

Mathematical Model of Drinking Epidemic

Abstract

We use *SHTR* model that describes the dynamics of non-drinkers (S), heavy drinkers (H), drinkers in treatment (T) and recovered drinkers (R) to study the dynamics of drinking as epidemic. We discussed the existence and stability of the drinking free and endemic equilibria. The reproductive number of drinking free equilibrium is asymptotically stable if $R_0 < 1$ and that of the endemic equilibrium is unstable if $R_0 > 1$. In the stability analysis of the equilibria, the drinking free equilibrium is asymptotically stable and the endemic equilibrium is unstable. Furthermore, Numerical simulations was conducted to confirm our analytic results. Through sensitivity analysis of the reproductive number, it was obvious that reducing the contact rate between the non-drinkers and heavy drinkers, increasing the number of drinkers that go into treatment and educating drinkers to refrain from drinking can be useful in combating the drinking epidemic.

Keywords: *Equilibrium Points, drinking free equilibrium, Endemic Equilibrium, Reproductive Number, Global Stability, Lyapunov function.*

1. Introduction

Alcohol is the world's third leading cause of ill health and premature death, after low birth weight and unsafe sex. In Europe, alcohol is also the third leading risk factor for disease and mortality after tobacco and high blood pressure [1,2]. Excessive drinking is not only harmful to personal health, but also leads to a range of negative social effects such as violence, antisocial and criminal behavior.

Long-term alcoholism will produce negative change in the brain, such as tolerance and physical dependence. Alcohol damages almost all parts of the body and contributes to a number of human disease including liver cirrhosis, pancreatitis, heart disease, sexual dysfunction, and eventually death [3,4]. For most individuals, the development of the colorectal cancer (CRC) is sporadic, and one of the risk factors include excessive alcohol intake [5]. Damage to the central and peripheral nervous systems can occur from sustained alcohol consumption [6,7,9]. Additionally, heavy drinking in women have been found to have a negative effect on the reproductive functioning [8,9]. The World Health Organization reports that the harmful use of alcohol causes approximately 3.3 million deaths every year (or 5.9% of all the global deaths), and 5.1% of the global burden of disease is attributed to alcohol consumption. Although there had been many attempts to reduce the problem, alcohol by young people had persisted and in some cases increased over the past several years [3,10].

Mathematical models could mimic the process of drinking and provide useful tools to analyze the spread and control of drinking behavior. Several different mathematical models for drinking had been formulated and studied [11]. In 2008, [12] proposed a mathematical model to study the dynamics of campus drinking as an epidemiological model. They divided

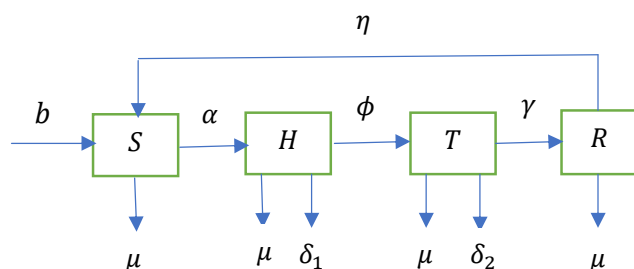
39 their population into three classes: non-drinkers (N), social drinkers (S) and problem drinkers
 40 (P). According to their results, the reproductive numbers were not sufficient to predict
 41 whether drinking behavior would persist on campus and that the pattern of recruiting new
 42 members played a significant role in the reduction of campus alcohol problems [3]. Another
 43 model which is relevant to this research is that of [13]. They presented a model to investigate
 44 the global property of a drinking model with public health educational campaigns. They
 45 divided their population into five compartments namely susceptible drinkers (S), educational
 46 drinkers (E), alcoholic drinkers (A), temporary removed drinkers (R), and quit drinkers (Q).
 47 They derived the global stability and the basic reproductive number of their model. Their
 48 results show that the public health educational campaigns of drinking individuals can slow
 49 down the drinking dynamics. Also, Xian et al [11] presented a quit drinking model taking
 50 into account of permanent quit drinker compartment and relapse, and global stability of
 51 equilibria was obtained [13].

52 In this paper, we use a modified SDTRS model of [16] to model alcoholism as epidemic. In
 53 their model, it was possible for a heavy drinker in treatment to become a heavy drinker
 54 without passing through the recovery compartment. Whereas we assume that heavy drinkers
 55 in treatment will only become heavy drinkers again after passing through the recovery and
 56 susceptible compartments respectively. The rest of the paper is organized as follows: In
 57 section 2, we present the model description and assumptions. Stability analysis of the
 58 drinking- free and endemic equilibria is discussed in section 3. In Section 4, we use the
 59 numerical example to show the dynamical behaviour of our results. In Section 5, we
 60 performed the sensitivity analysis of the basic reproductive number of the model, Section 6 is
 61 made up of discussion of our results. Finally, we give our conclusion.

62 2. Mathematical Model

63 2.1 Model Description

64 We formulate a mathematical model and divide the population into four compartments: non-
 65 drinkers (S), heavy drinkers (H), drinkers in treatment (T), and recovered drinkers (R). The
 66 interaction between the four drinking states are shown in the schematic diagram in figure1,
 67 below



73 Figure 1: Schematic diagram of the four drinking classes in the model.

75 2.2 Model Assumptions

76 The following assumptions were made in the model:

- 77 (i) the drinking epidemic occurs in a closed environment.
 78 (ii) sex, race and social status do not affect the probability of becoming a heavy drinker.
 79 (iii) heavy drinking is transmitted to non- drinkers when they are in contact with heavy
 80 drinkers
 81 (iv) the members mix homogeneously (have the same interaction to the same degree).
 82 (v) drinkersin treatment may only become heavy drinkers again after passing through the
 83 recovery and susceptible compartments respectively and
 84 (vi) drinkerswho have stopped drinking enter into recovery compartment.
 85 The drinking epidemic is modelled using the system of nonlinear Differential Equations.

86

$$87 \quad \frac{dS}{dt} = b - \alpha SH - \mu S + \eta R \quad (1)$$

$$88 \quad \frac{dH}{dt} = \alpha SH - (\mu + \delta_1 + \phi)H \quad (2)$$

$$89 \quad \frac{dT}{dt} = \phi H - (\mu + \delta_2 + \gamma)T \quad (3)$$

$$90 \quad \frac{dR}{dt} = \gamma T - (\mu + \eta)R \quad (4)$$

91

92 Where

93 b = recruitment rate of S

94 α =transmission rate from S to H

95 η =transmission rate from R to S

96 μ =natural death rate

97 δ_1 =drinking induced death rate of H

98 δ_2 = drinking induced death rate of T

99 ϕ = proportion of drinkers entering T compartment and

100 γ = recovered rate of T

101 We assume that the system of nonlinear differential equations (1) – (4) has positive initial
 102 conditions, then every solution $(S(t), H(t), T(t), R(t))$ of (1) – (4) has the positive
 103 properties, that is, $S(t) \geq 0, H(t) \geq 0, T(t) \geq 0$ and $R(t) \geq 0$.

104 We consider the following equations:

105
$$N(t) = S(t) + H(t) + T(t) + R(t) \quad (5)$$

106 and this implies that

$$\frac{dN}{dt} = \frac{dS}{dt} + \frac{dH}{dt} + \frac{dT}{dt} + \frac{dR}{dt}$$

107
$$\frac{dN}{dt} = b - \mu N - (\delta_1 H + \delta_2 T) \leq b - \mu N \quad (6)$$

108 From (6), it follows that:

109
$$\lim_{t \rightarrow \infty} \text{Sup } N(t) \leq \frac{b}{\mu}.$$

110 Hence, the feasible region of the system (1) – (4) is given by the set

111
$$\Gamma = \{(S, H, T, R) \in R_+^4 : S + H + T + R \leq \frac{b}{\mu}\},$$

112 is positively invariant set for the system(1) – (4). We will consider the dynamical behavior
113 of the system (1) – (4) in the set Γ .

114 Table 1: Model Parameters at drinking free equilibrium

Parameter	Description	Value	Source
b	recruitment rate of S	0.4	[15]
α	transmission rate from S to H	0.7	[15]
η	transmission rate from R to S	0.1	[15]
μ	natural death rate	0.25	[15]
δ_1	drinking induced death rate of H	0.35	[15]
δ_2	drinking induced death rate of T	0.3	[15]
γ	recovered rate of T	0.09	[15]
ϕ	proportion of drinkers entering T compartment	0.7	[15]

115

116 Table 2: Model Parameters at endemic equilibrium

117

118

119

120

Parameter	Description	Value	Source
b	recruitment rate of S	0.4	[15]
α	transmission rate from S to H	0.7	[15]
η	transmission rate from R to S	0.01	[15]
μ	natural death rate	0.025	[15]
δ_1	drinking induced death rate of H	0.035	[15]
δ_2	drinking induced death rate of T	0.03	[15]
γ	recovered rate of T	0.1	[15]
ϕ	proportion of drinkers entering T compartment	0.5	[15]

121

122 3. Model Analysis

123 3.1 Drinking-free Equilibrium and Basic Reproduction Number

124 In this section, we consider the drinking -free equilibrium $E_0 = (\frac{b}{\mu}, 0, 0, 0)$. That is a situation
 125 where there is no drinking problem. We analyze the stability of the drinking-free equilibrium
 126 by considering the linearized system of ODE's (1) – (4), taking the Jacobian matrix and
 127 obtained

$$128 \quad J(S, H, T, R) = \begin{bmatrix} -(\alpha H + \mu) & -\alpha S & 0 & \eta \\ \alpha H & \alpha S - (\mu + \delta_1 + \phi) & 0 & 0 \\ 0 & \phi & -(\mu + \delta_2 + \gamma) & 0 \\ 0 & 0 & \gamma & -(\mu + \eta) \end{bmatrix} \quad (7)$$

129 The local stability of the equilibrium may be determined from the Jacobian matrix (7). We
 130 denote the drinking-free equilibrium by the trivial equilibrium $(1, 0, 0, 0)$ and obtained:

$$131 \quad J(E_0) = \begin{bmatrix} -\mu & -\alpha & 0 & \eta \\ 0 & \alpha - (\mu + \delta_1 + \phi) & 0 & 0 \\ 0 & \phi & -(\mu + \delta_2 + \gamma) & 0 \\ 0 & 0 & \gamma & -(\mu + \eta) \end{bmatrix} \quad (8)$$

132 From the characteristic equation of $J(S, 0, 0, 0)$, the following eigenvalues were obtained:
 133 $\lambda_1 = -\mu, \lambda_2 = \alpha - (\mu + \delta_1 + \phi), \lambda_3 = -(\mu + \delta_2 + \gamma)$ and $\lambda_4 = -(\mu + \eta)$. It can be seen
 134 that $\lambda_1, \lambda_3, \lambda_4$ are real and negative. We know that $R_0 < 1$, this implies that $\alpha <$
 135 $(\mu + \delta_1 + \phi)$ and hence λ_2 is therefore real and negative. This means that the system (1) –
 136 (4) is asymptotically stable at $J(1, 0, 0, 0)$.

137 The basic reproduction number (R_0), is given by

$$138 \quad R_0 = \frac{\alpha}{(\mu + \delta_1 + \phi)} \quad (9)$$

139 **Theorem 1:** The drinking-free equilibrium $E_0 (\frac{b}{\mu}, 0, 0, 0)$ of the system (1) – (4) is
 140 asymptotically stable if $R_0 < 1$ and unstable if $R_0 > 1$.

3.2 Endemic Equilibrium

We evaluate the equilibrium points of the ODE (1) – (4) by setting the right -hand side of equation (1) – (4) to zero and then solve for S^* , H^* , T^* and R^* . We obtained:

$$S^* = \frac{\mu + \delta_1 + \phi}{\alpha}, H^* = \frac{\mu(\mu + \delta_1 + \phi) - \alpha b}{\alpha[\eta - (\mu + \delta_1 + \phi)]}, T^* = \frac{\phi[\mu(\mu + \delta_1 + \phi) - \alpha b]}{\alpha(\mu + \delta_2 + \gamma)[\eta - (\mu + \delta_1 + \phi)]}, \text{ and}$$

$$R^* = \frac{\gamma\phi[\mu(\mu + \delta_1 + \phi) - \alpha b]}{\alpha(\mu + \eta)(\mu + \delta_2 + \gamma)[\eta - (\mu + \delta_1 + \phi)]}. \quad (10)$$

We now consider the case when $R_0 > 1$. At the endemic equilibrium, all the four drinking states are present in the population. The steady states consider conditions under which all four drinking states can coexist in the equilibrium. We represent $E^* = (S^*, H^*, T^*, R^*)$ as endemic equilibrium of the system (1) – (4) and $(S^* \neq 0, H^* \neq 0, T^* \neq 0, R^* \neq 0)$. We know from equation (7) that:

$$J(E^*) = \begin{bmatrix} -(\alpha H + \mu) & -\alpha S & 0 & \eta \\ \alpha H & \alpha S - (\mu + \delta_1 + \phi) & 0 & 0 \\ 0 & \phi & -(\mu + \delta_2 + \gamma) & 0 \\ 0 & 0 & \gamma & -(\mu + \eta) \end{bmatrix} \quad (11)$$

Substituting the equilibrium points in (10) into equation (11), we obtain

$$J(E^*) = \begin{bmatrix} -\left[\alpha \left[\frac{\mu(\mu + \delta_1 + \phi) - \alpha b}{\alpha[\eta - (\mu + \delta_1 + \phi)]}\right] + \mu\right] & -\alpha \left[\frac{\mu + \delta_1 + \phi}{\alpha}\right] & 0 & \eta \\ \alpha \left[\frac{\mu(\mu + \delta_1 + \phi) - \alpha b}{\alpha[\eta - (\mu + \delta_1 + \phi)]}\right] & \alpha \left[\frac{\mu + \delta_1 + \phi}{\alpha}\right] - (\mu + \delta_1 + \phi) & 0 & 0 \\ 0 & \phi & -(\mu + \delta_2 + \gamma) & 0 \\ 0 & 0 & \gamma & -(\mu + \eta) \end{bmatrix} \quad (12)$$

Let

$$A_{11} = -\left[\alpha \left[\frac{\mu(\mu + \delta_1 + \phi) - \alpha b}{\alpha[\eta - (\mu + \delta_1 + \phi)]}\right] + \mu\right], A_{12} = -\alpha \left[\frac{\mu + \delta_1 + \phi}{\alpha}\right], A_{13} = 0, A_{14} = \eta,$$

$$A_{21} = \alpha \left[\frac{\mu(\mu + \delta_1 + \phi) - \alpha b}{\alpha[\eta - (\mu + \delta_1 + \phi)]}\right], A_{22} = \alpha \left[\frac{\mu + \delta_1 + \phi}{\alpha}\right] - (\mu + \delta_1 + \phi), A_{23} = 0, A_{24} = 0,$$

$$A_{31} = 0, A_{32} = \phi, A_{33} = -(\mu + \delta_2 + \gamma), A_{34} = 0$$

$$A_{41} = 0, A_{42} = 0, A_{43} = \gamma, A_{44} = -(\mu + \eta)$$

Substituting A_{ij} into (12), we obtain

$$J(E^*) = \begin{bmatrix} A_{11} & A_{12} & A_{13} & A_{14} \\ A_{21} & A_{22} & A_{33} & A_{24} \\ A_{31} & A_{32} & A_{33} & A_{34} \\ A_{41} & A_{42} & A_{41} & A_{44} \end{bmatrix} \quad (13)$$

The characteristic equation of (13) can be computed as follows

$$|J(E^*) - \lambda I| = \begin{vmatrix} A_{11} - \lambda & A_{12} & 0 & A_{14} \\ A_{21} & A_{22} - \lambda & 0 & 0 \\ 0 & A_{32} & A_{33} - \lambda & 0 \\ 0 & 0 & A_{43} & A_{44} - \lambda \end{vmatrix} = 0$$

$$\lambda^4 + (A_{11} + A_{22} + A_{33} + A_{44})\lambda^3 + (A_{12}A_{21} - A_{33}A_{44} - A_{11}A_{44} - A_{22}A_{44} - A_{11}A_{33} - A_{22}A_{33} - A_{11}A_{22})\lambda^2 + (A_{11}A_{33}A_{44} + A_{22}A_{33}A_{44} + A_{11}A_{22}A_{44} + A_{11}A_{22}A_{33} - A_{12}A_{21}A_{33} - A_{12}A_{21}A_{44})\lambda + (A_{12}A_{21}A_{33}A_{44} + A_{14}A_{21}A_{32}A_{43} - A_{11}A_{22}A_{33}A_{44}) = 0$$

We can write the characteristic equation above as:

$$\lambda^4 + C_1\lambda^3 + C_2\lambda^2 + C_3\lambda + C_4 = 0 \quad (14)$$

Where:

$$C_1 = A_{11} + A_{22} + A_{33} + A_{44}$$

$$C_2 = A_{12}A_{21} - A_{33}A_{44} - A_{11}A_{44} - A_{22}A_{44} - A_{11}A_{33} - A_{22}A_{33} - A_{11}A_{22}$$

$$C_3 = A_{11}A_{33}A_{44} + A_{22}A_{33}A_{44} + A_{11}A_{22}A_{44} + A_{11}A_{22}A_{33} - A_{12}A_{21}A_{33} - A_{12}A_{21}A_{44}$$

$$C_4 = A_{12}A_{21}A_{33}A_{44} + A_{14}A_{21}A_{32}A_{43} - A_{11}A_{22}A_{33}A_{44}$$

Using the Routh –Hurwitz criterion [15, 18]. It can be seen that all eigenvalues of the characteristic equation (14) has negative real part if and only if:

$$C_1 > 0, C_4 > 0, C_1C_2 - C_3 > 0, (C_1C_2 - C_3)C_3 - C_1^2C_4 > 0 \quad (15)$$

Theorem2: E^* is asymptotically stable if and only if inequalities (15) is satisfied.

3.5 Global Stability of the Equilibrium points

3.5.1 Global stability of the drinking free equilibrium.

We prove the global stability when $\alpha \leq \mu$

Theorem 3: The global stability E_0 is asymptotically stable in the region

177 $\Gamma = \left\{ (S, H, T, R) \in R_+^4 : S + H + T + R \leq \frac{b}{\mu} \right\}$ if $\alpha \leq \mu$ (note that $\alpha \leq \mu$ implies $R_0 < 1$).

178

179 **Proof:** it should be noted that $S < 1$ in Γ for time $(t) > 1$. Consider the Lyapunov
180 function:

$$L = S + H$$

$$\frac{dL}{dt} = (\alpha S - \mu - \delta_1)H - (\mu - \delta_2 + \gamma)T$$

$$\leq (\alpha - \mu - \delta_1)H - (\mu - \delta_2)T$$

181 $\frac{dL}{dt} < 0$ for $\alpha \leq \mu$ and $\frac{dL}{dt} = 0$ only if $H = 0$ and $T = 0$. Therefore, the only trajectory of

182 the system in which $\frac{dL}{dt} = 0$ is E_0 . Hence, Lasalle's invariance principle, E_0 is globally

183 asymptotically stable in Γ [14,17,18,19].

184

185 *3.5.2 Global Stability of the Endemic Equilibrium (E^*).*

186 We determine the global stability of the endemic equilibrium in this section, by using the
187 first three equations of the system (1) – (4) that is:

$$\begin{aligned} \frac{dS}{dt} &= b - \alpha SH - \mu S + \eta R \\ \frac{dH}{dt} &= \alpha SH - (\mu + \delta_1 + \phi)H \\ \frac{dT}{dt} &= \phi H - (\mu + \delta_2 + \gamma)T \end{aligned} \quad (16)$$

189 in the region $\Gamma^* = \{(S, H, T) \in R_+^3 : S + H + T \leq 1, S > 0, H \geq 0, T \geq 0\}$, Γ^* is positively
190 invariant, i.e. every solution of the model (16), with initial conditions in Γ^* remains there for
191 time $(t > 0)$.

192 We also consider

193 $\Gamma^{**} = \left\{ (S, H, T) : S + \left(\frac{\mu + \delta_1}{\mu} \right) H + \left(\frac{\mu + \delta_2 + \gamma}{\mu} \right) T = 1, S > 0, H \geq 0, T \geq 0 \right\}$ where $\Gamma^{**} \subset \Gamma^*$, Γ^{**}
194 is positively invariant, $E^* \in \Gamma^*$ and $b = \mu$.

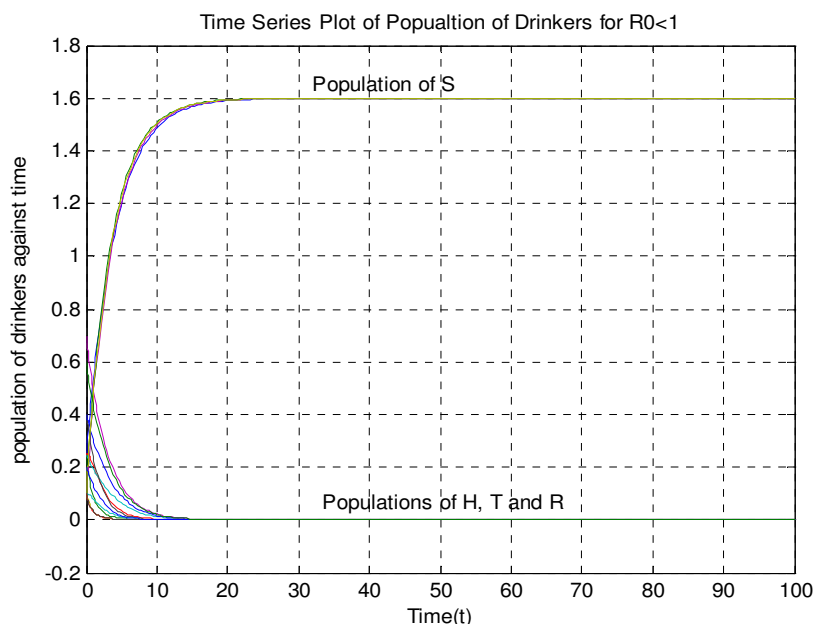
195 **Theorem 4:** The endemic equilibrium point E^* of model (16) is globally asymptotically
196 stable if $R_0 > 1$ (This means that $0 \leq \alpha$).

197 **Proof:** From **theorem 1**, if $R_0 > 1$ in Γ^{**} , then E_0 is unstable. Also Γ^{**} is positively
198 invariant subset of Γ^* and the ω -limit set of each solution of model (16) is a single point in

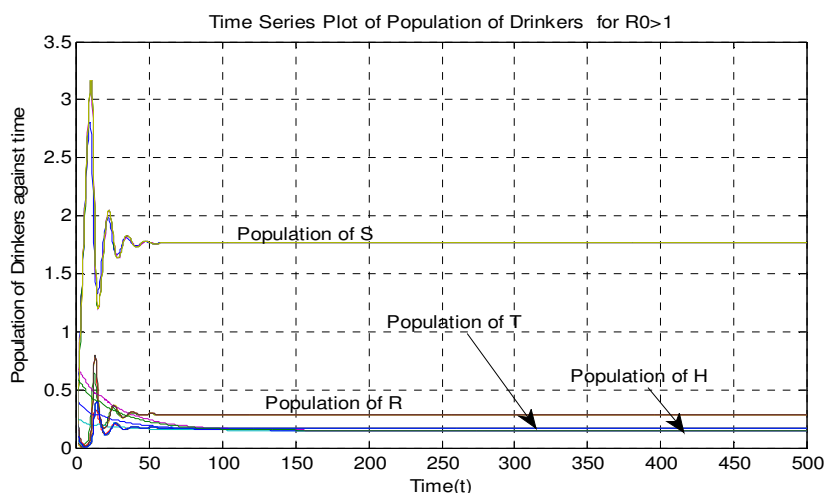
199 Γ^{**} since there is no periodic solutions, homoclinic loops and oriented phase polygons inside
 200 Γ^{**} if $\emptyset \leq \alpha$. Therefore E^* is globally asymptotically stable [14]

201 4. Numerical Example

202 In this section, we use numerical simulations to show the dynamical behaviour of our results,
 203 by assuming that our total population is 100% and choose $S = 0.50, H = 0.25, T = 0.15$
 204 and $R = 0.1$. The other parameters that would be used in this section are displayed in Table 1
 205 and Table 2 respectively.



206
 207 Fig. 2. When $R_0 = 0.5385$, only non-drinkers exist. The populations of heavy drinkers,
 208 drinkers in treatment and recovered drinkers, approaches zero and reaches disease free
 209 equilibrium
 210



211
 212 Fig. 3. When $R_0 = 1.25$, all the distinct drinking classes coexist and therefore approaches
 213 endemic equilibrium.

5. Sensitivity analysis of the basic reproductive numbers

We investigate the nature of the model by conducting sensitivity analysis of the reproductive number (R_0).

(a) At the drinking –free equilibrium, $\alpha = 0.7$, $\delta_1 = 0.35$, $\phi = 0.7$ and $\mu = 0.25$

$R_0 = 0.5385 < 1$.

(i) If the value of α is increased to any figure greater than 1.31 and the values of ϕ , μ , δ_1 are maintained the same, $R_0 > 1$.

(ii) If the value of ϕ is reduced to 0.068 and the values of α , δ_1 and μ are maintained the same, $R_0 > 1$.

(b) At the endemic equilibrium, $\alpha = 0.7$, $\delta_1 = 0.035$, $\mu = 0.025$, and $\phi = 0.5R_0 = 1.25 > 1$.

(i) If α is reduced to 0.5 and μ , δ_1 , ϕ are maintained the same, $R_0 < 1$.

(ii) If the values of α , μ and δ_1 are maintained at 0.7 and 0.025 and 0.035 and ϕ is increased to 0.8, $R_0 < 1$.

6. Discussion of Results

We use *SHTR* model to study the dynamics of drinking as an epidemic. We discussed the existence and stability of drinking free and endemic equilibria and performed the sensitivity analysis of the reproductive numbers. Based on the data in Table 1, the basic reproductive number of the drinking free equilibrium is estimated to be $R_0 = 0.5385 < 1$. This implies that only non-drinkers population is present ($S = 1.6$) and the heavy drinkers, drinkers in treatment and recovered drinkers' populations reduces to zero ($H = 0, T = 0, R = 0$). This means that the model is asymptotically stable at $R_0 < 1$ and satisfies Theorem 1. This has been verified numerically in Fig. 2. In the stability analysis of the drinking free equilibrium, the eigenvalues are $\lambda_1 = -0.25$, $\lambda_2 = -0.6$, $\lambda_3 = -0.64$ and $\lambda_4 = -0.35$. This also indicates that the drinking free equilibrium is asymptotically stable.

Considering the situation when $R_0 > 1$, the reproductive number of the endemic equilibrium is estimated to be $R_0 = 1.25 > 1$ using the data in Table 2. This shows the situation in which the non-drinkers, heavy drinkers, drinkers in treatment and recovered drinkers coexist in the population [$(S^*, H^*, T^*, R^*) = (0.8, 0.6909, 2.2287, 6.3678)$]. This indicates the existence of drinking problem in the population. People with drinking problem will continue to transform more non-drinkers into heavy drinkers and the drinking free equilibrium becomes unstable at

$R_0 > 1$. This is in line with our analytical results and has also been verified numerically in Figure 3.

At the sensitivity analysis of the basic reproductive number of the endemic equilibrium, if α is reduced from 0.7 to 0.5 and μ, δ_1, ϕ maintained the same, $R_0 < 1$. Furthermore, if ϕ is increased from 0.5 to 0.8, $R_0 < 1$. Also $R_0 > 1$ at the drinking -free equilibrium, if either α is increased to any figure greater than 1.31 or ϕ is reduced below 0.1.

7. Conclusion

Our model show that, drinking epidemic cannot only be controlled by reducing the contact rate between the non-drinkers and heavy drinkers but also increasing the number of drinkers that go into treatment and educating drinkers to refrain from drinking can be useful in combating the epidemic.

References

- [1] Kaithuru P. N., Stephen, A., 2015. Alcoholism and its impact on Work Force: A case of Kenya Meteorological station, Nairobi. J. Alcohol Drug Depend. 3(2), 1-4.
- [2] World Health Organization, 2012. Alcohol in the European Union: Consumption, Harm and Policy Approaches. Copenhagen: WHO.
- [3] Huo, H.-F., Chen, Y.-L., Xiang, H., 2017. Stability of a binge drinking model with delay. J. Biol. Dyn. 11(1), 210-225.
- [4] Glavas, M. M., Weinberg, J., 2006. Stress alcohol consumption and the hypothalamic-pituitary-adrenal axis, in Nutrients, Stress and Medical Disorders, Humana Press, New York, 165-183.
- [5] Cunningham, D., Atkin, W., Lenz, H.J., Lynch, H.T., Minsky, B., Nordinger, B., Starling, N., 2010. Colorectal Cancer. Lancet, 375(9719), 1030-1047.
- [6] Muller, D., Koch, R., et al., 1985. Neurophysiologic findings in chronic abuse. Psychiatr Neurol Med Psychol, 37(3), 129-132.
- [7] Testino, G., 2008. Alcoholic diseases in hepato-gastroenterology: a point of view. Hepato-Gastroenterology, 255(82-83), 371-377.
- [8] Blume, L., Nielson, N., et al., 1998. Alcoholism and alcohol of abuse among women:

- 283 report of the council of scientific affairs. *Journal of women's Health*, 7(7), 861-870.
- 284
- 285 [9] Bhunu, C. P., 2012. A mathematical analysis of alcoholism. *World Journal of modeling*
- 286 and *Simulation* 8(2), 124-134.
- 287
- 288 [10] Wechler, H., Lee, J. E., Nelson, T. F., Kuo, M. C., 2002. Underage college students
- 289 drinking Behavior, access to alcohol, and the influence of deterrence policies. *J. Amer.*
- 290 *College Health* 50(5), 223-236.
- 291
- 292 [11] Xiang, H., Song, N., Huo, H. F., 2016. Modelling effects of public health educational
- 293 campaigns on drinking dynamics. *J. Biol. Dyn.* 10 (1), 164-178.
- 294
- 295 [12] Manthey, J. L., Aidoo, A. Y., Ward, K. Y., 2008. Campus drinking: an epidemiological
- 296 model. *J. Biol. Dyn* 2(3), 346-356.
- 297
- 298 [13] Hiang, H., Zhu, C. C., Huo, H. F., Stability of a quit drinking model with relapse, *J.*
- 299 *Biomath. in press*.
- 300
- 301 [14] Alkhudhari, Z., Al-Sheikh, S., Al-Tuwairqi, S., 2014. Stability analysis of a giving up
- 302 smoking model. *Int. J. Appl. Res.* 3(6), 168-177.
- 303
- 304 [15] Swarnali, S., Samanta, G. P., 2013. Drinking as an Epidemic: A Mathematical Model
- 305 with Dynamics Behaviour. *J. Appl. Math and Informatics.* 31, 1-25.
- 306
- 307 [16] Mancuso, M. L., 2016. A Mathematical Model for Alcoholism Epidemic. <[http://](http://ecommons.udayton.edu/stander_posters/758)
- 308 ecommons.udayton.edu/stander_posters/758>.
- 309
- 310 [17] Cai, L., Li, X., Ghosh, M., Guo, B., 2009. Stability analysis of an HIV/AIDS epidemic
- 311 model with treatment. *J. Comput. and Appl. Math.* 229, 313-323.
- 312
- 313 [18] Hirsch, M. W., Smale, S., Devaney, R. L., 1974. Differential equations, dynamical
- 314 systems and an introduction to chaos. Elsevier Academic, press, New York.
- 315
- 316 [19] Busenberg, S., van den Driessche, P., 1990. Analysis of a disease transmission model
- in a population with varying size. *J. Math. Biol.* 28, 257-270.