

SOME E-CORDIAL LABELINGS ON EXTENDED DUPLICATE GRAPH OF COMB GRAPH

K. Sutha, K. Thirusangu, S. Bala

Department of Mathematics

S. I. V. E. T. College, Gowrivakkam, Chennai – 600 073.

E-mail: sutharavindran@yahoo.com; kthirusangu@gmail.com; balasankarasubbu@yahoo.com

Abstract

A graph labeling is a mapping that carries a set of graph elements onto a set of numbers called labels(usually the set of integers). In this paper we prove the existence of graph labeling such as E- cordial, total E- cordial, product E-cordial , total product E-cordial, Prime E- cordial and E_3 - cordial labeling for the extended duplicate graph of Comb graph by presenting algorithms.

AMS Subject Classification : 05C78

Keywords: Graph labeling, Comb graph , Duplicate graph.

1 . Introduction

Graph labeling is an interesting area of research in graph theory which was introduced by Rosa [6] in 1967 . It is an assignment of integers to the vertices or edges or both subject to certain conditions. Yilmaz and Cahit [10] considered a weaker version of edge graceful called E-cordial and E_k - cordial and establish some interesting theorems for these graphs.

Definition 1.1

If there exists a function $f : E \rightarrow \{0,1\}$ for a graph $G(V,E)$ such that the induced function f^* on V defined by $f^*(v_i) = \{\sum f(v_i v_j) \mid v_i v_j \in E\} \pmod{2}$ satisfies the property that the number of edges labeled with '0' and the number of edges labeled with '1' differ by at most one and number of vertices labeled with '0' and the number of vertices labeled with '1' differ by at most one is said to admit E-cordial labeling or E- cordial graph. It admits total E-cordial labeling if it satisfies the property that the number of edges and vertices labeled with '0' and the number of edges and vertices labeled with '1' differ by at most one.

Definition 1.2

Let G be a graph with vertex set V and edge set E . An edge labeling $f: E \rightarrow Z_3$ where $Z_3 = \{0,1,2\}$ induces a vertex labeling $f^*: V \rightarrow Z_3$ defined by $f^*(v_i) = \{ \sum f(v_i v_j) \mid v_i v_j \in E \} \pmod{3}$ for all vertex $v \in V$. If the number of vertices labeled with 0,1,2 differ by at most one with each other and the number of edges labeled with 0,1,2 differ by at most 1 with each other, it is called E_3 - cordial labeling or E_3 - cordial graph. An E_3 - cordial labeling is said to be total E_3 - cordial labeling of G if the total number of vertices and edges labeled with 0,1,2 are differ by at most one with each other. Such graph G is called total E_3 - cordial graph.

In [7] Thirusanguet.al., have introduced the concept of extended duplicate graph.

Definition 1.3

Duplicate graph of a path graph $G(V,E)$ is $DG=(V_1,E_1)$ where the vertex set $V_1 = V \cup V'$ and $V \cap V' = \Phi$ and $f: V \rightarrow V'$ is bijective (for $v \in V$, we write $f(v) = v'$ for convenience) and the edge set E_1 of DG is defined as follows: The edge $v_i v_j$ is in E if and only if both $v_i v_j'$ and $v_i' v_j$ are edges in E_1 . Clearly duplicate graph is disconnected. In order to make it as connected, add an edge $v_i v_i'$ for any i . This graph is called the Extended Duplicate Graph of a path graph G .

Definition 1.4

A Graph $G(V,E)$ is said to admit product E-cordial labeling if there exists a function $f: E \rightarrow \{0,1\}$ such that the induced function f^* on V defined by $f^*(v_i) = \{ \prod f(v_i v_j) \mid v_i v_j \in E \}$ satisfies the property that the number of edges labeled with '0' and the number of edges labeled with '1' differ by at most one and the number of vertices labeled with '0' and the number of vertices labeled with '1' differ by at most one. A graph that admits product E-cordial labeling is called product E-cordial. It is said to admit total product E-cordial labeling if it satisfies the property that the number of edges and vertices labeled with '0' and the number of edges and vertices labeled with '1' differ by at most one.

They proved that the extended duplicate graph $EDG(P_m)$ is E- cordial ,total E- cordial, product E- cordial , total product E-cordial, E_3 - cordial and total E_3 - cordial.

In [1] Bala and Thirusangu establish the concept of Product E- cordial and Prime E- cordial labelings.

Definition 1.5

A prime E-cordial labeling of a graph G with edge set E is a function $f: E \rightarrow \{1, 2, 3, \dots, q\}$ such that the induced function $f^*: V \rightarrow N$ defined as $f^*(v_i)$ is assigned the

label 1, if $\gcd(f(v_i v_j), f(v_i v_k)) = 1$ and 0, if $\gcd(f(v_i v_j), f(v_i v_k)) > 1$, then the number of edges labeled with '0' and '1' differ by at most 1.

It was shown that the extended triplicate graph $ETG(P_m)$ admits Product E- cordial and Prime E- cordial.

In[8] Singh et.al., proved some results on Comb graph.

Definition 1.6

Let P_{m+1} be a path graph. **Comb graph** is defined as $P_{m+1} \odot (m+1)K_1$. It has $2m+2$ vertices and $2m+1$ edges.

Motivated by the above study in this paper, we prove that the extended duplicate graph of Comb graph admits E- cordial, total E- cordial, product E-cordial , total product E-cordial, prime E-cordial , E_3 - cordial and total E_3 - cordial labeling by presenting algorithms.

2. Main Result

In this section, we prove the existence of E-cordial, total E- cordial, product E-cordial and total product E- cordial prime E-cordial, E_3 -cordial and total E_3 - cordial labeling for the extended duplicate graph of a Comb graph by presenting algorithms.

Definition (Structure of extended duplicate graph of comb graph).

Let $G(V,E)$ be a comb graph . $DG(\text{comb}) = (V_1, E_1)$ be the duplicate graph of combgraph with $4m+4$ vertices and $4m+2$ edges.

Denote the vertex set as $V_1 = \{v_1, v_2, \dots, v_{2m+2}, v_1', v_2', \dots, v_{2m+2}'\}$ and the edge set as $E_1 = \{v_i v_{i+1}' \cup v_i' v_{i+1} \text{ for } 1 \leq i \leq m\} \cup \{v_i' v_{m+i+1} \cup v_i v_{m+i+1}' \text{ for } 1 \leq i \leq m+1\}$. Clearly $DG(\text{comb})$ is disconnected. The extended duplicate graph of a comb, $EDG(\text{Comb})$ is obtained from $DG(\text{comb})$ by adding the edges (i) $v_1 v_{m+1}$ and $v_{m+2}' v_{2m+2}'$ if $m \equiv 1 \pmod{2}$. (ii) $v_1 v_{m+1}'$ and $v_{m+2}' v_{2m+2}$ if $m \equiv 0 \pmod{2}$.

Thus the extended duplicate graph of comb graph has $4m+4$ vertices and $4m+4$ edges.

Algorithm 2.1

Procedure : (E-cordial labeling for extended duplicate graph of comb graph).

// assignment of labels to the edges

for $i = 1$ to m do

{ $v_i v_{i+1}' \leftarrow 0$, $v_i' v_{i+1} \leftarrow 1$
}

end for

for $i=1$ to $m+1$ do

{ $v_i v_{m+i+1}' \leftarrow 0$, $v_i' v_{m+i+1} \leftarrow 1$
}

end for

if $m \equiv 0 \pmod{2}$

$v_1 v_{m+1}' \leftarrow 1$; $v_{m+2}' v_{2m+2} \leftarrow 0$

if $m \equiv 1 \pmod{2}$

$v_1 v_{m+1} \leftarrow 1$; $v_{m+2}' v_{2m+2}' \leftarrow 0$

end procedure

output :edge labeled extended duplicate graph of comb graph .

Theorem 2.1

The Extended duplicate graph of comb graph , $EDG(\text{comb})$, $m \geq 2$ is E-cordial.

Proof :

From the construction of Extended duplicate graph of comb graph , it is clear that $EDG(\text{comb}) (V,E)$ graph has $4m + 4$ vertices and $4m + 4$ edges.

In order to label the edges, define a function $f : E \rightarrow \{0, 1\}$ as given in algorithm 2.1. Clearly, the number of edges labeled with '1' is $2m + 2$ and number of edges labeled with '0' is $2m + 2$. Thus the number of edges labeled with '0' and '1' differ by at most one.

To obtain the labels for vertices, define the induced map $f^* : V \rightarrow \mathbb{N}$ such that $f^*(v) = [\sum_{u \in N(v)} f(uv)] \pmod{2}$.

The labels for the vertices are given below.

For $i=1$ to m , $f^*(v_i)$ takes the value 1 and $f^*(v_i')$ takes the value 0. $f^*(v_{m+i+1})$ receives the label 1 and $f^*(v_{m+i+1}')$ receives the label 0 if 'i' varies from 1 to $m+1$.

For all even m , we have $f^*(v_{m+1})$ as 1 and $f^*(v_{m+1}')$ as 0 and for odd m , $f^*(v_{m+1})$ as 1 and $f^*(v_{m+1}')$ as 0.

Thus the number of vertices labeled with '1' is $2m + 2$ and '0' is $2m+2$.

Hence the number of vertices labeled with '1' and '0' differ by at most one.

Therefore $EDG(\text{comb})$ graph is E-cordial.

Theorem 2.2

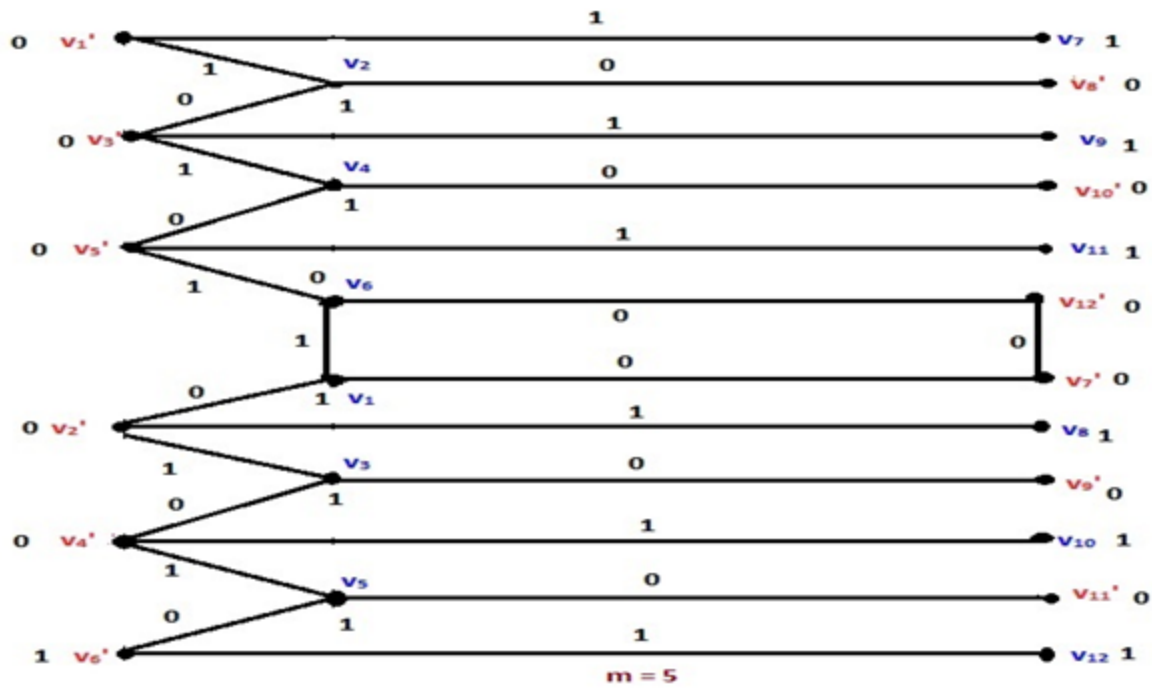
The Extended duplicate graph of comb graph, $EDG(\text{comb})$, $m \geq 2$ is total E-cordial.

Proof :

By theorem 2.1, the number of edges labeled with '1' and '0' are same as $2m + 2$. The number of vertices labeled with '1' and '0' are same as $2m + 2$. Thus the total number of vertices and edges labeled with '1' is $4m+4$ and '0' is $4m+4$ respectively. Hence the total number of vertices and edges labeled with '1' and the total number of vertices and edges labeled with '0' are differ by at most one. Therefore the Extended duplicate graph of comb graph is total E-cordial.

Example 2.1

E- cordial labeling of EDG(comb) graph for $m=5$ and $m=6$ are given in figure 1 and figure 2 respectively.

**Figure : 1**

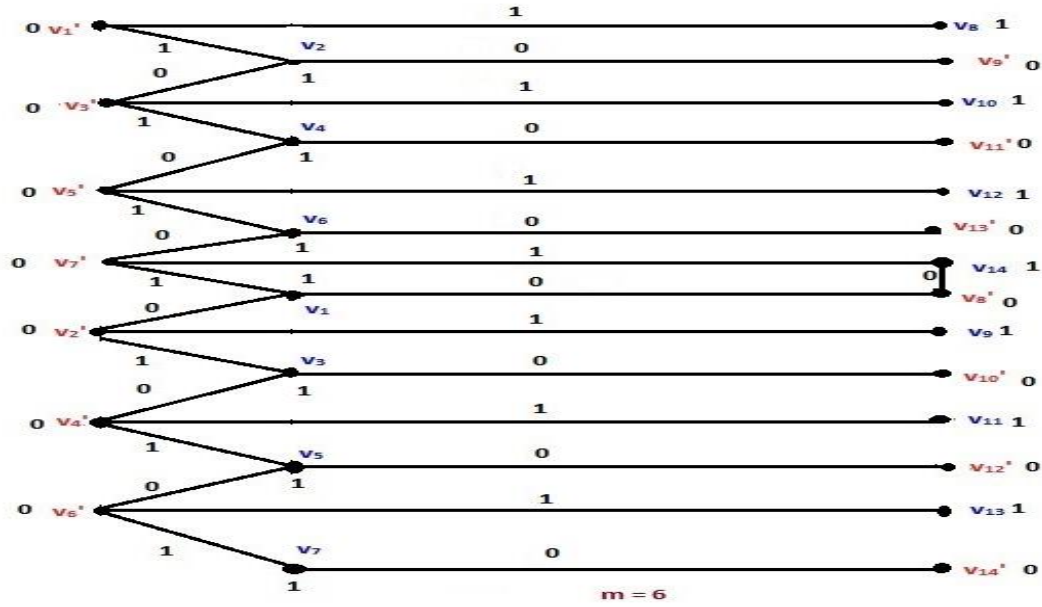


Figure : 2

Algorithm 2. 2

Procedure :(product E-cordial labeling for extended duplicate graph of comb graph).

// assignment of labels to the edges

$v_m'v_{m+1}$, $v_mv_{m+1}' \leftarrow 1$

for $i = 1$ to $m - 1$ do

$$\left\{ \begin{array}{l} v_i v_{i+1}' \leftarrow \begin{cases} 0 & \text{if } i \equiv 1 \pmod{2} \\ 1 & \text{otherwise} \end{cases} \\ v_i' v_{i+1} \leftarrow \begin{cases} 0 & \text{if } i \equiv 0 \pmod{2} \\ 1 & \text{otherwise} \end{cases} \end{array} \right.$$

end for

for $i = 1$ to m do

$$\left\{ \begin{array}{l} v_i v_{m+i+1}' \leftarrow \begin{cases} 0 & \text{if } i \equiv 1 \pmod{2} \\ 1 & \text{otherwise} \end{cases} \\ v_i' v_{m+i+1} \leftarrow \begin{cases} 0 & \text{if } i \equiv 0 \pmod{2} \\ 1 & \text{otherwise} \end{cases} \end{array} \right.$$

end for

if $m \equiv 0(\text{mod } 2)$

$$V_{m+1}V_{2m+2}' \leftarrow 1$$

$$V_1V_{m+1}', V_{m+1}'V_{2m+2}, V_{m+2}'V_{2m+2} \leftarrow 0$$

if $m \equiv 1(\text{mod } 2)$

$$V_{m+1}'V_{2m+2} \leftarrow 1$$

$$V_1V_{m+1}, V_{m+1}V_{2m+2}', V_{m+2}'V_{2m+2}' \leftarrow 0$$

end procedure

output : edge labeled extended duplicate graph of comb graph .

Theorem 2.3

The Extended duplicate graph of comb graph , $\text{EDG}(\text{comb})$, $m \geq 2$ is product E-cordial.

Proof :

From the construction of extended duplicate graph of comb graph, it is clear that $\text{EDG}(\text{comb})(V, E)$ has $4m + 4$ vertices and $4m + 4$ edges.

In order to label the edges, define a function $f : E \rightarrow \{0, 1\}$, using algorithm 2.2

Clearly,

The number of edges labeled with '1' = $2m+2$.

The number of edges labeled with '0' = $2m+2$.

Thus the number of edges labeled with '1' and '0' differ by at most one.

In order to get the label for vertices, define the induced map $f^* : V \rightarrow \mathbb{N}$ such that $f^*(v) = [\prod_{u \in N(v)} f(uv)]$.

The vertex labels are obtained as follows :

Let $i = 1$ to m . For all odd values of 'i', $f^*(v_i)$ and $f^*(v_{m+i+1}')$ receives the label '0' and is '1' if 'i' is even. Also $f^*(v_i')$ and $f^*(v_{m+i+1})$ takes the value '1' if 'i' is odd and is '0' if 'i' is even.

$f^*(v_{m+1})$ and $f^*(v_{2m+2}')$ receives the label '1' if 'm' is even and the label '0' if 'm' is odd.

$f^*(v_{m+1}')$ and $f^*(v_{2m+2})$ receives the label '0' if 'm' is even and the label '1' if 'm' is odd.

Thus the number of vertices labeled with '1' is $2m + 2$ '0' is $2m+2$.

Hence the number of vertices labeled with '1' and '0' differ by at most one.

Therefore $\text{EDG}(\text{comb})$ graph is product E-cordial.

The Extended duplicate graph of comb graph , $EDG(\text{comb})$, $m \geq 2$ is total product E-cordial.

Proof :

By theorem 2.3, the number of edges labeled with '1' and '0' are same as $2m + 2$. The number of vertices labeled with '1' and '0' are also same as $2m + 2$. Thus the total number of vertices and edges labeled with '1' is $4m+4$ and '0' is $4m+4$ respectively.

Hence the total number of vertices and edges labeled with ‘1’ and the total number of vertices and edges labeled with ‘0’ differ by at most one.

Therefore the extended duplicate graph of comb graph is total product E- cordial.

Example 2.2

Product E- cordial labeling of EDG(comb) graph for $m=5$ and $m=6$ are given in figure 3 and figure 4 respectively.

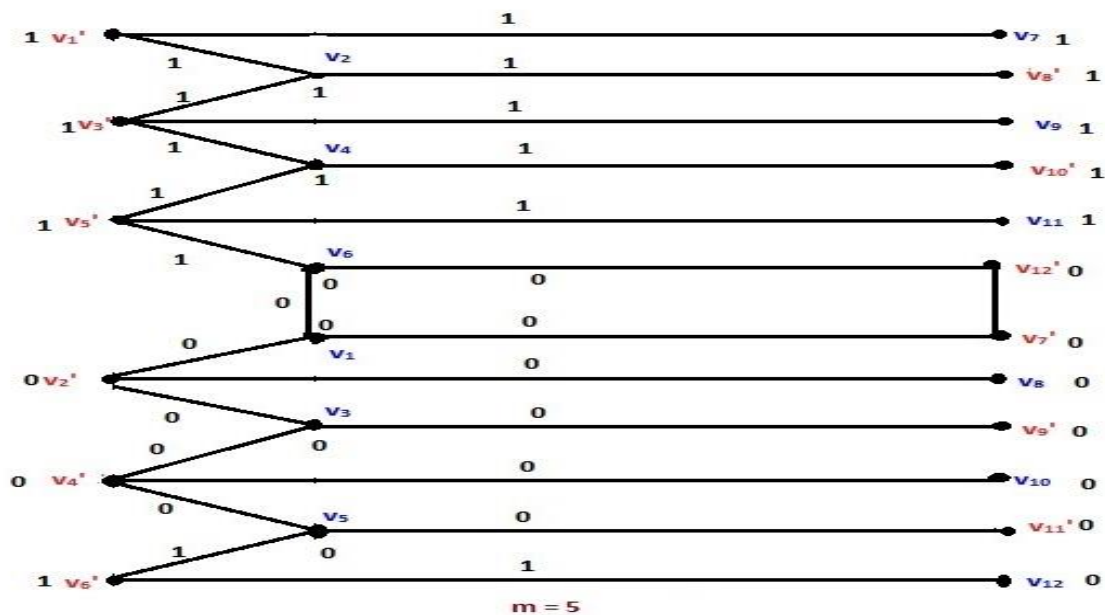


Figure : 3

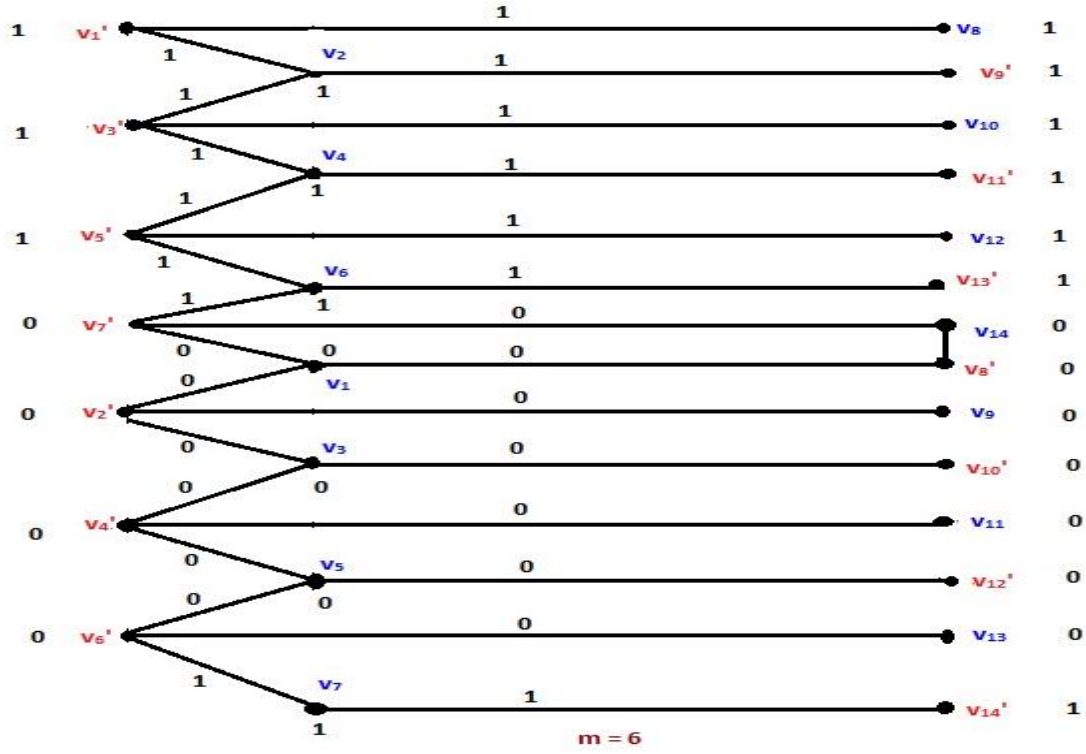


Figure : 4

Algorithm 2. 3

Procedure :(prime E-cordial labeling for extended duplicate graph of comb graph).

// assignment of labels to the edges

$v_1v_2' \leftarrow 4$; $v_1v_{m+2}' \leftarrow 6$

for $i = 2$ to m do

{ $v_i v_{i+1}' \leftarrow \begin{cases} 4i+2 & \text{if } i \equiv 1(\text{mod } 2) \\ 4i+3 & \text{otherwise} \end{cases}$

$v_i v_{m+i+1}' \leftarrow \begin{cases} 4i & \text{if } i \equiv 1(\text{mod } 2) \\ 4i+1 & \text{otherwise} \end{cases}$

}

end for

for $i = 1$ to m do

{ $v_i' v_{i+1} \leftarrow \begin{cases} 4i+3 & \text{if } i \equiv 1(\text{mod } 2) \\ 4i+2 & \text{otherwise} \end{cases}$

$$v_i v_{m+i+1} \leftarrow \begin{cases} 4i+1 & \text{if } i \equiv 1 \pmod{2} \\ 4i & \text{otherwise} \end{cases}$$

end for

if $m \equiv 0 \pmod{2}$

$$v_{m+1} v_{2m+2} \leftarrow 1, \quad v_1 v_{m+1}' \leftarrow 2, \quad v_{m+2} v_{2m+2} \leftarrow 3, \quad v_{m+1} v_{2m+2}' \leftarrow 4m+4$$

if $m \equiv 1 \pmod{2}$

$$v_{m+1} v_{2m+2}' \leftarrow 1, \quad v_1 v_{m+1} \leftarrow 2; \quad v_{m+2} v_{2m+2}' \leftarrow 3, \quad v_{m+1} v_{2m+2} \leftarrow 4m+4$$

end procedure

output : edge labeled extended duplicate graph of comb graph.

Theorem 2.5

The Extended duplicate graph of comb graph admits prime E- cordial labeling.

Proof :

EDG(comb) (V,E) graph has $4m+4$ vertices and $4m+4$ edges.

In order to label the edges , define a function $f: E \rightarrow \{1,2,3,\dots,4m+4\}$ as given in algorithm 2.3. Clearly all the $4m+4$ edges are labeled.

Let $V_1 = \{v_{m+2}, v_{m+3}, \dots, v_{2m+1}, v_{m+2}', v_{m+3}', \dots, v_{2m+1}'\}$ and $V_2 = V - V_1$.

The induced map $f^*: V \rightarrow N$ is defined such that,

$$\text{For all } v \in V_2, f^*(v) = \begin{cases} 1 & \text{if } \gcd(f(e)) = 1 \text{ where } e \in N(v) \\ 0 & \text{otherwise.} \end{cases}$$

$$\text{For all } v \in V_1, f^*(v) = \begin{cases} 0 & \text{if } f(e) \equiv 0 \pmod{2} \text{ where } e \in N(v) \\ 1 & \text{otherwise} \end{cases}$$

By the induced function , the vertex labels are obtained as follows:

Let $i = 1$ to $m+1$, $f^*(v_i')$ and $f^*(v_{m+i+1})$ receives the label '1' if 'i' is odd and '0' if 'i' is even. Also for all odd values of 'i' $f^*(v_i)$ and $f^*(v_{m+i+1}')$ takes the value '0' and is '1' if 'i' is even.

Thus the number of vertices labeled with '1' is $2m+2$ and '0' is also $2m+2$.

In both the cases, the number of vertices labeled with '1' and '0' is same as $2m+2$.

Hence the number of vertices labeled with '1' and '0' differ by at most '1'

Therefore EDG (comb) graph is prime E – cordial.

Example 2.3

Prime E- cordial labeling of EDG (comb) graph for $m=5$ and $m=6$ are given in figure 5 and figure 6 respectively.

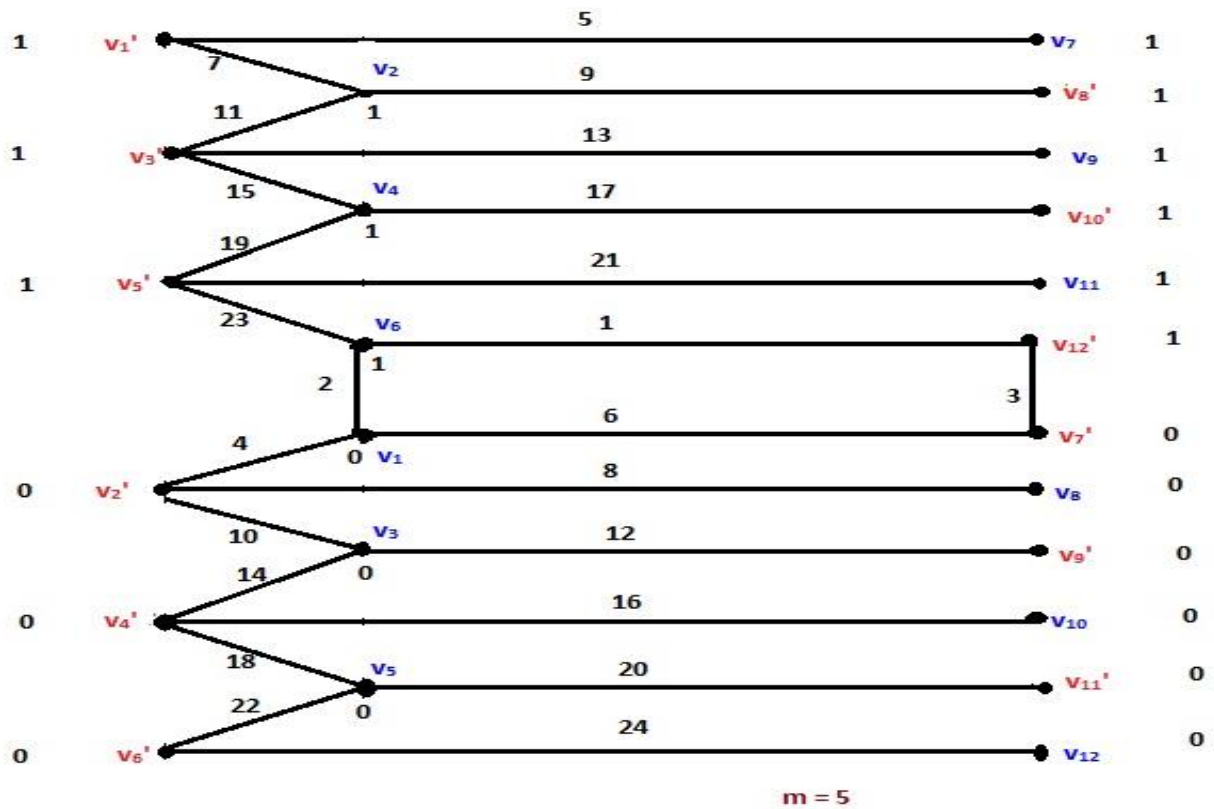


Figure : 5

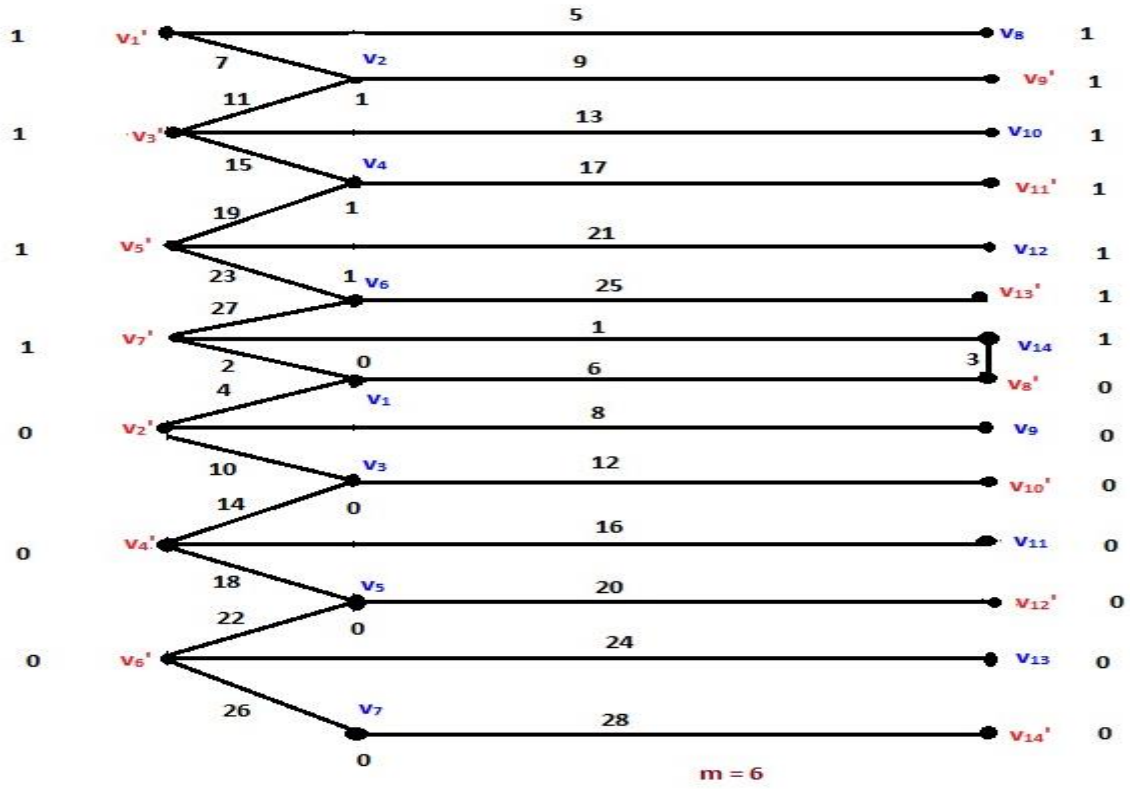


Figure : 6

Algorithm 2. 4

Procedure : (E_3 -cordial labeling for extended duplicate graph of comb graph).

// assignment of labels to the edges

$v_1 v_{m+2}' \leftarrow 0$, $v_1 v_2'$, $v_1' v_{m+2} \leftarrow 2$.

if $m = 2n$

$v_1 v_{m+1}'$, $v_{m+1}' v_{2m+2}$, $v_{m+2}' v_{2m+2} \leftarrow 0$

$v_{m+1} v_{2m+2}' \leftarrow 2$

$v_1' v_2 \leftarrow \begin{cases} 0 & \text{if } m = 3n+1 \\ 1 & \text{otherwise} \end{cases}$

$v_m v_{m+1}' \leftarrow \begin{cases} 0 & \text{if } m = 3n \\ 1 & \text{if } m = 3n+1 \\ 2 & \text{if } m = 3n-1 \end{cases}$

$$v_m'v_{m+1} \leftarrow \begin{cases} 0 & \text{if } m = 3n \\ 1 & \text{if } m = 3n-1 \\ 2 & \text{if } m = 3n+1 \end{cases}$$

for $i = 2$ to $m-1$ do

{

$$v_i v_{i+1}' \leftarrow \begin{cases} 0 & \text{if } i \in \{6n, 6n-1\} \\ 1 & \text{if } i \in \{6n-2, 6n-3\} \\ 2 & \text{if } i \in \{6n-4, 6n-5\} \end{cases}$$

$$v_i' v_{i+1} \leftarrow \begin{cases} 0 & \text{if } i \in \{6n, 6n-5\} \\ 1 & \text{if } i \in \{6n-4, 6n-3\} \\ 2 & \text{if } i \in \{6n-2, 6n-1\} \end{cases}$$

}

end for

for $i = 2$ to m do

$$\{ \quad v_i v_{m+i+1}' \leftarrow \begin{cases} 0 & \text{if } i \in \{6n, 6n-5\} \\ 1 & \text{if } i \in \{6n-4, 6n-1\} \\ 2 & \text{if } i \in \{6n-2, 6n-3\} \end{cases}$$

$$v_i' v_{m+i+1} \leftarrow \begin{cases} 0 & \text{if } i \in \{6n-2, 6n-3\} \\ 1 & \text{if } i \in \{6n-4, 6n-1\} \\ 2 & \text{if } i \in \{6n, 6n-5\} \end{cases}$$

}

end for

if $m = 2n + 1$

$$v_1 v_{m+1}, v_{m+1} v_{2m+2}', v_{m+2}' v_{2m+2}' \leftarrow 0$$

$$v_1' v_2 \leftarrow \begin{cases} 1 & \text{if } m = 3n+1 \\ 0 & \text{otherwise} \end{cases}$$

$$v_m' v_{m+1} \leftarrow \begin{cases} 1 & \text{if } m = 3n \\ 2 & \text{otherwise} \end{cases}$$

$$v_{m+1}v_{2m+2}' \leftarrow \begin{cases} 1 & \text{if } m = 3n+1 \\ 2 & \text{otherwise} \end{cases}$$

$$v_m v_{m+1}' \leftarrow \begin{cases} 0 & \text{if } m = 3n-1 \\ 1 & \text{if } m = 3n \\ 2 & \text{if } m = 3n+2 \end{cases}$$

for $i = 2$ to $m-1$ do

$$\{ \quad v_i v_{i+1}' \leftarrow \begin{cases} 0 & \text{if } i \in \{6n-1, 6n\} \\ 1 & \text{if } i \in \{6n-3, 6n-2\} \\ 2 & \text{if } i \in \{6n-4, 6n-5\} \end{cases}$$

$$v_i v_{i+1}' \leftarrow \begin{cases} 0 & \text{if } i \in \{6n, 6n-5\} \\ 1 & \text{if } i \in \{6n-3, 6n-4\} \\ 2 & \text{if } i \in \{6n-1, 6n-2\} \end{cases}$$

}

end for

for $i = 2$ to m do

$$\{ \quad v_i v_{m+i+1}' \leftarrow \begin{cases} 0 & \text{if } i \in \{6n, 6n-5\} \\ 1 & \text{if } i \in \{6n-1, 6n-4\} \\ 2 & \text{if } i \in \{6n-3, 6n-2\} \end{cases}$$

$$v_i v_{m+i+1}' \leftarrow \begin{cases} 0 & \text{if } i \in \{6n-3, 6n-2\} \\ 1 & \text{if } i \in \{6n-1, 6n-4\} \\ 2 & \text{if } i \in \{6n, 6n-5\} \end{cases}$$

}

end for

end procedure

output :edge labeled extended duplicate graph of comb graph.

Theorem 2.6

The Extended duplicate graph of comb graph, EDG (comb) graph $m \geq 2$, admits E_3 -cordial labeling.

Proof:

EDG (comb) (V, E) graph has $4m+4$ vertices and $4m+4$ edges.

To label the edges, define a function $f : E \rightarrow \{0, 1, 2\}$ as given in algorithm 2. 4.

Denote $m_i(f) = \{e \in E / f(e) = i\}$.

Clearly the edge labels are as follows:

If $m = 3n$, the number of edges labeled with 0 is $4m+6 / 3$ and labeled with 1 and 2 are same as $4m+3/3$.

If $m = 3n - 1$, the number of edges labeled with 0, 1 and 2 are same as $4m+4/3$.

If $m = 3n + 1$, the number of edges labeled with 1 is $4m+2 / 3$ and labeled with 0 and 2 are same as $4m+5/3$.

Now all the $4m+4$ edges are labeled.

Thus for all $i, j \in \{0, 1, 2\}$; $|m_i(f) - m_j(f)| \leq 1$.

The induced function $f^*: V \rightarrow \{0, 1, 2\}$ is defined such that,

$$f^*(v_i) = \{\sum f(v_i v_j) / v_i v_j \in E\} \pmod{3} \text{ for all vertex } v \in V$$

Denote $n_i(f^*) = \{v \in V: f^*(v) = i\}$.

Using the induced function, the vertex labels are obtained as follows:

$f^*(v_1)$ and $f^*(v_{m+2})$ receives the label '2' and $f^*(v_{m+2}')$ as '0'. If $m = (6n-1)$ or $(6n-2)$ or $(6n-3)$ $f^*(v_2)$ receives the value '0' otherwise it takes the value '1'.

For $i = 3$ to m , $f^*(v_i)$ is '0' if $i \in \{6n-1, 6n-4\}$ and is '1' if $i \in \{6n-3, 6n-2\}$. $f^*(v_i)$ takes the value '2' if $i \in \{6n, 6n-5\}$.

For $i = 2$ to $m-1$, $f^*(v_i')$ receives the label '0' if $i \in \{6n-3, 6n-2\}$ and receives 1

If $i \in \{6n-1, 6n-4\}$. Also $f^*(v_i')$ takes the value '2' if $i \in \{6n, 6n+1\}$.

For $i = 2$ to m , $f^*(v_{m+i+1})$ receives '0' and $f^*(v_{m+i+1}')$ receives '2' if $i \in \{6n-3, 6n-2\}$.

$f^*(v_{m+i+1})$ and $f^*(v_{m+i+1}')$ takes the value 1 if $i \in \{6n-1, 6n-4\}$ Also $f^*(v_{m+i+1})$ receives '2' and $f^*(v_{m+i+1}')$ receives '0' if $i \in \{6n, 6n+1\}$.

In case of $m \equiv 1 \pmod{2}$,

$f^*(v_{2m+2}')$ receives '0'.

If $m = 3n$, $f^*(v_{m+1})$ receives '1', $f^*(v_1')$ and $f^*(v_{2m+2})$ is 2; $f^*(v_{m+1}')$ and $f^*(v_m')$ is 0.

If $m = 3n - 1$, $f^*(v_{m+1})$, $f^*(v_1')$, $f^*(v_{2m+2})$, and $f^*(v_{m+1}')$ receives 2 and $f^*(v_m')$ receives '1'.

If $m = 3n + 1$, $f^*(v_{m+1})$ is 2; $f^*(v_1')$ and $f^*(v_{m+1}')$ receives '0'; $f^*(v_{2m+2})$ and $f^*(v_m')$ receives '1'.

In case of $m \equiv 0 \pmod{2}$

$f^*(v_{2m+2})$ is '0' and $f^*(v_{2m+2}')$ is '2'.

If $m = 3n$, $f^*(v_m')$ and $f^*(v_{m+1})$ takes the value '2' and $f^*(v_1')$ and $f^*(v_{m+1}')$ is '0'.

If $m = 3n - 1$, $f^*(v_m')$ is 1 ; $f^*(v_{m+1})$ and $f^*(v_1')$ receives '0' and $f^*(v_{m+1}')$ is '2'.

If $m = 3n + 1$, $f^*(v_m')$ is 0 and $f^*(v_{m+1})$ and $f^*(v_{m+1}')$ receives the label '1' and $f^*(v_1')$ is '2'.

Thus we have ,

For $m = 3n$, the number of vertices labeled with 0 is $4m+6 / 3$ and labeled with 1 and 2 are same as $4m+3/3$.

For $m = 3n - 1$, the number of vertices labeled with 0 ,1 and 2 are same as $4m+4/3$.

For $m = 3n + 1$, the number of vertices labeled with 1 is $4m+2 / 3$ and labeled with 0 and 2 are same as $4m+5/3$.

Thus for all $i, j \in \{0, 1, 2\}$, $|n_i(f^*) - n_j(f^*)| \leq 1$.

Hence EDG (comb) graph admits E_3 – cordial labeling.

Theorem 2.7

The extended duplicate graph of comb graph admits total E_3 – cordial labeling.

Proof :

In theorem 2.6, the number of edges labeled with '0' is $4m+6 / 3$ and labeled with '1' and '2' are same as $4m+3 / 3$ for the case $m = 3n$. The number of edges labeled with 0 , 1 , 2 are same as $4m+4/3$ for the case $m = 3n - 1$. The number of edges labeled with '1' is $4m+2/3$ and '0' and '2' are same as $4m+5/3$. It was shown that the number of vertices labeled with '0' is $4m+6 / 3$ and labeled with '1' and '2' are same as $4m+3 / 3$ for the case $m = 3n$. The number of vertices labeled with 0 , 1 , 2 are same as $4m+4/3$ for the case $m = 3n - 1$. The number of vertices labeled with '1' is $4m+2/3$ and '0' and '2' are same as $4m+5/3$ and satisfies for all $i, j \in \mathbb{Z}_3$ $|\{m_i(f) + n_i(f^*)\} - \{m_j(f) + n_j(f^*)\}| \leq 1$.

Hence the extended duplicate graph of comb graph is total E_3 – cordial.

Example 2.4

E_3 -cordial labeling of EDG(comb) graph for $m=5$ and $m=6$ are given in figure 7 and figure 8 respectively.

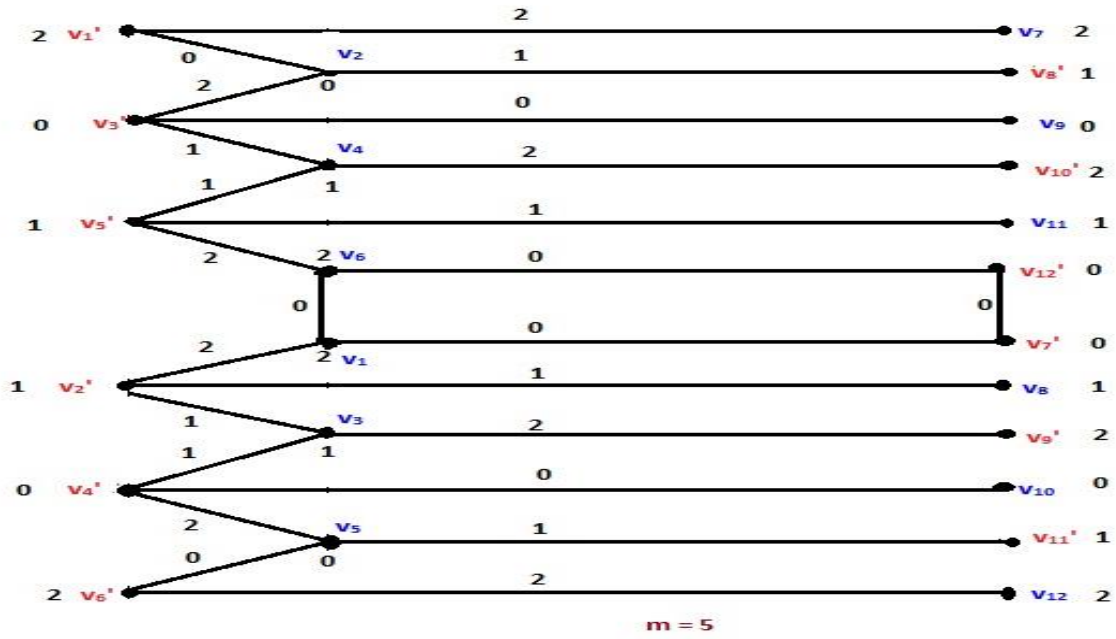


Figure : 7

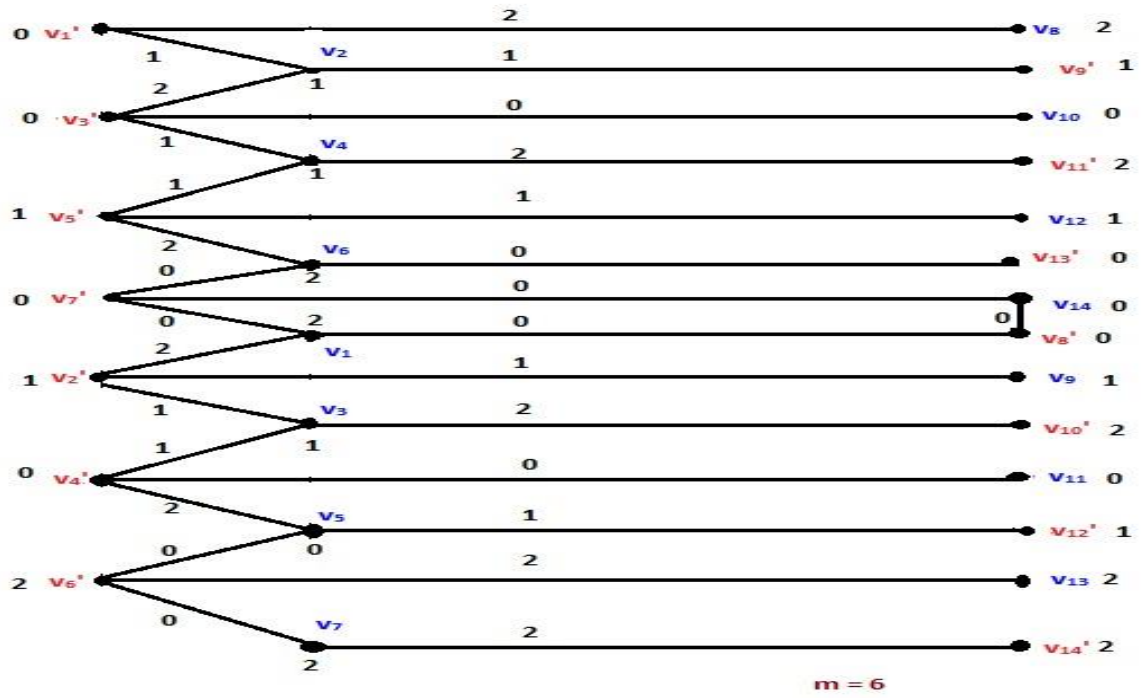


Figure : 8

Conclusion:

In this paper we have presented algorithms and proved that the extended duplicate graph of Comb graph admits E- cordial, total E- cordial, product E- cordial ,total product E-cordial, prime E- cordial, E_3 cordial, and total E_3 cordial labeling.

References:

1. Bala . E and Thirusangu . K. *E- cordial and Z_3 - Magic labeling in extended triplicate graph of the path*. General Mathematics Notes (GMN). (2011); 5; No. 2. 8-23
2. Bapat M.V and Limaye N.B, Edge- three cordial graphs arising from complete graphs, J.Combin.Math.Combin.Comput.,56(2006)147-169.
3. Bondy J.A and Murthy U.S.R. *Graph theory with applications* 1976.
4. Cahit . I and Yilmaz .R, E_3 -cordial graphs, Ars combin.,54(2000)119-127.
5. Gallian J.A. *A Dynamic survey of graph labeling. The Electronic Journal of combinatorics*.(2014); 17: # DS6.
6. Rosa. A, on certain valuations of a graph, Theory of graphs, International symposium, Rome, Gordon and Breach N.Y. at Dunod Paris, (1967): 349-355.
7. Selvam .B, Thirusangu .K and Ulaganathan P.P, On E_3 - cordial and total E_3 - cordial labelings for a New class of graphs, proceedings of the National conference on applications of mathematics and computer science at S.D.N.B. Vaishnav college, Chennai, India; (2012);115-118.
8. Singh G.S and Varkey T.K.M, Some results on prime labeling of graphs, preprint.
9. Tout .A, Daaboucy . A.N and Howalla . K, Primegraphs,Nat.Acad.Sci.Letters,11(1982) 365-368.
- 10 Yilmaz. R and Cahit. I. *E-Cordial graphs*. Ars Combin.1997; 46:251-266..