

COVARIANT ALMOST ANALYTIC VECTOR FIELD ON Q -QUASI UMBILICAL HYPERSURFACE OF A TRANS SASAKIAN MANIFOLD

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ABSTRACT. The present paper aims to study the axioms of covariant almost analytic vector field on Q -quasi umbilical hypersurface M of a trans-Sasakian manifold $\bar{M}$ with structure (ϕ, g, u, v, λ) and obtained the scalars a and b using 1-forms u, v covariant almost analytic for the hypersurface M to be totally umbilical and cylindrical.

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1. Introduction

The study of hypersurfaces with almost contact structures are important parts of differential geometry. It was 1967, M. Okumura [7] studied the totally umbilical hypersurfaces of a product Riemannian manifold and hypersurface of an almost r -paracontact Riemannian manifold studied by A. Bucki [2]. In 1970, S. I. Goldberg and K. Yano [3] studied the geometry of non-invariant hypersurfaces of almost contact manifolds. D. E. Blair [1] discussed the case of almost contact manifolds with Killing structure tensor. In 1989, D. Narain studied differential geometry of hypersurfaces with (f, g, u, v, λ) -structures [4]. Moreover, D. Narain and others also studied the non-invariant hypersurfaces of Sasakian, nearly Sasakian and para Sasakian manifold in [5,6,10] respectively. After that R. Prasad and M. M. Tripathi [9] studied the transversal hypersurfaces of Kenmotsu manifolds. In 1985, J. A. Oubina introduced a new class of almost contact metric manifold known as trans-Sasakian manifold [8]. This class contains α -Sasakian and β -Kenmotsu manifold. Thus motivated by the studies referred above in the present paper, we have studied the covariant almost analytic vector field on Q -quasi umbilical hypersurface of a trans-Sasakian manifold.

The present paper is organized as follows: In section 2, we give the brief introduction about the trans-Sasakian manifold. In section 3, the hypersurfaces of a trans-Sasakian manifold with (ϕ, g, u, v, λ) -structure have been discussed. In section 4, we express the Q -quasi umbilical hypersurface and in section 5, we obtained some results on covariant almost analytic vector field on Q -quasi umbilical hypersurface of a trans-Sasakian manifold.

2. PRELIMINARIES

Let $\bar{M}$ be a $(2n+1)$ dimensional manifold with almost contact metric structure $(\bar{\phi}, \bar{\xi}, \eta, \bar{g})$, where $\bar{\phi}$ is a $(1,1)$ tensor field, $\bar{\xi}$ is a vector field, η is a 1-form and $\bar{g}$ is a compatible Riemannian metric such that

$$(2.1) \quad \bar{\phi}^2 = -I + \eta \otimes \bar{\xi}, \quad \bar{\phi}\bar{\xi} = 0, \quad \eta \circ \bar{\phi} = 0, \quad \eta(\bar{\xi}) = 1,$$

$$(2.2) \quad \bar{g}(\bar{\phi}X, \bar{\phi}Y) = \bar{g}(X, Y) - \eta(X)\eta(Y),$$

$$(2.3) \quad \bar{g}(X, \bar{\phi}Y) = -\bar{g}(\bar{\phi}X, Y), \quad \bar{g}(X, \bar{\xi}) = \eta(X).$$

An almost contact metric structure $(\bar{\phi}, \bar{\xi}, \eta, \bar{g})$, is called an trans Sasakian manifold if

$$(2.4) \quad (\bar{\nabla}_X \bar{\phi})(Y) = \alpha(\bar{g}(X, Y)\bar{\xi} - \eta(Y)X) + \beta(\bar{g}(\bar{\phi}X, Y)\bar{\xi} - \eta(Y)\bar{\phi}X)$$

and it follows

$$(2.5) \quad \bar{\nabla}_X \bar{\xi} = -\alpha\bar{\phi}X + \beta(X - \eta(X)\bar{\xi}),$$

where $\bar{\nabla}$ denotes the Riemannian connection of the Riemannian metric $\bar{g}$, then the structure $(\bar{\phi}, \bar{\xi}, \eta, \bar{g})$ is called a trans-Sasakian manifold.

If we define $G(X, Y) = \bar{g}(\bar{\phi}X, Y)$, then we also find

$$(2.6) \quad G(X, Y) + G(Y, X) = 0,$$

$$(2.7) \quad G(X, \bar{\phi}Y) = G(Y, \bar{\phi}X),$$

$$(2.8) \quad G(\bar{\phi}X, \bar{\phi}Y) = G(X, Y).$$

3. HYPERSURFACE OF A TRANS-SASAKIAN MANIFOLD WITH (ϕ, g, u, v, λ) -STRUCTURE

Let us consider a $2n$ -dimensional manifold M embedded in $\bar{M}$ with embedding $i : M \rightarrow \bar{M}$. The map i induces a linear transformation map B , $B : T_p \rightarrow T_{i(p)}$. Let an affine unit normal N of M is in such way that $\bar{\phi}N$ is always tangent to the hypersurface and satisfying the linear transformations

$$(3.1) \quad \bar{\phi}BX = B\phi X + u(X)N,$$

$$(3.2) \quad \bar{\phi}N = -BU,$$

$$(3.3) \quad \xi = BV + \lambda N,$$

$$(3.4) \quad \eta(BX) = v(X),$$

where ϕ is $(1,1)$ type tensor; U, V are vector fields; u, v are 1-form and λ is a C^∞ -function. If $u \neq 0$, then M is called a non-invariant hypersurface of $\bar{M}$.

Operating (3.1), (3.2), (3.3) and (3.4) by $\bar{\phi}$ and using (2.1), (2.2), and (2.3) and taking tangent and normal parts separately, we get the following induced structures on M

$$(3.5) \quad \phi^2 X = -X + u(X)U + v(X)V,$$

$$(3.6) \quad u(\phi X) = \lambda v(X), \quad v(\phi X) = -\eta(N)u(X),$$

$$(3.7) \quad \phi U = -\eta(N)V, \quad \phi V = \lambda U,$$

$$(3.8) \quad u(U) = 1 - \lambda\eta(N), \quad u(V) = 0,$$

$$(3.9) \quad v(U) = 0, \quad v(V) = 1 - \lambda\eta(N)$$

and from (2.2) and (2.3), we get the induced metric g on M .

$$(3.10) \quad g(\phi X, \phi Y) = g(X, Y) - u(X)u(Y) - v(X)v(Y),$$

$$(3.11) \quad g(U, X) = u(X), \quad g(V, X) = v(X).$$

If we consider $\eta(N) = \lambda$, we get the following structures on M

$$(3.12) \quad \phi^2 = -I + u \otimes U + v \otimes V,$$

$$(3.13) \quad \phi U = \lambda V, \quad \phi V = \lambda U,$$

$$(3.14) \quad u \circ \phi = \lambda v, \quad v \circ \phi = -\lambda u,$$

$$(3.15) \quad u(U) = 1 - \lambda^2, \quad u(V) = 0,$$

$$(3.16) \quad v(U) = 0, \quad v(V) = 1 - \lambda^2.$$

A manifold M with a metric g satisfying (3.10), (3.11) and (3.12) is called manifold with (ϕ, g, u, v, λ) -structure. Let ∇ be the induced connection on the hypersurface M of the affine connection $\bar{\nabla}$ of $\bar{M}$.

Now the Gauss and Weingarten equations are given respectively by

$$(3.17) \quad \bar{\nabla}_{BX}BY = B\nabla_XY + h(X, Y)N,$$

$$(3.18) \quad \bar{\nabla}_{BX}N = BH X + w(X)N, \quad \text{where } g(HY, Z) = h(Y, Z).$$

Here h and H are the second fundamental tensor of type $(0, 2)$ and $(1, 1)$ and w is a 1-form.

Now differentiating (3.1), (3.2), (3.3) and (3.4) covariantly and using (3.17), (3.18) and (2.4) and again re-using (3.1), (3.2), (3.3) and (3.4), we get

$$(3.19) \quad (\nabla_Y \phi)(X) = \alpha \{v(X)Y - g(X, Y)V\} + \beta \{\lambda g(\phi X, Y)V - v(Y)\phi X\} \\ - h(X, Y)U - u(X)HY,$$

$$(3.20) \quad (\nabla_Y u)(X) = \alpha \lambda g(X, Y) + \beta \{\lambda g(\phi X, Y) - u(X)v(Y)\} + g(X, \phi Y) - h(X, \phi Y),$$

$$(3.21) \quad (\nabla_Y v)(X) = \alpha g(\phi Y, X) - \beta \{\lambda g(Y, X) - u(X)u(Y) - v(X)v(Y)\} + \lambda h(X, Y),$$

$$(3.22) \quad \nabla_Y U = \phi HY + \beta w(Y)U - \alpha \lambda Y,$$

$$(3.23) \quad \nabla_Y V = \alpha \phi Y + \lambda HY,$$

$$(3.24) \quad h(Y, V) = \alpha u(Y) + \beta \lambda w(X),$$

$$(3.25) \quad h(Y, U) = u(HY) + w(X).$$

Since $h(X, Y) = g(HX, Y)$, then from (2.3) and (3.25), we get

$$(3.26) \quad h(Y, U) = 0 \Rightarrow HU = 0.$$

4. Q -QUASI UMBILICAL HYPERSURFACE

M is called Quasi-umbilical hypersurface if

$$(4.1) \quad h(X, Y) = ag(X, Y) + bq(X)q(Y),$$

where a, b are scalar functions, q is a 1-form and if $g(Q, X) = q(X)$, where Q is a vector field, then M is called Q -quasi umbilical hypersurface. If $a = 0, b \neq 0$, then Q -quasi umbilical hypersurface M is called *cylindrical hypersurface*. If $a \neq 0, b = 0$, then Q -quasi-umbilical hypersurface M is called *totally umbilical* and if $a = 0, b = 0$, then Q -quasi umbilical hypersurface is *totally geodesic*. By using (4.1) in (3.19)-(3.25), we get the following:

$$(4.2) \quad (\nabla_Y \phi)(X) = \alpha \{v(X)Y - g(X, Y)V\} + \beta \{g(\phi X, Y)V - v(Y)\phi X\} \\ - \{ag(X, Y) + bq(X)q(Y)\}U - u(Y) \{aY + bq(Y)Q\},$$

$$(4.3) \quad (\nabla_Y u)(X) = \{(1 + a)g(\phi X, Y)\} - \beta \{\lambda g(\phi X, Y) - u(X)w(Y)\} \\ + \alpha \lambda g(X, Y) + bq(X)q(Y),$$

$$(4.4) \quad (\nabla_Y v)(X) = \alpha g(\phi Y, X) - \beta \{\lambda g(Y, X) - u(X)u(Y) - v(X)v(Y)\} \\ + \lambda \{ag(X, Y) + bq(X)q(Y)\},$$

$$(4.5) \quad \nabla_Y U = \beta w(Y)U + \{a\phi Y + bq(Y)Q\} - \alpha \lambda Y,$$

$$(4.6) \quad \nabla_Y V = \alpha \phi Y + \lambda \{aY + bq(Y)Q\},$$

$$(4.7) \quad h(Y, V) = ag(V, Y) + bq(V)q(Y),$$

$$(4.8) \quad |u(Q)|^2 = -\frac{a}{b}(1 - \lambda^2) + w(X).$$

Also from (3.15), (3.24) and (4.1), we get

$$(4.9) \quad w(U) = \frac{\alpha(1 - \lambda^2)}{\beta \lambda}$$

5. COVARIANT ALMOST ANALYTIC VECTOR FIELD ON Q -QUASI UMBILICAL HYPERSURFACE

1-form u and v are said to be covariant almost analytic if

$$(5.1) \quad u \{(\nabla_X \phi)(Y) - (\nabla_Y \phi)(X)\} = (\nabla_{\phi X} u)(Y) - (\nabla_X u)(\phi Y)$$

and

$$(5.2) \quad v \{(\nabla_X \phi)(Y) - (\nabla_Y \phi)(X)\} = (\nabla_{\phi X} v)(Y) - (\nabla_X v)(\phi Y).$$

Theorem 5.1. *On Q -quasi umbilical hypersurface M with (ϕ, g, u, v, λ) -structure of a trans-Sasakian manifold $\bar{M}$, if 1-form u be a covariant almost analytic, then we have*

$$(5.3) \quad \alpha \{v(Y)u(X) - v(X)u(Y)\} = -(2 + 2a + 2\beta)g(\phi X, \phi Y) - 2\alpha \lambda g(\phi X, Y) \\ - bq(\phi Y)q(\phi X) + \beta \{u(\phi Y)w(X) - u(Y)w(\phi X)\} \\ + 2bq(X)u(Y)u(Q).$$

Proof. From equation (4.2), we have

$$(\nabla_Y \phi)(X) = \alpha \{v(X)Y - g(X, Y)V\} + \beta \{g(\phi X, Y)V - v(Y)\phi X\} \\ - \{ag(X, Y) + bq(X)q(Y)\}U - u(X)\{aY + bq(Y)Q\}$$

and

$$(\nabla_X \phi)(Y) = \alpha \{v(Y)X - g(Y, X)V\} + \beta \{g(\phi Y, X)V - v(X)\phi Y\} \\ - \{ag(Y, X) + bq(Y)q(X)\}U - u(Y)\{aX + bq(X)Q\}.$$

Now from above equations we have

$$(\nabla_X \phi)(Y) - (\nabla_Y \phi)(X) = \alpha \{v(Y)X - v(X)Y\} + \beta \{v(Y)\phi X - v(X)\phi Y\} \\ - u(Y)\{aX + bq(X)Q\} + u(X)\{aY + bq(Y)Q\} \\ (5.4) \quad u\{(\nabla_X \phi)(Y) - (\nabla_Y \phi)(X)\} = \alpha \{v(Y)X - v(X)Y\} \\ + b\{u(X)q(Y) - q(X)u(Y)\}u(Q).$$

Also from equation (4.3)

$$(\nabla_X u)(Y) = -\{(1+a)g(\phi Y, X)\} + \beta \{\lambda g(\phi Y, X) - u(Y)w(X)\} \\ + \alpha \lambda g(Y, X) + bq(\phi Y)q(X)$$

replacing ϕ by ϕX and ϕY in the last equation, we find respectively

$$(\nabla_{\phi X} u)(Y) = -\{(1+a)g(\phi Y, \phi X)\} + \beta \{\lambda g(\phi Y, \phi X) - u(Y)w(\phi X)\} \\ + \alpha \lambda g(Y, \phi X) + bq(\phi Y)q(\phi X)$$

and

$$(\nabla_X u)(\phi Y) = -(1+a)g(\phi X, \phi Y) + bq(Y)q(X) - bu(Y)q(U)q(X) \\ - \beta u(\phi Y)w(X) - \alpha \lambda g(X, \phi Y).$$

From last two equations we find

$$(5.5) \quad (\nabla_{\phi X} u)(Y) - (\nabla_X u)(\phi Y) = -(2+2a+2\beta)g(\phi X, \phi Y) - bq(\phi Y)q(\phi X) \\ + bu(Y)q(U)q(X) + \beta \{u(\phi Y)w(X) - u(Y)w(\phi X)\} \\ - bq(Y)q(X) + \alpha \lambda g(X, \phi Y) - \alpha \lambda g(\phi X, Y)$$

and using equation (5.1) in (5.4) and (5.5), we get (5.3). $\square$

Corollary 5.2. *On Q -quasi umbilical hypersurface M with (ϕ, g, u, v, λ) -structure of a trans-Sasakian manifold $\bar{M}$, if 1-form u be a covariant almost analytic, then we also have*

$$(5.6) \quad 2\alpha\lambda^2 = 0, \quad \alpha \neq 0,$$

where λ is covariant constant along $\bar{M}$.

Proof.

$$\alpha(1-\lambda^2)^2 = -(2+2a+2\beta).0 + 2\alpha\lambda^2(1-\lambda^2) + \beta\lambda(1-\lambda^2)w(U) \\ \alpha(1-\lambda^2) = +2\alpha\lambda^2(1-\lambda^2) + \beta\lambda(1-\lambda^2)w(U) \\ \alpha(1-\lambda^2) = +2\alpha\lambda^2(1-\lambda^2) + \beta\lambda \left\{ \frac{\alpha(1-\lambda^2)}{\beta\lambda} \right\} \\ \alpha(1-\lambda^2) = 2\alpha\lambda^2 + \alpha(1-\lambda^2) \\ 2\alpha\lambda^2 = 0, \quad \alpha \neq 0,$$

which shows that λ is covariant constant along $\bar{M}$. □

Theorem 5.3. *On cylindrical hypersurface M with (ϕ, g, u, v, λ) -structure of a trans-Sasakian manifold $\bar{M}$ with covariant analytic vector fields U and V , we have*

$$(5.7) \quad \alpha \{v(Y)u(X) - v(X)u(Y)\} = -(2 + 2\beta)g(\phi X, \phi Y) - 2\alpha\lambda g(\phi X, Y) \\ - bq(\phi Y)q(\phi X) + \beta \{u(\phi Y)w(X) - u(Y)w(\phi X)\}.$$

Proof. Putting $a = 0$ in equation (5.3) and using $u(Q) = 0$, we get equation (5.7). □

Corollary 5.4. *On cylindrical hypersurface M with (ϕ, g, u, v, λ) -structure of a trans-Sasakian manifold $\bar{M}$ with covariant analytic vector fields U, V , we have*

$$(5.8) \quad \alpha u(Y) + (1 + \beta)v(Y) = 0,$$

$$(5.9) \quad q(Q) = 0.$$

Proof. Putting $X = V$ in equation (5.7), we get

$$\begin{aligned} -\alpha(1 - \lambda^2)u(Y) &= -(2 + 2\beta)g(\phi V, \phi Y) - 2\alpha\lambda g(\phi V, Y) \\ &\quad - bq(\phi Y)q(\phi V) + \beta \{u(\phi Y)w(V) - u(Y)w(\phi V)\} \\ -\alpha(1 - \lambda^2)u(Y) &= -(2 + 2\beta)v(Y)\lambda^2 - 2\alpha\lambda^2 u(Y) + \beta \{u(\phi Y)w(V) - u(Y)w(\phi V)\} \\ -\alpha(1 - \lambda^2)u(Y) &= -(2 + 2\beta)v(Y)\lambda^2 - 2\alpha\lambda^2 u(Y) + \beta u(\phi Y)w(V) - \beta \lambda u(Y)w(U) \\ \alpha(3\lambda^2 - 1)u(Y) &= -\beta \lambda u(Y) \left\{ \frac{\alpha(1 - \lambda^2)}{\beta \lambda} \right\} - (2 + 2\beta)v(Y)\lambda^2 \\ 2\alpha\lambda^2 u(Y) &= -(2 + 2\beta)v(Y)\lambda^2 \\ \alpha u(Y) + (1 + \beta)v(Y) &= 0. \end{aligned}$$

Also from equation (5.7), we get

$$u(X)V - v(X)U = -(2 + 2\beta)\phi X - 2\alpha\lambda\phi X - bqQq(\phi X) - bq(X)Q + \beta \{w(X)\phi U - w(\phi X)U\}$$

which on contracting with respect to X gives

$$0 = -bq(\phi^2 Q) - bq(Q) + \beta \{\lambda w(X) - \lambda w(V)\}$$

or

$$bq(Q) = (V\lambda) = 0,$$

since $b \neq 0$, so $q(Q) = 0$. □

Theorem 5.5. *On Q -quasi umbilical hypersurface M with (ϕ, g, u, v, λ) -structure of a trans-Sasakian manifold $\bar{M}$, if 1-form u is covariant almost analytic and M is totally umbilical, then we have*

$$(5.10) \quad \alpha \{v(Y)u(X) - v(X)u(Y)\} = -(2 + 2a + 2\beta)g(\phi X, \phi Y) - 2\alpha\lambda g(\phi X, Y) \\ + \beta \{u(\phi Y)w(X) - u(Y)w(\phi X)\}$$

Proof. Putting $b = 0$ in equation (5.3), we get equation (5.10). □

Corollary 5.6. *On Q -quasi umbilical hypersurface M with (ϕ, g, u, v, λ) -structure of a trans-Sasakian manifold $\bar{M}$, if 1-form u is covariant almost analytic and M is totally umbilical, then we have*

$$(5.11) \quad a = \left\{ \frac{\alpha(1-3\lambda^2)v(Y)}{\lambda} \right\} - \left\{ \frac{\alpha(1-3\lambda^2)}{\lambda} + 2(1+\beta) \right\} u(Y)$$

Proof. Put $X = U$ in equation (5.10), we get

$$\alpha(1-\lambda^2)v(Y) = -(2+2a+2\beta)v(\phi Y) + 2\alpha\lambda^2v(Y) + \beta \{u(\phi U)w(X) - u(Y)w(\phi U)\}$$

$$\alpha(1-\lambda^2)v(Y) = -(2+2a+2\beta)v(\phi Y) + 2\alpha\lambda^2v(Y) + \beta \left\{ u(\phi Y) \frac{\alpha(1-\lambda^2)}{\beta\lambda} \right\}$$

$$\alpha(1-\lambda^2)v(Y) - 2\alpha\lambda^2v(Y) = -(2+2a+2\beta)u(Y) + u(Y) \left\{ u(\phi Y) \frac{\alpha(1-\lambda^2)}{\lambda} \right\}$$

$$\alpha(1-3\lambda^2)v(Y) = u(Y) \{-2\lambda - 2a\lambda - 2\beta\lambda + \alpha(1-\lambda^2)\}$$

$$\alpha(1-3\lambda^2)v(Y) - \alpha(1-\lambda^2)u(Y) = -2u(Y) \{\lambda + a\lambda + \beta\lambda\}$$

$$a\lambda = \alpha(1-3\lambda^2)V - \alpha(1-\lambda^2)U + 2U\lambda + 2\beta\lambda U$$

which on further solving gives (5.11). □

Theorem 5.7. *On Q -quasi umbilical hypersurface M with (ϕ, g, u, v, λ) -structure of a trans-Sasakian manifold $\bar{M}$, if 1-form v is covariant almost analytic, then we have*

$$(5.12) \quad \alpha \{u(X)v(Y) - u(Y)v(X)\} = -2\alpha g(\phi X, \phi Y) + 2(\lambda + \beta)ag(\phi X, Y) \\ + \lambda b \{q(\phi X)q(Y) - q(X)q(\phi Y)\}$$

Proof. From equation (4.2), we get

$$(5.13) \quad (\nabla_X \phi)(Y) - (\nabla_Y \phi)(X) = \alpha \{v(Y)X - v(X)Y\} + \beta \{v(Y)\phi X - v(X)\phi Y\} \\ - u(Y) \{aX + bq(X)Q\} + u(X) \{aY + bq(Y)Q\} \\ (\nabla_X \phi)(Y) - (\nabla_Y \phi)(X) = \beta \{v(Y)\phi X - v(X)\phi Y\} + a \{u(X)v(Y) - u(Y)v(X)\}$$

From (4.3), we have

$$(\nabla_{\phi X} v)(Y) = (\lambda a + \beta)g(\phi X, Y) + \lambda \{u(X)v(Y) - v(X)u(Y)\}$$

and

$$(\nabla_X v)(\phi Y) = -\alpha \{g(X, Y) - u(X)u(Y) - v(X)v(Y)\} + (\lambda - \beta)g(X, \phi Y) \\ + \lambda \{v(X)u(Y) - u(X)v(Y)\}.$$

Therefore we have

$$(5.14) \quad (\nabla_{\phi X} v)(Y) - (\nabla_X v)(\phi Y) = -2\alpha g(\phi X, \phi Y) + 2(\lambda + \beta)ag(\phi X, Y) \\ + \lambda b \{q(\phi X)q(Y) - q(X)q(\phi Y)\}.$$

By using equation (5.2) in (5.13) and (5.14), we get (5.12). □

Theorem 5.8. *Let the 1-form v be a covariant almost analytic on Q -quasi umbilical hypersurface M with (ϕ, g, u, v, λ) -structure of a trans-Sasakian manifold $\bar{M}$ if it is also cylindrical, then we have*

$$(5.15) \quad \begin{aligned} 2g(\phi X, \phi Y) &= \lambda b \{q(\phi X)q(Y) - q(X)q(\phi Y)\} \\ &\quad - \beta \{v(Y)v(\phi X) - v(X)v(\phi Y)\} \end{aligned}$$

with $X \neq U$.

Proof. Putting $\alpha = 0$ in equation (5.12), we obtain equation (5.15). Further $X = U$, then from (5.15), we have

$$\begin{aligned} 2g(\phi U, \phi Y) &= \lambda b \{-\lambda q(V)q(Y) - q(U)q(\phi Y)\} \\ &\quad - \beta \{v(Y)v(\phi U)\} \\ -2\lambda g(V, \phi Y) + \beta g(V, Y)g(V, \phi U) &= 0 \end{aligned}$$

or

$$-2\lambda v(\phi Y) + \beta v(Y)V(\phi U) = 0,$$

since $v(\phi Y) \neq 0$ and $\lambda \neq 0$, therefore $X \neq U$.

□

Theorem 5.9. *Let the 1-form v be a covariant almost analytic on Q -quasi umbilical non-invariant hypersurface of M with (ϕ, g, u, v, λ) -structure of a trans-Sasakian manifold $\bar{M}$, if it is also totally umbilical, then we have*

$$(5.16) \quad \begin{aligned} \alpha \{u(X)v(Y) - v(X)u(Y)\} &= -2\alpha g(\phi X, \phi Y) + 2(\lambda + \beta)ag(\phi X, Y) \\ &\quad - \beta \{v(Y)v(\phi X) - v(X)v(\phi Y)\}, \end{aligned}$$

with $X \neq U$.

Proof. Putting $b = 0$ in equation (5.12), we get (5.14). If we take $X = U$ in (5.16), we get

$$\begin{aligned} a(1 - \lambda^2)v(Y) + 2(\lambda + \beta)av(Y) &= 2\alpha\lambda v(\phi Y) - \beta v(Y)u(\phi U) \\ a \{1 - \lambda^2 + 2(\lambda + \beta) + \beta u(\phi U)\} v(Y) &= -2\alpha\lambda v(\phi Y) \\ a \{\lambda^2 - 2(\lambda + \beta) - \beta U - 1\} v(Y) &= -2\alpha\lambda^2 U \end{aligned}$$

i.e. V and U are linearly dependent, which is a contradiction. Thus $X \neq U$. □

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