

Application of Quick Simplex Method(A new approach) On Two Phase Method

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ABSTRACT

In this article, we suggest a new approach while solving two phase simplex method. The method sometimes involves less iteration than in the Simplex Method or at the most an equal number because the method attempts to replace more than one basic variable simultaneously. While dealing with Two Phase Simplex Method a new method [7] (Quick Simplex Method) can be applied in Phase I and also in Phase II.

This has been illustrated by giving the solution of solving Two Phase Simplex Method problems. It is also shown that either the iterations required are the same or less but iterations required are never more than those of the Simplex Method.

KEY WORDS AND PHRASES : *basic feasible solution, optimum solution, simplex method, key determinant, constraints, net evaluation*

1. INTRODUCTION

There are two methods to obtain the solution of the linear programming problem. These methods can be classified as :

(i) The Graphical method (ii) Simplex method.

The linear programming has its own importance in obtaining the solution of a problem where two or more activities compete for limited resources.

Mathematically we have to maximize the objective function cx

subject to $Ax = b$ $x \geq 0$,

where

$x = n \times 1$ column vector

$A = m \times n$ coefficient matrix

$b = m \times 1$ column vector

$C = 1 \times n$ row vector

35 and the columns of A are denoted by $P_1, P_2, \dots, P_n$.
 36 There are various alternative approaches to solve Simplex methods and Two Phase Simplex
 37 Method [8], [9],[10],[12] .
 38 In this article, a new approach Quick Simplex method [7] is suggested while solving
 39 Two Phase Simplex Method. New method can be apply in Phase I and in Phase II
 40 wherever applicable. The method sometimes involves less iteration than in the simplex
 41 method or at the most an equal number because the method attempts to replace more than
 42 one basic variable simultaneously.
 43 It is known that the conventional simplex method is rather inconvenient in handling the
 44 degeneracy and cycling problems because here the choice of the vectors, entering and
 45 outgoing, plays an important role. The degeneracy occurs when there is a tie for outgoing
 46 vector. The possibility of cycling is crucial only if the current basic feasible solution has more
 47 than one variable zero. In the Simplex method when there arises a tie for entering the vector,
 48 the vector with the lowest index j , is selected.
 49 In our method the problem of tie in most of the degeneracy problems is solved. The
 50 powerfulness of our method lies in getting rid of the tie in the degeneracy problems. In such
 51 situations our technique positively is more powerful and handy as well.

52

53 **2. QUICK SIMPLEX METHOD :**

54

55 For the solution of any L.P.P. by simplex algorithm, the existence of an initial basic
 56 feasible solution is always assumed. The steps for the computation of an optimum
 57 solution are as follows:

58 Step 1) Check whether the objective functions of the given L.P.P. is maximized or
 59 minimized .If it is minimized, convert it into a problem of maximizing by using the result
 60 Minimum $z = -\text{Maximum}(-z)$

61 Step 2) Check whether RHS of the constraints, q_i ("i=1,2,...,m".)
 62 are non-negative. If any one q_i is negative, then multiply the corresponding inequalities of
 63 the constraints by (-1), so as to get all q_i (i=1,2,...,m.) non-negative.

64 Step 3) Convert all the inequalities of the constraints into equations by introducing slack and
 65 surplus variables in the constraints. Put the costs of these variables equal to zero. Artificial
 66 variables can be used if needed.

67 Step 4) Obtain an initial basic feasible solution to the problem in the form $X_B = B^{-1}Q$ and put it
 68 in the last column of the simplex table.

69 Step 5) Compute the net evaluations $z_j - c_j$ ($j=1,2,\dots, n$.) by using the relation

70 $z_j - c_j = C_B P_j - c_j$ where $P_j = B^{-1} a_j$

71 Here we assume that at least one $z_j - c_j$ is negative.

72 Step 6) Choose most negative net evaluation. Let it be $z_j - c_j$. Calculate the ratios θ_{rj}

73 for positive a_{rj} . If all the entries in j th column are negative then go for next most negative net

74 evaluation. If only one net evaluation say $z_j - c_j$ is negative and all the entries in the j th

75 column are negative, then the problem has unbounded solution and procedure ends there

76 otherwise goto the next step.

77 Step 7) Choose minimum θ_{rj} say θ_{kj} . Break the tie arbitrarily if any.

78 Step 8) Element of simplex table at the chosen element i.e. a_{kj} will be the pivotal element.

79 Step 9) If number of pivotal element is less than m , then check if any more negative net

80 evaluation is left. If it is so go to step 7 ignoring row and column of pivotal element otherwise

81 go to the next step.

82 Step 10) Variables corresponding to the columns of pivotal elements are the incoming

83 variables in the basis and they replace the variables corresponding the row of the pivotal

84 elements. Note the number of pivotal elements say r . Then define key determinant of order

85 $(r \times r)$ by using sub matrix of matrix A whose diagonal elements are the pivotal elements.

86 Step 11) Calculate entries in X_B column using new basis, formed by using vectors containing

87 all the pivotal elements.

88 Step 12) If all the entries are non-negative then the above replacement of r variables is

89 allowable and next step can be followed otherwise the variable corresponding to the

90 negative value of the X_B column should not be entered in the basis. In such case goto step

91 number 15.

92 Step 13) Determine entries in the new simplex table by replacing the r variables

93 simultaneously.

94 Step 14) Check the optimality of the solution using net evaluations.

95 a) If all net evaluations are non-negative then the solution is optimal and procedure ends

96 there.

97 b) If at least one net evaluation is negative go to step 6.

98 Step 15) Check if basis changes in case there was a tie, by choosing alternative variable in

99 the basis, otherwise instead of minimum θ_{rj} . Choose next minimum θ_{rj} for which $z_j - c_j$ is

100 negative and X_i was not in the basis. This should be done for all those variables for which

101 value of X_B column is obtained as negative. Now go to step 10 using this new basis.

102

103 **3. STATEMENT OF THE PROBLEM-I**

104

105 Use two phase simplex method to solve the following LPP

106 Maximize $Z = 5x_1 + 8x_2$

107 Subject to the constraints :

108 $3x_1 + 2x_2 \geq 3$

109 $x_1 + 4x_2 \geq 4$
 110 $x_1 + x_2 \leq 5$
 111 $x_1, x_2 \geq 0$

112

113 4. SOLUTION OF THE PROBLEM BY TWO PHASE SIMPLEX METHOD.

114

115 In this problem it is observed that an optimum basic feasible solution has been reached by
 116 conventional Simplex Method in three step in Phase I and again three steps in Phase II and
 117 the solution is $x_1=0$ and $x_2=5$, and Max $Z = 40$.

118

119 5. HERE WE APPLY QUICK SIMPLEX METHOD FOR PHASE-I

120

121 **Step (1):** (Initial table)

		0	0	0	-1	0	-1	0		R1	R2
C_B	X_B	P_1	P_2	P_3	P_4	P_5	P_6	P_7	X_B		
-1	x_5	$a_1=3$	$b_1=2$	-1	1	0	0	0	3	1	3/2
-1	x_6	$a_2=1$	$b_2=4$	0	0	-1	1	0	4	4	1
0	x_7	$a_3=1$	$b_3=1$	0	0	0	0	1	5	5	5
	$Z_j - C_j$	-4	-6	1	0	1	0	0			
		↑	↑		↓		↓				

122

123 Here we introduce P_1, P_2 Simultaneously and outgoing vectors are P_4, P_6

124 To find new values in X_B column.

125 Here we can find c_1^{**}, c_2^{**} and c_3^{**} using following formula.[7]

126

127 Column $X_B = \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix}$

128

$$c_1^{**} = \frac{(-1)^1 \begin{vmatrix} 2 & 3 \\ 4 & 4 \end{vmatrix}}{10} = 2/5, \quad c_2^{**} = \frac{(-1)^2 \begin{vmatrix} 3 & 3 \\ 1 & 4 \end{vmatrix}}{10} = 9/10,$$

129

$$c_3^{**} = \frac{\begin{vmatrix} 3 & 2 & 3 \\ 1 & 4 & 4 \\ 1 & 1 & 5 \end{vmatrix}}{10} = 37/10, \quad R = \begin{vmatrix} 3 & 2 \\ 1 & 4 \end{vmatrix} = 10$$

$$130 \quad \text{New } X_B = \begin{bmatrix} 2/5 \\ 9/10 \\ 37/10 \end{bmatrix}$$

131

$$132 \quad \text{Column } P_3 = \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} : -$$

133

$$c_1^{**} = \frac{(-1)^1 \begin{vmatrix} 2 & -1 \\ 4 & 0 \end{vmatrix}}{10} = -\frac{2}{5}, \quad c_2^{**} = \frac{(-1)^2 \begin{vmatrix} 3 & -1 \\ 1 & 0 \end{vmatrix}}{10} = 1/10,$$

$$c_3^{**} = \frac{\begin{vmatrix} 3 & 2 & -1 \\ 1 & 4 & 0 \\ 1 & 1 & 0 \end{vmatrix}}{10} = 3/10, \quad R = \begin{vmatrix} 3 & 2 \\ 1 & 4 \end{vmatrix} = 10$$

$$134 \quad \text{New } P_3 = \begin{bmatrix} -2/5 \\ 1/10 \\ 3/10 \end{bmatrix}$$

$$135 \quad \text{Column } P_4 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} : -$$

136

$$c_1^{**} = \frac{(-1)^1 \begin{vmatrix} 2 & 1 \\ 4 & 0 \end{vmatrix}}{10} = 2/5, \quad c_2^{**} = \frac{(-1)^2 \begin{vmatrix} 3 & 1 \\ 1 & 0 \end{vmatrix}}{10} = -1/10$$

137

$$138 \quad \text{New Column } P_4 = \begin{bmatrix} 2/5 \\ -1/10 \\ -3/10 \end{bmatrix}$$

$$139 \quad \text{Column } P_5 = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix} : -$$

$$c_1^{**} = \frac{(-1)^1 \begin{vmatrix} 2 & 0 \\ 4 & -1 \end{vmatrix}}{10} = 1/5, \quad c_2^{**} = \frac{(-1)^2 \begin{vmatrix} 3 & 0 \\ 1 & -1 \end{vmatrix}}{10} = -3/10,$$

$$c_3^{**} = \frac{\begin{vmatrix} 3 & 2 & 0 \\ 1 & 4 & -1 \\ 1 & 1 & 0 \end{vmatrix}}{10} = 1/10, \quad R = \begin{vmatrix} 3 & 2 \\ 1 & 4 \end{vmatrix} = 10$$

$$140 \quad \text{New Column } P_5 = \begin{bmatrix} 1/5 \\ -3/10 \\ 1/10 \end{bmatrix}$$

141 Column $P_6 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$: -

142

$$c_1^{**} = \frac{(-1)^1 \begin{vmatrix} 2 & 0 \\ 4 & 1 \end{vmatrix}}{10} = -1/5, \quad c_2^{**} = \frac{(-1)^2 \begin{vmatrix} 3 & 0 \\ 1 & 1 \end{vmatrix}}{10} = 3/10$$

143

$$c_3^{**} = \frac{\begin{vmatrix} 3 & 2 & 0 \\ 1 & 4 & 1 \\ 1 & 1 & 0 \end{vmatrix}}{10} = -1/10, \quad R = \begin{vmatrix} 3 & 2 \\ 1 & 4 \end{vmatrix} = 10$$

144 New Column $P_6 = \begin{bmatrix} -1/5 \\ 3/10 \\ -1/10 \end{bmatrix}$

145 Column $P_7 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$: -

$$c_1^{**} = \frac{(-1)^1 \begin{vmatrix} 2 & 0 \\ 4 & 0 \end{vmatrix}}{10} = 0, \quad c_2^{**} = \frac{(-1)^2 \begin{vmatrix} 3 & 0 \\ 1 & 0 \end{vmatrix}}{10} = 0,$$

146

$$C_3^{**} = \frac{\begin{vmatrix} 3 & 2 & 0 \\ 1 & 4 & 0 \\ 1 & 1 & 1 \end{vmatrix}}{10} = 1 \quad R = \begin{vmatrix} 3 & 2 \\ 1 & 4 \end{vmatrix} = 10$$

147 New Column $P_7 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

148 So we get direct third simplex table using above formulae.

149 **Step (3):** Introduce P_1 and drop P_4

		0	0	0	-1	0	-1	0	
C_B	X_B	P_1	P_2	P_3	P_4	P_5	P_6	P_7	X_B
0	x_1	1	0	-2/5	2/5	1/5	-1/5	0	2/5
0	x_2	0	1	1/10	-1/10	-3/10	3/10	0	9/10
0	x_7	0	0	3/10	-3/10	1/10	-1/10	1	37/10

	$z_j - c_j$	0	0	0	1	0	1	0	
--	-------------	---	---	---	---	---	---	---	--

150

151 Since all $z_j - c_j \geq 0$, we go to **PHASE-II**

152 **Step (4):**

		5	8	0		0		0		R1	R2
C_B	X_B	P_1	P_2	P_3	P_4	P_5	P_6	P_7	X_B		
5	x_1	1	0	$b_1 = -2/5$		$a_1 = 1/5$		0	$2/5$	-ve	2
8	x_2	0	1	$b_2 = 1/10$		$a_2 = 3/10$		0	$9/10$	9	-ve
0	x_7	0	0	$b_3 = 3/10$		$a_3 = 1/10$		1	$37/10$	$37/3$	37
	$z_j - c_j$	0	0	-6/5		-7/5		0			
		↓	↓	↑		↑					

153

154 Here we introduce P_3 and P_5 simultaneously and outgoing vectors are P_2 and P_1 .

155 To find new values in X_B column.

156 Here we can find c_1^{**} , c_2^{**} and c_3^{**} using following formula.[7]

157

158 .

159

160 Column $X_B = \begin{bmatrix} 2/5 \\ 9/10 \\ 37/10 \end{bmatrix}$

$$c_1^{**} = \frac{(-1)^1 \begin{vmatrix} -2/5 & 2/5 \\ 1/10 & 9/10 \end{vmatrix}}{-1/10} = -4, \quad c_2^{**} = \frac{(-1)^2 \begin{vmatrix} 1/5 & 2/5 \\ -3/10 & 9/10 \end{vmatrix}}{-1/10} = -3$$

161

$$c_3^{**} = \frac{\begin{vmatrix} 1/5 & -2/5 & 2/5 \\ -3/10 & 1/10 & 9/10 \\ 1/10 & 3/10 & 37/10 \end{vmatrix}}{-1/10} = 5, \quad R = \begin{vmatrix} 1/5 & -2/5 \\ -3/10 & 1/10 \end{vmatrix} = -1/10$$

162 New $X_B = \begin{bmatrix} -4 \\ -3 \\ 5 \end{bmatrix}$

163 Here we found entries in X_B Column are negative so here we have to apply conventional

164 simplex method in step 4 only.

165 **Step (4):** Introduce P_5 and drop P_1

166

		5	8	0		0		0		Ratio
C_B	X_B	P_1	P_2	P_3	P_4	P_5	P_6	P_7	X_B	
0	x_5	5	0	-2		1		0	2	2
8	x_2	3/2	1	-1/2		0		0	3/2	-ve
0	x_7	-1/2	0	1/2		0		1	7/2	37
	$Z_j - C_j$	7	0	-4		0		0		
				↑				↓		

167

168 **Step (5):** Introduce P_3 and drop P_7

		5	8	0		0		0	
C_B	X_B	P_1	P_2	P_3	P_4	P_5	P_6	P_7	X_B
0	x_5	3	0	0		1		4	16
8	x_2	1	1	0		0		1	5
0	x_3	-1	0	1		0		2	7
	$Z_j - C_j$	3	0	0		0		8	

169

170 Since all $Z_j - C_j \geq 0$, an optimum basic feasible solution has been reached.

171 $x_1=0$ and $x_2=5$, and Max $Z = 40$.

172 It is observed that we reached to solution in PHASE -1 in one step only by Quick Simplex
 173 Method while in conventional Simplex Method we reached to solution after 2 step. here we
 174 got advantage of Quick Simplex Method.

175

176 6. STATEMENT OF THE PROBLEM-II

177 Use two phase simplex method to solve the following LPP

178 Maximize $Z = -2x_1 - x_2$

179 Subject to the constraints :

180 $3x_1 + x_2 \geq 3$

181 $4x_1 + 3x_2 \geq 6$

182 $x_1 + 2x_2 \geq 3, \quad x_j \geq 0$

183 **7. PROBLEM MODIFIED IN THE STANDARD FORM:**

184 $\text{Max } Z = 0x_1 + 0x_2 + 0x_3 - x_4 + 0x_5 - x_6 + 0x_7 - x_8$

185 Subject to the constraints :

186 $3x_1 + x_2 - x_3 + x_4 = 3,$

187 $4x_1 + 3x_2 - x_5 + x_6 = 6,$

188 $x_1 + 2x_2 - x_7 + x_8 = 3, \quad x_j \geq 0$

189 **8. SOLUTION TO THE PROBLEM USING SIMPLEX METHOD:**

190 In this problem it is observed that an optimum basic feasible solution has been reached by
 191 conventional Simplex Method in four step in phase I and again in one step in phase II
 192 and solution is $x_2 = 3/5$ and $x_2 = 6/5$ and $\text{Max } Z = -12/5$.

193 **9. SOLUTION TO THE PROBLEM USING QUICK SIMPLEX METHOD:**

194 **PHASE-I**

195 **Step (1):** (Initial table)

196

		0	0	0	-1	0	-1	0	-1		R1	R 2
C _B	X _B	P ₁	P ₂	P ₃	P ₄	P ₅	P ₆	P ₇	P ₈	X _B		
-1	x ₄	a₁=3	b ₁ =1	-1	1	0	0	0	0	3	1	3
-1	x ₆	a ₃ =4	b ₃ =3	0	0	-1	1	0	0	6	3/2	2
-1	x ₈	a ₂ =1	b₂=2	0	0	0	0	-1	1	3	3	3/2
	Z _j - C _j	-8	-6	1	0	1	0	1	0			
		↑	↑		↓				↓			

197

198

199 Here we introduce P₁ and P₂ simultaneously and outgoing vectors are P₄ and P₈

200 To find new values in X_B column. Using above formulae we find

201 New $X_B = \begin{bmatrix} 3/5 \\ 0 \\ 6/5 \end{bmatrix}$

202 Similarly We find other entries.

203 **Step(2):**

		0	0	0	-1	0	-1	0	-1	
C_B	X_B	P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8	X_B
0	x_1	1	0	$-2/5$	$2/5$	0	0	$1/5$	$-1/5$	$3/5$
0	x_6	0	0	1	-1	-1	1	1	-1	0
0	x_2	0	1	$1/5$	$-1/5$	0	0	$-3/5$	$3/5$	$6/5$
	$Z_j - C_j$	0	0	0	1	0	1	0	1	

204 Since all $z_j - c_j \geq 0$ we go to **PHASE-II**

206 **Step (2):**

		-2	-1	0	-1	0	-1	0	-1	
C_B	X_B	P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8	X_B
-2	x_1	1	0	$-2/5$	$2/5$	0	0	$1/5$	$-1/5$	$3/5$
-1	x_6	0	0	1	-1	-1	1	1	-1	0
-1	x_2	0	1	$1/5$	$-1/5$	0	0	$-3/5$	$3/5$	$6/5$
	$Z_j - C_j$	2	0	$-2/5$	$2/5$	1	0	$-4/5$	$4/5$	
							↓	↑		

207 **Step (3):** Introducing P_7 and dropping P_6

		-2	-1	0	-1	0	-1	0	-1		
C_B	X_B	P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8	X_B	
-2	x_1	1	0	$-3/5$	$3/5$	0	0	0	0	$3/5$	
0	x_7	0	0	1	-1	-1	1	1	-1	0	

-1	x_2	0	1	$4/5$	$-4/5$	$-3/5$	$3/5$	0	0	$6/5$	
	$z_j - c_j$	0	0	$2/5$	$3/5$	$3/5$	$2/5$	0	1		

209

210 Since all $z_j - c_j \geq 0$, an optimum basic feasible solution has been reached.

211

$x_1 = 3/5$ and $x_2 = 6/5$ and Max $Z = -12/5$.

212

213 In this problem it is observed that we reached a solution in PHASE -I in one step only by
 214 Quick Simplex Method, to which we reached after 3 steps when conventional Simplex
 215 Method is used. Hence the number of iterations required are reduced by Quick Simplex
 216 Algorithm Methodology. This method found to be more convenient when introducing more
 than one vector simultaneously.

217

10 STATEMENT OF THE PROBLEM-III

218

219 Use two phase simplex method to solve the following LPP

220

Minimize $z = 3x_1 - 2x_2 + 4x_3$

221

Subject to the constraints :

222

$3x_1 + 5x_2 + 4x_3 \geq 7$

223

$6x_1 + x_2 + 3x_3 \geq 4$

224

$7x_1 - 2x_2 - x_3 \leq 10$

225

$x_1 - x_2 + 5x_3 \geq 3$

226

$4x_1 + 7x_2 - 2x_3 \geq 2$

227

$x_j \geq 0$

228

Max $z = -3x_1 + 2x_2 - 4x_3$

229

Subject to the constraints :

230

$3x_1 + 5x_2 + 4x_3 \geq 7$

231

$6x_1 + x_2 + 3x_3 \geq 4$

232

$7x_1 - 2x_2 - x_3 \leq 10$

233

$x_1 - x_2 + 5x_3 \geq 3$

234

$4x_1 + 7x_2 - 2x_3 \geq 2$

235

$x_j \geq 0$

236

11 SOLUTION OF THE PROBLEM BY TWO PHASE SIMPLEX METHOD.

237

PHASE -I

238

Maximize $z = 0x_1 + 0x_2 + 0x_3 + 0x_4 - x_5 + 0x_6 - x_7 + x_8 + x_9 - x_{10} + x_{11} - x_{12}$

239

Subject to the constraints :

240

$3x_1 + 5x_2 + 4x_3 - x_4 + x_5 = 7$

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E-mail address: xyz@abc.com.

$$6x_1 + x_2 + 3x_3 - x_6 + x_7 = 4$$

$$7x_1 - 2x_2 - x_3 + x_8 = 10$$

$$x_1 - x_2 + 5x_3 - x_9 + x_{10} = 3$$

$$4x_1 + 7x_2 - 2x_3 - x_{11} + x_{12} = 2$$

$$x_j \geq 0$$

246

247 12. SOLUTION TO THE PROBLEM USING SIMPLEX METHOD:

248 In this problem it is observed that an optimum basic feasible solution has been reached by
249 conventional Simplex Method in five step in phase I and again in three step in phase II .

250 and solution is $x_1=0$ and $x_2=1$, $x_3=1$ and Max $Z = -2$.

251 13. SOLUTION TO THE PROBLEM USING QUICK SIMPLEX METHOD:

252 **Step (1):** (Initial table)

253

		0	0	0	0	-1	0	-1	0	0	-1	0	-1		Ratio1	Ratio2	Ratio3
C_B	X_B	P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8	P_9	P_{10}	P_{11}	P_{12}	x_B			
-1	x_5	$a_4=3$	$b_4=5$	$c_4=4$	-1	1	0	0	0	0	0	0	0	7	7/3	7/5	1.75
-1	x_7	$a_1=6$	$b_1=1$	$c_1=3$	0	0	-1	1	0	0	0	0	0	4	2/3	4	1.33
0	x_8	$a_5=7$	$b_5=-2$	$c_5=-1$	0	0	0	0	1	0	0	0	0	10	10/7	-5	-10
-1	x_{10}	$a_2=1$	$b_2=-2$	$c_2=5$	0	0	0	0	0	-1	1	0	0	3	3	-1.5	0.6
-1	x_{12}	$a_3=4$	$b_3=7$	$c_3=-2$	0	0	0	0	0	0	0	-1	1	2	1/2	0.28	-1
	$z_j - c_j$	-14	-11	-10	1	0	1	0	0	1	0	1	0				
		↑	↑	↑				↓			↓		↓				

254

255

256

257

258 Here we introduce P_1, P_2, P_3 simultaneously and outgoing vectors are P_7, P_{12}, P_{10} .

259 To find new values in X_B column.

260 Here we can find $d_1^{***}, d_2^{***}, d_3^{***}$ and d_4^{***} using following formula.

$$261 \quad R = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}, \quad d_1^{***} = \begin{vmatrix} b_1 & c_1 & d_1 \\ b_2 & c_2 & d_2 \\ b_3 & c_3 & d_3 \end{vmatrix} / R$$

262

$$263 \quad d_2^{***} = \begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix} / R, d_3^{***} = \begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix} / R$$

264

265

$$266 \quad d_4^{***} = \begin{vmatrix} a_1 & b_1 & c_1 & d_1 \\ a_2 & b_2 & c_2 & d_2 \\ a_3 & b_3 & c_3 & d_3 \\ a_4 & b_4 & c_4 & d_4 \end{vmatrix} / R$$

267

$$268 \quad \text{Column } X_B = \begin{bmatrix} 7 \\ 4 \\ 10 \\ 3 \\ 2 \end{bmatrix},$$

269

$$270 \quad R = \begin{vmatrix} 6 & 1 & 3 \\ 1 & -2 & 5 \\ 4 & 7 & -2 \end{vmatrix} = -119$$

$$271 \quad d_1^{***} = \begin{vmatrix} 1 & 3 & 4 \\ -2 & 5 & 3 \\ 7 & -2 & 2 \end{vmatrix} / R = 33/119, d_2^{***} = \begin{vmatrix} 6 & 4 & 3 \\ 1 & 3 & 5 \\ 4 & 2 & -2 \end{vmatrix} / R = 38/119,$$

$$272 \quad d_3^{***} = \begin{vmatrix} 6 & 1 & 4 \\ 1 & -2 & 3 \\ 4 & 7 & 2 \end{vmatrix} / R = 80/119,$$

$$273 \quad d_4^{***} = \begin{vmatrix} 6 & 1 & 3 & 4 \\ 1 & -2 & 5 & 3 \\ 4 & 7 & -2 & 2 \\ 3 & 5 & 4 & 7 \end{vmatrix} / R = 32/17, d_5^{***} = \begin{vmatrix} 6 & 1 & 3 & 4 \\ 1 & -2 & 5 & 3 \\ 4 & 7 & -2 & 2 \\ 7 & -2 & -1 & 10 \end{vmatrix} / R = 1115/119$$

274 Here we fill the entries according to the above table.

$$275 \quad \text{New } X_B = \begin{bmatrix} 32/17 \\ 33/119 \\ 1115/119 \\ 38/119 \\ 80/119 \end{bmatrix}$$

276 In this way we can find other entries of respective columns. Thus we reached directly to

277 fourth table of conventional simplex method.

Step (4):

		0	0	0	0	-1	0	-1	0	0	-1	0	-1		Ratio
C_B	X_B	P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8	P_9	P_{10}	P_{11}	P_{12}	x_B	
-1	x_5	0	0	0	-1	1	-11/17		0	29/17		22/17		32/17	32/29
0	x_3	0	0	1	0	0	15/119		0	-38/119		-13/119		80/119	-ve
0	x_8	0	0	0	0	0	276/119		1	-247/119		-144/119		1115/119	-ve
0	x_2	0	1	0	0	0	22/119		0	-24/119		-27/119		38/119	-ve
0	x_1	1	0	0	0	0	-31/119		0	23/119		11/119		33/119	33/23
	$z_j - c_j$	0	0	0	1	0	11/17		0	-29/17		-22/17			
						↓				↑					

278

Step (5): Introduce P_9 and drop P_5

		0	0	0	0	-1	0	-1	0	0	-1	0	-1	
C_B	X_B	P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8	P_9	P_{10}	P_{11}	P_{12}	x_B
0	x_9	0	0	0	-17/29		-11/29		0	1		22/29		32/29
0	x_3	0	0	1	-38/203		1/203		0	0		27/203		208/203
0	x_8	0	0	0	-247/203		311/203		1	0		74/203		2367/203
0	x_2	0	1	0	-24/203		22/203		0	0		-15/203		110/203
0	x_1	1	0	0	23/203		-38/203		0	0		-11/203		13/203
	$z_j - c_j$	0	0	0	0		0		0	0		0		

279

280

281

Since all $z_j - c_j \geq 0$, we go to PHASE-II

Step (6):

		-3	2	-4	0	-1	0	-1	0	0	-1	0	-1		Ratio
C_B	X_B	P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8	P_9	P_{10}	P_{11}	P_{12}	x_B	
0	x_9	0	0	0	-17/29		-11/29		0	1		22/29		32/29	16/11
-4	x_3	0	0	1	-38/203		1/203		0	0		27/203		208/203	208/27
0	x_8	0	0	0	-247/203		311/203		1	0		74/203		2367/203	2367/74
2	x_2	0	1	0	-24/203		22/203		0	0		-15/203		110/203	
-3	x_1	1	0	0	23/203		-38/203		0	0		-11/203		13/203	
	$z_j - c_j$	0	0	0	5/29		22/29		0	0		-15/29			
										↓		↑			

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285

Here we proceed as conventional simplex method to reach the solution as there is only one vector whose $z_j - c_j$ is negative.

Step (7): Introduce P_{11} and drop P_9

		-3	2	-4	0	-1	0	-1	0	0	-1	0	-1		Ratio
C_B	X_B	P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8	P_9	P_{10}	P_{11}	P_{12}	x_B	
0	x_{11}	0	0	0	-17/22		-1/2		0	29/22		1		16/11	-ve
-4	x_3	0	0	1	-13/154		1/14		0	-27/154		0		64/77	-ve
0	x_8	0	0	0	-72/77		12/7		1	-37/77		0		857/77	-ve
2	x_2	0	1	0	-27/154		1/14		0	15/154		0		50/77	-ve
-3	x_1	1	0	0	1/14		-3/14		0	1/14		0		1/7	2
	$z_j - c_j$	0	0	0	-5/22		1/2		0	15/22		0			
		↓			↑										

Step (8): Introduce P_4 and drop P_1

		-3	2	-4	0	-1	0	-1	0	0	-1	0	-1		Ratio
C_B	X_B	P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8	P_9	P_{10}	P_{11}	P_{12}	x_B	
0	x_{11}	119/11	0	0	0		-31/11		0	23/11		1		3	
-4	x_3	13/11	0	1	0		-2/11		0	-1/11		0		1	
0	x_8	144/11	0	0	0		-12/11		1	5/11		0		13	
2	x_2	27/11	1	0	0		5/11		0	3/11		0		1	
0	x_4	14	0	0	1		-3		0	1		0		2	
	$z_j - c_j$	35/11	0	0	0		2/11		0	10/11		0			

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289

290 Since all $z_j - c_j \geq 0$, an optimum basic feasible solution has been reached.

291 $x_1=0$ and $x_2=1$, $x_3=1$ and Max $Z = -2$.

292 CONCLUSION :

293 In the above problems it is observed that we reached to solution in PHASE -1 in one step
 294 only by quick simplex method .while in conventional simplex we reached to solution after 3
 295 or more steps. Such advantage occurs by following Quick Simplex Method[7] in solving
 296 many linear programming problems.

297 REFERENCES:

298

299 [1] Dr. Bharambe M.T., Simultaneous exchange of two basic variables in simplex
 300 method, J. Indian Soc. Stat. Oper. Res. Vol. XVII, No.1-4, Dec 1996.

301

302 [2] M.T. Bharambe, Mrs. N. N. Kasturiwale: Some practical experience with Karmarkar's
 303 Algorithm, J. Indian Soc. Stat. Oper. Res. Vol. XVI, No.1-4, Dec 1995.

304

305 [3] M.T. Bharambe, Mrs. N. N. Kasturiwale R R Dagaonkar: The Non-Basic Simplex
 306 Method, J. Indian Soc. Stat. Oper. Res. Vol. XIX, No.1-4, Dec 1998.

307

308 [4] M.T. Bharambe, R R Dagaonkar The Non-Basic Simplex Method for simple upper
 309 bounded linear programmes J. Indian Soc. Stat. Oper. Res. Vol. XX, No.1-4, Dec 1999.

310

- 311 [5] Dr. Paranjape S.R. The Simplex Method:Two variable replacement, Management
312 Science 12,135-141(1965)
313
314
- 315 [6] Mrs. N. V. Vaidya, Dr. Mrs. N.N .kasturiwale:Solving Game Problems Using a Quick
316 Simplex Algorithm a New Method,Int. J Latest Trend Math, (IJLTM) (E-ISSN-2049-2561)
317 Vol-4 No 1 March 2014,165-182.
318
- 319 [7] Mrs. N. V. Vaidya, Dr. Mrs. N.N .kasturiwale:Quick Simplex Algorithm for Optimal
320 Solution to the Linear Programming Problem along with Theoretical Proof of Formulae.,
321 Int. J Latest Trend Math, (IJLTM) (E-ISSN-2049- 2561) Vol-4 No 2 June, 2014,183- 200.
322
- 323 [8] Mrs. N. V. Vaidya, Dr. Mrs. N.N .kasturiwale: Comparison between various entering
324 vector criteria with quick simplex algorithm for optimal solution to the linear programming
325 problem.International Journal of Mathematics and Statistics Invention (IJMSI) E- ISSN:
326 2321 – 4767 P-ISSN: 2321 – 4759 www.ijmsi.org Vol 2 Issue 6 || June. 2014 || PP-17-36
327
- 328 [9] Khobragade N.W. and Khot P.G (2005): Alternative approach to the simplex method-II,
329 Acta Ciencia Indica, vol. xxx I M, No.3, 651,India.
330
- 331 [10] N. W. Khobragade , P.G.Khot: (2012): An Alternative Approach to the Simplex Method-1
332 Bulletin of Pure and Applied Sciences. Vol-23E(No.1) 2004;P.35-40.
333
- 334 [11] N. W. Khobragade , Navneet K. Lamba, P.G.Khot: Alternative Approach to solution to
335 Revised Simplex Method, Int. J. of Pure and Applied Mathematics, Vol. 52, No. 5,
336 2009,693-699.
337
- 338 [12] Mrs. N. V. Vaidya and N. W. Khobragade (2012):Optimum Solution to the Simplex
339 Method- An Alternative Approach Int. J Latest Trend Math Page No. 99 to 105 Vol.-2
340 No.3 Nov.2012
341
- 342 [13] Mrs. N. V. Vaidya and N. W. Khobragade :Solution of Game Problems Using New
343 Approach,International Journal of Engineering and Innovative Technology (IJEIT),
344 Volume 3, Issue 5,November 2013.
345
- 346 [14] Mrs. N.V. Vaidya , N. W. Khobragade and Navneet K. Lamba:Approximation algorithm
347 for optimal solution to the linear programming problem, Int. J. Mathematics in
348 Operational Research, Vol. 6, No. 2, 2014.
349
350
351
352
353
354
355
356