

MATHEMATICAL ANALYSIS OF PLANT DISEASE DISPERSION MODEL THAT INCORPORATES WIND STRENGTH AND INSECT VECTOR AT EQUILIBRIUM

Abstract

Numerous plant diseases caused by pathogens like bacteria, viruses, fungi protozoa and pathogenic nematodes are propagated through media such as water, wind and other intermediary carries called vectors, and are therefore referred to as vector borne plant diseases.

Insect vector borne plant diseases are currently a major concern due abundance of insects in the tropics which impacts negatively on food security, human health and world economies.

Elimination or control of which can be achieved through understanding the process of propagation via Mathematical modeling. However existing models are linear and rarely incorporates climate change parameters to improve on their accuracy.

This paper formulates and analyzes a dynamical nonlinear plant vector borne dispersion disease model that incorporates insect and plant population at equilibrium and wind as a parameter of climate change, to determine R_0 , local and global stability in addition to sensitivity analysis of the basic reproduction number R_0 .

Keywords: Basic reproduction number, sensitivity analysis, disease free equilibrium point (DFEP), endemic equilibrium point (EEP), local and global stability.

1. Introduction

Numerous plant diseases caused by pathogens namely: bacteria, viruses, fungi, protozoa and pathogenic nematodes are spread through media such as water, wind and other intermediary carriers called vectors, and are thus referred to as vector borne diseases (Agrios, 1997).

Diseases that spread from one host to another are called infectious diseases, most of which are very destructive to plants and animals. Vector borne plant diseases have important transmission properties which include: survival rate of the vector, reproduction rate of the vector, time of the year and vectors / insects activity level specifically biting rate, rate of development and reproduction of pathogen on the vector amongst others (Patz, 2008).

The most common vectors are insects such as aphids, whiteflies, plant hoppers and leafhoppers with aphids as the most important group. Aphids have been estimated to cause 70% of the vector borne plant diseases (Nault, 1997).

Vectors (insects) are susceptible to external environmental influences which include the climate change and affect the spread of plant diseases. The climate parameters that vary include temperature, precipitation (humidity) and wind.

Climate change could first affect disease directly by either decreasing or increasing the encounter rate between the vectors and hosts by changing the ranges of the two species.

36 Disease severity should be positively correlated with increase in virulence and aggressiveness
37 of the vector. These two effects on disease will be mediated by host resistance and encounter
38 rates which are potentially affected by climate change.

39 Mathematical models provide a powerful tool used to understand the dynamics of disease
40 spread through a population and in decision making in regards to disease prediction and
41 disease control (Zhou and Yao, 2014). Empirical models like regression models and
42 deterministic models with climate variables as predictors and epidemic parameters as response
43 variables are used to predict the success of the hosts and vectors across the range of conditions
44 with a possibility of extrapolation when the mechanism of the relationship is clearly
45 understood. Simulation models based on theoretical relationships are used to predict outcomes
46 under a range of scenario.

47 The research will develop a Mathematical model incorporating the climatic change to be used to
48 evaluate the dynamics of vector- borne diseases in plants. The climate change variables, temperature
49 and precipitation influence the biting rate (a) while wind will be factored as an agent to the
50 movement of the vector, emigration or immigration hence incorporated in the model as $(\pm \theta)$.
51 Climate induced death to insects will also be considered. Since insect have very a short lifespan, the
52 study developed a deterministic general model for the perennial plant disease with assumption that
53 vectors would reproduce very fast and attain equilibrium. The plant population was assumed to be
54 constant. Section 2 describes model development. Section 3 describes model analysis where also
55 basic reproduction will be determined using the next generation method and analytical global
56 stability by the Lyapunov method. Finally Section 4 expounds results.

57 **2. Model Development.**

58 The model development begins with model description, model assumption, flow chart and
59 listing of model equations.

60 **2.1 Model description**

61 Let $N_V(t)$ and $N_P(t)$ be the total population of the vectors and plants respectively. The plant
62 population is subdivided into three compartment classes: susceptible plant class $S_P(t)$, exposed
63 plant class $E_P(t)$ and infected plant class $I_P(t)$. The vector population is subdivided into two
64 compartment classes: susceptible vector class $S_V(t)$ and infected plant class $I_V(t)$

65 The recruitment rate of plants and vectors are given by μ_p and π respectively. The rates at
66 which $E_P(t)$ recover naturally to $S_P(t)$ is given by ω . Climate induced death occur at a rate δ in
67 $S_V(t)$ and $I_V(t)$ respectively. The constant natural death rate in subclasses $S_P(t)$, $E_P(t)$ and $I_P(t)$ is
68 given by μ_p . The rate at which $E_P(t)$ progresses to $I_P(t)$ is τ . The constant natural death rate in
69 subclasses $S_V(t)$ and $I_V(t)$ is given by μ_v . The immigration and emigration rate of vector is given
70 by θ , this implies that θ can be either be positive or negative. The infection rates for plants and
71 vectors are β_1 and β_2 respectively while the vector biting rate is given by a .

72 **2.2 Model assumptions**

73 The model was formulated under the following assumptions;

- 74 i. Homogeneous mixing of the plants and vectors.

- 75 ii. It is possible for exposed plants to recover from natural immunity.
- 76 iii. The plant population is constant while vector population is not constant.
- 77 iv. Rate of immigration and emigration is constant and can be estimated.
- 78 v. Vector population is assumed to increase very fast and then attain equilibrium.
- 79 vi. Effects of competitions from other vectors are assumed to be insignificant.
- 80 vii. The susceptible plants are infected when they come into contact with infected insect
- 81 vectors.
- 82 viii. Susceptible insect vectors become infected only when they get in contact with an
- 83 infected plant and exposed plants. There is no vertical infection through birth and from
- 84 insect to insect directly.

85 Modification parameter k is such that $k \geq 1$, implying that the climate factors increases force of

86 infection

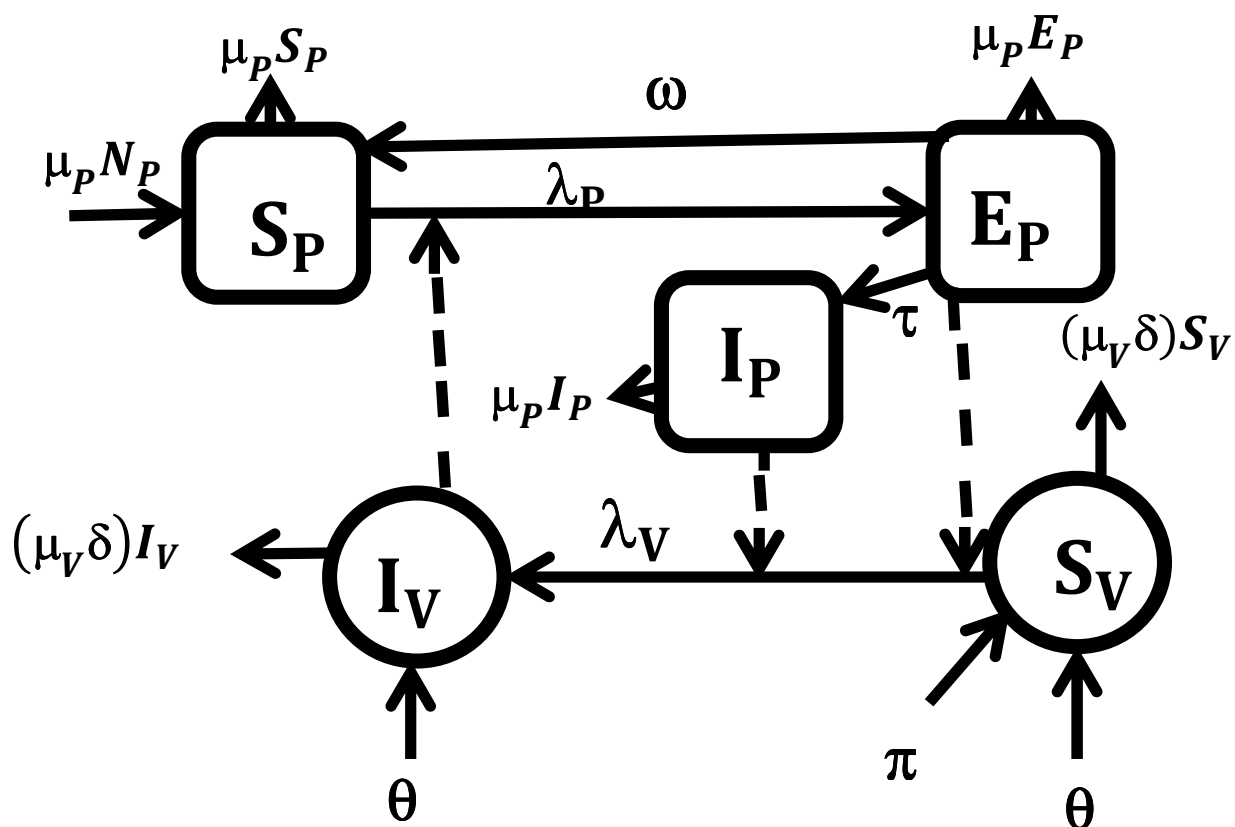
87 2.3 Flow chart diagram and the model equations

88 We considered the five state variables and parameters described in section 2.1 model

89 assumptions listed in section 2.2 to represent the model flow chart diagram in the figure 1 in the

90 next page;

91 **Figure 1: Flow chart diagram of the $SEISI$ model**



95 The model was described by the following differential equations draw from the flow diagram

$$96 \quad \frac{dS_P}{dt} = \mu_P N_P + \omega E_P - (k\lambda_P + \mu_P) S_P \quad (1),$$

$$\frac{dE_P}{dt} = k\lambda_P S_P - (\omega + \tau + \mu_P) E_P \quad (2),$$

$$97 \quad \frac{dI_P}{dt} = \tau E_P - \mu_P I_P \quad (3),$$

$$\frac{dS_V}{dt} = \pi N_V \pm \theta S_V - (k\lambda_V + \delta + \mu_V) S_V \quad (4),$$

$$\frac{dI_V}{dt} = k\lambda_V S_V \pm \theta I_V - (\delta + \mu_V) I_V \quad (5),$$

98 where

$$N_V(t) = S_V(t) + I_V(t), N_P(t) = S_P(t) + E_P(t) + I_P(t), \lambda_P = a\beta_1 I_V \text{ and } \lambda_V = a\beta_2 (E_P + \eta I_P).$$

99 The insects have very short life span, they reproduce within a very short time and therefore it is
100 plausible to assume the dynamics of the insects analyzed at steady states. We equate equations

101 (4 – 5) to zero to obtain the expressions below at steady state,

$$N_V^* = S_V^* + I_V^*; S_V^* = \frac{\pi N_V^*}{k\lambda_V + \delta + \mu_V \pm \theta}; I_V^* = \frac{k\lambda_V \pi N_V^*}{(k\lambda_V + \delta + \mu_V \pm \theta)(\delta + \mu_V \pm \theta)};$$

$$\lambda_P^* = \frac{a\beta_1 k\lambda_V \pi N_V^*}{(k\lambda_V + \delta + \mu_V \pm \theta)(\delta + \mu_V \pm \theta)}$$

102 After substituting λ_P^* at steady state, the system of equations(1 – 5) reduces to;

$$103 \quad \frac{dS_P}{dt} = \mu_P N_P + \omega E_P - \left(\frac{a\beta_1 K^2 \pi N_V^* \lambda_V}{(k\lambda_V + \delta + \mu_V \pm \theta)(\delta + \mu_V \pm \theta)} + \mu_P \right) S_P \quad (6),$$

$$\frac{dE_P}{dt} = \frac{a\beta_1 K^2 \pi N_V^* \lambda_V S_P}{(k\lambda_V + \delta + \mu_V \pm \theta)(\delta + \mu_V \pm \theta)} - (\omega + \tau + \mu_P) E_P \quad (7),$$

$$104 \quad \frac{dI_P}{dt} = \tau E_P - \mu_P I_P \quad (8),$$

105 With initial conditions as, $S_P(0) = (S_P)_0$, $E_P(0) = (E_P)_0$ and $I_P(0) = (I_P)_0$. The sum of system
106 of equations[(6) – (8)] which is the rate of change of total population

107 is given by $\frac{dN_P}{dt} = 0$.

108 3. Model analysis

109 We analyzed the model by proving various theorems and carrying out algebraic computation
110 dealing with different attributes.

111

3.1. Positivity and boundedness of the solutions

We proved positivity and boundedness by stating and proving the theorem below.

Theorem1: The region Z given by $Z = \{S_P(t), E_P(t), I_P(t) \in \mathbb{R}_+^3 ; N_P = C\}$

is positively invariant and attracting with respect to model system [(6) – (8)],

Proof.

Let $\{S_P(t), E_P(t)$ and $I_P(t)\}$ be any solutions of the system with non-negative initial conditions

$\{S_P(0) \geq 0, E_P(0) \geq 0, I_P(0) \geq 0\}$.

Since, $\frac{dS_P}{dt} = \mu_P N_P + \omega E_P - \left(\frac{a\beta_1 K^2 \pi N_V^* \lambda_V}{(k\lambda_V + \delta + \mu_V \pm \theta)(\delta + \mu_V \pm \theta)} + \mu_P \right) S_P$, it follows that

$\frac{dS_P}{dt} = - \left(\frac{a\beta_1 K^2 \pi N_V^* \lambda_V}{(k\lambda_V + \delta + \mu_V \pm \theta)(\delta + \mu_V \pm \theta)} + \mu_P \right) S_P$. On integration, we obtained

$S_P(t) = e^{\int_0^t - \left(\frac{a\beta_1 K^2 \pi N_V^* \lambda_V(s)}{(k\lambda_V(s) + \delta + \mu_V \pm \theta)(\delta + \mu_V \pm \theta)} + \mu_P \right) dt} \geq 0$. Clearly, $S_P(t) = e^{\int_0^t - \left(\frac{a\beta_1 K^2 \pi N_V^* \lambda_V(s)}{(k\lambda_V(s) + \delta + \mu_V \pm \theta)(\delta + \mu_V \pm \theta)} + \mu_P \right) dt}$ is a non-negative function of t , thus $S_P(t)$ stays positive.

The positivity of $E_P(t)$ and $I_P(t)$ was proved along the same lines hence considering equation (7) $\frac{dE_P}{dt} \geq -(\omega + \tau + \mu_P) E_P$ on integrating $E_P(t) = C_1 e^{-(\omega + \tau + \mu_P)t}$, where C_1 is a constant of integration. Applying initial condition we obtain $C_1 = E_P(0)$. Hence $E_P(t) = E_P(0) e^{-(\omega + \tau + \mu_P)t}$ and $E_P(t) = E_P(0) e^{-(\omega + \tau + \mu_P)t} \geq 0$.

Similarly, $I_P(t) = I_P(0) e^{-\mu_P t} \geq 0$.

Taking the time derivative of our total population along its solution path gives

$$\frac{dN_P}{dt} = \frac{dS_P}{dt} + \frac{dE_P}{dt} + \frac{dI_P}{dt} = 0,$$

This implies that, $N_P(t) = C$, where C is the constant of integration.

Hence, $\lim_{t \rightarrow \infty} N(t) = C$

This proves the boundedness of the solutions inside Z . This implies that all the solutions of our system [(6) – (8)], starting in Z and will remain in Z for all $t \geq 0$. Thus Z is positively invariant and attracting, and hence it is sufficient to consider the dynamics of our system in Z . This completes the proof.

3.2 Disease-free equilibrium point (DFEP)

The disease-free equilibrium point (DFEP) of the system [(6) – (8)], was obtained by setting all the E_P and I_P classes to zero to obtain $\mu N_P - \mu S_P^0 = 0$ which yields, $S_P^0 = N_P$

The DFEP for our system is given by $E^0 = (S_P^0, E_P^0, I_P^0) = (N_P, 0, 0)$.

140 The DFEP (E^0) is the infection free equilibrium point of the system[(6) – (8)], which indicates
141 that in absence of insects, the system[(6) – (8)] will consist of one compartment class

142 3.3. The basic reproduction number (R_0)

143 We used the next-generation matrix method to determine the basic reproduction number (R_0)
144 of the model (Carlos Castillo-Chavez, 2001). Using the notation f for a matrix of new infections
145 terms and v for the matrix of the remaining transfer of infection terms in our system, we
146 obtained

$$f = \begin{pmatrix} \frac{a\beta_1 K^2 \pi N_V^* \lambda_V S_P}{(k\lambda_V + \delta + \mu_V \pm \theta)(\delta + \mu_V \pm \theta)} \\ 0 \end{pmatrix} \text{ and } v = \begin{pmatrix} (\omega + \tau + \mu_P)E_P \\ -\tau E_P + \mu_P I_P \end{pmatrix}.$$

147 We then obtained the matrices F and V by finding the Jacobian matrices of f and v evaluated at
148 DFEP respectively

$$F = \begin{pmatrix} \frac{a^2 \beta_1 \beta_2 K^2 \pi N_V^* N_P}{(\delta + \mu_V \pm \theta)^2} & \frac{a^2 \beta_1 \beta_2 K^2 \pi N_V^* \eta N_P}{(\delta + \mu_V \pm \theta)^2} \\ 0 & 0 \end{pmatrix} \text{ and } V = \begin{pmatrix} \omega + \tau + \mu_P & 0 \\ -\tau & \mu_P \end{pmatrix}.$$

149 We computed V^{-1} the inverse of V hence $V^{-1} = \begin{pmatrix} \frac{1}{\omega + \tau + \mu_P} & 0 \\ \frac{\tau}{\mu_P(\omega + \tau + \mu_P)} & \frac{1}{\mu_P} \end{pmatrix}.$

150 Multiplying the matrices F and V^{-1} we obtained,

$$FV^{-1} = \begin{pmatrix} \frac{a^2 \beta_1 \beta_2 K^2 \pi N_V^* N_P}{(\delta + \mu_V \pm \theta)^2(\omega + \tau + \mu_P)} + \frac{a^2 \beta_1 \beta_2 K^2 \pi N_V^* \eta \tau N_P}{\mu_P(\delta + \mu_V \pm \theta)^2(\omega + \tau + \mu_P)} & \frac{a^2 \beta_1 \beta_2 K^2 \pi N_V^* \eta N_P}{\mu_P(\delta + \mu_V \pm \theta)^2} \\ 0 & 0 \end{pmatrix}.$$

151 The eigenvalues of the matrix (FV^{-1}) are

$$q(1) = 0 \text{ and } q(2) = \frac{a^2 \beta_1 \beta_2 K^2 \pi N_V^* N_P}{(\delta + \mu_V \pm \theta)^2(\omega + \tau + \mu_P)} + \frac{a^2 \beta_1 \beta_2 K^2 \pi N_V^* \eta \tau N_P}{\mu_P(\delta + \mu_V \pm \theta)^2(\omega + \tau + \mu_P)}.$$

152 The basic reproduction number(R_0) is given by the spectral radius ζ (the dominant eigenvalue)
153 of the matrix FV^{-1} denoted by $\zeta(FV^{-1})$ hence

$$R_0 = \zeta(FV^{-1}) = \frac{a^2 \beta_1 \beta_2 K^2 \pi N_V^* N_P}{(\delta + \mu_V \pm \theta)^2(\omega + \tau + \mu_P)} \left\{ 1 + \frac{\eta \tau}{\mu_P} \right\}.$$

154 The basic reproduction number(R_0) is the average number of susceptible plants which can be
155 infected by an infected insect in absence of interventions.

156 3.4. Existence of Endemic Equilibrium point for the model (EEP)

157 We state and prove the following theorem

158 **Theorem 2:** A positive endemic equilibrium exists whenever $R_0 > 1$.

159 **Proof**

160 Let the endemic equilibrium point be $E^* = (S_P^*, E_P^*, I_P^*)$ and the force of infection at equilibrium
 161 point be $\lambda_V^* = a\beta_2(E_P^* + \eta I_P^*)$. The system of equation (6)– (8) were equated to zero then solved
 162 in terms of λ^* to obtain;

$$163 \quad \frac{dS_P}{dt} = \mu_P N_P + \omega E_P - \left(\frac{a\beta_1 K^2 \pi N_V^* \lambda_V}{[k\lambda_V(\delta + \mu_V \pm \theta) + (\delta + \mu_V \pm \theta)^2]} + \mu_P \right) S_P \quad (6),$$

$$\frac{dE_P}{dt} = \frac{a\beta_1 K^2 \pi N_V^* \lambda_V S_P}{[k\lambda_V(\delta + \mu_V \pm \theta) + (\delta + \mu_V \pm \theta)^2]} - (\omega + \tau + \mu_P) E_P \quad (7),$$

$$S_P^* = \frac{\mu_P N_P [k\lambda_V^* (\delta + \mu_V \pm \theta) + (\delta + \mu_V \pm \theta)^2]}{a\beta_1 K^2 \pi N_V^* \lambda_V^* + \mu_P [k\lambda_V^* (\delta + \mu_V \pm \theta) + (\delta + \mu_V \pm \theta)^2]} + \frac{[k\lambda_V^* (\delta + \mu_V \pm \theta) + (\delta + \mu_V \pm \theta)^2] \omega E_P^*}{a\beta_1 K^2 \pi N_V^* \lambda_V^* + \mu_P [k\lambda_V^* (\delta + \mu_V \pm \theta) + (\delta + \mu_V \pm \theta)^2]}$$

$$E_P^* = \frac{a\beta_1 K^2 \pi N_V^* \lambda_V^* \mu_P N_P}{(\omega + \tau + \mu_P) \{a\beta_1 K^2 \pi N_V^* \lambda_V^* + \mu_P [k\lambda_V^* (\delta + \mu_V \pm \theta) + (\delta + \mu_V \pm \theta)^2]\} - a\beta_1 K^2 \pi N_V^* \lambda_V \omega},$$

$$I_P^* = \frac{\tau a\beta_1 K^2 \pi N_V^* \lambda_V^* \mu_P N_P}{\mu_P [(\omega + \tau + \mu_P) \{a\beta_1 K^2 \pi N_V^* \lambda_V^* + \mu_P (k\lambda_V^* (\delta + \mu_V \pm \theta) + (\delta + \mu_V \pm \theta)^2)\} - a\beta_1 K^2 \pi N_V^* \lambda_V \omega]},$$

$$\lambda_V^* = \frac{a^2 \beta_2 \beta_1 K^2 \pi N_V^* \lambda_V^* \mu_P N_P}{(\omega + \tau + \mu_P) \{a\beta_1 K^2 \pi N_V^* \lambda_V^* + \mu_P [k\lambda_V^* (\delta + \mu_V \pm \theta) + (\delta + \mu_V \pm \theta)^2]\} - a\beta_1 K^2 \pi N_V^* \lambda_V \omega} \left(1 + \frac{\tau \eta}{\mu_P} \right)$$

$$\lambda_V^* = \frac{\lambda_V^* \mu_P R_0 (\delta + \mu_V \pm \theta)^2 (\omega + \tau + \mu_P)}{(\omega + \tau + \mu_P) \{a\beta_1 K^2 \pi N_V^* \lambda_V^* + \mu_P [k\lambda_V^* (\delta + \mu_V \pm \theta) + (\delta + \mu_V \pm \theta)^2]\} - a\beta_1 K^2 \pi N_V^* \lambda_V \omega}$$

164

$$R_0 = \frac{a^2 \beta_1 \beta_2 K^2 \pi N_V^* N_P}{(\delta + \mu_V \pm \theta)^2 (\omega + \tau + \mu_P)} \left\{ 1 + \frac{\eta \tau}{\mu_P} \right\}$$

165 $\lambda_V^* = 0$ Corresponds to DFE

$$1 = \frac{\mu_P R_0 (\delta + \mu_V \pm \theta)^2 (\omega + \tau + \mu_P)}{(\omega + \tau + \mu_P) \{a\beta_1 K^2 \pi N_V^* \lambda_V^* + \mu_P [k\lambda_V^* (\delta + \mu_V \pm \theta) + (\delta + \mu_V \pm \theta)^2]\} - a\beta_1 K^2 \pi N_V^* \lambda_V \omega}$$

166

$$\lambda_V^* (\omega + \tau + \mu_P) \{a\beta_1 K^2 \pi N_V^* + \mu_P [k(\delta + \mu_V \pm \theta)]\} - a\beta_1 K^2 \pi N_V^* \omega = \mu_P R_0 (\delta + \mu_V \pm \theta)^2 (\omega + \tau + \mu_P) - \mu_P (\omega + \tau + \mu_P) [(\delta + \mu_V \pm \theta)^2]$$

$$\lambda_V^* = \frac{\mu_P(\delta + \mu_V \pm \theta)^2(\omega + \tau + \mu_P)(R_0 - 1)}{(\omega + \tau + \mu_P)\{a\beta_1 K^2 \pi N_V^* + \mu_P[k(\delta + \mu_V \pm \theta)]\} - a\beta_1 K^2 \pi N_V^* \omega}$$

167 Since $(\omega + \tau + \mu_P)\{a\beta_1 K^2 \pi N_V^* + \mu_P[k(\delta + \mu_V \pm \theta)]\} - a\beta_1 K^2 \pi N_V^* \omega > 0$, the condition necessary
 168 and sufficient for $\lambda_V^* > 0$ is $\mu_P(\delta + \mu_V \pm \theta)^2(\omega + \tau + \mu_P)(R_0 - 1) > 0$ that is $R_0 > 1$. That
 169 completes the proof

170 The endemic equilibrium point (EEP) for our system $E^* = (S_P^*, E_P^*, I_P^*)$ is given by

$$S_P^* = \frac{\mu_P N_P [k\lambda_V^* (\delta + \mu_V \pm \theta) + (\delta + \mu_V \pm \theta)^2]}{a\beta_1 K^2 \pi N_V^* \lambda_V^* + \mu_P [k\lambda_V^* (\delta + \mu_V \pm \theta) + (\delta + \mu_V \pm \theta)^2]} + \frac{[k\lambda_V^* (\delta + \mu_V \pm \theta) + (\delta + \mu_V \pm \theta)^2] \omega E_P^*}{a\beta_1 K^2 \pi N_V^* \lambda_V^* + \mu_P [k\lambda_V^* (\delta + \mu_V \pm \theta) + (\delta + \mu_V \pm \theta)^2]},$$

$$E_P^* = \frac{a\beta_1 K^2 \pi N_V^* \lambda_V^* \mu_P N_P}{(\omega + \tau + \mu_P)\{a\beta_1 K^2 \pi N_V^* \lambda_V^* + \mu_P [k\lambda_V^* (\delta + \mu_V \pm \theta) + (\delta + \mu_V \pm \theta)^2]\} - a\beta_1 K^2 \pi N_V^* \lambda_V^* \omega},$$

$$I_P^* = \frac{\tau a\beta_1 K^2 \pi N_V^* \lambda_V^* \mu_P N_P}{\mu_P [(\omega + \tau + \mu_P)\{a\beta_1 K^2 \pi N_V^* \lambda_V^* + \mu_P (k\lambda_V^* (\delta + \mu_V \pm \theta) + (\delta + \mu_V \pm \theta)^2)\} - a\beta_1 K^2 \pi N_V^* \lambda_V^* \omega]},$$

171 Where,

$$\lambda_V^* = \frac{\mu_P(\delta + \mu_V \pm \theta)^2(\omega + \tau + \mu_P)(R_0 - 1)}{(\omega + \tau + \mu_P)\{a\beta_1 K^2 \pi N_V^* + \mu_P[k(\delta + \mu_V \pm \theta)]\} - a\beta_1 K^2 \pi N_V^* \omega}$$

172

173 3.5. Local Stability of the disease free equilibrium point (DFE).

174 To determine the local stability of the disease free equilibrium point we stated and proved the
 175 following theorem.

176 **Theorem 3:** The DFEP of the system [(6) – (8)] is locally asymptotically stable $R_0 < 1$ and
 177 unstable otherwise.

178 Proof

179 To establish the local stability of the system [(6) – (8)], we used the Jacobian of the model
 180 evaluated at E_*^0 . Stability of this steady state was then determined based on the signs
 181 eigenvalues of the corresponding Jacobian which were functions of the model parameters. We
 182 let

$$E^0 = (S_P^0, E_P^0, I_P^0) = (N_P, 0, 0)$$

183 The Jacobian matrix evaluated at disease free equilibrium point E^0 is obtained as

$$\lambda_V = a\beta_2(E_P + \eta I_P)$$

$$J(E^0) = \begin{pmatrix} -\frac{a\beta_1 K^2 \pi N_V^* N_P}{(\delta + \mu_V \pm \theta)^2} + \mu_P & -\frac{a^2 \beta_1 \beta_2 K^2 \pi N_V^* N_P}{(\delta + \mu_V \pm \theta)^2} + \omega & -\frac{a^2 \beta_1 \beta_2 \eta K^2 \pi N_V^* N_P}{(\delta + \mu_V \pm \theta)^2} \\ 0 & \frac{a^2 \beta_1 \beta_2 K^2 \pi N_V^* N_P}{(\delta + \mu_V \pm \theta)^2} - (\omega + \tau + \mu_P) & \frac{a^2 \beta_1 \beta_2 \eta K^2 \pi N_V^* N_P}{(\delta + \mu_V \pm \theta)^2} \\ 0 & \tau & -\mu_P \end{pmatrix}$$

184 Solving the equation

$$185 |J(E^0) - q(i)H| = 0,$$

186 Where H is the identity matrix and , $i = 1(1)3$ are the eigenvalues. Using the Mathematica
187 software we obtained the following eigenvalues

$$q(1) = -\frac{a\beta_1 K^2 \pi N_V^* N_P}{(\delta + \mu_V \pm \theta)^2} + \mu_P,$$

$$q(2) = \frac{1}{2}(-\tau - \omega - 2\mu_P + \frac{R_0 \mu_P (\tau + \omega + \mu_P)}{\eta \tau + \mu_P} - \sqrt{(\tau + \omega)^2 + \frac{2((-1 + 2\eta)\tau - \omega)R_0 \mu_P (\tau + \omega + \mu_P)}{\eta \tau + \mu_P} + \frac{R_0^2 \mu_P^2 (\tau + \omega + \mu_P)^2}{(\eta \tau + \mu_P)^2}}),$$

$$q(3) = \frac{1}{2}(-\tau - \omega - 2\mu_P + \frac{R_0 \mu_P (\tau + \omega + \mu_P)}{\eta \tau + \mu_P} + \sqrt{(\tau + \omega)^2 + \frac{2((-1 + 2\eta)\tau - \omega)R_0 \mu_P (\tau + \omega + \mu_P)}{\eta \tau + \mu_P} + \frac{R_0^2 \mu_P^2 (\tau + \omega + \mu_P)^2}{(\eta \tau + \mu_P)^2}})$$

188

189 Clearly, $q(1) < 0$.

190 We used Mathematica software to make R_0 the subject of the expressions below to show that
191 $q(2)$ and $q(3)$ are both less than zero, that is,

$$\frac{1}{2}(-\tau - \omega - 2\mu_P + \frac{R_0 \mu_P (\tau + \omega + \mu_P)}{\eta \tau + \mu_P}) < \pm \sqrt{(\tau + \omega)^2 + \frac{2((-1 + 2\eta)\tau - \omega)R_0 \mu_P (\tau + \omega + \mu_P)}{\eta \tau + \mu_P} + \frac{R_0^2 \mu_P^2 (\tau + \omega + \mu_P)^2}{(\eta \tau + \mu_P)^2}}$$

192 Hence the conditions necessary and sufficient for $q(2)$ and $q(3)$ is $R_0 < 1$. This completed the
193 proof.

194 3.6. Global stability of the disease free point.

195 To prove the global stability, we stated and proved the following theorem.

Theorem 4: The conditions necessary and sufficient for the DFE to be globally asymptotically stable in Lyapunov sense are $\left\{\frac{\mu_P}{\tau} + \eta\right\} \leq 0$ or $\frac{a\beta_1 K^2 \pi N_V^* S_P^0 I_P}{(k(E_P + \eta I_P) + \delta + \mu_V \pm \theta)(\delta + \mu_V \pm \theta)} \left\{\frac{\mu_P}{\tau} + \eta\right\} < \mu_P \frac{(S_P - S_P^0)^2}{S_P} + \mu_P I_P + I_P \frac{\mu_P^2}{\tau} + \mu_P N_P \frac{S_P^0}{S_P} + \omega E_P \frac{S_P^0}{S_P}$ and unstable otherwise

Proof

We proposed the following Lyapunov function for the system [(6) – (8)]

$$L(S_P, E_P, I_P) = S_P - S_P^0 - S_P^0 \ln \frac{S_P}{S_P^0} + E_P + X_1 I_P \quad (9)$$

$L(S_P, E_P, I_P)$ is positive definite and satisfies the conditions;

$$L(S_P^0, E_P^0, I_P^0) = 0 \text{ and } L(S_P, E_P, I_P) > 0$$

For $\frac{dL(S_P, E_P, I_P)}{dt}$ to be negative definite, it must satisfy

$$\frac{dL(S_P^0, E_P^0, I_P^0)}{dt} = 0 \text{ and } \frac{dL(S_P, E_P, I_P)}{dt} < 0$$

Where, X_1 and X_2 were positive constants to be determined at DFE point

$$E^0 = (S_P^0, E_P^0, I_P^0) = (N_P, 0, 0)$$

the system [(6) – (8)] satisfy, $S_P^0 = N_P$,

The time derivative of the Lyapunov function is obtained as,

$$\frac{dL(S_P, E_P, I_P)}{dt} = \left(1 - \frac{S_P^0}{S_P}\right) \frac{dS_P}{dt} + X_1 \left(1 - \frac{E_P^0}{E_P}\right) \frac{dE_P}{dt} + X_2 \frac{dI_P}{dt}, \quad (10)$$

On substituting for $\frac{dS_P}{dt}$, $\frac{dE_P}{dt}$ and $\frac{dI_P}{dt}$ in equations (6) – (8) we obtain;

$$\begin{aligned} \frac{dL(S_P, E_P, I_P)}{dt} = & \left(1 - \frac{S_P^0}{S_P}\right) \left\{ \mu_P N_P + \omega E_P - \left(\frac{a\beta_1 K^2 \pi N_V^* \lambda_V}{(k\lambda_V + \delta + \mu_V \pm \theta)(\delta + \mu_V \pm \theta)} + \mu_P \right) S_P \right\} \\ & + \left\{ \frac{a\beta_1 K^2 \pi N_V^* \lambda_V S_P}{(k\lambda_V + \delta + \mu_V \pm \theta)(\delta + \mu_V \pm \theta)} - (\omega + \tau + \mu_P) E_P \right\} + X_1 \{ \tau E_P - \mu_P I_P \}, \end{aligned}$$

At DFE $\mu_P N_P = \mu_P S_P^0$

$$\begin{aligned} \frac{dL(S_P, E_P, I_P)}{dt} = & -\mu_P \frac{(S_P - S_P^0)^2}{S_P} - (\tau + \mu_P) E_P + X_1 \{ \tau E_P - \mu_P I_P \} - \mu_P N_P \frac{S_P^0}{S_P} - \omega E_P \frac{S_P^0}{S_P} \\ & + \left(\frac{a\beta_1 K^2 \pi N_V^* (E_P + \eta I_P)}{(k(E_P + \eta I_P) + \delta + \mu_V \pm \theta)(\delta + \mu_V \pm \theta)} \right) S_P^0 \end{aligned}$$

Setting E_P to zero we obtain

$$-(\tau + \mu_P) + \frac{a\beta_1 K^2 \pi N_V^* S_P^0}{(k(E_P + \eta I_P) + \delta + \mu_V \pm \theta)(\delta + \mu_V \pm \theta)} + X_1 \tau = 0.$$

$$X_1 = 1 + \frac{\mu_P}{\tau} - \frac{a\beta_1 K^2 \pi N_V^* S_P^0}{\tau(k(E_P + \eta I_P) + \delta + \mu_V \pm \theta)(\delta + \mu_V \pm \theta)}$$

211 Then

$$\begin{aligned} \frac{dL(S_P, E_P, I_P)}{dt} &= -\mu_P \frac{(S_P - S_P^0)^2}{S_P} - \mu_P I_P \left\{ 1 + \frac{\mu_P}{\tau} - \frac{a\beta_1 K^2 \pi N_V^* S_P^0}{\tau(k(E_P + \eta I_P) + \delta + \mu_V \pm \theta)(\delta + \mu_V \pm \theta)} \right\} \\ &\quad - \mu_P N_P \frac{S_P^0}{S_P} - \omega E_P \frac{S_P^0}{S_P} + \frac{a\beta_1 K^2 \pi N_V^* \eta I_P S_P^0}{(k(E_P + \eta I_P) + \delta + \mu_V \pm \theta)(\delta + \mu_V \pm \theta)}, \\ \frac{dL(S_P, E_P, I_P)}{dt} &= -\mu_P \frac{(S_P - S_P^0)^2}{S_P} - \mu_P I_P - I_P \frac{\mu_P^2}{\tau} - \mu_P N_P \frac{S_P^0}{S_P} - \omega E_P \frac{S_P^0}{S_P} \\ &\quad + \frac{a\beta_1 K^2 \pi N_V^* S_P^0 I_P}{(k(E_P + \eta I_P) + \delta + \mu_V \pm \theta)(\delta + \mu_V \pm \theta)} \left\{ \frac{\mu_P}{\tau} + \eta \right\}, \end{aligned}$$

212 The conditions necessary and sufficient for $\frac{dL(S_P, E_P, I_P)}{dt} < 0$ are

213 $\left\{ \frac{\mu_P}{\tau} + \eta \right\} \leq 0$ or $+\frac{a\beta_1 K^2 \pi N_V^* S_P^0 I_P}{(k(E_P + \eta I_P) + \delta + \mu_V \pm \theta)(\delta + \mu_V \pm \theta)} \left\{ \frac{\mu_P}{\tau} + \eta \right\} < \mu_P \frac{(S_P - S_P^0)^2}{S_P} + \mu_P I_P + I_P \frac{\mu_P^2}{\tau} + \mu_P N_P \frac{S_P^0}{S_P} +$

214 $\omega E_P \frac{S_P^0}{S_P}$. This completes the proof.

215 3.7 Bifurcation Analysis.

216 This bifurcation was explored using the Centre Manifold theory (Junli et al. 2011). The change of
217 variables was made first for simplicity. Let $S_P = y_1$, $E_P = y_2$ and $I_P = y_4$, so that,

218 $N_P = y_1 + y_2 + y_3$.

$$R_0 = \frac{a^2 \beta_1 \beta_2 K^2 \pi N_V^* N_P}{(\delta + \mu_V \pm \theta)^2 (\omega + \tau + \mu_P)} \left\{ 1 + \frac{\eta \tau}{\mu_P} \right\}.$$

219 Further, by using vector notation, $y = (y_1, y_2, y_3)^T$, the insect vector disease model [(6) –
220 (8)] was written in the form $\frac{dy}{dt} = F(y)$, with $F = (p_1, p_2, p_3)^T$, as follows:

221 $\dot{y}_1 = p_1 = \mu_P N_P + \omega y_2 - \left(\frac{a\beta_1 K^2 \pi N_V^* \lambda_V^{**}}{(k\lambda_V^{**} + \delta + \mu_V \pm \theta)(\delta + \mu_V \pm \theta)} + \mu_P \right) y_1,$
222 (11)

223 $\dot{y}_2 = p_2 = \frac{a\beta_1 K^2 \pi N_V^* \lambda_V^{**} y_1}{(k\lambda_V^{**} + \delta + \mu_V \pm \theta)(\delta + \mu_V \pm \theta)} - (\omega + \tau + \mu_P) y_2$ (12)

224 $\dot{y}_3 = p_3 = \tau y_2 - \mu_P y_3$ (13)

225 With $\lambda_V^{**} = a\beta_2(y_2 + \eta y_3)$

226 The method entailed evaluating the Jacobian of the system [(9) – (11)] at the disease free
227 equilibrium point, by $J(E_*^0)$, has eigenvalues $\left(-\frac{(\omega + \tau + \mu_P)\mu_P}{a^2 \beta_2 (\mu_P + \eta \tau)} + \mu_P, -\frac{(\omega + \tau + \mu_P)\eta \tau + \eta \tau \mu_P + \mu_P^2}{\mu_P + \eta \tau}, 0 \right)$,
228 denoted by $J(E_*^0)$. This gave:

$$J(E_*^0) = \begin{pmatrix} -\frac{a\beta_1^* K^2 \pi N_V^* N_P}{(\delta + \mu_V \pm \theta)^2} + \mu_P & -\frac{a^2 \beta_1^* \beta_2 K^2 \pi N_V^* N_P}{(\delta + \mu_V \pm \theta)^2} + \omega & -\frac{a^2 \beta_1^* \beta_2 \eta K^2 \pi N_V^* N_P}{(\delta + \mu_V \pm \theta)^2} \\ 0 & \frac{a^2 \beta_1^* \beta_2 K^2 \pi N_V^* N_P}{(\delta + \mu_V \pm \theta)^2} - (\omega + \tau + \mu_P) & \frac{a^2 \beta_1^* \beta_2 \eta K^2 \pi N_V^* N_P}{(\delta + \mu_V \pm \theta)^2} \\ 0 & \tau & -\mu_P \end{pmatrix}$$

229 We considered the case where $R_0 = 1$. Suppose, further, that $\beta_1 = \beta_1^*$ is chosen as a bifurcation
230 parameter. Solving for β^* from $R_C^* = 1$ gives

$$\beta_1^* = \frac{(\delta + \mu_V \pm \theta)^2 (\omega + \tau + \mu_P) \mu_P}{a^2 \beta_2 K^2 \pi N_V^* N_P (\mu_P + \eta \tau)}$$

231 Using Mathematica software the Jacobian of $\frac{dy}{dt} = F(y)$ at the disease free equilibrium point,
232 with $\beta = \beta^*$, denoted by $J(E_*^0)$, has eigenvalues $\left(-\frac{(\omega + \tau + \mu_P) \mu_P}{a^2 \beta_2 (\mu_P + \eta \tau)} + \mu_P, -\frac{(\omega + \tau + \mu_P) \eta \tau + \eta \tau \mu_P + \mu_P^2}{\mu_P + \eta \tau}, 0 \right)$.
233 We obtained one zero eigenvalue and two negative eigenvalues hence, the Centre Manifold
234 theory was used to analyze the dynamics of the model (Junli Liu, et.al 2011).

235 **Theorem 5:** Castillo-Chavez and Song, considered the following general system of ordinary
236 differential equations with a parameter β_1^*

$$\frac{dy}{dt} = f(y, \beta_1^*), f: R^n \times R \rightarrow R^n \text{ and } f \in C^2(R^n \times R),$$

237 where 0 is an equilibrium point of the system (that is, $f(y, \beta_1^*) \equiv 0$ for all β_1^*) and

238 $A = D_y f(0, 0) = \left(\frac{\delta p_i}{\delta y_j}(0, 0) \right)$, is the linearization matrix of the system around the equilibrium 0
239 with β_1^* evaluated at 0;

240 Zero is a simple eigenvalue of A and all other eigenvalues of A have negative real parts;

241 Matrix A has a right eigenvector u and a left eigenvector v corresponding to the zero eigenvalue.

242 Let p_k be the kth component of p, $r = \sum_{k,i,j=1}^n v_k u_i u_j \frac{\partial^2 p_k}{\partial y_i \partial y_j}(0, 0)$ and $q =$

243 $\sum_{k,i,j=1}^n v_k u_i \frac{\partial^2 p_k}{\partial y_i \partial \beta_1^*}(0, 0)$, then the local dynamics of the system around the equilibrium point
244 (0,0) is totally determined by the signs of r and q.

245 Particularly when:

246 $r > 0$ and $q > 0$, when $\beta_1^* < 0$ with $|\beta_1^*| \ll 1$, (0,0), is locally asymptotically stable and there
247 exists a positive unstable equilibrium; when $0 < \beta_1^* \ll 1$, (0,0) is unstable and there exists a
248 negative and locally asymptotically stable equilibrium.

249 $r < 0$ and $q < 0$, when $\beta_1^* < 0$ with $|\beta_1^*| \ll 1$, (0,0) is unstable; when $0 < \beta_1^* \ll 1$, (0,0) is
250 asymptotically stable and there exists a positive unstable equilibrium.

251 $r < 0$ and $q > 0$, when $\beta^* < 0$ with $|\beta_1^*| \ll 1$, $(0,0)$ is unstable, and there exists a negative and
 252 locally asymptotically stable equilibrium; when $0 < \beta^* \ll 1$, $(0,0)$ is stable and there exists a
 253 positive unstable equilibrium.

254 $r > 0$ and $q < 0$, when β_1^* changes from negative to positive, $(0,0)$ changes its stability from
 255 stable to unstable. On the other hand a negative equilibrium becomes positive and locally
 256 asymptotically stable.

257 Eigenvectors of J_{β^*} : For the case, when $R_0 = 1$ it can be shown that the Jacobian $[J(E_*)]$ at $\beta_1 =$
 258 β_1^* (denoted by J_{β^*}) has a right eigenvector given by $u = [u_1, u_2, u_3]^T$, where,

259

$$u_1 = \frac{-\frac{a^2\beta_1^*\beta_2K^2\pi N_V^*N_P u_2}{(\delta+\mu_V\pm\theta)^2} - \frac{a^2\beta_1^*\beta_2\eta K^2\pi N_V^*N_P u_3}{(\delta+\mu_V\pm\theta)^2} + \omega u_2}{\frac{a\beta_1^*K^2\pi N_V^*N_P}{(\delta+\mu_V\pm\theta)^2} - \mu_P} < 0, u_2 = u_2 > 0, u_3 = \frac{\tau u_2}{\mu_P} > 0,$$

260 Further, J_{β^*} has a left eigenvector ($v = [v_1, v_2, v_3]$), where,

$$v_1 = 0, v_2 = v_2 > 0, v_3 = \frac{\frac{a^2\beta_1^*\beta_2\eta K^2\pi N_V^*N_P v_2}{(\delta+\mu_V\pm\theta)^2}}{\mu_P} > 0.$$

261 Since ($v_1 = 0$), we only need to compute the partial derivatives of p_2 and p_3 (at the disease free
 262 equilibrium point). For the system [(9) – (11)] the associated non-zero partial derivative of f_3
 263 (at the disease free equilibrium) is given by

$$264 \frac{\partial^2 p_2}{\partial y_1 \partial y_2} = \frac{\partial^2 p_2}{\partial p_2 \partial y_1} = \frac{a^2\beta_1\beta_2K^2\pi N_V^*}{(\delta+\mu_V\pm\theta)^2} \text{ and } \frac{\partial^2 p_3}{\partial y_1 \partial y_3} = \frac{\partial^2 p_3}{\partial y_3 \partial y_1} = \frac{a^2\beta_1\beta_2\eta K^2\pi N_V^*}{(\delta+\mu_V\pm\theta)^2}.$$

265 This implies that,

$$r = \sum_{i,j=1}^3 v_2 u_i u_j \frac{\partial^2 p_2}{\partial x_i \partial x_j},$$

$$r = \left\{ v_2 u_1 u_2 \frac{a^2\beta_1\beta_2K^2\pi N_V^*}{(\delta+\mu_V\pm\theta)^2} + v_2 u_2 u_1 \frac{a^2\beta_1\beta_2K^2\pi N_V^*}{(\delta+\mu_V\pm\theta)^2} + v_2 u_1 u_3 \frac{a^2\beta_1\beta_2\eta K^2\pi N_V^*}{(\delta+\mu_V\pm\theta)^2} \right. \\ \left. + v_2 u_3 u_1 \frac{a^2\beta_1\beta_2\eta K^2\pi N_V^*}{(\delta+\mu_V\pm\theta)^2} \right\}.$$

266 Since u_1 are less than zero, it follows that,

$$r = 2v_2 \left\{ u_1 u_2 \frac{\partial^2 p_2}{\partial y_1 \partial y_2} + u_1 u_3 \frac{\partial^2 p_2}{\partial y_1 \partial y_3} \right\} < 0.$$

267 Also,

$$y_2 = p_2 = \frac{a\beta_1 K^2 \pi N_V^* \lambda_V^{**} y_1}{(k\lambda_V^{**} + \delta + \mu_V \pm \theta)(\delta + \mu_V \pm \theta)} - (\omega + \tau + \mu_P)y_2 \quad (10),$$

268 with, $\lambda_V^{**} = a\beta_2(y_2 + \eta y_3)$

$$269 \quad q = \sum_{k,i,j=1}^n v_k u_i \frac{\partial^2 p_k}{\partial y_i \partial \beta^*} (0,0)$$

$$\frac{\partial^2 p_2}{\partial y_2 \partial \beta_1^*} = \frac{a^2 \beta_2 K^2 \pi N_V^* N_P}{(\delta + \mu_V \pm \theta)^2}; \quad \frac{\partial^2 p_2}{\partial y_3 \partial \beta_1^*} = \frac{a^2 \beta_2 K^2 \pi \eta N_V^* N_P}{(\delta + \mu_V \pm \theta)^2}$$

$$q = \sum_{i=1}^3 v_2 u_i \frac{\partial^2 p_2}{\partial y_i \partial \beta_1^*} = v_2 u_2 \frac{\partial^2 p_2}{\partial y_2 \partial \beta_1^*} + v_2 u_3 \frac{\partial^2 p_2}{\partial y_3 \partial \beta_1^*},$$

$$q = 2v_2 \left\{ u_2 \frac{a^2 \beta_2 K^2 \pi N_V^* N_P}{(\delta + \mu_V \pm \theta)^2} + u_3 \frac{a^2 \beta_2 K^2 \pi \eta N_V^* N_P}{(\delta + \mu_V \pm \theta)^2} \right\}.$$

270 Since v_2, u_2 and u_3 were greater than zero it followed that, $q > 0$.

271 Hence, it followed (from Theorem 3 above) that when $\beta_1^* < 0$ with $|\beta_1^*| \ll 1$, $(0,0)$ is unstable,

272 and there exists a negative and locally asymptotically stable equilibrium; when $0 < \beta_1^* \ll$

273 1 , $(0,0)$ is stable and there exists a positive unstable equilibrium.

274 3.8. Global stability of the Endemic Equilibrium point (EEP)

275 To determine global stability we stated and proved the following theorem.

276 **Theorem 6:** The conditions necessary and sufficient for the EEP to be globally asymptotically

277 stable in Lyapunov sense that, is $\frac{dK(S_P, E_P, I_P)}{dt} \leq 0$ is $P > Q$ and unstable otherwise, where,

$$\begin{aligned} P = \mu_P \frac{(S_P - S_P^*)^2}{S_P} + \left\{ 1 + \frac{\mu_P}{\tau} \right\} \mu_P I_P + \frac{a\beta_1 K^2 \pi N_V^* (E_P + \eta I_P) S_P}{(k\lambda_V + \delta + \mu_V \pm \theta)(\delta + \mu_V \pm \theta)} \\ + \left(\frac{a^2 \beta_1 \beta_2 K^2 \pi N_V^* (E_P^* + \eta I_P^*)}{(ka\beta_2 (E_P^* + \eta I_P^*) + \delta + \mu_V \pm \theta)(\delta + \mu_V \pm \theta)} \right) S_P^* \frac{S_P^*}{S_P} + \omega \frac{S_P^*}{S_P} E_P + \left\{ 1 + \frac{\mu_P}{\tau} \right\} \mu_P I_P^* \\ + \frac{\mu_P}{\tau} \tau \frac{I_P^* E_P}{I_P}, \end{aligned}$$

278

$$\begin{aligned} Q = \frac{a^2 \beta_1 \beta_2 K^2 \pi N_V^* S_P^*}{(ka\beta_2 (E_P + \eta I_P) + \delta + \mu_V \pm \theta)(\delta + \mu_V \pm \theta)} \left\{ E_P + \eta I_P \frac{S_P E_P^*}{E_P} + \omega \frac{S_P^* E_P^*}{S_P} + \left\{ 1 + \frac{\mu_P}{\tau} \right\} \mu_P I_P^* \right. \\ \left. + (\tau + \mu_P) E_P^* + \frac{a^2 \beta_1 \beta_2 K^2 \pi N_V^* (E_P^* + \eta I_P^*) S_P^*}{(ka\beta_2 (E_P^* + \eta I_P^*) + \delta + \mu_V \pm \theta)(\delta + \mu_V \pm \theta)} \right\} \end{aligned}$$

279 **Proof**

280 We proposed the following Lyapunov function,

$$K(S_P, E_P, I_P) = S_P - S_P^* - S_P^* \ln \frac{S_P}{S_P^*} + \left(E_P - E_P^* - E_P^* \ln \frac{E_P}{E_P^*} \right) + X_1 \left(I_P - I_P^* - I_P^* \ln \frac{I_P}{I_P^*} \right),$$

$$E^* = (S_P^*, E_P^*, I_P^*)$$

281 where, X_1 is a positive constant to be determined. The Lyapunov function $K(S_P, E_P, I_P)$ satisfies
 282 the conditions, $K(S_P^*, E_P^*, I_P^*) = 0$ and $K(S_P, E_P, I_P) > 0$, hence it is positive definite. For
 283 $\frac{dK(S_P, E_P, I_P)}{dt}$ to be negative definite, it must satisfy,

$$\frac{dK(S_P^*, E_P^*, I_P^*)}{dt} = 0 \quad \text{and} \quad \frac{dK(S_P, E_P, I_P)}{dt} < 0.$$

284 Determining the time derivative of the lyapunov equation we obtain,

$$K(S_P^*, E_P^*, I_P^*) = 0 \text{ and } K(S_P, E_P, I_P)$$

$$\frac{dK(S_P, E_P, I_P)}{dt} = \left(1 - \frac{S_P^*}{S_P} \right) \frac{dS_P}{dt} + \left(1 - \frac{E_P^*}{E_P} \right) \frac{dE_P}{dt} + X_1 \left(1 - \frac{I_P^*}{I_P} \right) \frac{dI_P}{dt},$$

285 Substituting for $\frac{dS_P}{dt}$, $\frac{dE_P}{dt}$ and $\frac{dI_P}{dt}$.

$$\begin{aligned} \frac{dK(S_P, E_P, I_P)}{dt} = & \left(1 - \frac{S_P^*}{S_P} \right) \left\{ \mu_P N_P + \omega E_P - \left(\frac{a\beta_1 K^2 \pi N_V^* \lambda_V}{(k\lambda_V + \delta + \mu_V \pm \theta)(\delta + \mu_V \pm \theta)} + \mu_P \right) S_P \right\} \\ & + \left(1 - \frac{E_P^*}{E_P} \right) \left\{ \frac{a\beta_1 K^2 \pi N_V^* \lambda_V S_P}{(k\lambda_V + \delta + \mu_V \pm \theta)(\delta + \mu_V \pm \theta)} - (\omega + \tau + \mu_P) E_P \right\} + X_1 \left(1 - \frac{I_P^*}{I_P} \right) \\ & \{ \tau E_P - \mu_P I_P \} \end{aligned}$$

286 The endemic equilibrium point $E^* = (S_P^*, E_P^*, I_P^*)$ for the system satisfies equations (6) - (8)

287 $\mu_P N_P = -\omega E_P^* + \left(\frac{a\beta_1 K^2 \pi N_V^* \lambda_V^*}{(k\lambda_V^* + \delta + \mu_V \pm \theta)(\delta + \mu_V \pm \theta)} + \mu_P \right) S_P^*,$

$$\frac{a\beta_1 K^2 \pi N_V^* \lambda_V^* S_P^*}{(k\lambda_V^* + \delta + \mu_V \pm \theta)(\delta + \mu_V \pm \theta)} = (\omega + \tau + \mu_P) E_P^*,$$

288 $\tau E_P^* = \mu_P I_P^* \text{ and } \lambda_V^* = a\beta_2 (E_P^* + \eta I_P^*).$

$$\begin{aligned}
\frac{dK(S_P, E_P, I_P)}{dt} &= \left(1 - \frac{S_P^*}{S_P}\right) \left\{ \mu_P S_P^* - \mu_P S_P - \omega E_P^* + \left(\frac{a\beta_1 K^2 \pi N_V^* \lambda_V^*}{(k\lambda_V^* + \delta + \mu_V \pm \theta)(\delta + \mu_V \pm \theta)} \right) S_P^* + \omega E_P \right. \\
&\quad \left. - \left(\frac{a\beta_1 K^2 \pi N_V^* a\beta_2 (E_P + \eta I_P)}{(ka\beta_2 (E_P + \eta I_P) + \delta + \mu_V \pm \theta)(\delta + \mu_V \pm \theta)} \right) S_P \right\} \\
&\quad + \left(1 - \frac{E_P^*}{E_P}\right) \left\{ \frac{a\beta_1 K^2 \pi N_V^* a\beta_2 (E_P + \eta I_P) S_P}{(ka\beta_2 (E_P + \eta I_P) + \delta + \mu_V \pm \theta)(\delta + \mu_V \pm \theta)} - (\omega + \tau + \mu_P) E_P \right\} \\
&\quad + X_1 \left(1 - \frac{I_P^*}{I_P}\right) \left\{ \tau E_P - \mu_P I_P \right\} \frac{dK(S_P, E_P, I_P)}{dt} \\
&= -\mu_P \frac{(S_P - S_P^*)^2}{S_P} - \omega E_P^* + \omega E_P + \frac{a^2 \beta_1 \beta_2 K^2 \pi N_V^* (E_P^* + \eta I_P^*) S_P^*}{(ka\beta_2 (E_P^* + \eta I_P^*) + \delta + \mu_V \pm \theta)(\delta + \mu_V \pm \theta)} \\
&\quad + \frac{a^2 \beta_1 \beta_2 K^2 \pi N_V^* (E_P + \eta I_P) S_P^*}{(ka\beta_2 (E_P + \eta I_P) + \delta + \mu_V \pm \theta)(\delta + \mu_V \pm \theta)} \\
&\quad - \frac{a^2 \beta_1 \beta_2 K^2 \pi N_V^* (E_P + \eta I_P) S_P}{(ka\beta_2 (E_P + \eta I_P) + \delta + \mu_V \pm \theta)(\delta + \mu_V \pm \theta)} \\
&\quad + \frac{a^2 \beta_1 \beta_2 K^2 \pi N_V^* (E_P + \eta I_P) S_P}{(ka\beta_2 (E_P + \eta I_P) + \delta + \mu_V \pm \theta)(\delta + \mu_V \pm \theta)} - (\omega + \tau + \mu_P) E_P + X_1 \tau E_P \\
&\quad - X_1 \mu_P I_P - \frac{a\beta_1 K^2 \pi N_V^* \lambda_V}{(k\lambda_V + \delta + \mu_V \pm \theta)(\delta + \mu_V \pm \theta)} \frac{S_P E_P^*}{E_P} \\
&\quad - \left(\frac{a^2 \beta_1 \beta_2 K^2 \pi N_V^* (E_P^* + \eta I_P^*)}{(ka\beta_2 (E_P^* + \eta I_P^*) + \delta + \mu_V \pm \theta)(\delta + \mu_V \pm \theta)} \right) S_P^* \frac{S_P^*}{S_P} - \omega \frac{S_P^*}{S_P} E_P - X_1 \tau \frac{I_P^* E_P}{I_P} \\
&\quad + \omega \frac{S_P^* E_P^*}{S_P} + X_1 \mu_P I_P^* + (\omega + \tau + \mu_P) E_P^*,
\end{aligned}$$

$$\begin{aligned}
\frac{dK(S_P, E_P, I_P)}{dt} &= -\mu_P \frac{(S_P - S_P^*)^2}{S_P} + \{-(\tau + \mu_P) + X_1 \tau\} E_P \\
&\quad + \frac{a^2 \beta_1 \beta_2 K^2 \pi N_V^* (E_P^* + \eta I_P^*) S_P^*}{(ka\beta_2 (E_P^* + \eta I_P^*) + \delta + \mu_V \pm \theta)(\delta + \mu_V \pm \theta)} - X_1 \mu_P I_P \\
&\quad + \frac{a^2 \beta_1 \beta_2 K^2 \pi N_V^* S_P^*}{(ka\beta_2 (E_P + \eta I_P) + \delta + \mu_V \pm \theta)(\delta + \mu_V \pm \theta)} \{E_P + \eta I_P\} \\
&\quad - \frac{a\beta_1 K^2 \pi N_V^* (E_P + \eta I_P) S_P}{(k\lambda_V + \delta + \mu_V \pm \theta)(\delta + \mu_V \pm \theta)} \frac{S_P E_P^*}{E_P} \\
&\quad - \left(\frac{a^2 \beta_1 \beta_2 K^2 \pi N_V^* (E_P^* + \eta I_P^*)}{(ka\beta_2 (E_P^* + \eta I_P^*) + \delta + \mu_V \pm \theta)(\delta + \mu_V \pm \theta)} \right) S_P^* \frac{S_P^*}{S_P} - \omega \frac{S_P^*}{S_P} E_P - X_1 \tau \frac{I_P^* E_P}{I_P} \\
&\quad + \omega \frac{S_P^* E_P^*}{S_P} + X_1 \mu_P I_P^* + (\tau + \mu_P) E_P^*,
\end{aligned}$$

289 Setting E_P to zero we obtained the following equation,

$$-(\tau + \mu_P) + X_1 \tau = 0,$$

$$X_1 = \left\{1 + \frac{\mu_P}{\tau}\right\}.$$

$$\begin{aligned} \frac{dK(S_P, E_P, I_P)}{dt} = & -\mu_P \frac{(S_P - S_P^*)^2}{S_P} + \frac{a^2 \beta_1 \beta_2 K^2 \pi N_V^* (E_P^* + \eta I_P^*) S_P^*}{(k a \beta_2 (E_P^* + \eta I_P^*) + \delta + \mu_V \pm \theta)(\delta + \mu_V \pm \theta)} \left\{ 1 - \frac{S_P^*}{S_P} \right\} - \left\{ 1 + \frac{\mu_P}{\tau} \right\} \mu_P I_P \\ & + \frac{a^2 \beta_1 \beta_2 K^2 \pi N_V^* S_P^*}{(k a \beta_2 (E_P + \eta I_P) + \delta + \mu_V \pm \theta)(\delta + \mu_V \pm \theta)} \{E_P + \eta I_P\} \\ & - \frac{a \beta_1 K^2 \pi N_V^* (E_P + \eta I_P) S_P}{(k \lambda_V + \delta + \mu_V \pm \theta)(\delta + \mu_V \pm \theta)} \frac{S_P E_P^*}{E_P} - \omega \frac{S_P^*}{S_P} E_P - \left\{ 1 + \frac{\mu_P}{\tau} \right\} \tau \frac{I_P^* E_P}{I_P} + \omega \frac{S_P^* E_P^*}{S_P} \\ & + \left\{ 1 + \frac{\mu_P}{\tau} \right\} \mu_P I_P^* + (\tau + \mu_P) E_P^*, \end{aligned}$$

$$\begin{aligned} \frac{dK(S_P, E_P, I_P)}{dt} = & -\mu_P \frac{(S_P - S_P^*)^2}{S_P} - \left\{ 1 + \frac{\mu_P}{\tau} \right\} \mu_P I_P - \frac{a \beta_1 K^2 \pi N_V^* (E_P + \eta I_P) S_P}{(k \lambda_V + \delta + \mu_V \pm \theta)(\delta + \mu_V \pm \theta)} \\ & - \left(\frac{a^2 \beta_1 \beta_2 K^2 \pi N_V^* (E_P^* + \eta I_P^*)}{(k a \beta_2 (E_P^* + \eta I_P^*) + \delta + \mu_V \pm \theta)(\delta + \mu_V \pm \theta)} \right) S_P^* \frac{S_P^*}{S_P} - \omega \frac{S_P^*}{S_P} E_P - \left\{ 1 + \frac{\mu_P}{\tau} \right\} \tau \frac{I_P^* E_P}{I_P} \\ & + \frac{a^2 \beta_1 \beta_2 K^2 \pi N_V^* S_P^*}{(k a \beta_2 (E_P + \eta I_P) + \delta + \mu_V \pm \theta)(\delta + \mu_V \pm \theta)} \{E_P + \eta I_P\} \frac{S_P E_P^*}{E_P} \\ & + \omega \frac{S_P^* E_P^*}{S_P} + \left\{ 1 + \frac{\mu_P}{\tau} \right\} \mu_P I_P^* + (\tau + \mu_P) E_P^* \\ & + \frac{a^2 \beta_1 \beta_2 K^2 \pi N_V^* (E_P^* + \eta I_P^*) S_P^*}{(k a \beta_2 (E_P^* + \eta I_P^*) + \delta + \mu_V \pm \theta)(\delta + \mu_V \pm \theta)}, \end{aligned}$$

290

291 Let,

$$\begin{aligned} P = & \mu_P \frac{(S_P - S_P^*)^2}{S_P} + \left\{ 1 + \frac{\mu_P}{\tau} \right\} \mu_P I_P + \frac{a \beta_1 K^2 \pi N_V^* (E_P + \eta I_P) S_P}{(k \lambda_V + \delta + \mu_V \pm \theta)(\delta + \mu_V \pm \theta)} \\ & + \left(\frac{a^2 \beta_1 \beta_2 K^2 \pi N_V^* (E_P^* + \eta I_P^*)}{(k a \beta_2 (E_P^* + \eta I_P^*) + \delta + \mu_V \pm \theta)(\delta + \mu_V \pm \theta)} \right) S_P^* \frac{S_P^*}{S_P} + \omega \frac{S_P^*}{S_P} E_P + \left\{ 1 + \frac{\mu_P}{\tau} \right\} \tau \frac{I_P^* E_P}{I_P}, \end{aligned}$$

$$\begin{aligned} Q = & \frac{a^2 \beta_1 \beta_2 K^2 \pi N_V^* S_P^*}{(k a \beta_2 (E_P + \eta I_P) + \delta + \mu_V \pm \theta)(\delta + \mu_V \pm \theta)} \left\{ E_P + \eta I_P \frac{S_P E_P^*}{E_P} + \omega \frac{S_P^* E_P^*}{S_P} + \left\{ 1 + \frac{\mu_P}{\tau} \right\} \mu_P I_P^* \right. \\ & \left. + (\tau + \mu_P) E_P^* + \frac{a^2 \beta_1 \beta_2 K^2 \pi N_V^* (E_P^* + \eta I_P^*) S_P^*}{(k a \beta_2 (E_P^* + \eta I_P^*) + \delta + \mu_V \pm \theta)(\delta + \mu_V \pm \theta)} \right\}, \end{aligned}$$

$$\frac{dK(S_P, E_P, I_P)}{dt} = -P + Q,$$

292 The conditions necessary and sufficient for the EEP to be globally asymptotically stable in

293 Lyapunov sense that is $\frac{dK(S_P, E_P, I_P)}{dt} \leq 0$ is $P > Q$. This completes the proof.

294 3.9 : SENSITIVITY ANALYSIS

295 The analytical sensitivity analysis of R_0 with various parameters was obtained by partial

296 differentiation as follows,

$$\begin{aligned}\frac{dR_0}{d\omega} &= -\frac{a^2\beta_1\beta_2K^2\pi N_V^*N_P}{(\delta + \mu_V \pm \theta)^2(\omega + \tau + \mu_P)^2} \left\{1 + \frac{\eta\tau}{\mu_P}\right\} < 0; \quad \frac{dR_0}{d\delta} \\ &= -\frac{2a^2\beta_1\beta_2K^2\pi N_V^*N_P}{(\delta + \mu_V \pm \theta)^3(\omega + \tau + \mu_P)^2} \left\{1 + \frac{\eta\tau}{\mu_P}\right\} < 0\end{aligned}$$

297 When $\theta > 0$, $\frac{dR_0}{d\theta} = -\frac{a^2\beta_1\beta_2K^2\pi N_V^*N_P}{(\delta + \mu_V + \theta)^2(\omega + \tau + \mu_P)^2} \left\{1 + \frac{\eta\tau}{\mu_P}\right\} < 0$ and when $\theta < 0$ then

$$\frac{dR_0}{d\theta} = \frac{a^2\beta_1\beta_2K^2\pi N_V^*N_P}{(\delta + \mu_V + \theta)^2(\omega + \tau + \mu_P)^2} \left\{1 + \frac{\eta\tau}{\mu_P}\right\} > 0; \quad \frac{dR_0}{d\tau} = \frac{a^2\beta_1\beta_2K^2\pi N_V^*N_P(\eta\omega + \mu_P[\eta - 1])}{\mu_P(\delta + \mu_V \pm \theta)^2(\omega + \tau + \mu_P)^2} > 0$$

$$\begin{aligned}\frac{dR_0}{d\beta_1} &= \frac{a^2\beta_2K^2\pi N_V^*N_P}{(\delta + \mu_V \pm \theta)^2(\omega + \tau + \mu_P)} \left\{1 + \frac{\eta\tau}{\mu_P}\right\} > 0; \quad \frac{dR_0}{d\beta_2} = \frac{a^2\beta_1K^2\pi N_V^*N_P}{(\delta + \mu_V \pm \theta)^2(\omega + \tau + \mu_P)} \left\{1 + \frac{\eta\tau}{\mu_P}\right\} \\ &> 0\end{aligned}$$

$$\begin{aligned}\frac{dR_0}{dK} &= 2 \frac{a^2\beta_1\beta_2K\pi N_V^*N_P}{(\delta + \mu_V \pm \theta)^2(\omega + \tau + \mu_P)} \left\{1 + \frac{\eta\tau}{\mu_P}\right\} > 0; \quad \frac{dR_0}{da} = 2 \frac{a\beta_1\beta_2K^2\pi N_V^*N_P}{(\delta + \mu_V \pm \theta)^2(\omega + \tau + \mu_P)} \left\{1 + \frac{\eta\tau}{\mu_P}\right\} \\ &> 0\end{aligned}$$

298 The parameters which are greater than one are directly proportional to R_0 while those that are
299 less than zero are inversely proportional to R_0 . Climatic factors affect immigration or
300 emigration of insects vectors, their biting rate, infection rates and death induced rates.

301 4. Results and discussion

302 The expression for basic reproduction number(R_0) obtained in model analysis estimates the
303 number of secondary exposed plants and/or infectious plant that would be obtained if one
304 infectious plant or exposed plant is introduced into the completely susceptible plant population.
305 If $R_0 > 1$, the disease would invade the plant population and reach endemic equilibrium
306 point(EEP)and should worry stakeholders. If $R_0 < 1$,the disease would die out.If $R_0 = 1$, the
307 number of infected plants would remain constant.

308 The DFEP was found to be locally stable whenever $R_0 < 1$, which means that any starting values
309 are near DFEP, they would move toward DFE asymptotically over time. The conditions
310 necessary for global stability of DFEP and EEP were obtained, inferring that if those conditions
311 are satisfied, any values they take would move toward equilibrium points asymptotically over
312 time.

313 5. Conclusion

314 In this paper, we formulated a non-linear deterministic model for plant vector borne diseases by
315 incorporating temperature and precipitation under the biting rate and wind as an agent of
316 immigration and emigration, obtained the conditions for positivity and boundedness, local and
317 global stability of the solution.

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381 **Appendix 1: List of Variables and Parameters**

Variables	Descriptions
$N_V(t)$	The total population of the vectors
$N_P(t)$	The total population of the plants
$S_P(t)$,	The population of susceptible plant
$E_P(t)$	The population of exposed plant
$I_P(t)$.	The population of infected plant
$S_V(t)$	The population of susceptible vectors
$I_V(t)$	The population of infected vectors

Parameters	Descriptions
π	The recruitment rate of vectors

μ_p	The recruitment rate of plants
ω	The rates at which $E_p(t)$ recover naturally to $S_p(t)$.
δ	The rate at which climate induced death occur in both $S_v(t)$ and $I_v(t)$
μ_p	The constant natural death rate in subclasses $S_p(t)$, $E_p(t)$ and $I_p(t)$
τ	The rate at which $E_p(t)$ progresses to $I_p(t)$.
μ_v	The constant natural death rate in subclasses $S_v(t)$ and $I_v(t)$
θ	The immigration and emigration rate of vector (this implies that the rate can be either be positive or negative).
β_1	The infection rates for plants
β_2	The infection rates for vectors
a	The vector biting rate

λ_p	The number of individual plants which become infected at given time(force of infection of plants)
λ_v	The number of individual plants which become infected at given time(force of infection of plants)

382 k The rate of change of force of infection due to climatic factors. Assumption is $K \geq 1$

383 **Appendix 2: Definition of terms**

Term	Definition
Susceptible population	The populations who are free of infection but are at a risk of contracting the infection.
Exposed population	The populations that has contracted the infection but are at a lower risk of transmitting it and/or with no signs of infection
Infected population	The population with the disease causing pathogen and capable of transmitting the infection to other plants or vectors on contact
Basic reproduction number/ratio	The number of cases one generates on average over the course of infectious period in a completely susceptible population
Bifurcation	It occurs when a smooth change is made to parameter values causes the system to change its behavior
Disease free equilibrium point	It is infection free point
Endemic equilibrium point	The stationary point at which the disease has completely invaded the population

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