

Fuzzy graceful labelling for complete bipartite graphs

Abstract

The concept of fuzzy graceful labelling is introduced. A graph which admits a fuzzy graceful labelling is called a fuzzy graceful graph. Fuzzy graceful labelled graphs are becoming an increasingly useful family of mathematical models for a broad range of applications. In this paper the concept of fuzzy graceful labelling is applied to complete bipartite graphs. Also we discussed the edge and vertex gracefulness of some complete bipartite graphs.

Key words: Fuzzy labelling, Graceful labelling, Fuzzy graceful labelling, Fuzzy edge- vertex graceful labelling, Fuzzy bipartite graph.

AMS Mathematics Subject classification: 05C, 94D

Introduction

Zadeh introduced the fuzzy set as a class of object with a continuum of grades of membership. In contrast to classical crisp sets where a set is defined by either membership or non-membership, the fuzzy approach relates to a grade of membership between $[0, 1]$, defined in terms of the membership function of a fuzzy number. Fuzzy relation on a set was first defined by Zadeh in 1965[15] and the first definition of a fuzzy graph was introduced by Kaufmann in 1973 [5] and the structure of fuzzy graphs was developed by Azriel Rosenfeld in 1975[13]. Fuzzy graphs have many more applications in modelling real time systems where the level of information inherent in the system varies with different levels of precision. The concept of a graceful labelling has been introduced by Rosa in 1966[12].

This note is a further contribution on fuzzy labelling. Fuzzy labelling for some fuzzy complete bipartite graphs which satisfy fuzzy graceful labelling based on few conditions.

Preliminaries and Main Results

Definition: 1

Let U and V be two sets. Then ρ is said to be a fuzzy relation from U into V if ρ is a fuzzy set of $U \times V$.

Definition: 2

A fuzzy graph $G = (\sigma, \mu)$ is a pair of functions $\sigma : V \rightarrow [0, 1]$ and $\mu : V \times V \rightarrow [0, 1]$, where for all $u, v \in V$, we have $\mu(u, v) \leq \sigma(u) \wedge \sigma(v)$.

Definition: 3

A labelling of a graph is an assignment of values to the vertices and edges of a graph.

Definition: 4

35 A graceful labelling of a graph G with q edges is an injection $f: V(G) \rightarrow \{0, 1, 2, \dots, q\}$ such that when each
36 edge $xy \in E(G)$ is assigned the label $|f(x) - f(y)|$, all of the edge labels are distinct.

37 **Definition: 5**

38 A graph $G = (V, E)$ is said to be a fuzzy graceful labelling graph if $\sigma: V \rightarrow [0, 1]$ and $\mu: V \times V \rightarrow [0, 1]$ is
39 bijective such that the membership value of edges and vertices are distinct and $\mu(u, v) < \sigma(u) \wedge \sigma(v)$ for all
40 $u, v \in V$.

41
42 **Definition: 6**

43 A graph $G = (V(G), E(G))$ is said to be **Bipartite** if and only if there exists a partition $V(G) = V_1 \cup V_2$ and
44 $V_1 \cap V_2 = \emptyset$. Hence all edges share a vertex from both set V_1 and V_2 and there are no edges formed between two
45 vertices in the set V_1 and there are no edges formed between the two vertices in V_2 .

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47 **Definition: 7**

48 A bipartite graph with fuzzy labelling is called a fuzzy bipartite graph.

49 **Definition: 8**

50 A graph $G = (V(G), E(G))$ is said to be **Complete Bipartite** if and only if there exists a partition $V(G) = V_1 \cup$
51 V_2 and $V_1 \cap V_2 = \emptyset$ so that all edges share a vertex from both set V_1 and V_2 and all possible edges that join
52 vertices from set V_1 to set V_2 are drawn.

53 We denote a complete bipartite graph as $K_{m,n}$ where m refers to the number of vertices in subset V_1 and n
54 refers to the number of vertices in subset V_2 .

55 **Definition: 9**

In a fuzzy complete bipartite graph if all vertices(edges) are distinct then it is called fuzzy vertex(edges) graceful labelling bipartite graph.

Definition : 10

For any graph G , we define if all the points of G have the same degree r that is $\delta(G) = \Delta(G) = r$ then in this case G is called a regular graph of degree r .

56 **Theorem**

57 Every Fuzzy Complete Bipartite graph $K_{m,n}$ is a fuzzy graceful labelling graph.

58 **Proof:**

59 Let $K_{m,n}$ be a complete bipartite graph with vertex group $V = \{V_1, V_2\}$ such that $V_1 = \{u_1, u_2, u_3, \dots, u_m\}$ and $V_2 =$
60 $\{v_1, v_2, v_3, \dots, v_n\}$, where $m \geq 2$ and $n \geq 2$.

61 Let $V \rightarrow [0, 1]$ such that if the starting point of the vertex may be assigned $nm/100$, where $2 \leq m, n \leq 24$ and
62 $nm/1000$, where $m, n \geq 2$ then the fuzzy complete bipartite graph we get would be a Fuzzy Graceful Labelling
63 Complete Bipartite graph.

64

65

66 **Case (i): $m=n$, where $2 \leq m, n \leq 7$**

67 Let us take $\sigma(u_1) = mn/100$

68 If we assign the value of the edge

69 $\mu(u_1, v_n) = |\sigma(u_1) - \sigma(v_n)| = 0.01$,

70 Then the fuzzy labelling will continue in the following order such as

71 $\mu(u_1, v_n) = 0.01$

72 $\mu(u_1, v_{n-1}) = 0.02$, etc.,

73 According to this order of fuzzy labelling, we get

74 $\mu(u_1, v_j) = [i^2 + (n-j)]/100$, where $i=1, j=1, 2, \dots, n, 2 \leq n \leq 7$

75 $\mu(u_2, v_j) = [i^2 + (j+(n-4))]/100$, where $i=2, j=1, 2, \dots, n, 2 \leq n \leq 7$

76 $\mu(u_3, v_j) = [i^2 + (j+(2n-9))]/100$, where $i=3, j=1, 2, \dots, n, 3 \leq n \leq 7$

77 $\mu(u_4, v_j) = [i^2 + (j+(3n-16))]/100$, where $i=4, j=1, 2, \dots, n, 4 \leq n \leq 7$, etc.,

78 $\mu(u_{n-1}, v_j) = [i^2 + (j-1)]/100$, where $i=n-1, j=1, 2, \dots, n, 2 \leq n \leq 7$

79 $\mu(u_n, v_j) = [i^2 + (j-n)]/100$, where $i=n, j=1, 2, \dots, n, 2 \leq n \leq 7$

80 **Case(ii): $m < n, 2 \leq m \leq 6$ and $3 \leq n \leq 24$**

81 Let us take $\sigma(u_1) = mn/100$

82 If we assign the value of the edge

83 $\mu(u_1, v_n) = |\sigma(u_1) - \sigma(v_n)| = 0.01$,

84 Then the fuzzy labelling will continue in the following order such as

85 $\mu(u_1, v_n) = 0.01$

86 $\mu(u_1, v_{n-1}) = 0.02$, etc.,

87 According to this order of fuzzy labelling, we get Fuzzy Graceful Labelling Complete Bipartite graph based on
88 the following cases:

89 **Sub case (i): $m=2, 3 \leq n \leq 24$**

90 $\mu(u_1, v_j) = [i^2 + (n-j)]/100$, where $i=1, j=1, 2, \dots, n$

91 $\mu(u_2, v_j) = [i^2 + (j+(n-2m))]/100$, where $i=2, j=1, 2, \dots, n$

92 **Sub case (ii): $m=3, 4 \leq n \leq 16$**

93 $\mu(u_1, v_j) = [i^2 + (n-j)]/100$, where $i=1, j=1, 2, \dots, n$

94 $\mu(u_2, v_j) = [i^2 + (j+(n-(m+1)))]/100$, where $i=2, j=1, 2, \dots, n$

95 $\mu(u_3, v_j) = [i^2 + (j+(2n-3m))]/100$, where $i=3, j=1, 2, \dots, n$

96 **Sub case (iii): $m=4, 5 \leq n \leq 12$**

- 97 $\mu(u_1, v_j) = [i^2 + (n-j)]/100$, where $i=1$, $j=1,2,...,n$
- 98 $\mu(u_2, v_j) = [i^2 + (j + (n-m))]/100$, where $i=2$, $j=1,2,...,n$
- 99 $\mu(u_3, v_j) = [i^2 + (j + (2n - (2m+1)))]/100$, where $i=3$, $j=1,2,...,n$
- 100 $\mu(u_4, v_j) = [i^2 + (j + (3n - 4m))]/100$, where $i=4$, $j=1,2,...,n$
- 101 **Sub case (iv): $m=5$, $6 \leq n \leq 9$**
- 102 $\mu(u_1, v_j) = [i^2 + (n-j)]/100$, where $i=1$, $j=1,2,...,n$
- 103 $\mu(u_2, v_j) = [i^2 + (j + (n - (m-1)))]/100$, where $i=2$, $j=1,2,...,n$
- 104 $\mu(u_3, v_j) = [i^2 + (j + (2n - (2m-1)))]/100$, where $i=3$, $j=1,2,...,n$
- 105 $\mu(u_4, v_j) = [i^2 + (j + (3n - (3m+1)))]/100$, where $i=4$, $j=1,2,...,n$
- 106 $\mu(u_5, v_j) = [i^2 + (j + (4n - 5m))]/100$, where $i=5$, $j=1,2,...,n$
- 107 **Sub case (v): $m=6$, $7 \leq n \leq 8$**
- 108 $\mu(u_1, v_j) = [i^2 + (n-j)]/100$, where $i=1$, $j=1,2,...,n$
- 109 $\mu(u_2, v_j) = [i^2 + (j + (n - (m-2)))]/100$, where $i=2$, $j=1,2,...,n$
- 110 $\mu(u_3, v_j) = [i^2 + (j + (2n - (2m-3)))]/100$, where $i=3$, $j=1,2,...,n$
- 111 $\mu(u_4, v_j) = [i^2 + (j + (3n - (3m-2)))]/100$, where $i=4$, $j=1,2,...,n$
- 112 $\mu(u_5, v_j) = [i^2 + (j + (4n - (4m+1)))]/100$, where $i=5$, $j=1,2,...,n$
- 113 $\mu(u_6, v_j) = [i^2 + (j + (5n - 6m))]/100$, where $i=6$, $j=1,2,...,n$
- 114 **Case (iii): $m > n$, $3 \leq m \leq 24$ and $2 \leq n \leq 6$**
- 115 **Sub case (i): $m=3$, $n=2$**
- 116 $\mu(u_1, v_j) = [i^2 + (n-j)]/100$, where $i=1$, $j=1,2,...,n$
- 117 $\mu(u_2, v_j) = [i^2 + (j-2)]/100$, where $i=2$, $j=1,2,...,n$
- 118 $\mu(u_3, v_j) = [i^2 + (j-5)]/100$, where $i=3$, $j=1,2,...,n$
- 119 **Sub case (ii): $m=4$, $n=2,3$**
- 120 $\mu(u_1, v_j) = [i^2 + (n-j)]/100$, where $i=1$, $j=1,2,...,n$
- 121 $\mu(u_2, v_j) = [i^2 + (j - (m-n))]/100$, where $i=2$, $j=1,2,...,n$
- 122 $\mu(u_3, v_j) = [i^2 + (j - (2m - (2n-1)))]/100$, where $i=3$, $j=1,2,...,n$
- 123 $\mu(u_4, v_j) = [i^2 + (j - (3m - (3n-4)))]/100$, where $i=4$, $j=1,2,...,n$
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- 126 **Sub case (iii): $m=5$, $2 \leq n \leq 4$**

127 $\mu(u_1, v_j) = [i^2 + (n-j)]/100$, where $i=1$, $j=1,2,\dots,n$

128 $\mu(u_2, v_j) = [i^2 + (j - (m - (n+1)))]/100$, where $i=2$, $j=1,2,\dots,n$

129 $\mu(u_3, v_j) = [i^2 + (j - (2m - (2n+1)))]/100$, where $i=3$, $j=1,2,\dots,n$

130 $\mu(u_4, v_j) = [i^2 + (j - (3m - (3n+1)))]/100$, where $i=4$, $j=1,2,\dots,n$

131 $\mu(u_5, v_j) = [i^2 + (j - (4m - (4n+5)))]/100$, where $i=5$, $j=1,2,\dots,n$

132 **Sub case (iv): $m=6$, $2 \leq n \leq 5$**

133 $\mu(u_1, v_j) = [i^2 + (n-j)]/100$, where $i=1$, $j=1,2,\dots,n$

134 $\mu(u_2, v_j) = [i^2 + (j - (m - (n+2)))]/100$, where $i=2$, $j=1,2,\dots,n$

135 $\mu(u_3, v_j) = [i^2 + (j - (2m - (2n+3)))]/100$, where $i=3$, $j=1,2,\dots,n$

136 $\mu(u_4, v_j) = [i^2 + (j - (3m - (3n+2)))]/100$, where $i=4$, $j=1,2,\dots,n$

137 $\mu(u_5, v_j) = [i^2 + (j - (4m - (4n+1)))]/100$, where $i=5$, $j=1,2,\dots,n$

138 $\mu(u_6, v_j) = [i^2 + (j - (5m - (5n+6)))]/100$, where $i=6$, $j=1,2,\dots,n$

139 **Sub case (v): $m=7$, $2 \leq n \leq 6$**

140 $\mu(u_1, v_j) = [i^2 + (n-j)]/100$, where $i=1$, $j=1,2,\dots,n$

141 $\mu(u_2, v_j) = [i^2 + (j - (m - (n+3)))]/100$, where $i=2$, $j=1,2,\dots,n$

142 $\mu(u_3, v_j) = [i^2 + (j - (2m - (2n+5)))]/100$, where $i=3$, $j=1,2,\dots,n$

143 $\mu(u_4, v_j) = [i^2 + (j - (3m - (3n+5)))]/100$, where $i=4$, $j=1,2,\dots,n$

144 $\mu(u_5, v_j) = [i^2 + (j - (4m - (4n+3)))]/100$, where $i=5$, $j=1,2,\dots,n$

145 $\mu(u_6, v_j) = [i^2 + (j - (5m - (5n+1)))]/100$, where $i=6$, $j=1,2,\dots,n$

146 $\mu(u_7, v_j) = [i^2 + (j - (6m - (6n+7)))]/100$, where $i=7$, $j=1,2,\dots,n$

147 In all the above cases Fuzzy Complete Bipartite graph $K_{m,n}$ is satisfied the conditions of fuzzy graceful
148 labelling graph.

149 **Remarks:**

150 The fuzzy complete bipartite graph will be a Fuzzy Graceful Labelling Complete Bipartite graph for the
151 following conditions also.

152 (i) When $m=8$ and $2 \leq n \leq 6$

153 (ii) When $m=9$ and $2 \leq n \leq 5$

154 (iii) When $m=10$ and $2 \leq n \leq 4$

155 (iv) When $m=11$ and $2 \leq n \leq 4$

156 (v) When $m=12$ and $2 \leq n \leq 4$

157 (vi) When $m=13$ and $n = 2,3$

158 (vii) When $m=14$ and $n = 2,3$

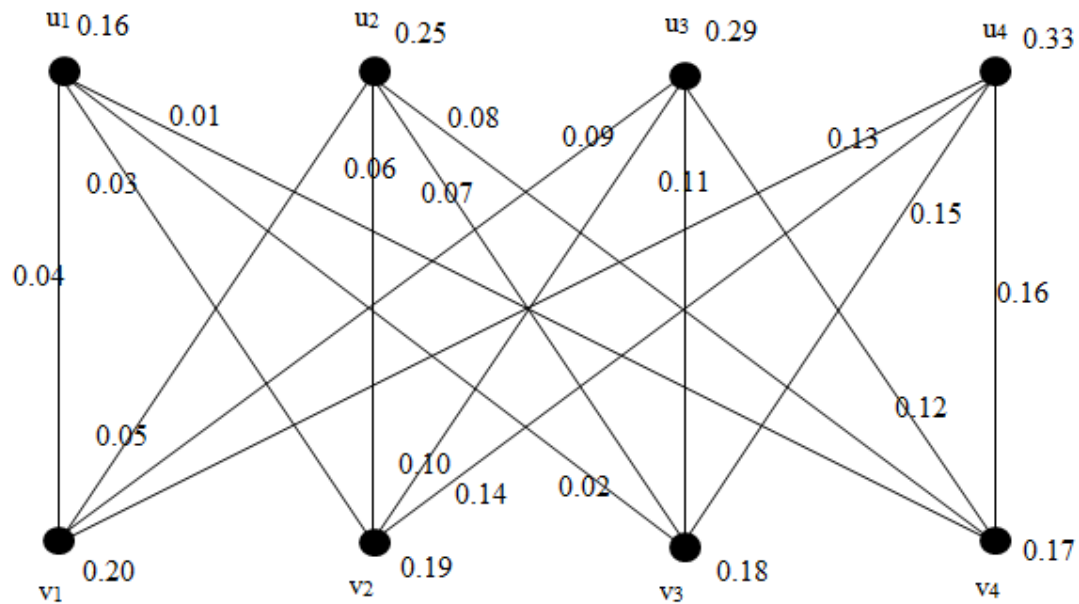
159 (viii) When $m=15$ and $n = 2,3$

160 (ix) When $m=16$ and $n = 2,3$

161 (x) When $m=17$ and $n = 2$

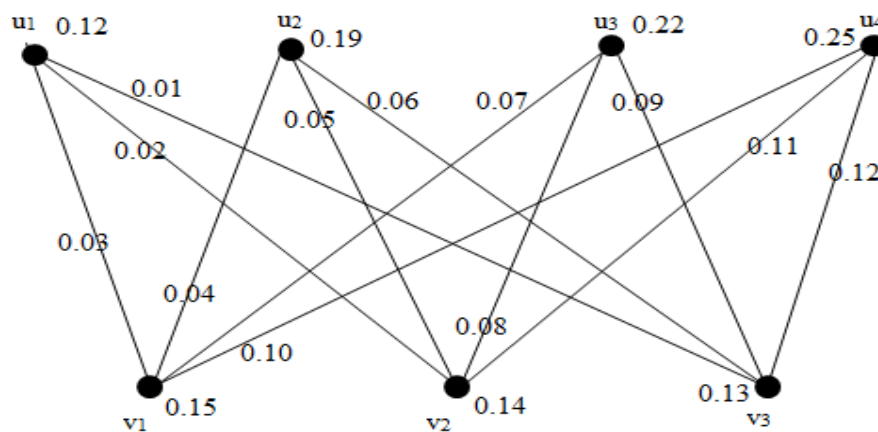
- 162 (xi) When $m=18$ and $n=2$
- 163 (xii) When $m=19$ and $n=2$
- 164 (xiii) When $m=20$ and $n=2$
- 165 (xiv) When $m=21$ and $n=2$
- 166 (xv) When $m=22$ and $n=2$
- 167 (xvi) When $m=23$ and $n=2$
- 168 (xvii) When $m=24$ and $n=2$

169 **Example: 1** when $m = n$



In this graph, $m=4$ and $n=4$

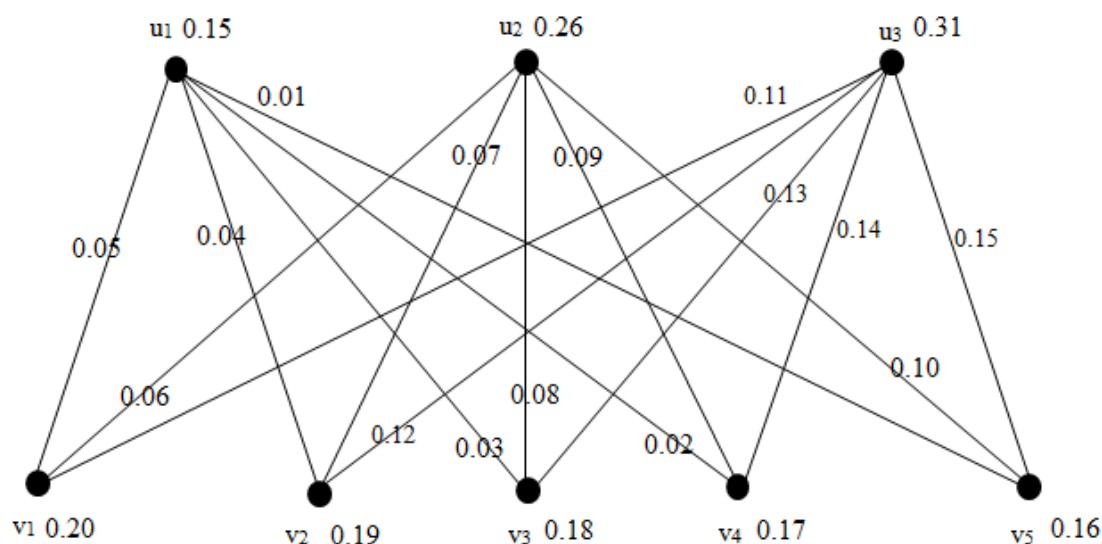
173 **Example: 2** when $m > n$



In this graph, $m=4$ and $n=3$

176 **Example: 3** when $m < n$

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In this graph, $m=3$ and $n=5$

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Results:

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1. Every Fuzzy Complete Bipartite graph $K_{m,n}$ is a fuzzy regular graph when $m=n$.

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2. Every Fuzzy Complete Bipartite graph $K_{m,n}$ is not a fuzzy regular graph when $m>n$ and $m<n$

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Conclusion:

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In this paper the concept of fuzzy edge(vertex) graceful labelling for complete bipartite graph has been introduced. We plan to extend our research work to fuzzy graceful labelling for even vertex and odd edges of complete bipartite graph.

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