

Original Research Article

A NEW DECISION MAKING METHOD ON INTERVAL VALUED FUZZY SOFT MATRIX (IVFSM)

Abstract

In this paper, we study basic definitions of IVFSM with some properties and prove commutative laws, associative laws and De-Morgan laws by using And-Operation and Or-Operation on IVFSM. We propose a new decision making method on IVFSM named as “interval valued fuzzy soft max-min decision making method” (IVFSMmDM) with the help of interval valued fuzzy soft max-min decision making function. Finally, we apply this method for decision making by using hypothetical data.

Key words: *Interval valued fuzzy set (IVFS), interval valued soft set (IVSS), interval valued fuzzy soft set (IVFSS), IVFSM, IVFSMmDM.*

1 Introduction

In recent time, classical methods are used to solve different problems which are faced in medical sciences, engineering, social sciences etc. Many theories like fuzzy set theory, probability theory, rough set theory etc are used to solve these uncertainties, but these theories have their own problems which were attensioned by Zadeh. First of all (Zadeh) in [1] initiated the concept of fuzzy sets by the extension of classical notion on sets. Now a day fuzzy set theory is rapidly progressing, but in particular cases we face some limitations such as how to adjust the membership functions in this theory. A new concept of SS introduced for the solution of these limitations (Molodtsov) [2].

In these days, mathematician plays a vital role in the SS fuzzification. After fuzzification of SS a new theory introduced which is known as FSS with different types and properties (Maji et al.) [3]. The SS theory was reviewed [4] and used this theory for decision making, and also introduced different types of SS with examples and defined some operations such as And-operation, Or-operation, union, intersection, complement etc on SS [5]. After some time the work on SS was extended by using the definition of parameterization reduction [6] and used it in decision making problem. Some new notions on SS [7] (Ali et al.) are introduced, such as restricted union, restricted intersection, restricted difference and extended intersection with examples and properties.

Generalized FSS [8] was proposed with some properties and used (Majumdar et al.) generalized FSS for decision making problem and for the diagnosis of Pneumonia. (Neog and Sut) Some propositions were proposed on fuzzy soft union and intersection with proof and examples on FSS and verify the De Morgan Laws for a family of fuzzy soft sets with examples [9].

Matrix representation play a vital role in engineering and science, but sometimes classical matrices fails to solve the problems. (Yang and Chenli ji) FSM [10] was introduced for the solution of those problems which can not be handled by classical matrices by the product of FSM and used it for decision making. The product of soft matrices and fuzzy soft matrices are introduced with operations and some properties [11, 12] (Cagman, N. and Enginoglu, S) and also proposed new decision making methods which are known as soft max–min decision making method and fuzzy soft max–min decision making method. (Bohra et al.) further advanced the FSM theory and its applicability in different fields of life in 2012 and also used it for decision making by product of fuzzy soft matrices [13].

The idea of IVFSS is introduced [14] by combining the IVFS and SS, (Yang et al.) IVFSS is used for decision making and define And-operation, Or-operation also proved De Morgan laws, associative laws and distributive laws on IVFSS. (Chetia et al.) IVFSS is used for medical diagnosis in 2010 [15]. (Rajarajeshwari and Dhanalakshmi) A new concept is proposed by the combination of IVFSS and soft matrices with examples and different properties which is called IVFSM [16].

By the extension of IVFSM (Sarala and Prabhavathi) [17] initiated the Sanchez's approach for diagnosis of Dengue and Chikangunya also introduced union and intersection of IVFSM. IVFSM is used for check the performance of different provinces of a country [18]. FSM and IVFSM are used for decision making and redefined the product of IVFSM [19] also compare the result of FSM and IVFSM for decision making and observed that the FSM is more appropriate for decision making.

In this paper, we proved commutative laws, associative laws and De-Morgan laws by using And-Operation and Or-Operation of IVFSM also defined IVFSMmDM with the help of interval valued fuzzy soft max-min decision function. We construct an algorithm for IVFSMmDM in this paper and use this method for decision making.

2 PRELIMINARIES:

In this section, we review some concepts and definitions of IVFSM with examples, which will be needed in the sequel.

M represents the universal set and E represents the set of parameters in this whole paper.

Definition 2.1 [2]

A pair (F, A) is called a SS over M if A is any subset of E, and there exist a mapping from A to P(M) is F, P(M) is the parameterized family of subsets of the M but not a set.

Definition 2.2 [1]

A fuzzy set A in M is characterized by a membership function $f_A(y_i)$ which associates with each object of M in the interval [0, 1], with the value of $f_A(y_i)$ where y_i representing the grade of membership of y in A.

Definition 2.3 [3]

A pair (F, A) is called FSS over M, and there exist a mapping from A to P(M) is F, P(M) is the collection of fuzzy subsets of M.

Definition 2.4 [10]

A pair (f_A, E) is called a SS over M if A is any subset of E. Then a subset R_A of $M \times E$ is defined as

$R_A = \{(m, y) : y \in A, m \in f_A(y)\}$, is the relation form of (f_A, E) the characteristic function of R_A is written by

$$\chi_{RA} : U \times E \rightarrow \{0, 1\}, \quad \chi_{RA}(m, y) = \begin{cases} 1, & (m, y) \in R_A \\ 0, & (m, y) \notin R_A \end{cases}$$

It can be written in matrix form such as.

$$[a_{ij}]_{m \times n} = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix}$$

This is called an SM of the SS (f_A, E) over M of order $m \times n$.

Definition 2.5 [10]

A pair (F, A) is called FSS in the fuzzy soft class (M, E). Then (F, A) is represented in a matrix form such as

$$A_{m \times n} = [a_{ij}]_{m \times n} \text{ or } A = [a_{ij}] \quad (i = 1 \rightarrow m), (j = 1 \rightarrow n)$$

Where

$$a_{ij} = \begin{cases} \mu_j(b_j) & \text{if } y_j \in A \\ 0 & \text{if } y_j \notin A \end{cases}.$$

Example 2.1

Let $M = \{Z_1, Z_2, Z_3, Z_4\}$ be a universal set and $E = \{y_1, y_2, y_3, y_4\}$ be the set of parameters,

$A = \{y_3, y_4\} \subseteq E$ then FSS can be written as

$$(F, A) = \{F(y_3) = \{(Z_1, 0.51), (Z_2, 0.26), (Z_3, 0.56), (Z_4, 0.28)\},$$

$$F(y_4) = \{(Z_1, 0.25), (Z_2, 0.51), (Z_3, 0.49), (Z_4, 0.23)\}\}$$

This FSS can be represented in FSM form such as

$$(F, A) = \begin{matrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \end{matrix} \begin{bmatrix} 0.0 & 0.0 & 0.51 & 0.25 \\ 0.0 & 0.0 & 0.26 & 0.51 \\ 0.0 & 0.0 & 0.56 & 0.49 \\ 0.0 & 0.0 & 0.28 & 0.23 \end{bmatrix}$$

Definition 2.6 [14]

A pair (F, A) is called IVFSS over M where F is a mapping such that

$$F: A \rightarrow I^M$$

Where I^M represent the all interval valued fuzzy subsets (IVFSbS) of M .

Definition 2.7 [16]

A pair (F, A) is called IVFSS over M , where F is a mapping such that

$F: A \rightarrow I^M$, where I^M represent the all IVFSbS of M . Then the IVFSS can be expressed in

matrix form as

$$A_{m \times n} = [a_{ij}]_{m \times n}$$

Or

$$A = [a_{ij}] \quad (i = 1 \rightarrow m), (j = 1 \rightarrow n)$$

Where

$$a_{ij} = \begin{cases} [\mu_{jL}(b_i), \mu_{jU}(b_i)] & \text{if } y_j \in A \\ [0, 0] & \text{if } y_j \notin A \end{cases}$$

Where $[\mu_{jL}(b_i), \mu_{jU}(b_i)]$

Represent the membership of b_i in the IVFS $F(y_j)$.

Example 2.2

Let $M = \{V_1, V_2, V_3, V_4\}$ be a universal set and $E = \{y_1, y_2, y_3, y_4\}$ be the set of parameters.

Consider

$$A = \{y_3, y_4\} \subseteq E \quad F: A \rightarrow P(M)$$

Then IVFSS (F, A) is

$$(F, A) = \{F(y_3) = \{(V_1, [0.5, 0.6]), (V_2, [0.2, 0.3]), (V_3, [0.5, 0.6]), (V_4, [0.2, 0.3])\},$$

$$F(y_4) = \{(V_1, [0.2, 0.3]), (V_2, [0.5, 0.6]), (V_3, [0.4, 0.5]), (V_4, [0.2, 0.3])\}\}$$

This IVFSS can be represented in IVFSM such as

$$(F, A) = \begin{matrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{matrix} \begin{bmatrix} [0.0, 0.0] & [0.0, 0.0] & [0.5, 0.6] & [0.2, 0.3] \\ [0.0, 0.0] & [0.0, 0.0] & [0.2, 0.3] & [0.5, 0.6] \\ [0.0, 0.0] & [0.0, 0.0] & [0.5, 0.6] & [0.4, 0.5] \\ [0.0, 0.0] & [0.0, 0.0] & [0.2, 0.3] & [0.2, 0.3] \end{bmatrix}$$

Definition 2.8 [16]

A and B are two IVFS-matrices then A is said to be IVFS sub matrix of B if $\mu_{AL} \leq \mu_{BL}$ and $\mu_{AU} \leq \mu_{BU}$ $\forall i$ and j , it is denoted by $A \subseteq B$

Example 2.3

Let A, B $\subseteq E$ such that $A \subseteq B$

$A = \{y_3, y_4\}$ and $B = \{y_2, y_3, y_4\}$

Then

$(F, A) = \{F(y_3) = \{(V_1, [0.3, 0.4]), (V_2, [0.2, 0.3]), (V_3, [0.5, 0.6]), (V_4, [0.2, 0.3])\},$

$F(y_4) = \{(V_1, [0.2, 0.3]), (V_2, [0.3, 0.4]), (V_3, [0.4, 0.5]), (V_4, [0.2, 0.3])\}\}$

$(F, B) = \{F(y_2) = \{(V_1, [0.4, 0.5]), (V_2, [0.6, 0.7]), (V_3, [0.8, 0.9]), (V_4, [0.3, 0.4])\},$

$F(y_3) = \{(V_1, [0.3, 0.4]), (V_2, [0.2, 0.3]), (V_3, [0.5, 0.6]), (V_4, [0.2, 0.3])\},$

$F(y_4) = \{(V_1, [0.2, 0.3]), (V_2, [0.3, 0.4]), (V_3, [0.4, 0.5]), (V_4, [0.2, 0.3])\}\}$

These IVFSS can be represented in IVFSM such as

$$(F, A) = \begin{matrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{matrix} \begin{bmatrix} [0.0, 0.0] & [0.0, 0.0] & [0.3, 0.4] & [0.2, 0.3] \\ [0.0, 0.0] & [0.0, 0.0] & [0.2, 0.3] & [0.3, 0.4] \\ [0.0, 0.0] & [0.0, 0.0] & [0.5, 0.6] & [0.4, 0.5] \\ [0.0, 0.0] & [0.0, 0.0] & [0.2, 0.3] & [0.2, 0.3] \end{bmatrix}$$

And

$$(F, B) = \begin{matrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{matrix} \begin{bmatrix} [0.0, 0.0] & [0.4, 0.5] & [0.3, 0.4] & [0.2, 0.3] \\ [0.0, 0.0] & [0.6, 0.7] & [0.2, 0.3] & [0.3, 0.4] \\ [0.0, 0.0] & [0.8, 0.9] & [0.5, 0.6] & [0.4, 0.5] \\ [0.0, 0.0] & [0.3, 0.4] & [0.2, 0.3] & [0.2, 0.3] \end{bmatrix}$$

Where $(F, A) \subseteq (F, B)$

Definition 2.9 [16]

$A = [a_{ij}]$ be an IVFSM of order $m \times n$, then transpose of IVFSM can be defined as $A^T = [a_{ji}]$ of order $n \times m$,

Where

$[a_{ij}] = [\mu_{jL}(b_i), \mu_{jU}(b_i)], (i = 1 \rightarrow m) \text{ and } (j = 1 \rightarrow n).$

Example 2.4

Consider A be IVFSM of order 2×2

$$A = \begin{bmatrix} [0.2, 0.5] & [0.4, 0.6] \\ [0.2, 0.4] & [0.3, 0.8] \end{bmatrix} \text{ then}$$

$$A^T = \begin{bmatrix} [0.2, 0.5] & [0.2, 0.4] \\ [0.4, 0.6] & [0.3, 0.8] \end{bmatrix}$$

Definition 2.10 [16]

The addition of two IVFS-matrices A and B is conformable if their order is same, it is defined as

$$A+B = [\max (\mu_{AL}, \mu_{BL}), \max (\mu_{AU}, \mu_{BU})] \square i \text{ and } j.$$

Example 2.5

A and B are two IVFS-matrices.

$$A = \begin{bmatrix} [0.2, 0.5] & [0.4, 0.6] \\ [0.2, 0.4] & [0.3, 0.8] \end{bmatrix} \text{ and}$$

$$B = \begin{bmatrix} [0.4, 0.7] & [0.7, 0.8] \\ [0.3, 0.6] & [0.7, 0.9] \end{bmatrix}$$

Then

$$A+B = \begin{bmatrix} [0.4, 0.7] & [0.7, 0.8] \\ [0.3, 0.6] & [0.7, 0.9] \end{bmatrix}$$

Definition 2.11 [17]

The subtraction of two IVFS-matrices A and B is conformable if their order is same, it is defined as

$$A-B = [\min (\mu_{AL}, \mu_{BL}), \min (\mu_{AU}, \mu_{BU})] \square i \text{ and } j.$$

Example 2.6

A and B are two IVFS-matrices.

$$A = \begin{bmatrix} [0.2, 0.5] & [0.4, 0.6] \\ [0.2, 0.4] & [0.3, 0.8] \end{bmatrix} \text{ and}$$

$$B = \begin{bmatrix} [0.4, 0.7] & [0.7, 0.8] \\ [0.3, 0.6] & [0.7, 0.9] \end{bmatrix}$$

Then

$$A-B = \begin{bmatrix} [0.2, 0.5] & [0.4, 0.6] \\ [0.2, 0.4] & [0.3, 0.8] \end{bmatrix}$$

Definition 2.12 [19]

Let $A = [a_{ij}]$ and $B = [b_{ij}]$ are two IVFS-matrices of order $m \times n$ and $n \times p$ respectively then their product defined as

$$A*B = [c_{ik}]_{m \times p} = [\max (\mu_{ALj} * \mu_{BLj}), \max (\mu_{AUj} * \mu_{BUj})] \square i, j, k.$$

OR

$$A*B = c_{ij} = \sum_{k=1}^n (a_{ik} * b_{ik}), i = 1, 2 \dots m \text{ and } j = 1, 2 \dots p$$

$$A*B = [c_{ik}]_{m \times p} = [\sum_{k=1}^n (a_{ikL} * b_{ikL}), \sum_{k=1}^n (a_{ikU} * b_{ikU})], i = 1, 2 \dots m \text{ and } k = 1, 2 \dots p \square i, j, k.$$

Example 2.7

Consider A and B are two IVFS-matrices.

$$A = \begin{bmatrix} [0.2, 0.5] & [0.4, 0.6] \\ [0.2, 0.4] & [0.3, 0.8] \end{bmatrix} \text{ and}$$

$$B = \begin{bmatrix} [0.4, 0.7] & [0.7, 0.8] \\ [0.3, 0.6] & [0.7, 0.9] \end{bmatrix}$$

Then

$$A * B = \begin{bmatrix} [0.2, 0.5] & [0.4, 0.6] \\ [0.2, 0.4] & [0.3, 0.8] \end{bmatrix} * \begin{bmatrix} [0.4, 0.7] & [0.7, 0.8] \\ [0.3, 0.6] & [0.7, 0.9] \end{bmatrix}$$

$$A * B = \begin{bmatrix} [0.08, 0.35] + [0.12, 0.36] & [0.14, 0.40] + [0.28, 0.54] \\ [0.08, 0.28] + [0.09, 0.48] & [0.14, 0.32] + [0.21, 0.72] \end{bmatrix}$$

$$A * B = \begin{bmatrix} [0.12, 0.36] & [0.28, 0.54] \\ [0.09, 0.48] & [0.21, 0.72] \end{bmatrix}$$

Definition 2.13 [19]

$A = [a_{ij}]$ and $B = [b_{ik}]$ are two IVFSM of same order $m \times n$ then And-product is defined as

$\Lambda: A \times B \rightarrow C_{m \times n}^2$, $[a_{ij}]_{m \times n} \wedge [b_{ik}]_{m \times n} = [c_{ip}]_{m \times n}^2$ where

$c_{ip} = [\min(\mu_{ALj}, \mu_{BLj}), \min(\mu_{AUj}, \mu_{BUj})]$ • i, j, k . such that $p = n(j-1) + k$.

Example 2.8

Consider A and B are two IVFS-matrices.

$$A = \begin{bmatrix} [0.2, 0.5] & [0.4, 0.6] \\ [0.2, 0.4] & [0.3, 0.8] \end{bmatrix}_{2 \times 2} \text{ and}$$

$$B = \begin{bmatrix} [0.4, 0.7] & [0.7, 0.8] \\ [0.3, 0.6] & [0.7, 0.9] \end{bmatrix}_{2 \times 2}$$

Then

$$A \wedge B = \begin{bmatrix} [0.2, 0.5] & [0.4, 0.6] \\ [0.2, 0.4] & [0.3, 0.8] \end{bmatrix} \wedge \begin{bmatrix} [0.4, 0.7] & [0.7, 0.8] \\ [0.3, 0.6] & [0.7, 0.9] \end{bmatrix}$$

$$A \wedge B = [c_{ip}]_{2 \times 4} = \begin{bmatrix} [0.2, 0.5] & [0.2, 0.5] & [0.4, 0.6] & [0.4, 0.6] \\ [0.2, 0.4] & [0.2, 0.4] & [0.3, 0.6] & [0.3, 0.8] \end{bmatrix}$$

Definition 2.14 [19]

$A = [a_{ij}]$ and $B = [b_{ik}]$ are two IVFS-matrices of same order $m \times n$ then Or-product is defined as

$V: A \times B \rightarrow C_{m \times n}^2$, $[a_{ij}]_{m \times n} \vee [b_{ik}]_{m \times n} = [c_{ip}]_{m \times n}^2$ where

$c_{ip} = [\max(\mu_{ALj}, \mu_{BLj}), \max(\mu_{AUj}, \mu_{BUj})]$ • i, j, k . such that $p = n(j-1) + k$.

Example 2.9

Consider A and B are two IVFS-matrices.

$$A = \begin{bmatrix} [0.2, 0.5] & [0.4, 0.6] \\ [0.2, 0.4] & [0.3, 0.8] \end{bmatrix} \text{ and}$$

$$B = \begin{bmatrix} [0.4, 0.7] & [0.7, 0.8] \\ [0.3, 0.6] & [0.7, 0.9] \end{bmatrix}$$

Then

$$A \vee B = \begin{bmatrix} [0.2, 0.5] & [0.4, 0.6] \\ [0.2, 0.4] & [0.3, 0.8] \end{bmatrix} \vee \begin{bmatrix} [0.4, 0.7] & [0.7, 0.8] \\ [0.3, 0.6] & [0.7, 0.9] \end{bmatrix}$$

$$A \vee B = [c_{ip}]_{2 \times 4} = \begin{bmatrix} [0.4, 0.7] & [0.7, 0.8] & [0.4, 0.7] & [0.7, 0.8] \\ [0.3, 0.6] & [0.7, 0.9] & [0.3, 0.8] & [0.7, 0.9] \end{bmatrix}$$

De Morgan laws, commutative laws, domination laws and involution laws are hold in And-product and Or-product of IVFSM, the proof of these laws can be seeing in [19].

Definition 2.15 [12]

C_{ip} be an FSM of order $m \times n^2$, where $I_k = \{p: \exists i, c_{ip} \neq 0, (k-1)n < p \leq kn\}$ for all $k \in I = \{1, 2, \dots, n\}$. Then fs-max-min decision function, denoted Mm, is defined as follows

$$Mm: FSM_{m \times n^2} \rightarrow FSM_{m \times 1},$$

$$Mm [c_{ip}] = [d_{i1}] = [\max \{t_{ik}\}]$$

216 Where

$$217 \quad t_{ik} = \begin{cases} \min_{p \in I_k} \{c_{ip}\} & \text{if } I_k \neq 0 \\ 0.0 & \text{if } I_k = 0 \end{cases}$$

218 The one column fs-matrix $M_m [c_{ip}]$ is called max-min decision fs-matrix.

219 **Definition 2.16 [12]**

220 M is a universe and $[d_{il}]$ is a max-min decision fs-matrix. Then optimum fuzzy set on M is
221 defined as

$$222 \quad \text{Opt } [d_{il}] (M) = \{d_{il} / m_i : m_i \in M, d_{il} \neq 0\}$$

223 Now, using above definitions we can construct an FSMmDM method by the following

224

225 **3 Operations on IVFSM**

226 In this section, we prove the De-Morgan laws, Associative laws and commutative laws by
227 using the definitions of And-Operation and Or-Operation of IVFSM.

228 **Definition 3.1 [20]**

229 Let $A = [a_{ij}]_{m \times n}$ and $B = [b_{ij}]_{m \times n}$ are two IVFS-matrices then their OR-operation is defined as

$$230 \quad A \vee B = [c_{ij}]_{m \times n}, \text{ where } c_{ij} = [\max(\mu_{AL}, \mu_{BL}), \max(\mu_{AU}, \mu_{BU})] \text{ for all } i, j.$$

231 **Proposition: 3.1 (Commutative law)**

$$232 \quad A \vee B = B \vee A$$

233 **Proof:**

234 Let $A = [a_{ij}] = [\mu_{AL}, \mu_{AU}]$ and $B = [b_{ij}] = [\mu_{BL}, \mu_{BU}]$ are two IVFS-matrices of order $m \times n$, than

$$\begin{aligned} 235 \quad \text{L.H.S} &= A \vee B = [\mu_{AL}, \mu_{AU}] \vee [\mu_{BL}, \mu_{BU}] \\ 236 &= [\max(\mu_{AL}, \mu_{BL}), \max(\mu_{AU}, \mu_{BU})] \\ 237 &= [\max(\mu_{BL}, \mu_{AL}), \max(\mu_{BU}, \mu_{AU})] \\ 238 &= B \vee A \\ 239 &= \text{R.H.S} \end{aligned}$$

240 **Proposition: 3.2 (Associative law)**

$$241 \quad (A \vee B) \vee C = A \vee (B \vee C)$$

242 **Proof:**

243 Let $A = [a_{ij}] = [\mu_{AL}, \mu_{AU}]$, $B = [b_{ij}] = [\mu_{BL}, \mu_{BU}]$ and $C = [c_{ij}] = [\mu_{CL}, \mu_{CU}]$ are IVFS-matrices
244 of order $m \times n$, than

$$\begin{aligned} 245 \quad \text{L.H.S} &= (A \vee B) \vee C = ([\mu_{AL}, \mu_{AU}] \vee [\mu_{BL}, \mu_{BU}]) \vee [\mu_{CL}, \mu_{CU}] \\ 246 &= ([\max(\mu_{AL}, \mu_{BL}), \max(\mu_{AU}, \mu_{BU})] \vee [\mu_{CL}, \mu_{CU}]) \\ 247 &= [\max(\mu_{AL}, \mu_{BL}, \mu_{CL}), \max(\mu_{AU}, \mu_{BU}, \mu_{CU})] \\ 248 &= [\mu_{AL}, \mu_{AU}] \vee ([\max(\mu_{BL}, \mu_{CL}), \max(\mu_{BU}, \mu_{CU})]) \end{aligned}$$

$$= A \vee (B \vee C)$$

$$= \mathbf{R.H.S}$$

Definition 3.2 [20]

Let $A = [a_{ij}]_{m \times n}$ and $B = [b_{ij}]_{m \times n}$ are two IVFS-matrices then AND-operation is defined as

$A \wedge B = [c_{ij}]_{m \times n}$, where $c_{ij} = [\min(\mu_{AL}, \mu_{BL}), \min(\mu_{AU}, \mu_{BU})]$ for all i, j .

Proposition: 3.3 (Commutative law)

$$A \wedge B = B \wedge A$$

Proof:

Let $A = [a_{ij}] = [\mu_{AL}, \mu_{AU}]$ and $B = [b_{ij}] = [\mu_{BL}, \mu_{BU}]$ are two IVFS-matrices of order $m \times n$, than

$$\mathbf{L.H.S} = A \wedge B = [\mu_{AL}, \mu_{AU}] \wedge [\mu_{BL}, \mu_{BU}]$$

$$= [\min(\mu_{AL}, \mu_{BL}), \min(\mu_{AU}, \mu_{BU})]$$

$$= [\min(\mu_{BL}, \mu_{AL}), \min(\mu_{BU}, \mu_{AU})]$$

$$= B \wedge A$$

$$= \mathbf{R.H.S}$$

Proposition: 3.4 (Associative law)

$$(A \wedge B) \wedge C = A \wedge (B \wedge C)$$

Proof:

Let $A = [a_{ij}] = [\mu_{AL}, \mu_{AU}]$, $B = [b_{ij}] = [\mu_{BL}, \mu_{BU}]$ and $C = [c_{ij}] = [\mu_{CL}, \mu_{CU}]$ are IVFS-matrices of order $m \times n$, than

$$\mathbf{L.H.S} = (A \wedge B) \wedge C = ([\mu_{AL}, \mu_{AU}] \wedge [\mu_{BL}, \mu_{BU}]) \wedge [\mu_{CL}, \mu_{CU}]$$

$$= ([\min(\mu_{AL}, \mu_{BL}), \min(\mu_{AU}, \mu_{BU})] \wedge [\mu_{CL}, \mu_{CU}])$$

$$= [\min(\mu_{AL}, \mu_{BL}, \mu_{CL}), \min(\mu_{AU}, \mu_{BU}, \mu_{CU})]$$

$$= [\mu_{AL}, \mu_{AU}] \wedge ([\min(\mu_{BL}, \mu_{CL}), \min(\mu_{BU}, \mu_{CU})])$$

$$= A \wedge (B \wedge C)$$

$$= \mathbf{R.H.S}$$

Theorem: 3.1 (De Morgan laws)

Let $C = [c_{ij}] = [\mu_{CL}, \mu_{CU}]$ and $D = [d_{ij}] = [\mu_{DL}, \mu_{DU}]$ are two IVFS-matrices of order $m \times n$, than

$$(1) (C \vee D)^C = C^C \wedge D^C$$

$$(2) (C \wedge D)^C = C^C \vee D^C$$

Proof: 1

$$\mathbf{L.H.S} = (C \vee D)^C$$

$$= ([\mu_{CL}, \mu_{CU}] \vee [\mu_{DL}, \mu_{DU}])^C$$

$$= [\max(\mu_{CL}, \mu_{DL}), \max(\mu_{CU}, \mu_{DU})]^C$$

$$= [1 - \max(\mu_{CU}, \mu_{DU}), 1 - \max(\mu_{CL}, \mu_{DL})]$$

$$\begin{aligned}
 &= [\min (1- \mu_{CU}, 1- \mu_{DU}), \min (1- \mu_{CL}, 1- \mu_{DL})] \\
 &= C^C \wedge D^C \\
 &= \mathbf{R.H.S}
 \end{aligned}$$

Proof: 2

$$\begin{aligned}
 \mathbf{L.H.S} &= (C \wedge D)^C \\
 &= ([\mu_{CL}, \mu_{CU}] \wedge [\mu_{DL}, \mu_{DU}])^C \\
 &= [\min (\mu_{CL}, \mu_{DL}), \min (\mu_{CU}, \mu_{DU})]^C \\
 &= [1- \min (\mu_{CU}, \mu_{DU}), 1- \min (\mu_{CL}, \mu_{DL})] \\
 &= [\max (1- \mu_{CU}, 1- \mu_{DU}), \max (1- \mu_{CL}, 1- \mu_{DL})] \\
 &= C^C \vee D^C \\
 &= \mathbf{R.H.S}
 \end{aligned}$$

Similarly we can prove other related laws by using the above definitions.

4. IVFS-Max-Min Decision Making

In this section, we construct an IVFS-max-min decision making (IVFSMmDM) method by using IVFS-max-min decision function which is defined here. The method selects optimum alternatives from the set of the alternatives.

Definition 4.1

Let $[c_{ip}] \in \text{IVFSM}_{m \times n}^2$, $I_k = \{P: \exists i, c_{ip} \neq 0, (k-1)n < P \leq kn\} \square k \in I = \{1, 2, 3, \dots, n\}$.

Then IVFS-max-min decision function denoted by Mm and defined as

$Mm: \text{IVFSM}_{m \times n}^2 \rightarrow \text{IVFSM}_{m \times 1}$

$Mm [c_{ip}] = [d_{i1}] = [\max \{t_{ik}\}]$

Where

$$t_{ik} = \begin{cases} \min_{p \in I_k} \{c_{ip}\} & \text{if } I_k \neq 0 \\ [0.0, 0.0] & \text{if } I_k = 0 \end{cases}$$

Where $c_{ip} = [\min (\mu_{ALj}, \mu_{BLj}), \min (\mu_{AUj}, \mu_{BUj})] \square i, j, k$. such that $p = n(j-1) + k$

Which is one column IVFSM $Mm [c_{ip}]$ is called max-min decision IVFSM.

Definition 4.2

$M = \{m_1, m_2, m_3, \dots, m_n\}$ be a universal set and max-min $[c_{ip}] = [d_{i1}]$. Then optimum fuzzy

set on U is defined as

$\text{Opt} [d_{i1}] (M) = \{d_{i1}/m_i; m_i \in M, d_{i1} \neq 0\}$ which is called optimum IVFS on M.

Now by the help of this definition we construct an algorithm for IVFSMmDM.

Algorithm:

1. Select the appropriate subsets from the set of parameters,
2. Construct the IVFSM by selected set of parameters,
3. Take the product of the IVFS-matrices by And product,
4. Find max-min decision IVFSM,
5. Finally we find optimum IVFS on M.

Application:

Let $M = \{Q, P, K, S\}$ be a universal set which contains any four cities of an country the world.

Let $E = \{y_1, y_2, y_3, y_4\}$ be the set of parameter where each y_i stand for Education, health, local bodies and development program, respectively.

Example 4.1

Two friends A and B are wants to live in any city of M, if both friends have their own set of parameters. Then we select a city on the basis of the sets of friend's parameters by using the IVFSmDM as follows.

Suppose $M = \{Q, P, K, S\}$ be a universal set and $E = \{y_1, y_2, y_3, y_4\}$ be a set of parameters.

Step 1

First of all A and B both friends chose the set of parameters,

$A = \{y_1, y_2, y_4\}$ and $B = \{y_1, y_2, y_3\}$.

Step 2

Now we construct the IVFS-matrices according to selected parameters of both friends.

$$A = [a_{ij}] = \begin{bmatrix} [0.4, 0.5] & [0.3, 0.4] & [0.0, 0.0] & [0.2, 0.3] \\ [0.7, 0.8] & [0.6, 0.6] & [0.0, 0.0] & [0.5, 0.6] \\ [0.8, 0.8] & [0.7, 0.8] & [0.0, 0.0] & [0.4, 0.5] \\ [0.3, 0.4] & [0.3, 0.4] & [0.0, 0.0] & [0.2, 0.3] \end{bmatrix}$$

$$B = [b_{ik}] = \begin{bmatrix} [0.6, 0.7] & [0.4, 0.5] & [0.3, 0.4] & [0.0, 0.0] \\ [0.7, 0.8] & [0.6, 0.7] & [0.2, 0.3] & [0.0, 0.0] \\ [0.8, 0.9] & [0.8, 0.9] & [0.5, 0.6] & [0.0, 0.0] \\ [0.4, 0.5] & [0.3, 0.4] & [0.2, 0.3] & [0.0, 0.0] \end{bmatrix}$$

Step 3

Now we find product of IVFS-matrices by using And-product

$$[a_{ij}] \wedge [b_{ik}] = \begin{bmatrix} [0.4, 0.5] & [0.3, 0.4] & [0.0, 0.0] & [0.2, 0.3] \\ [0.7, 0.8] & [0.6, 0.6] & [0.0, 0.0] & [0.5, 0.6] \\ [0.8, 0.8] & [0.7, 0.8] & [0.0, 0.0] & [0.4, 0.5] \\ [0.3, 0.4] & [0.3, 0.4] & [0.0, 0.0] & [0.2, 0.3] \end{bmatrix} \wedge$$

$$\begin{bmatrix} [0.6, 0.7] & [0.4, 0.5] & [0.3, 0.4] & [0.0, 0.0] \\ [0.7, 0.8] & [0.6, 0.7] & [0.2, 0.3] & [0.0, 0.0] \\ [0.8, 0.9] & [0.8, 0.9] & [0.5, 0.6] & [0.0, 0.0] \\ [0.4, 0.5] & [0.3, 0.4] & [0.2, 0.3] & [0.0, 0.0] \end{bmatrix}$$

$$[a_{ij}] \otimes [b_{ik}] = \begin{bmatrix} [4, 5] & [4, 5] & [3, 4] & [0, 0] & [3, 4] & [3, 4] & [3, 4] & [0, 0] & [0, 0] & [0, 0] & [0, 0] & [0, 0] & [2, 3] & [2, 3] & [2, 3] & [0, 0] \\ [7, 8] & [6, 7] & [2, 3] & [0, 0] & [6, 6] & [6, 6] & [2, 3] & [0, 0] & [0, 0] & [0, 0] & [0, 0] & [0, 0] & [5, 6] & [5, 6] & [2, 3] & [0, 0] \\ [8, 8] & [8, 8] & [5, 6] & [0, 0] & [7, 8] & [7, 8] & [5, 6] & [0, 0] & [0, 0] & [0, 0] & [0, 0] & [0, 0] & [4, 5] & [4, 5] & [4, 5] & [0, 0] \\ [3, 4] & [3, 4] & [2, 3] & [0, 0] & [3, 4] & [3, 4] & [2, 3] & [0, 0] & [0, 0] & [0, 0] & [0, 0] & [0, 0] & [2, 3] & [2, 3] & [2, 3] & [0, 0] \end{bmatrix}$$

Step 4

To calculate Mm $([a_{ij}] \wedge [b_{ik}]) = [d_{il}]$, in this step we find d_{il} for all $i=1, 2, 3, 4$. First of all we find d_{11} .

$$d_{11} = \max \{t_{1k}\} = \max \{t_{11}, t_{12}, t_{13}, t_{14}\} \text{ where } k \in \{1, 2, 3, 4\}$$

For d_{11} we also find t_{1k} for all $k \in \{1, 2, 3, 4\}$. When $k = 1$ and $n = 4$ then t_{11} is $I_1 = \{P: c_{iP} \neq 0, 0 < P \leq 4\}$ and when we find t_{12} then $k = 2$, $n = 4$ and $I_2 = \{P: c_{iP} \neq 0, 4 < P \leq 8\}$. Similarly for $k = 3$, $I_3 = \{P: c_{iP} \neq 0, 8 < P \leq 12\}$ and for $k = 4$, $I_4 = \{P: c_{iP} \neq 0, 12 < P \leq 16\}$.

$t_{11} = \min \{c_{11}, c_{12}, c_{13}\}$, where $c_{ip} = [\min \{\mu_{ALJ}, \mu_{BLJ}\}, \min \{\mu_{AUJ}, \mu_{BUJ}\}]$

$t_{11} = \min \{[0.4, 0.5], [0.4, 0.5], [0.3, 0.4]\}$

$t_{11} = [0.3, 0.4]$

Similarly we obtain other values

$t_{12} = \min \{c_{15}, c_{16}, c_{17}\} = \min \{[0.3, 0.4], [0.3, 0.4], [0.3, 0.4]\}$

$t_{12} = [0.3, 0.4]$

$t_{13} = [0.0, 0.0]$

$t_{14} = \min \{c_{213}, c_{214}, c_{215}\} = \min \{[0.2, 0.3], [0.2, 0.3], [0.2, 0.3]\}$

360 $t_{14} = [0.2, 0.3]$
 361 So
 362 $d_{11} = \max \{t_{11}, t_{12}, t_{13}, t_{14}\} = \max \{[0.3, 0.4], [0.3, 0.4], [0.0, 0.0], [0.2, 0.3]\}$
 363 **$d_{11} = [0.3, 0.4]$**
 364 Similarly we find d_{21}, d_{31}, d_{41} .
 365 $d_{21} = \max \{t_{2k}\} = \max \{t_{21}, t_{22}, t_{23}, t_{24}\}$ where $k \in \{1, 2, 3, 4\}$
 366 $t_{21} = \min \{c_{21}, c_{22}, c_{23}\} = \min \{[0.7, 0.8], [0.6, 0.7], [0.2, 0.3]\}$
 367 $t_{21} = [0.2, 0.3]$
 368 $t_{22} = \min \{c_{25}, c_{26}, c_{27}\} = \min \{[0.6, 0.6], [0.6, 0.6], [0.2, 0.3]\}$
 369 $t_{22} = [0.2, 0.3]$
 370 $t_{23} = [0.0, 0.0]$
 371 $t_{24} = \min \{c_{213}, c_{214}, c_{215}\} = \min \{[0.5, 0.6], [0.5, 0.6], [0.2, 0.3]\}$
 372 $t_{24} = [0.2, 0.3]$
 373 So
 374 $d_{21} = \max \{t_{21}, t_{22}, t_{23}, t_{24}\} = \max \{[0.2, 0.3], [0.2, 0.3], [0.0, 0.0], [0.2, 0.3]\}$
 375 **$d_{21} = [0.2, 0.3]$**
 376 $d_{31} = \max \{t_{3k}\} = \max \{t_{31}, t_{32}, t_{33}, t_{34}\}$ where $k \in \{1, 2, 3, 4\}$
 377 $t_{31} = \min \{c_{31}, c_{32}, c_{33}\} = \min \{[0.8, 0.8], [0.8, 0.8], [0.5, 0.6]\}$
 378 $t_{31} = [0.5, 0.6]$
 379 $t_{32} = \min \{c_{35}, c_{36}, c_{37}\} = \min \{[0.7, 0.8], [0.7, 0.8], [0.5, 0.6]\}$
 380 $t_{32} = [0.5, 0.6]$
 381 $t_{33} = [0.0, 0.0]$
 382 $t_{34} = \min \{c_{313}, c_{314}, c_{315}\} = \min \{[0.4, 0.5], [0.4, 0.5], [0.4, 0.5]\}$
 383 $t_{34} = [0.4, 0.5]$
 384 So
 385 $d_{31} = \max \{t_{31}, t_{32}, t_{33}, t_{34}\} = \max \{[0.5, 0.6], [0.5, 0.6], [0.0, 0.0], [0.4, 0.5]\}$
 386 **$d_{31} = [0.5, 0.6]$**
 387 Finally we find d_{41}
 388 $d_{41} = \max \{t_{4k}\} = \max \{t_{41}, t_{42}, t_{43}, t_{44}\}$ where $k \in \{1, 2, 3, 4\}$
 389 $t_{41} = \min \{c_{41}, c_{42}, c_{43}\} = \min \{[0.3, 0.4], [0.3, 0.4], [0.2, 0.3]\}$
 390 $t_{41} = [0.2, 0.3]$
 391 $t_{42} = \min \{c_{45}, c_{46}, c_{47}\} = \min \{[0.3, 0.4], [0.3, 0.4], [0.2, 0.3]\}$
 392 $t_{42} = [0.2, 0.3]$
 393 $t_{43} = [0.0, 0.0]$
 394 $t_{44} = \min \{c_{413}, c_{414}, c_{415}\} = \min \{[0.2, 0.3], [0.2, 0.3], [0.2, 0.3]\}$
 395 $t_{44} = [0.2, 0.3]$
 396 So
 397 $d_{41} = \max \{t_{41}, t_{42}, t_{43}, t_{44}\} = \max \{[0.2, 0.3], [0.2, 0.3], [0.0, 0.0], [0.2, 0.3]\}$
 398 **$d_{41} = [0.2, 0.3]$**
 399 Finally we obtain the IVFSM by IVFSMmDM.

$$400 \quad Mm ([a_{ij}] \wedge [b_{ik}]) = [d_{i1}] = \begin{bmatrix} d_{11} \\ d_{21} \\ d_{31} \\ d_{41} \end{bmatrix}$$

$$401 \quad [d_{i1}] = \begin{bmatrix} [0.3, 0.4] \\ [0.2, 0.3] \\ [0.5, 0.6] \\ [0.2, 0.3] \end{bmatrix}$$

402 Finally we obtain an optimum IVFS on M according to above matrix.

$$403 \quad \text{Opt } Mm ([a_{ij}] \wedge [b_{ik}]) (M) = \{[0.3, 0.4]/Q, [0.2, 0.3]/P, [0.5, 0.6]/K, [0.2, 0.3]/S\}$$

Where K is optimum city of the world for living of A and B .

Similarly we can apply Or-product for other problems.

5 CONCLUSIONS

In this paper, we studied IVFSM and proved the commutative laws, associative laws and De-Morgan laws by using And-Operation and Or-Operation on IVFSM. We introduced a new decision making method with the help of interval valued fuzzy soft max-min decision function and applied this method for decision making by taking hypothetical data.

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