



SDI FINAL EVALUATION FORM 1.1

PART 1:

Journal Name:	British Journal of Mathematics & Computer Science
Manuscript Number:	Ms_BJMCS_33945
Title of the Manuscript:	Juxtaposition operation to obtain Latin squares
Type of Article:	Short Communication

PART 2:

FINAL EVALUATOR'S comments on revised paper (if any)	Authors' response to final evaluator's comments
This is a revision of a paper that I reviewed earlier for this journal. It deals with an implementation into a C program of the generation of Latin square by means of two methods (<i>juxtaposition</i> and <i>Kronecker's product</i>), both of them derived from the direct product of two or more Latin squares. The presentation of the paper has slightly been improved from the earlier version, taking on board some (but not all) of the reviewer comments. Particularly, some examples and references, which improve the understanding of the paper, have been included. Nevertheless, I do not agree with the way in which the authors have dealt with one of the main aspects to be revised. This is the fact that both methods described by the authors are not new and this should be explicitly remarked throughout the paper. In my previous review, I mentioned the reference of Dénes and Keedwell as a bedside monograph in the theory of Latin squares from which the authors may develop the notation that they use in their work. I mean, the idea is not to remark the similarity with the concept introduced by the author (see the next-to-last paragraph in Section 2 ("In a similar way but using ...")), but to make use of the conventional concept of direct product of Latin squares referred in the known handbook of Dénes and Keedwell. Particularly, I mentioned that - The <i>Kronecker's product</i> $A1 \circ A2$ is the direct product $A2 \times A1$- - The <i>juxtaposition</i> $A1 * \dots * A1$ (n times) consists of the direct product of $A1$ with the cyclic Latin $12 \dots (n-1)n$ $n1 \dots (n-2)(n-1)$	Thank you very much for comments, All of them are taking into account In the paper, we have explained that our operations can be deduced from the constructions made by Dénes and Keedwell The name juxtaposition has been changed by cyclic-union In the introduction, it has been specified that the work is intended to support students interested in this subject The second paragraph in Section 3 has been removed. All minor comments also has been taking into account The last example has been rewritten in a more readable form the reader can distinguish the columns of the Latin square



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23...n1

In any case, imagine that we call $L(n)$ to the previous cyclic Latin square. Keeping in mind the notation of the authors, what I am saying is that the *juxtaposition* $A1*...An$ is the *Kronecker's product* $L(n) \times A1$. In this regard, observe that Kronecker's product should be described in the paper before than juxtaposition, but not later, as the authors do. In this regard, there are not two distinct operations described in the paper, but a main operation (Kronecker's product) and a particular case (juxtaposition).

The advantage of using the reference of Dénes and Keedwell is that it can also be used to describe the *juxtaposition* $A1*...*Ar$ as the so-called non-uniform product, as I explained in my previous review. In this regard, the authors have remarked in their feedback that $A1*...*Ar$ is different than $A1x...x Ar$, because the former is of order nr and the latter of order n^r . I agree with their comment, but this does not contradict my observation. What I am saying is that the juxtaposition is the non-uniform product of r Latin rectangles by using the direct (or Kronecker's) product. The order of the resulting Latin square is nr , because, in fact, it is exactly the juxtaposition product.

In any case, apart from all these observation, remark that it is not a good idea at all to call this product "*juxtaposition*", because in Latin square theory, the juxtaposition of two Latin squares is done with pair of cells in the same row and column and it is explicitly used to introduce and develop the concept of orthogonal Latin squares. This fact should be remarked in the paper. That is, the so-called juxtaposition operation by the authors is distinct from the classical juxtaposition of Latin squares.

I also mentioned in my previous remark that the authors should not put the main goal of their paper in the introduction of the operation of the methods, but in their programming in C language, that is, in Section 3. To do it, they must expose the code for general order (or **at least a pseudocode for the general case**). Otherwise, the paper is only a description of two known methods for constructing a new Latin square and the code of a very simple example.

Following referee instructions, the Latin square of the final example has been rewritten to make it look better. The initial image was directly extracted from the computer, as the result of the machine using the algorithm. It was for that reason that I had put the image in this form. It is true that with the new script it looks better.



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Another comment that the authors have not clarified is the fact that they consider the Kronecker's product and the juxtaposition product as two methods to optimize the generation of random Latin squares. Where is the randomization of the methods? The two exposed constructions are deterministic and none randomness appears. If none explicit method of randomization is used, then the second paragraph in Section 3 should be removed.

The last observation from my original review that the authors have not considered is to make clearer the Latin square that appears in Section 4. If they prefer to make use of the bitwise method introduced by Yordzhev, the former should write the Latin square at least as the latter does in the Section 4 of his paper. That is, not to include a figure (which in this case is in fact blurred), but an array from which the reader can distinguish the columns of the Latin square.

Some other minor revisions to be taking into account:

- The word "Latin" must be always capitalized. Revise it throughout the text.
- The authors have improved the first paragraph of the second page of the original version of the paper, but it is still wrong. They have indicated that *"the number of Latin squares is not known for order $n \geq 10$ "*.
- Revise the references. Distinct typos appear therein.