

A De-noising Algorithm of Wavelet Threshold Based on a New Threshold Function

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ABSTRACT

Hard threshold function is discontinuity in the threshold point, and soft threshold function have a constant error between the estimated coefficient of wavelet and the original coefficient of wavelet. To solve this problem, an improved threshold function is presented, which is continuous and high-order-differential in the threshold point. Simulation results show that this function is better than the other two functions.

Keywords: Wavelet threshold; Image de-noising; Improved methods; threshold function;

1. INTRODUCTION

Image signal is often mingled with noise in transmission process, which has impact to the subsequent signal processing in a way, so image signal de-noising is very meaningful. Common de-noising methods are mathematical morphological method, adaptive smoothing, random method, filter method, wavelet de-noising method. Because wavelet has characteristics of low entropy, multi-resolution, decorrelation and flexibility of the selection, wavelet de-noising has become the most widely used method in the application [1,13]. Wavelet de-noising method is generally divided into three categories: the first method is correlation de-noising method[2], that is, to calculate the correlation of the wavelet coefficients between adjacent scales, then eliminate unqualified coefficients, finally

reconstruct the signal; The second method is wavelet modulus maxima de-noising method [3]. The propagation characteristics of signal and noise are different in each scale of wavelet transform, so as long as remove the modular maximum value from noise, and keep the corresponding modulus maxima of signal, and then reconstruct the wavelet coefficients, we can recover signal and remove noise. The third category is wavelet threshold method [4]. The wavelet coefficients of signal contain important information and its amplitude is large, but the number of wavelet coefficients is less. Instead, the wavelet coefficients of noise are uniformly distributed and its amplitude is low, but the number of wavelet coefficients is major. So if the absolute value of the smaller coefficient is set to zero, then retain the absolute value of larger coefficient, and reconstruct signal to de-noising.

Nowadays, there are many traditional methods in the field of image de-noising. Traditional methods can filter noise, but at the same time they make the image details blurred. Therefore, it is significant to further improve the existing method. Different algorithms have different mathematical basis, and the effect of image de-noising is also different. By studying the different algorithms to improve the de-noising effect is very meaningful too. Based on the above situation, this paper mainly studied the wavelet threshold de-noising method.

The main structures of this article are as follows: the principle of wavelet threshold de-noising will be introduced in second section. In third section, the method of hard threshold and soft threshold method will be presented, and their disadvantages will be discussed. In order to solve the problems existing in traditional threshold function, an improved threshold function is presented. MATLAB simulation experiment is used to prove the effectiveness and superiority of improved function in fourth section. The fifth section is the summary.

2. WAVELET THRESHOLD DE-NOISING

Wavelet threshold method is a typical noise suppression method based on non-parametric model. After wavelet decomposition, the energy of image itself is mainly distributed in the low resolution scale function, and the energy of noise is still distributed evenly in the low resolution scale function and all the wavelet coefficients. In the transformational domain, the spatial correlation of image is reduced and energy is more concentrated, and energy distribution of noise is constant. Based on this feature, wavelet de-noising technology retains or shrinks large wavelet coefficients and remove small wavelet coefficients, which called the wavelet threshold method.

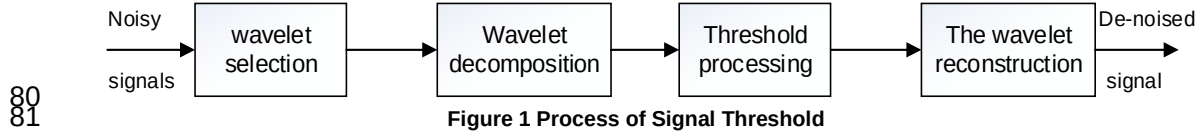
Signal based on wavelet threshold de-noising process can be divided into the following steps[5-7]:

Step1. Select the appropriate wavelet basis and wavelet decomposition layers, the calculation of wavelet decomposition coefficient with noise signal.

Step2. Select the appropriate threshold, for the high frequency coefficient threshold processing.

Step3. According to the processed wavelet coefficients reconstruct signal.

79 The specific processes are as shown in Figure 1.



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83

84 2.1 Traditional threshold function

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86 It is one of the most important problems in wavelet de-noising algorithm that how to select
87 threshold function, which is related to the quality of signal de-noising. The traditional
88 threshold de-noising method can be divided into the hard-threshold de-noising method and
89 the soft-threshold de-noising method. The expression is shown as following.

90

91 The expression of hard-threshold function is

92

$$93 \quad \hat{w}_{j,k} = \begin{cases} w_{j,k} & |w_{j,k}| \geq \lambda \\ 0 & |w_{j,k}| < \lambda \end{cases} \quad (2.1)$$

94

95 The expression of soft-threshold function is

96

$$97 \quad \hat{w}_{j,k} = \begin{cases} \text{sgn}(w_{j,k})[|w_{j,k}| - \lambda] & |w_{j,k}| \geq \lambda \\ 0 & |w_{j,k}| < \lambda \end{cases} \quad (2.2)$$

98

99 Where $w_{j,k}$ stands for the wavelet coefficients, $\hat{w}_{j,k}$ stands for the wavelet coefficients after
100 treatment. λ is the threshold .

101

102 Although hard and soft threshold de-noising method proposed has been widely used in the
103 actual process of image de-noising, but it is still insufficient[8-9]. In the hard threshold
104 method, the wavelet coefficients processed by the threshold are discontinuous at the two

105 points λ and $-\lambda$, therefore, which may lead to the reconstructed signal prone to
106 Pseudo-Gibbs phenomenon and a certain degree of oscillation will directly lead to the
107 increasing error. In soft threshold function, the continuity of estimated wavelet coefficients is
108 good on the whole and doesn't produce additional shocks, but there is a constant deviation
109 between $\hat{w}_{j,k}$ and $w_{j,k}$ when $|\hat{w}_{j,k}| \geq \lambda$. This will cause a deviation between the
110 reconstructed signal and the real signal. In addition, the derivative of soft threshold function is
111 discontinuous. But in actual applications, it is often processed by the first derivative and even
112 higher order derivative, so soft threshold function has some limitations.

113

114 2.2 Threshold selection

115

116 The selection of wavelet threshold is related to the effect of de-noising. If the selection of
117 threshold value is too low, the de-noising effect is not ideal; On the contrary, if the selection of
118 threshold is too higher, some important edge detail characteristics of image will be filtered to
119 remove while removing noise and the image is too smooth[10].

120

121 The expression of Traditional threshold selection function (see [11]) is

122

$$123 \quad \lambda = \sigma \sqrt{2 \log N} \quad (2.3)$$

124

125 Where σ is the standard deviation of the noise, and N stands for image size.

126

127

128 3. THE IMPROVED-THRESHOLD FUNCTION

129

130 Soft and hard threshold de-noising methods, although it has been widely applied and
131 achieved well results in practice, but these two kinds of method itself have some potential
132 disadvantages. For example, high-order-differential does not exist in the soft threshold
133 function at the threshold point, the hard threshold function is not continuous at the threshold
134 point, and these will have a certain impact on the de-noising effect. In order to overcome the
135 shortcomings of the two methods above, an improved method based on them is proposed.
136 The function can be adjusted by 3 parameters, which can effectively reduce the constant
137 error between this signal and the original signal. It not only inherits the advantages of the soft
138 and hard threshold, but also overcomes their shortcomings, and achieves the desired
139 purpose.

140

141 Its function is as followed:

142

$$143 \quad \hat{w}_{j,k} = \begin{cases} \operatorname{sgn}(w_{j,k})[a(|w_{j,k}| - \lambda + \frac{\lambda}{2\beta+1}) + (1-a)\sqrt{w_{j,k}^2 - \lambda^2}] + b \sin(\frac{\pi}{2\lambda} w_{j,k}) & |w_{j,k}| \geq \lambda \\ 0 & |w_{j,k}| < \lambda \end{cases} \quad (3.1)$$

144 Where, a 、 β and b are variable parameters, the function is continuous at the threshold, and

145 the high order derivative of the function is continuous when $|w_{j,k}| > \lambda$. $\hat{w}_{j,k}$ will be more and

146 more close to $w_{j,k}$ when $|w_{j,k}|$ continues to increase.

147

Proof:

148

149 (1) If $w_{j,k} = \lambda$, we get

$$\begin{aligned} \hat{w}_{j,k} &= \operatorname{sgn}(w_{j,k})[a(|w_{j,k}| - \lambda + \frac{\lambda}{2\beta+1}) + (1-a)\sqrt{w_{j,k}^2 - \lambda^2}] + b \sin(\frac{\pi}{2\lambda} w_{j,k}) = \frac{a\lambda}{2\beta+1} + b \\ 150 \quad &= \hat{w}_{j,k} \Big|_{w_{j,k} \rightarrow \lambda} = \lim_{w_{j,k} \rightarrow \lambda} [\frac{a(w_{j,k})^{2\beta+1}}{(2\beta+1)\lambda^{2\beta}} + b \sin(\frac{\pi}{2\lambda} w_{j,k})] = \frac{a\lambda}{2\beta+1} + b \end{aligned}$$

151

152 This function is continuous at λ , and the same can be proved at $-\lambda$, that is, the function is
153 continuous at the threshold.

154

155 (2) If $w_{j,k} > \lambda$, we have

$$\begin{aligned} 156 \quad (\hat{w}_{j,k})' &= \{\operatorname{sgn}(w_{j,k})[a(w_{j,k} - \lambda + \frac{\lambda}{2\beta+1}) + (1-a)\sqrt{w_{j,k}^2 - \lambda^2}] + b \sin(\frac{\pi}{2\lambda} w_{j,k})\}' \\ &= a + \frac{(1-a)w_{j,k}}{\sqrt{w_{j,k}^2 - \lambda^2}} + \frac{b\pi}{2\lambda} \cos(\frac{\pi}{2\lambda} w_{j,k}) \end{aligned}$$

157

158 The derivative of the function is continuous when $w_{j,k} > \lambda$, and the same can be proved

159 when $w_{j,k} < -\lambda$, that is, the derivative of the function is continuous when $|w_{j,k}| > \lambda$. Obviously

160 the high order derivative of the function is continuous when $|w_{j,k}| > \lambda$.

161 (3)

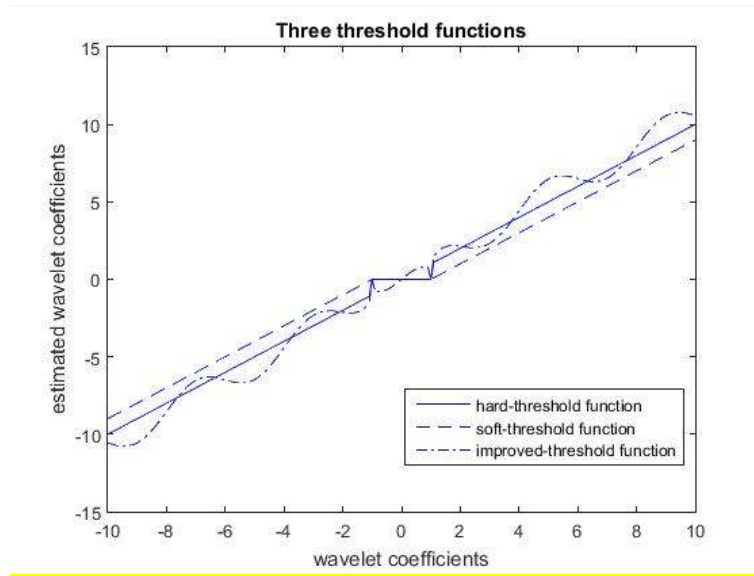
$$\begin{aligned}
 \lim_{w_{j,k} \rightarrow +\infty} \frac{\hat{w}_{j,k}}{w_{j,k}} &= \lim_{w_{j,k} \rightarrow +\infty} \frac{a(w_{j,k} - \lambda + \frac{\lambda}{2\beta+1}) + (1-a)\sqrt{w_{j,k}^2 - \lambda^2} + b \sin(\frac{\pi}{2\lambda} w_{j,k})}{w_{j,k}} \\
 &= \lim_{w_{j,k} \rightarrow +\infty} [a + (1-a) \frac{\sqrt{w_{j,k}^2 - \lambda^2}}{w_{j,k}} + \frac{b}{w_{j,k}} \sin(\frac{\pi}{2\lambda} w_{j,k})] \\
 &= \lim_{w_{j,k} \rightarrow +\infty} [a + (1-a) \sqrt{1 - \frac{\lambda^2}{w_{j,k}^2}} + 0] = 1
 \end{aligned}$$

163 $\hat{w}_{j,k}$ is equal to $w_{j,k}$ when $|w_{j,k}|$ tend to ∞ .

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Figure 2 Three kinds of threshold function curves

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171 Figure 2 is the hard threshold function, soft threshold function and the improved threshold
 172 function of the curve. Where, $\lambda = 1, \beta = 1, a = 2, b = 1$.

173

174 As we can see from figure2, hard-threshold function does not have continuity in the threshold
 175 point, high-order-differential doesn't exist in soft-threshold function at the threshold point, but
 176 the improved threshold function is continuous and high-order-differential at the threshold
 177 point , which overcomes the shortcomings of the traditional threshold function, and achieves
 178 the desired results.

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182 4. Experimental Results

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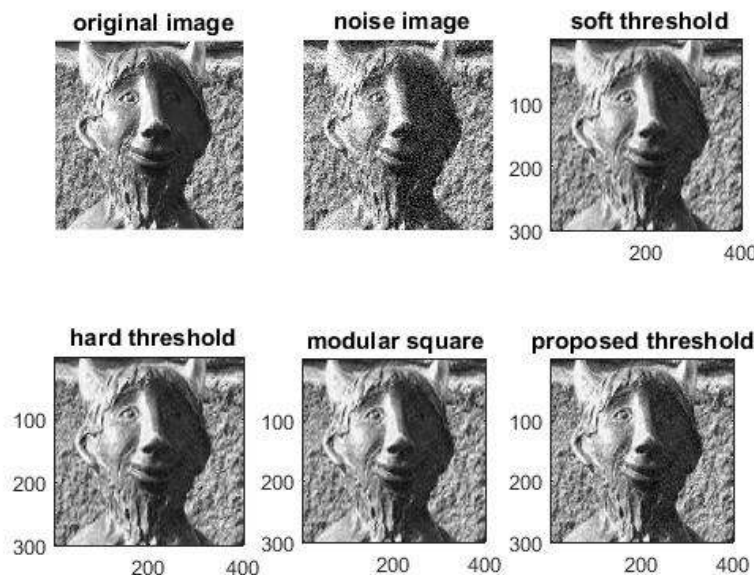
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185 In order to show the new threshold function de-noising effect, this article used the hard
 186 threshold function, soft threshold function , modular square function (see [12])and the
 187 improved threshold function based on Matlab to remove the noise of images of devil,
 188 Catherine and wflower which contained 0.1 Gaussian noise by simulation experiments.We all
 189 choose "rbio6.8" wavelet to 2 layer wavelet decomposition that contain noise image, and
 190 $\lambda = \sqrt{2 \log N}, \beta = 1$.

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192

193 Experimental results are shown in Figure 3, 4, 5. The data in experimental simulation are
 shown in Table 1.

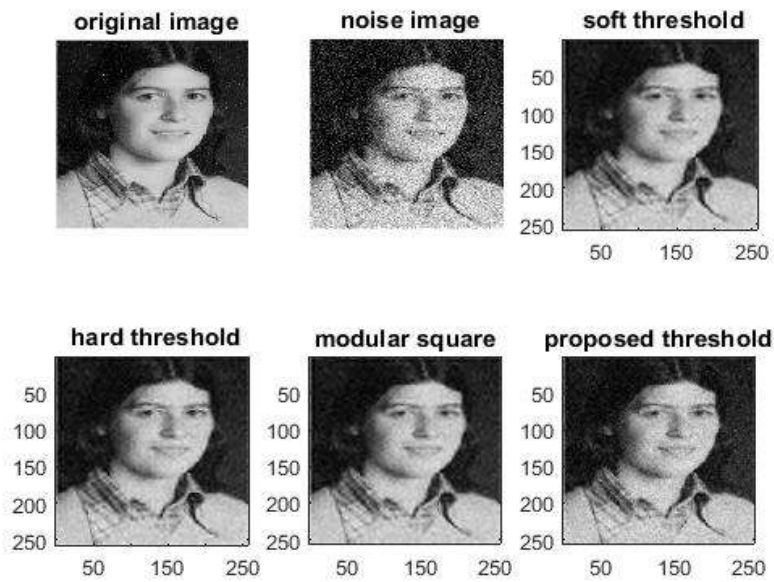


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195 Figure 3 devil de-noising effect chart($a=-0.8, \beta=1, b=20$)

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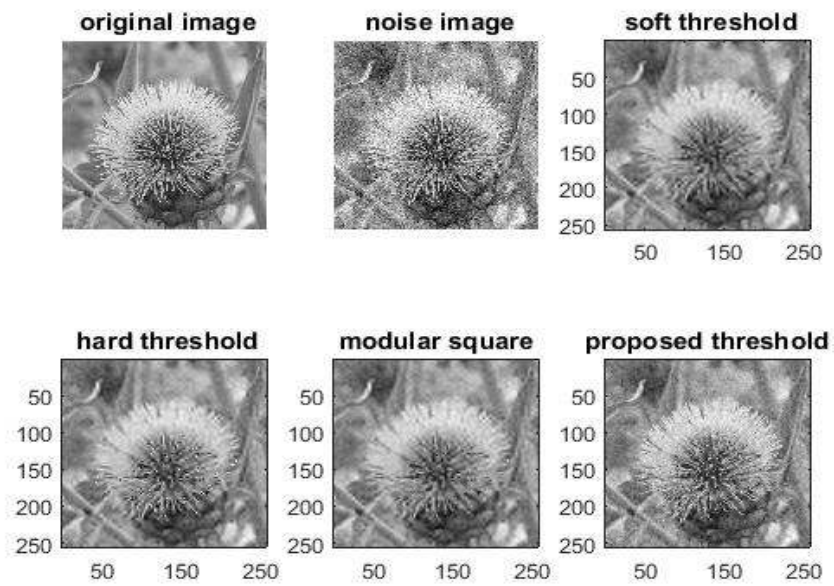
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Figure 4 Catherine de-noising effect chart($a=-0.8$, $\beta=1$, $b=7$)



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Figure 5 wflower de-noising effect chart($a=-1.8$, $\beta=1$, $b=22$)

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Table 1 . Comparison of four kinds of threshold function de-noising

date	signal	soft threshold	hard threshold	modular square	proposed threshold
MSE	devil	402.2432	383.8176	385.3568	243.7073
	Catherine	195.6408	191.4148	190.9599	155.1403
	wflower	482.2878	445.7197	453.2064	270.6761
PSNR	devil	22.0859	22.2896	22.2722	24.2621
	Catherine	25.2162	25.3110	25.3214	26.2236
	wflower	21.2977	21.6402	21.5678	23.8063

205

206 The devil signal β value de-noising effect comparison are show in Table 2

207

β	0.5	1	1.5	2	2.5	3
MSE	456.2730	243.7073	308.3281	280.6986	308.4620	296.9577
PSNR	21.5386	24.2621	23.2336	23.6884	23.2388	23.4039

208 **Table 2. β comparison of the parameters**

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210

211 The devil signal α value de-noising effect comparison are show in Table 3

212

213 **Table 3. α comparison of the parameters**

α	-1.2	-1	-0.8	-0.6	-0.4	-0.2
MSE	245.6282	245.3365	243.5469	249.7632	250.0588	251.7874
PSNR	22.2280	24.2332	24.2650	24.1555	24.1504	24.1205

214

215 The devil signal b value de-noising effect comparison are show in Table 4

216

217 **Table4. b comparison of the parameters**

b	14	17	20	23	26	29
MSE	253.6766	246.7555	244.3815	245.6854	247.4836	248.0187
PSNR	24.0880	24.2081	24.2501	24.2270	24.1953	24.1860

218

219 As we can see from Figure 3~5, the new threshold function image on the visual effect is
220 better than soft threshold function, hard threshold function and modular square function. This
221 function preserves more details of original images too. Table 1 quantitatively describes the
222 superiority. The new threshold function has been improved by 8.97% compared to the other
223 functions when de-noising signal is devil. The new threshold function has been improved by
224 3.84% compared to the other functions when de-noising signal is Catherine. And the new
225 threshold function has been improved by 10.5% compared to the other functions when
226 de-noising signal is wflower. It can be very intuitive to get that the PSNR is the highest, and
227 the MSE is the least.

228

229 Through table 2, we can see that the PSNR is the highest, and the MSE is the least when β
230 is equal to 1. That is to say, the optimal values of the parameters of the devil signal is $\beta = 1$. In
231 the same way, we can know that the optimal values of the other two parameters of the devil
232 signal is $a = -0.8, b = 20$. In a word, we can determine the optimal values of the three
233 parameters of the devil signal is $a = -0.8, \beta = 1, b = 20$.

234

235 In summary, new threshold function is not only effectively removed Gaussian noise, but also
236 is better to keep the image edge information, improving the peak signal to noise ratio of the
237 image.

238

239 5. CONCLUSION

240

241 A novel wavelet threshold de-noising function proposed in this article which can adjust its
242 parameter. Through mathematical analysis, we demonstrate the superiority of this
243 function. And the experimental results show that the de-noising effect of this function is better
244 than the soft threshold function, the hard threshold function and the modular square
245 function. Therefore, the improved method is a practical and effective method to remove the
246 noise of the images.

247

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