Original Research Article

Dynamics of Water Hyacinth in a Fishery System with Delay Effect

ABSTRACT

This paper analyses the dynamics of a fishery system in an aquatic environment that consists of two zones: water hyacinth zone and free zone. Fish harvesting is allowed in both the zones and fish migration is allowed from water hyacinth zone to free zone and not back. This paper presents dynamics of the stability when discrete time delay is incorporated in the fish death rate due to oxygen depletion and water pollution caused by water hyacinth. It is shown that the time delay can cause a switch from stable state to unstable state and there by Hopf-bifurcation occurs. Numerical simulations are carried out to validate the analytical findings.

Key words: Water hyacinth, Fishery, Stability, Time delay, Hopf-bifurcation.

1. INTRODUCTION

Water hyacinth (*Eichhornia crassipes*) is one of the aquatic plants in the nature though it is native to Brazil, globally wide spread and creates a nuisance in the aquatic environment. Once these plants are introduced into the natural environment they rapidly increase in coverage because of the highest growth rate in saltwater or fresh water. It doubles in number in a times range between 6-18 days, for these reasons hyacinth has earned nicknames such as “the weed from hell” and “the beautiful devil”[3] .

Fish provides a good source of high-quality protein, and it also contains many vitamins and minerals. For this reason, it is consumed as a food by many species including human through out the world. So the study of existence for Fish population is very much essential in our society. Quantitative models of fish population include simple models that consider only the biomass of the population processes, such as growth and recruitment. These are often adjusted to incorporate the impact of external effects, such as predation, competition for food supply and other environmental factors.

The water hyacinth effect greatly influence the fish industry, its pollution appears in different levels ranging from low to moderate and high. The hyacinth mats impacts significantly on fishing activities due to increase in time to access the fishing ground. The mats also had detrimental effects by blocking light, severely reducing oxygen levels and allowing poisonous gases such as hydrogen sulfide and ammonia. This result in loss of aquatic biodiversity [10].Already many researchers [2], [4], [5], [6] and [7] links water hyacinth pollution to fish production. In this they were discussed the effect of water hyacinth in fish growth.

To protect the fishery population, having much threat from water hyacinth, adopting two equal zonal systems (in terms of area and volume of water) is better idea. A zone having no influence of water hyacinth, called hyacinth free zone, is formed and fishing is taking place in both zones. It should be noted

that we are considering the same fish species in both zones. In this present paper it is assumed that the density of fish is directly proportional to the abundance of fish at time (t) and the catch ability coefficient decreases as the abundance level of water hyacinth increases. Further it was assumed that water hyacinth abundance leads to biological effects such as fish death and fish migration. The oxygen depletion and the environmental pollution cause by the growth of water hyacinth will not have immediate impact on the death of the fish population, but it will have some time lag. The effect of this time lag on the dynamics of this system is presented in this population.

2. MATHEMATICAL MODEL

To formulate a mathematical representation of water hyacinth model the following notations are being used. Let W(t) and F(t) represents the biomass densities of fish population in water hyacinth zone and free zone of water hyacinth respectively at any time't'. ρ denote the rate at which water hyacinth reduces fish catch; ψ denote fish migration rate from water hyacinth zone to free zone of water hyacinth; r and s denotes fish death rates due to oxygen depletion and water pollution due to water hyacinth; c and e denote the catchability coefficient and harvesting efforts in both the zones; α and β denote the intrinsic growth rates of fish population in both the zones respectively; m and n denote carrying capacity of fish population in both the zones. Further both the variables W and F are non-negative and the parameters are assumed to be non-negative.

The mathematical formulation of water hyacinth model is represented by the following system of first order non linear ordinary differential equation:

$$\begin{aligned} \frac{dW}{dt} &= \alpha W\left(1-\frac{W}{m}\right)-(1-\rho)ceW-(\psi+r+s)W \\ \frac{dF}{dt} &= \beta F\left(1-\frac{F}{n}\right)+\psi W-ceF \end{aligned} \tag{2.1}$$

Now incorporating time delay (τ) in fish death rate due to oxygen depletion and water pollution caused by water hyacinth , the equation (2.1) becomes

$$\begin{aligned} \frac{dW}{dt} &= \alpha W\left(1-\frac{W}{m}\right)-(1-\rho)ceW-\psi W-rW(t-\tau)-sW(t-\tau) \\ \frac{dF}{dt} &= \beta F\left(1-\frac{F}{n}\right)+\psi W-ceF \end{aligned} \tag{2.2}$$

3. EQUILIBRIUM ANALYSIS

The equilibrium points of the system (2.1) and (2.2) are the solutions of the steady state equations.

$$\alpha W\left(1-\frac{W}{m}\right)-(1-\rho)ceW-(\psi+r+s)W=0$$

$$\beta F\left(1-\frac{F}{n}\right)+\psi W-ceF=0$$

The possible equilibrium points are

$E_1(0,0)$ (In the absence of both the zones)

$E_2(\gamma,0)$ (In the presence of water hyacinth zone)

$E_3(0,\delta)$ (In the presence of free zone of water hyacinth)

$E_4(\gamma^*,\delta^*)$ (In the presence of both the zones i.e., the interior equilibrium)

Case (i): The population is extinct and this trivial steady state always exists.

Case (ii): if γ is positive solution of $\frac{dW}{dt}=0$ then

$$\gamma=\frac{m}{\alpha}\left[\rho ce+(\alpha-ce-\psi-r-s)\right]$$

This positive steady state exists only when

$$\rho ce+\alpha>ce+\psi+r+s \tag{3.1}$$

Case (iii): if δ is the positive solution of $\frac{dF}{dt}=0$ then

$$\delta=\frac{n}{\beta}[\beta-ce]$$

This positive steady state exists only when

$$\beta>ce \tag{3.2}$$

Case (iv): if (γ^*,δ^*) are the positive solutions of $\frac{dW}{dt}=0$ and $\frac{dF}{dt}=0$ then

$$\gamma^*=\frac{m}{\alpha}[\rho ce+\alpha-ce-\psi-r-s]$$

$$\delta^{*2}+G\delta^*+H=0 \tag{3.3}$$

Where G $=-\frac{n}{\beta}[\beta-ce]$

H= $-\frac{\psi mn}{\alpha\beta}\left[\alpha+\rho ce-(ce+\psi+r+s)\right]$

From the biological point of view we only interested on the interior equilibrium $E_4(\gamma^*,\delta^*)$.

Let $W=\gamma-\gamma^*, F=\delta-\delta^*$ be the perturbed variables

After removing the non-linear terms we obtain the linearaized system corresponding to (2.2) is

$$\frac{d\gamma}{dt}=\left[\alpha-\frac{2\alpha\gamma^*}{m}-(1-\rho)ce-\psi-(r+s)e^{-\lambda\tau}\right]\gamma$$
$$\frac{d\delta}{dt}=\psi\gamma+\left[\beta-\frac{2\beta\delta^*}{n}-ce\right]\delta \tag{3.4}$$

The characteristic equation of the linear system is given by

$$\Delta(\lambda,\tau)=\lambda^2+P\lambda+Q+e^{-\lambda\tau}(R\lambda+T)=0 \tag{3.5}$$

Where P = -A-D; Q = AD;

R = r + s; T = -D(r + s)

And $A=\alpha-\frac{2\alpha\gamma^*}{m}-(1-\rho)ce-\psi$ B= ψ D= $\beta-\frac{2\beta\delta^*}{n}-ce$

Now we discuss the stability of the interior equilibrium point in the presence and absence of delay.

4. STABILITY ANALYSIS

4.1 Stability analysis in the absence of delay

The characteristic equation of the model (2.2) is $\lambda^2+X\lambda+Y=0$ (4.1)

Where X= P + R

Y= Q + T

Here X > 0 and Y >0

So the Eigen values of the characteristic equation are either real and negative or complex conjugate with negative real parts.

Hence the system (2.1) is locally asymptotically stable.

THEOREM 1: The system (2.1) is locally asymptotically stable at $E_4(\gamma^*,\delta^*)$ if the equation (4.1) has both the roots with negative real parts.

NUMERICAL ANALYSIS OF THE MODEL IN THE ABSENCE OF DELAY

Re arrangement of equation (2.1), a technique applied by [2], gives

$$\frac{dW}{dt}=-W\left(\frac{\alpha}{m}W+\psi+r+s+ce-(\alpha+\rho ce)\right)$$

$$\frac{dF}{dt}=-F\left(\frac{\beta}{n}F+ce-\beta\right)+\psi W$$

In this mathematical model, the parameters $\alpha,\beta,\rho,\psi,r,s,c,m,n$ are assumed to be positive constants and $0<c<1$ and $0<\rho<1$.

Fish catch in water hyacinth zone = $ceW - \rho ceW$

$= (1-\rho)ceW$.

When $\rho = 1$, then no fish catch in water hyacinth zone.

If $\psi + r + s + ce - (\alpha + \rho ce) > 0$ then $\frac{dW}{dt} < 0$ then the system is collapsed. Hence the condition that $\psi + r + s + ce - (\alpha + \rho ce) < 0$ is imposed. Similarly $ce - \beta > 0$ then $\frac{dF}{dt} < 0$

Therefore, for the existence of the system $ce - \beta < 0$ is imposed

Hence we require $\psi + r + s + ce < \alpha + \rho ce$ and $ce < \beta$.

Hence throughout our analysis in this work we assume that

$\alpha + \rho ce - (\psi + r + s + ce) > 0$ and $\beta - ce > 0$.

This is explain it (3.1) and (3.2).

Example 1: Consider the following parameters for the model (2.1)

$m = 600; n = 300; e = 10; c = 0.004; \alpha = 1.5; \beta = 0.6; \rho = 0.32; \psi = 0.19; r = 0.45; s = 0.36;$

$W(0) = 30; F(0) = 40$

Equilibrium points for the system (2.1) - (2.2) are (189.125, 333.80)

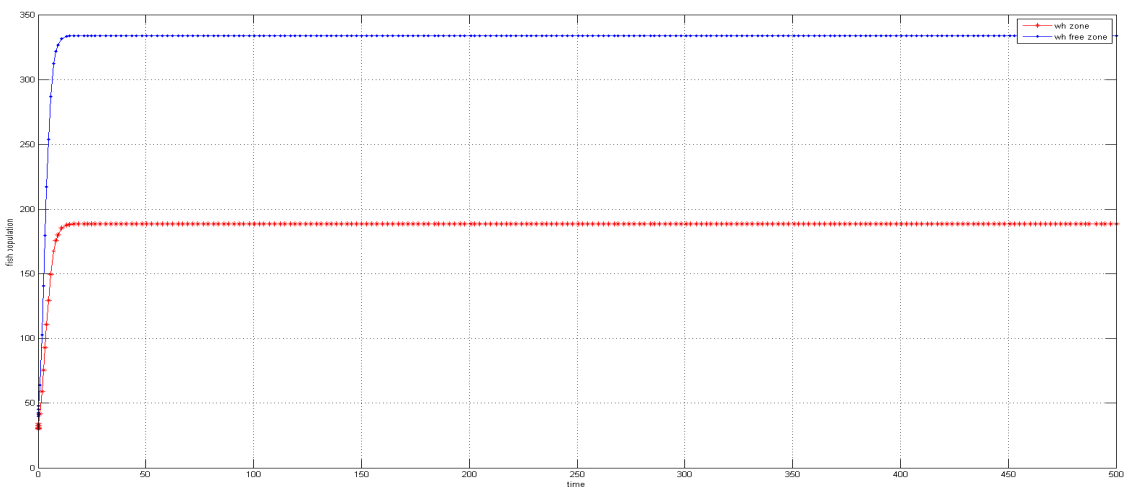

Fig. 1

Fig.1. Shows the time series evolution of fish population in the two zones showing stable oscillation of the population towards $E_4(\gamma^*, \delta^*)$

4.2 Stability analysis in the presence of delay

Let $\lambda(\tau)=\theta(\tau)+i\eta(\tau)$ be a root of the characteristic equation (3.5).

Let τ be a particular value of the delay such that $\theta(\tau)=0,\eta(\tau)>0$

Substituting $\lambda=i\eta$ in (3.5) we get,

$$E_4\left(\gamma^*,\delta^*\right)\tau\geq 0P^2-2Q-R^2Q^2-T^2\eta\lambda=i\eta \quad \left[-\eta^2+i\eta P+Q\right]+\left[Ri\eta+T\right]e^{-i\eta\tau}=0$$

$$\left[-\eta^2+i\eta P+Q\right]+\left[Ri\eta+T\right]\left[\cos\eta\tau+i\sin\eta\tau\right]=0$$

Separating the real and imaginary parts we get,

$$\eta^2-Q=T\cos\eta\tau+R\eta\sin\eta\tau \tag{4.2}$$

$$P\eta=-R\eta\cos\eta\tau+T\sin\eta\tau \tag{4.3}$$

Squaring and adding (4.2)-(4.3), we get

$$\eta^4+\eta^2(\mathrm{P}^2-2\mathrm{Q}-\mathrm{R}^2)+\left(Q^2-T^2\right)=0$$

The above equation can be written in the form of

$$\eta^4+K\eta^2+L=0 \tag{4.4}$$

Where K= P^2-2Q-R^2 : L= Q^2-T^2

Case 1: If K= P^2-2Q-R^2 >0 then the equation (4.4) does not have any real solutions.

THEOREM 2: If K>0, L>0 then the equilibrium point $E_4\left(\gamma^*,\delta^*\right)$ is locally asymptotically stable for all $\tau\geq 0$.

PROOF: For $\tau=0$, $E_4\left(\gamma^*,\delta^*\right)$ is locally asymptotically stable from THEOREM 1.

When $\tau>0$, by case 1 the equation (4.4) does not have any real solution
i.e., there exist no real η as a solution for the equation (4.4).

Hence no real $\lambda=i\eta$ (η is real) will be a solution to the equation (3.5).

It is obvious that the equilibrium point $E_4\left(\gamma^*,\delta^*\right)$ is locally asymptotically stable for all $\tau\geq 0$.

Case 2: If K > 0, L <0 then the equation (4.4) have a unique positive root, it is η_0^2 and let the corresponding τ be τ_0.

Case 3: if K < 0, L > 0 and K^2-4L > 0 then the equation (4.4) have two positive roots. Let them be $\eta_\pm^2$ and the corresponding τ is $\tau^\pm$.

Eliminating $\sin\theta\tau$ from (4.2)-(4.3)

$$\cos\eta\tau = \frac{\eta^2\left(T-PR\right)-QT}{T^2+\eta^2R^2}$$

$$\tau_z = \frac{1}{\eta}\arccos\left[\frac{\eta^2\left(T-PR\right)-QT}{T^2+\eta^2R^2}\right]+\frac{2\pi z}{\eta} \tag{4.5}$$

Where z = 0, 1, 2…, etc.

5. HOPF-BIFURCATION ANALYSIS

In this section, investigate the effect of the time delay on bifurcations of the system.

Now differentiating equation (3.5) with respect to τ ,

$$\left[2\lambda+P+\mathrm{Re}^{-\lambda\tau}-\tau\left(R\lambda+T\right)e^{-\lambda\tau}\right]\frac{d\lambda}{d\tau}=\lambda e^{-\lambda\tau}\left(R\lambda+T\right)$$

$$\left[\frac{d\lambda}{d\tau}\right]^{-1}=\frac{2\lambda+P}{-\lambda\left(\lambda^2+p\lambda+Q\right)}+\frac{R}{\lambda\left(R\lambda+T\right)}-\frac{\tau}{\lambda}$$

$$\left[\frac{d\lambda}{d\tau}\right]^{-1}=\mathrm{Re}\left[\frac{d\lambda}{d\tau}\right]^{-1}=\left[\frac{2\left(\eta^2-Q\right)+P^2}{\eta^4+\left(P^2-2Q\right)\eta^2+Q^2}-\frac{R^2}{T^2+\eta^2R^2}\right] \tag{5.1}$$

$$=\left[\frac{2\eta^2+P^2-2Q-R^2}{T^2+\eta^2R^2}\right] \tag{5.2}$$

Thus $sign\left[\frac{d}{d\tau}\left(\mathrm{Re}\,\lambda\right)\right]_{\lambda=i\eta}=sign\left[\mathrm{Re}\left(\frac{d\lambda}{d\tau}\right)^{-1}\right]$

$$=sign\left[\frac{2\eta^2+\left(P^2-2Q-R^2\right)}{T^2+\eta^2R^2}\right] \tag{5.3}$$

Thus η may be η_0 or $\eta_\pm$.

The above said positive roots , either from case 2 or from case 3, satisfy all the equations from (4.2)-(4.5).

THEOREM 3: The system (2.2) is locally asymptotically stable at $E_4\left(\gamma^*,\delta^*\right)$ if K >0 and L < 0 for all $\tau<\tau_0$ and is unstable for all $\tau>\tau_0$ and hopf-bifurcation occurs at $\tau=\tau_0$.

PROOF: From equation (5.3) we have

$$sign\left[\frac{d}{d\tau}\left(\mathrm{Re}\,\lambda\right)\right]_{\lambda=i\eta_0}=sign\left[\mathrm{Re}\left(\frac{d\lambda}{d\tau}\right)^{-1}\right]_{\lambda=i\eta_0}$$

$$= sign\left[\frac{2\eta^2 + \left(P^2 - 2Q - R^2\right)}{T^2 + \eta^2 R^2}\right]$$

It is clear that

$$sign\left[\frac{d}{d\tau}\left(\operatorname{Re}\lambda\right)\right]_{\lambda=i\eta_0} = sign\left[\operatorname{Re}\left(\frac{d\lambda}{d\tau}\right)^{-1}\right]_{\eta=\eta_0,\tau=\tau_0} > 0 \tag{5.4}$$

this signifies that there exists Eigen value with negative real part for $\tau < \tau_0$ and there exist Eigen value with positive real part for $\tau > \tau_0$ .More over the condition of hopf-bifurcation is then satisfied yielding the required periodic solution.

THEOREM 4: The system (2.2) is locally asymptotically stable at $E_4\left(\gamma^*, \delta^*\right)$ when $\tau \in [\tau_0^+, \tau_0^-) \cup \left(\tau_0^-, \tau_1^+\right) \cup ... \cup \left(\tau_{i-1}^-, \tau_i^+\right)$ if K < 0 , L >0 and $K^2 - 4L > 0$ and it is unstable when $\tau \in \left[\tau_0^+, \tau_0^-\right) \cup \left(\tau_1^+, \tau_1^-\right) \cup ... \cup \left(\tau_{i-1}^+, \tau_{i-1}^-\right)$,for some positive integer i. Therefore there are bifurcations at the equilibrium point $E_4\left(\gamma^*, \delta^*\right)$ when $\tau = \tau_z^{\pm}$, z=0,1,2,...

PROOF: From equation (5.3) we have

$$sign\left[\frac{d}{d\tau}\left(\operatorname{Re}\lambda\right)\right]_{\lambda=i\eta_+} = sign\left[+\frac{\sqrt{K^2 - 4L}}{\left\{\left(-\eta_+^2 + Q\right)^2 + P^2\eta_+^2\right\}\left\{T^2 + R^2\eta_+^2\right\}}\right]$$

Therefore, $\left[\frac{d}{d\tau}\left(\operatorname{Re}\lambda\right)\right]_{\eta=\eta_+,\tau=\tau_n^+} > 0$.

Again, $sign\left[\frac{d}{d\tau}\left(\operatorname{Re}\lambda\right)\right]_{\lambda=i\eta_-} = sign\left[-\frac{\sqrt{K^2 - 4L}}{\left\{\left(-\eta_-^2 + Q\right)^2 + P^2\eta_-^2\right\}\left\{T^2 + R^2\eta_-^2\right\}}\right]$

Therefore, $\left[\frac{d}{d\tau}\left(\operatorname{Re}\lambda\right)\right]_{\eta=\eta_-,\tau=\tau_n^-} < 0$ $\left[\frac{d}{d\tau}\left(\operatorname{Re}\lambda\right)\right]_{\eta=\eta_-,\tau=\tau_n^-} < 0$

Hence the transversality conditions are satisfied.

This completes the proof.

NUMERICAL ANALYSIS OF THE MODEL IN THE PRESENCE OF DELAY:

Example 2: Consider the following parameters for the model (2.2)

$$m = 600; n = 300; e = 10; c = 0.004; \alpha = 1.5; \beta = 0.6; \rho = 0.32; \psi = 0.19; r = 0.45; s = 0.36;$$
$$\mathrm{W}(0) = 180; \mathrm{F}(0) = 330;$$

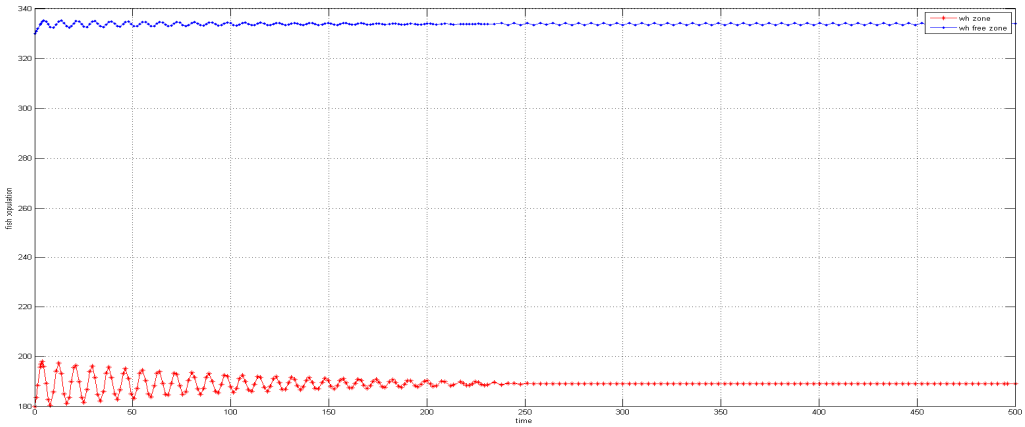

Fig.2

Fig.2. shows the time series evolution of fish population in the two zones showing stable oscillation of the population towards $E_4\left(\gamma^*,\delta^*\right) delay = 1.53(<1.549)$

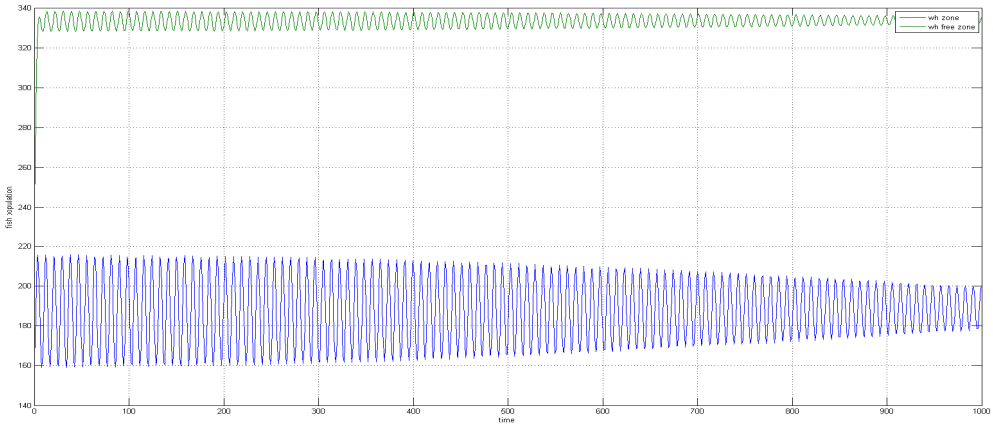

Fig.3

Fig.3. Shows the time series evolution of fish population in the two zones showing unstable oscillation of the population towards $E_4\left(\gamma^*,\delta^*\right) delay = 1.549(\tau_0)$

Example 3: Consider the following parameters for the model (2.2)

$$m = 600; n = 300; e = 10; c = 0.004; \alpha = 1.5; \beta = 0.6; \rho = 0.33; \psi = 0.20;$$
$$r = 0.46; s = 0.37; \mathrm{W}(0) = 180; F(0) = 330.$$

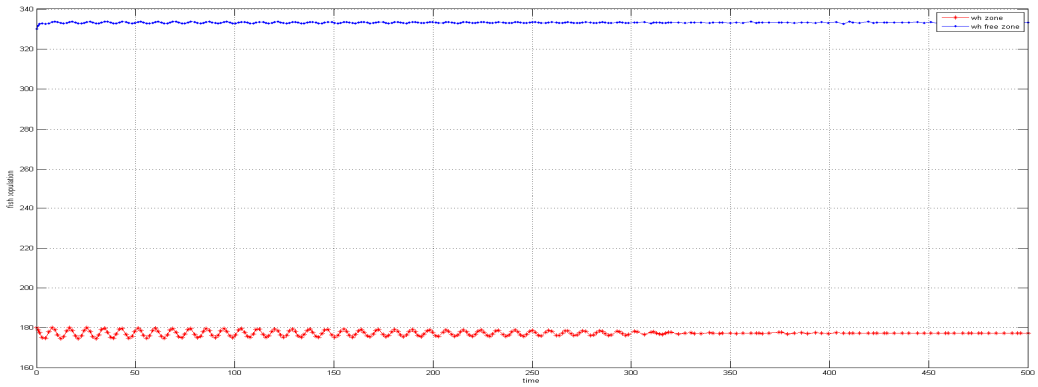

Fig.4

Fig.4. Shows the time series evolution of fish population in the two zones showing stable oscillation of the population towards $E_4\left(\gamma^*,\delta^*\right) delay = 1.48(<1.495)$

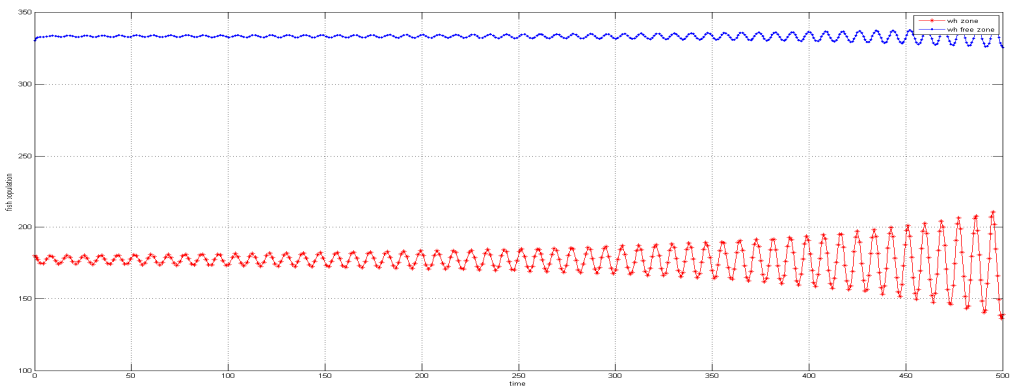

Fig.5

Fig.5. Shows the time series evolution of fish population in the two zones showing unstable oscillation of the population towards $E_4\left(\gamma^*,\delta^*\right) \quad delay = 1.495(\tau_0)$

From our analysis it is observed that slight increments in the model parameters associated with water hyacinth the value of τ_0 decreases (see example 2 and example 3).

6. CONCLUSIONS

In this paper we investigated the effect of water hyacinth on the equilibrium of fish biomass densities. The equilibrium of the model was analyzed.

1. The study has proved that water hyacinth have severe impact on fish stock.
2. Slight increments on the model parameters associated with water hyacinth completely changed the equilibrium of the model.
3. Fish population in the water hyacinth zone was decreasing in the equilibrium.

4. It is observed that the delay of certain dimensions can induce instability oscillations via hopf bifurcation with switching of stability.
5. Water hyacinth may lead to fish extinction; therefore efforts should be done to eradicate it with whatever means which are environment friendly.

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