n operations increase grows faster than any power of $\ln(n)$.

Practical computations either run out of time or run out of memory.

The information definition connects its entropy-physical origin with encoding, memory and energy cost.

Applying the Jarzynski equality (JE) of irreversible thermodynamic transition [49] to conversion energy in information, and using result of its experimental verification [50], leads to the JE form

$$e^{\Delta F/k_B\theta} < e^{W/k_B\theta} \Rightarrow \gamma, 0 \leq \gamma \leq 2, \quad (5.1)$$

where ΔF is increment of free energy needed to produce energy W , γ is parameter of the verification, which uses the experimental control; at $\gamma = 1$, the JE satisfies exactly.

The thermodynamic process satisfying the JE for all its states sequence holds the stable irreversible thermodynamics.

Quantity of information I equivalent to energy W is $I_w = k_B\theta I$. Taking logarithm from both side of

$$e^{\Delta F/k_B\theta} = \gamma \quad (5.1a) \text{ leads to } \Delta F / k_B \theta I = \ln \gamma / I. \quad (5.1b)$$

Applying (5.1b) to the impulse which gets information $I_{\delta t}$ during the cutting time interval δt with influx of potential energy

$\Delta F = \Delta F_{\delta t}$ corresponds to JE in form

$$\Delta F_{\delta t} (k_B T \times I_{\delta t}) = \ln \gamma / I_{\delta t} \quad (5.2)$$

The equivalence of JE (5.1a) for the information transition requires

$$I_{\delta t} = [1], \quad (5.3)$$

where $[1]$ is a unit of information per impulse to compensate for the Maxwell Demon (DM) energy by the time of transmission. Thus, to satisfy DM, the information producing by each impulse time interval should be invariant holding constant unit (Bit, Nat) in (5.3). It confirms that the initial extremal principle (EP)(1.1.5a) satisfies the JE for impulse information transition, or vice versa, each impulse time interval enables encoding invariant unit of information (3.6). Or, the EP follows from the JF in physical process whose interactive time interval is equivalent of the impulse information cutting from the correlation carrying energy. The cutting correlation's time intervals hold the information equivalent of this energy, and any real time of interval of interaction brings the entropy equivalent of energy $\Delta F_{\delta t}$ which compensates for the DM while producing information during the interaction. In a cutting Markov process, whose sequence of cuts satisfy the EP, each impulse can encode the cutting correlation, and all information of cutoff correlations of this process can be encoded in *information process*. The last cutting impulse encodes the process total information in that process IPF. Such an impulse natural encoding merges memory with the time of memorizing information and compensates the cutting cost by running time intervals of encoding. The information process, preserving the invariant cutting information, holds invariant irreversible thermodynamics of its information dynamics.

5.1. Specifics of interacting curvatures of step-up and step-down actions at memorizing a bit

Each impulse (Fig.1A) step-down action has negative curvature $-K_{e1}$ corresponding attraction, step-up reaction has positive curvature $+K_{e3}$ corresponding repulsion, the middle part of the impulse having a negative curvature transfers the attraction between these parts.

In probing virtual observation [41], the rising Baey's probabilities increase reality of interactions bringing energy.

When an external process interacts with the entropy impulse, it injects energy capturing the entropy of impulse' ending step-up action (Sec.2). The inter-action with other (an internal) process generates its impulse' step-down reaction, modeling 0-1 bit (Fig.1A, B).

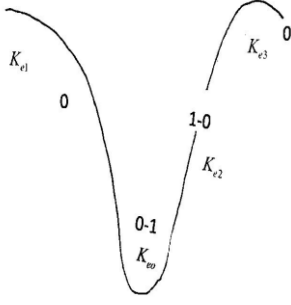


Figure 1A

A virtual impulse (Fig.1A) starts step-down action with probability 0 of its potential cutting part; the impulse middle part has a transitional impulse with transitive logical 0-1; the step-up action changes it to 1-0 holding by the end interacting part 0, which, after the inter-active step-down cut, transforms the impulse entropy to information bit.

In Fig. 1B, the impulse Fig. 1A, starting from instance 1 with probability 0, transits at instance 2 during interaction to the interacting impulse with negative curvature $-K_{e1}$ of this impulse step-down action, which is opposite to curvature $+K_{e3}$ of ending the step-up action ($-K_{e1}$ is analogous to that at beginning the impulse Fig.1A). The opposite curved interaction provides a time-space difference (a barrier) between 0 and 1 actions, necessary for creating the Bit. The interactive impulse' step-down ending state memorizes the Bit when the interactive process provides Landauer's energy with maximal probability (certainty) 1.

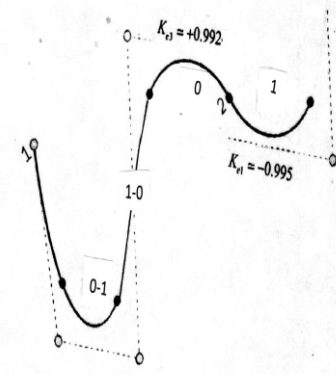


Figure 1B

The step-up action of an external (natural) process' curvature $+K_{e3}$ is equivalent of potential entropy $e_o = 0.01847 \text{ Nat}$ which carries entropy $\ln 2$ of the impulse total entropy 1 Nat.

The interacting step-down part of internal process impulse' invariant entropy 1 Nat has potential entropy $1 - \ln 2 = e_1$. Actually, this step-down opposite interacting action brings entropy -0.25 Nat with anti-symmetric impact -0.025 Nat which carries the impulse wide $\sim -0.05 \text{ Nat}$ [41] with the total $\sim -0.3 \text{ Nat}$ that equivalent to $-e_1$. Thus, during the impulse interaction, the initial energy-entropy $W_o = k_B \theta_o e_o$ changes to

$W_1 = -k_B \theta_1 e_1$, since the interacting parts of the impulses have opposite-positive and negative curvatures accordingly; the first one repulses, the second attracts the energies. The internal process needs minimal entropy $e_{10} = \ln 2$ for erasing the Bit which corresponds Landauer's energy $W = k_B \theta \ln 2$.

If the internal interactive process accepts this Bit by memorizing (through erasure), it should deliver the Landauer energy compensating the difference of these energies-entropy: $W_o - W_1 = W$ in balance form

$$k_B \theta_o e_o + k_B \theta_1 e_1 = k_B \theta \ln 2. \quad (6.1)$$

Assuming the interactive process supplies the energy W at moment t_1 of appearance of the interacting Bit, we get $k_B \theta_1(t_1) = k_B \theta(t_1)$. That brings (6.1) to form

$$k_B \theta_o 0.01847 + k_B \theta (1 - \ln 2) = k_B \theta \ln 2, \theta_o / \theta = (2 \ln 2 - 1) / 0.01847 = 20.91469199. \quad (6.2)$$

The opposite curved interaction decreases the ratio of above temperatures on $\ln 2 / 0.0187 - (2 \ln 2 - 1) / 0.01847 = 16.61357983$, with ratio $(2 \ln 2 - 1) / \ln 2 \cong 0.5573$.

External impulse with maximal entropy density $e_{do} = 1 / 0.01847 = 54.14185$ interacting with internal curved impulse transfers minimal entropy density $e_{d1} = \ln 2 / 0.01847 = 37.52827182$.

Ratio of these densities $k_d = e_{do} / e_{d1} = 1.44269041$ equals

$$k_d = 1/\ln 2. \quad (6.3)$$

Here the interacting curvature, enclosing entropy density (6.3), lowers the initial energy and the related temperatures (6.40B) in the above ratio. From that follow

Conditions creating a natural bit memorized in interacting curved impulse

Necessary conditions:

1. The impulse should create asymmetrical 0-1 action.
2. The process interacting with a natural process should possess the Landauer energy at ending the 1-action.

These conditions satisfy

1. Opposite non equal asymmetrical *curvatures* of 0-1 actions in the interactive transition;
2. The interacting process delivering Landauer energy when the natural action 1 hits.

Sufficient conditions

1. The interacting impulse should hold invariant measure [1] of entropy 1 Nat whose the topological metric π preserves the impulse opposite curvatures;
2. The interactive curved transition requires to keep entropy ratio $1/\ln 2$. •

Compared to other interactions for generating a bit, the opposite curved interaction lowers potential energy needed for the asymmetry.

Energy which delivers the internal process will erase the entropy of both attracting and repulsive movements covering energy of the both movements that are ending at the impulse stopping states. The erased impulse total cutoff entropy is memorizes as equivalent information, encoding the impulse Bit in the impulse ending state.

Recent results [51] prove that physical process, which holds the invariant entropy measure for each phase space volume (for example, minimal phase volume $v_{eo} \cong 1.242$ per a process dimension [41, Sec.III.1) characterized by the above topological invariant, satisfies Second Thermodynamic Law.

The last follows from the impulse' max-min mini-max law under its stepdown-stepup actions, which generate the invariant [1] Nat time-space measure with topological metric π .

Energy W , which delivers the internal process, will erase the entropy of both attracting and repulsive movements covering energy of the both movements that are ending at the impulse stopping states.

The erased impulse total cutoff entropy is memorizes as equivalent information, encoding the impulse Bit in the impulse ending state.

The ending observer's probing logic, which capture entropy increment (3. 3.3a), Sec.3.3 moving along negative curvature of its last up impulse and overcoming the gap acquires the equal information (3.3.6) that compensates for the movements logical cost.

Thus, the attractive logics of an invariant impulse, converting its entropy to information within the impulse, performs function of *logical Demon Maxwell* (DM) in the microprocess. (More details are in [41, Sec.8]).

Topological transitivity at the curving interactions

The impulse of the external process holds its 1 Nat transitive entropy until its ending curved part interacts, creating information bit during the interaction.

Theoretically, when a cutting maximum of entropy reaches a minimum at the end of the impulse, the interaction can occur, converting the entropy to information by getting energy from the interactive process. The invariant' topological transitivity has a duplication point (transitive base) where one dense form changes to its conjugated form during orthogonal transition of hitting time.

During the transition, the invariant holds its measure (Fig.1) preserving its total energy, while the densities of these energies are changing.

The topological transition separates (on the transitive base) both primary dense form and its conjugate dense form, while this transition turns the conjugated form to orthogonal.

At the turning moment, a jump of the time curvature switches to a space curvature Fig.1B with rising a space waves [52, Sec.3.6.2.].

This is what real DM does using for that an energy difference of the forms temperature [43].

The time intervals of the curved interaction

If the external space action curves the internal interactive part, the joint interactive time-space measures the last interactive impact. But if the internal curvature of $K_{e1} \approx -0.995037$ enclosing $-0.025Nat$ presents only by moment t_{o1} before an interaction, then interacting time-space interval measures the difference of these intervals $|t_{o1}| - t_o \triangleq 0.0250 - 0.01847 = 0.0653Nat$.

For that case, the internal curved inter-action attracts the energy of external interactive action until internal process binds Landauer's energy W by moment t_1 .

If No part presents at t_o and Yes part arises by t_1 , then the internal impulse spends $1 - \ln 2 + \ln 2 = 1$ Nat on creating and memorizing a bit.

If the external process hits the internal process having energy W , the inter-action of this process brings that energy by moment t_1 as the reaction, which carries W to hold the bit. The same energy will erase the bit and memorize it according to (6.2-6.3). The internal impulse spends ~ 1 Nat on creating and memorizing a bit while its curving part holds $(1 - \ln 2) \cong 0.3Nat$, since the curved topology of interacting impulses decreases the needed energy ratio to (6.3).

Thus, time interval $t_o - t_1$ creates the bit and performs the DP function.

Multiple interactions generate a code of the interacting process at the following conditions:

Necessary:

1. Each impulse holds an invariant probability –entropy measure, satisfying the natural Bit conditions. For elementary 0-1 impulse such entropy $S = \ln 2$ generates single Bit ~ 0.7 Nat, while 0.3 Nat connects elementary bits in the multiple code.
 2. The impulse interactive process, which delivers such code, must be a part of a real physical process that keeps this invariant entropy-energy measure. That process memorizes the bit and creates information process of multiple encoded bits, which build the process information dynamic structure.
- For example, a water, cooling interacting drops of a hot oils in the found ratio of temperatures, enables spending an external hot energy on its chemical components to encode other chemical structures, or the water kinetic energy will carry the accepting multiple drops' bits as an arising information dynamic flow.

Sufficient:

1. Building the multiple Bits code requires increasing the impulse information density in three times with each following impulse acting on the interacting process [52]. Such physical process generating the code should supply it with the needed energy.
2. To create a code of the bits, each interactive impulse, produced a Bit, should follow three impulses measure π , i.e. frequency of interactive impulse should be $f = 1/3 \pi \approx 0.1061$. •

Interval 3π gives opportunity to joint three bits' impulses in a triplet [52], as elementary macro unit of the encoding information process, and combats the noise, redundancies from both internal and external natural processes.

Multiplication mass M on curvature K_{e2} of the impulse equals to relative density $\text{Nat}/\text{Bit}=1.44$ which determine $M=1.44/K_{e2}$. At $K_{e2}=0.993362$, we get a relative mass $M=1.452335645$.

The multiple curving interactions create topological bits code, which sequentially forms moving spiral structure [41]. Therefore, the curving interaction dynamically encodes bits in natural process code developing the code information structure of the interacting information process.

How to find an invariant energy measure, which each bit encloses starting the DM?

Since its minimal energy is $W = k_B \theta \ln 2$, it's possible to find such temperature θ_1^o that is equal to inverse value of k_B . If the interacting process carries this temperature, then its minimal energy holds $W_1^o = \ln 2$ at $\theta_1^o = 1/k_B$, which is equal to the bits' time-space Nat entropy invariant.

Let us evaluate θ_1^o at $k_B = 8617 \times 10^{-5} \text{ eV/K}$ and Kelvin temperature $K = 20/293 = 0.0682259386^{\circ\text{C/K}}$ equivalent to $20^{\circ\text{C}}$. Then $\theta_1^o = 588.19 \times 10^5 / \text{eV}$.

If we assume that this primary energy brings eV amount equivalent to quanta of light $e_q = 1240 \text{ eVnm}$, $1 \text{ nm} = 10^{-9} \text{ m}$, then $\theta_1^o = 588.19 \times 10^5 \times 1.240 \times 10^3 / e_q \times 10^{-9} \text{ m} \cong 72.9356^{\circ\text{C/m}} / e_q$. Or each quant should bring temperature' density $\theta_1^o = 72.9356^{\circ\text{C/m}}$, which is reasonably real.

With this θ_1^o , the interacting impulse will bring energy $W_1^o = \ln 2$ to create its bit. Following balance eqs, the external process at this θ_1^o should have temperature $\theta_o^o = 20.91469199\theta_1^o = 1525,42^{\circ\text{C/m}}$ brought with a quant. This energy would hold an invariant impulse $|1|_M = 1 \text{ Nat}$ with metric π , or each such impulse has entropy density $1 \text{ Nat} / \pi$. The interacting impulse has minimal density energy equivalent to $\ln 2 / \pi = 0.22$ at temperature θ_1^o .

The variety of impulse $\downarrow\uparrow$ physical interactions unites this impulse information code, which EF-IPF integrates in the encoding information process, generally describing various natural interactive processes via a universal code logic. The energy quantity (power) of a specific interaction limits the universal code length by the final Bit's information density; the energy quality, evaluated by the energy entropy, limits the entropy probability when of the code length starts.

Result [51] shows that the entropy's invariant measure is consistent with the second law of thermodynamics, naturally creating an increasing gradient of diffusion density, while an entropy decreases in the time evolution of the uncertainty; it physically verifies potential natural encoding a self-organized information process. Possibility of natural increase of correlations illustrate experimental results [53], [54], more are cited in [41]. That supports natural encoding the cutting correlations.

Markov process, modeling natural interaction, whose cutting sequence satisfies the EP, may naturally encode these interactions in related information process, which the IPF encloses in its Feller kernel.

6. Estimation of information hidden by interstates' connection of the diffusion process

The evaluated information effect of losing the functional's bound information at the cutoff moments, according to (1.3.5), holds amount of 0.5 Nats ($\sim 0.772 \text{ bits}$). These cutoff in the form of $\delta^i[\tau_k]$ -function, applied $k = m$ times to each of the process dimension $i = 1, \dots, n$, bring total information

$$I_c \cong 0.772m \times n \text{ bits.}$$

Thus, the process functional's information measures (1.1.10), (4.4.5, 4.4.8) encloses I_c bits more, compared to the information measure, applied separately to each of the process $m \times n$ states (during the same time). This result is applicable to a comparative information evaluation of the divided and undivided fractions of an information process, measured by corresponding EF, where each two bits of undivided process' pair contains 0.772 bits more hidden information, measured by the functional. That means that information *process* holds more information than any divided number of its fractions, and the considered entropy functional, measured this process, evaluates the quantity of information that *connect* these fractions. Moreover, by knowing this initially hidden information, one could determine which information is necessary to connect the data, being measured separately, toward composing a unit of some information process. According to the evaluation of an upper bound entropy per an English character (token) [55], its minimum is estimated by 1.75 bits, with the average amount between 4.66-7 bits per character. The evaluation includes the inner information bound by a character. At minimal entropy per symbol in 1 bit, a minimal symbol's bound information is 0.75, which is close to our evaluation at the cutoff.

7. Application to industrial technologies, listed by citations

The impulse cut-off method was *applied* in different solidification processes with impulse controls' automatic system [55]. This method reveals some unidentified phenomena-such as compulsive appearance centers of crystallization-indicators of generation information, integrated in the IPF during the impulse metal extraction (withdrawing).

(In such metallic alloys, the "up-hill diffusion, creating density gradients, is often observed" [51]).

The frequency of the impulse withdrawing computes and regulates the designed automatic system to reach a maximum of the IPF information indicators.

The experimental data of the industrial implemented system [56] and [57].

The automatic control regulator in the impulse frequency cutting movement was implemented for different electro- technological processes [58].

The examples of method applications in communications, biological and cognitive systems, others are in [59], [60], and [61]. Multiple results of the applications in various areas concentrate [62].

The computer program is in arXiv: 1307.0449.

Retinal Ganglion Cells are Eyes discrete impulse receptors interacting with observations and generating information which transmission integrates [63].

Encoding though natural chemical reactions connecting chemical molecules are in [64].

Experiments [65] confirm encoding coherent qubits in spinning electron locked in attractive "hole spin".

Other examples are quantum solar dots of semiconducting particles using for the information coding and retrieving images [66].

Experiments [67] demonstrates "a quantum transfer of information between the stationary qubit, encoded in the spin of the hole held captive in the crystal, and the flying qubit - the photon", traveling at the speed of light.

8. Analyzing the path functional in multi-dimensional observing process. Discussion. Conclusion

Initial Kolmogorov probability distribution $P[x(\omega)]$ and the following EF represent all n -dimensional Markovian model of the observing process. Probability of each process dimension are local for each its random ensemble being a part of whole process ensemble.

Total multi-dimensional probability integrates the conditional Kolmogorov's probabilities (Sec.1).

That is why the EF presents a potential information functional of the Markov process until the applied impulse control, carrying the cutoff contributions, transforms it to the informational path functional.

Markov random process is a source of each information contribution, whose entropy increment of cutting random states delivers information, hidden between these states' correlation. The finite restriction on the cutting function determines the discrete impulse's step-up and step-down actions between impulse cutoff $\delta_k = \tau_k^{+o} - \tau_k^{-o}$, which supply entropy hidden between impulses on τ_k^{-o} , cut it and transfers to τ_k^{+o} where the cutting entropy produces the equivalent physical information and memorizes it. Each interval $\Delta_t \rightarrow o(t)$ following next cutoff, delivers new hidden process' information.

Information is a physical entity, which distinguishes from entropy that is observer's virtual-imaginable. In the multi-dimensional diffusion process, the step-wise controls, acting on the process all dimensions, sequentially stops and starts the process, evaluating the multiple functional information. Impulses delta-function δu_t or discrete δu_{τ_k} implement transitional transformations, initiating the Feller kernels along the process and extracting total kernel information for n -dimensional process with m cuts off.

The maximal sum measures the interstates information connections held by the process along the trajectories during its real time $(T - s)$, which are hidden by the random process correlating states.

The EF functional information measure on trajectories is not covered by traditional Shannon entropy measure.

The dissolved element of the functional's correlation matrix at the cutoff moments provides independence of the cutting off fractions, leading to orthogonality of the correlation matrix for these cut off fractions.

Intervals between the impulses do not generate information, being imaginary-potential for getting information, since no real double controls are applying within these intervals.

The minimized increments of entropy functional between the cutoffs allow prediction each following cutoff with maximal conditional probability. Sum of information contributions, extracted from the EF, approaches its theoretical measure (I) which evaluates the upper limit of the sum.

Since the sum of additive fractions of the EF on the finite time intervals is less than EF, which is defined by the additive functional, the additive principle for a process' information, measured by the EF is *violated*.

Each k -cutoff "kills" its process dimension after moment τ_k^{+o} , creating process that balances killing at the same rate [34]. Then $k = n$, and condition (4.8) requires infinite process dimensions, continuing the processes balance creation.

The EF measure, taken along the process trajectories during time $(T - s)$, limits maximum of total process information, extracting its hidden cutoff information (during the same time), and brings more information than Shannon traditional information measure for multiple states of the process.

The limited time integration and the last cutting correlation bind the EF last cut.

Maximum of the process cutoff information, extracting its total hidden information, approaches the EF information measure. Or total physical information, collected by IPF in the infinite dimensional Markov diffusion process, is finite. Since rising process dimension up to $n \rightarrow \infty$ increases number of the dimensional kernels, information of the cutting off kernels grows, and when the EF transforms to the IPF kernels, the IPF finally measures all kernels finite information.

Total process uncertainty-entropy, measured by the EF is also finite on the finite time of its transition to total process certainty-information, measured by the IPF.

The EF integrates this time (4.8a) along the trajectories.

The IPF formally defines the distributed actions of multi-dimensional delta-function on the EF via the multi-dimensional additive functional, which leads to analytical solution and representation by Furies series. The delta-impulse, generating spectrum of multiple frequencies, is source of experimental probabilities in formal theory [3].

Significance of natural encoding the IPF cutting impulses.

The standard unit of information bit generates any natural process through discrete (yes-no) inter-actions, which includes interactive macro impulses in classical physics and the elementary micro impulses of quantum interactions.

The impulse natural inter-action cuts information hidden in correlation extracting each bit hidden position, erases it at a cost of the cutting real time interval, and memorizes encoding information in the interacting physical state.

The impulse sequential natural encoding merges memory with the time of encoding, which minimizes that time.

The interactive impulse reveals information to be a phenomenon of interaction.

The natural encoding includes transitional logical memory, which satisfies Landauer's principle, and compensates for the cost of Maxwell's demon.

The paper multiple applications support and physically verify the reliability of presented results.

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