

SOME E-CORDIAL LABELINGS ON EXTENDED DUPLICATE GRAPH OF COMB GRAPH

Abstract

A graph labeling is a mapping that carries a set of graph elements onto a set of numbers called labels(usually the set of integers). In this paper we prove the existence of graph labeling such as E- cordial, total E- cordial, product E-cordial , total product E-cordial, Prime E- cordial and E_3 - cordial labeling for the extended duplicate graph of Comb graph by presenting algorithms.

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Keywords: Graph labeling, Comb graph , Duplicate graph.

1 . Introduction

Graph labeling is an emerging area of research in graph theory. It was introduced by Rosa in 1967 . It is an assignment of integers to the vertices or edges or both subject to certain conditions.In 1997 Yilmaz and Cahit [10] introduced a weaker version of edge graceful called E-cordial and E_k - cordial.They proved that the following graphs are E_3 - cordial:stars S_n iff $n \not\equiv 1(\text{mod } 3)$; K_n ($n \geq 3$); C_n ($n \geq 3$) .

The concept of product, total product and prime cordial labeling are introduced by Sundaram, Ponraj and Somasundaram [8] . They proved that the following graphs are product cordial: trees, unicyclic graphs of odd order; $K_{m,n}$ ($m,n > 2$), $P_m \times P_n$ ($m,n > 2$) and wheels are not product cordial.

K.Thirusangu, et al. [7], have introduced the concept of extended duplicate graph and they proved that $EDG(P_m)$ is E- cordial and also proved that the extended duplicate graph of a path P_m admits E- cordial ,total E- cordial, product E- cordial and total product E-cordial labeling.

In this paper, we prove that, the extended duplicate graph of Comb graph admits E- cordial, total E- cordial, product E-cordial , total product E-cordial, prime E-cordial and E_3 - cordial labeling by presenting algorithms.

2 Preliminaries

In this section, we give basic notions relevant to this paper.

Definition : 2.1

Let P_{m+1} be a path graph with $m+1$ vertices and m edges. Comb graph is defined as $P_{m+1} \odot (m+1)K_1$. It has $2m+2$ vertices and $2m+1$ edges.

Definition : 2.2

Let $G(V,E)$ be a Comb graph. A Duplicate graph of G is $DG=(V_1,E_1)$ where the vertex set $V_1 = V \cup V'$ and $V \cap V' = \emptyset$ and $f: V \rightarrow V'$ is bijective (for $v \in V$, we write $f(v) = v'$ for convenience) and the edge set E_1 of DG is defined as follows: The edge $v_i v_j$ is in E if and only if both $v_i v_j'$ and $v_i' v_j$ are edges in E_1 . Clearly duplicate graph is disconnected.

Definition : 2.3

A graph $G(V,E)$ is said to admit E-cordial labeling if there exists a function $f: E \rightarrow \{0,1\}$ such that the induced function f^* on V defined by $f^*(v_i) = \{\sum f(v_i v_j) \mid v_i v_j \in E\} \pmod{2}$ satisfies the property that the number of edges labeled with '0' and the number of edges labeled with '1' differ by at most one and number of vertices labeled with '0' and the number of vertices labeled with '1' differ by at most one. A graph that admits E-cordial labeling is called E-cordial. It admits total E-cordial labeling if it satisfies the property that the number of edges and vertices labeled with '0' and the number of edges and vertices labeled with '1' differ by at most one.

Definition : 2.4

A Graph $G(V,E)$ is said to admit product E-cordial labeling if there exists a function $f: E \rightarrow \{0,1\}$ such that the induced function f^* on V defined by $f^*(v_i) = \{\prod f(v_i v_j) \mid v_i v_j \in E\}$ satisfies the property that the number of edges labeled with '0' and the number of edges labeled with '1' differ by at most one and the number of vertices labeled with '0' and the number of vertices labeled with '1' differ by at most one. A graph that admits product E-cordial labeling is called product E-cordial. It is said to admit total product E-cordial

labeling if it satisfies the property that the number of edges and vertices labeled with '0' and the number of edges and vertices labeled with '1' differ by at most one.

Definition : 2.5

A prime E-cordial labeling of a graph G with edge set E is a function $f : E \rightarrow \{1, 2, 3, \dots, q\}$ such that the induced function $f^* : V \rightarrow N$ defined as $f^*(v_i)$ is assigned the label 1, if $\gcd(f(v_i v_j), f(v_i v_k)) = 1$ and 0, if $\gcd(f(v_i v_j), f(v_i v_k)) > 1$, then the number of edges labeled with '0' and '1' differ by at most 1.

Definition : 2.6

Let G be a graph with vertex set V and edge set E. An edge labeling $f : E \rightarrow Z_3$ where $Z_3 = \{0, 1, 2\}$ induces a vertex labeling $f^* : V \rightarrow Z_3$ defined by $f^*(v_i) = \left\{ \sum f(v_i v_j) \mid v_i v_j \in E \right\} \pmod{3}$ for all vertex $v \in V$. For $i \in Z_3$, let $m_i(f) = \{e \in E \mid f(e) = i\}$ and $n_i(f) = \{v \in V \mid f^*(v) = i\}$.

If $|m_i(f) - m_j(f)| \leq 1$ and $|n_i(f) - n_j(f)| \leq 1$, ($i \neq j$) for all $i, j \in Z_3$, it is called E_3 -cordial labeling. A graph G is called E_3 -cordial if it admits E_3 -cordial labeling. An E_3 -cordial labeling is said to be total E_3 -cordial labeling of G if for all $i, j \in Z_3 : |\{m_i(f) + n_i(f)\} - \{m_j(f) + n_j(f)\}| \leq 1$. A graph G is called total E_3 -cordial if it admits total E_3 -cordial labeling.

3. Main Result

In this section we prove the existence of E-cordial, total E-cordial, product E-cordial and total product E-cordial prime E-cordial and E_3 -cordial labeling for the extended duplicate graph of a Comb graph by presenting algorithms.

Definition:3.1 Structure of extended duplicate graph of comb graph.

Let $DG(\text{comb}) = (V, E)$ be the duplicate graph of comb graph. The extended duplicate graph of comb graph is obtained as follows: (i) Add the edges $v_1 v_{m+1}$ and $v_{m+2} v_{2m+2}'$ if $m \equiv 1 \pmod{2}$, (ii) Add $v_1 v_{m+1}'$ and $v_{m+2} v_{2m+2}$ to $DG(\text{comb})$ if $m \equiv 0 \pmod{2}$. Thus the vertex and edge set are as follows:

$$V = \{v_1, v_2, \dots, v_{2m+2}, v_1', v_2', \dots, v_{2m+2}'\}.$$

$$E = E_1 \cup E_2.$$

Where $E_1 = \{v_i v_{i+1}' \cup v_i' v_{i+1} \text{ for } 1 \leq i \leq m\} \cup \{v_i' v_{m+i+1} \cup v_i v_{m+i+1}' \text{ for } 1 \leq i \leq m+1\}$.

$$E_2 = \begin{cases} v_1 v_{m+1}, v_{m+2} v_{2m+2}' & \text{if } m \equiv 1 \pmod{2}. \\ \end{cases}$$

92 $\{ v_1 v_{m+1}', v_{m+2}' v_{2m+2} \}, \text{otherwise.}$

93 Thus the extended duplicate graph of comb graph has $4m+4$ vertices and $4m+4$ edges.

94 It is denoted by $\text{EDG}(\text{comb})$.

95 **Algorithm 1 :**

96 **Procedure :** (E-cordial labeling for extended duplicate graph of comb graph).

97 // assignment of labels to the edges

98 for $i=1$ to m do

99 $\{ v_i v_{i+1}' \leftarrow 0, v_i' v_{i+1} \leftarrow 1$

100 $\}$

101 end for

102 for $i=1$ to $m+1$ do

103 $\{ v_i v_{m+i+1}' \leftarrow 0, v_i' v_{m+i+1} \leftarrow 1$

104 $\}$

105 end for

106 **if** $m \equiv 0 \pmod{2}$

107 $v_1 v_{m+1}' \leftarrow 1; v_{m+2}' v_{2m+2} \leftarrow 0$

108 **if** $m \equiv 1 \pmod{2}$

109 $v_1 v_{m+1} \leftarrow 1; v_{m+2}' v_{2m+2}' \leftarrow 0$

110 end procedure

111 **output :** Edge labeled extended duplicate graph of comb graph .

112

113 **Theorem : 3.1**

114 Extended duplicate graph of comb graph , $\text{EDG}(\text{comb})$, $m \geq 2$ is E-cordial.

115 **Proof :**

116 From the construction of Extended duplicate graph of comb graph , it is clear that

117 $\text{EDG}(\text{comb})$ graph has $4m + 4$ vertices and $4m + 4$ edges.

118 In order to label the edges, define a function $f : E \rightarrow \{0, 1\}$ as given in algorithm

119 1. Clearly the number of edges labeled with '1' is $2m + 2$ and number of edges labeled

120 with '0' is $2m + 2$.

121 Thus the number of edges labeled with '0' and '1' is differ by atmost one.

122 For vertices, define the induced map $f^* : V \rightarrow \mathbb{N}$ such that $f^*(v) = [\sum_{u \in N(v)} f(uv)] \pmod{2}$

123 Thus the vertex labels are obtained as follows:

124 For $1 \leq i \leq m$, $f^*(v_i) = 1$, $f^*(v_i') = 0$

125 For $1 \leq i \leq m+1$, $f^*(v_{m+i+1}) = 1$, $f^*(v_{m+i+1}') = 0$

126 **If $m = 0(\text{mod } 2)$**

127 $f^*(v_{m+1}) = 1$: $f^*(v_{m+1}') = 0$

128 **If $m = 1(\text{mod } 2)$**

129 $f^*(v_{m+1}') = 1$: $f^*(v_{m+1}) = 0$

130 Thus the number of vertices labeled with '1' = $m + m + 1 + 1 = 2m + 2$

131 The number of vertices labeled with '0' = $m + m + 1 + 1 = 2m + 2$

132 Hence the number of vertices labeled with '1' and '0' differ by atmost one.

133 Therefore **EDG (comb)** graph is E-cordial.

134

135 **Theorem: 3.2**

136 Extended duplicate graph of comb graph, EDG (comb), $m \geq 2$ is total E-cordial.

137 **Proof :**

138 By theorem 3.2, we have the number of edges labeled with '1' and '0' are same as $2m + 2$.

139 The number of vertices labeled with '1' and '0' are same as $2m + 2$.

140 Thus the total number of vertices and edges labeled with '1' is $4m+4$ and '0' is $4m+4$

141 respectively. Hence the total number of vertices and edges labeled with '1' and the total number

142 of vertices and edges labeled with '0' are differ by atmost one.

143 Therefore the **Extended duplicate graph of comb graph** is total E-cordial

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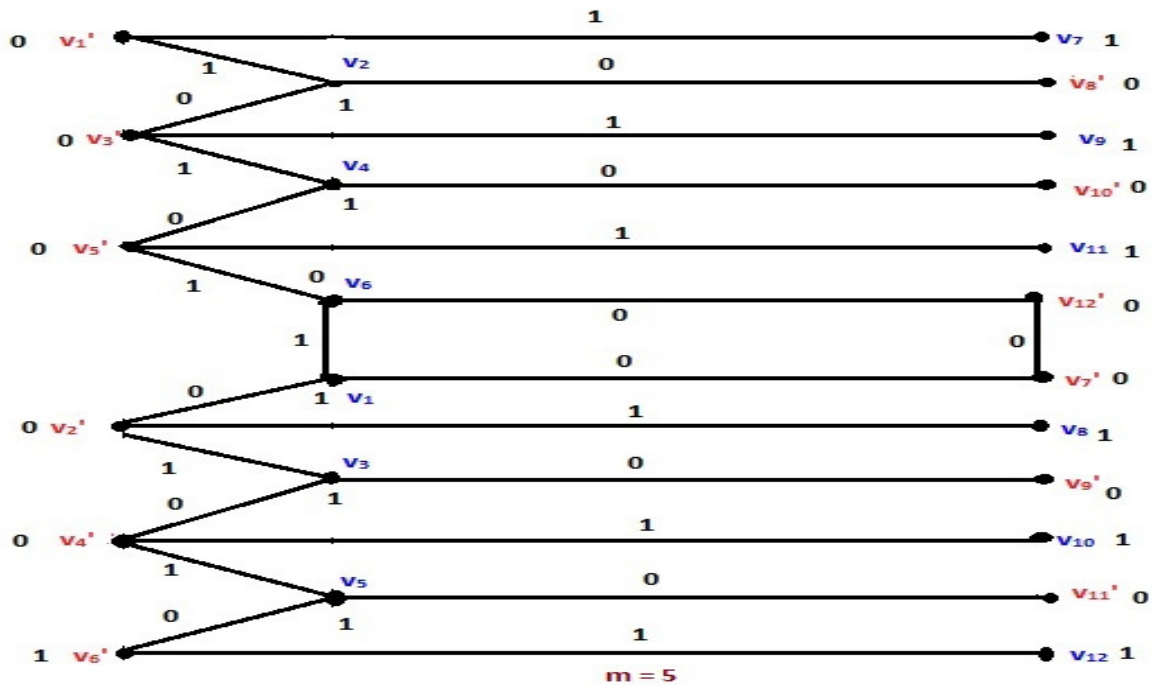
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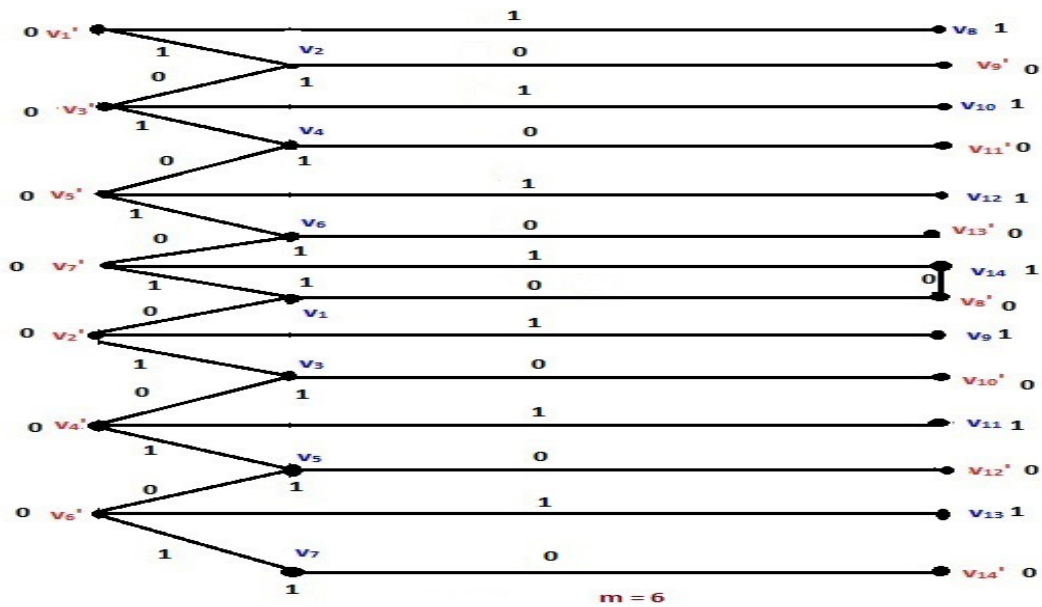
149 **Example : 3.1**

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E- cordial labeling of EDG(comb) graph for m=5 and m=6 are given below.



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155

156 Algorithm : 2

157 **Procedure :** (product E-cordial labeling for extended duplicate graph of comb graph).

158

159 // assignment of labels to edges

160 $v_m'v_{m+1} \quad , \quad v_mv_{m+1}' \quad \leftarrow \quad 1$

161 for $i = 1$ to $m - 1$ do

162 $\{ \quad v_iv_{i+1}' \quad \leftarrow \quad \begin{cases} 0 & \text{if } i \equiv 1 \pmod{2} \\ 1 & \text{otherwise} \end{cases}$

163 $\quad \quad \quad v_i'v_{i+1} \quad \leftarrow \quad \begin{cases} 0 & \text{if } i \equiv 0 \pmod{2} \\ 1 & \text{otherwise} \end{cases}$

166 $\}$

167 end for

168 for $i = 1$ to m do

169 $\{ \quad v_iv_{m+i+1}' \quad \leftarrow \quad \begin{cases} 0 & \text{if } i \equiv 1 \pmod{2} \\ 1 & \text{otherwise} \end{cases}$

171 $\quad \quad \quad v_i'v_{m+i+1} \quad \leftarrow \quad \begin{cases} 0 & \text{if } i \equiv 0 \pmod{2} \\ 1 & \text{otherwise} \end{cases}$

173 $\}$

174 end for

175 **if $m \equiv 0 \pmod{2}$**

176 $v_{m+1}v_{2m+2}' \quad \leftarrow \quad 1$

177 $v_1v_{m+1}', \quad v_{m+1}'v_{2m+2} \quad , \quad v_{m+2}'v_{2m+2} \quad \leftarrow \quad 0$

178 **if $m \equiv 1 \pmod{2}$**

179 $v_{m+1}'v_{2m+2} \quad \leftarrow \quad 1$

180 $v_1v_{m+1} \quad , \quad v_{m+1}v_{2m+2}' \quad , \quad v_{m+2}'v_{2m+2}' \quad \leftarrow \quad 0$

181 end procedure

182 **output :** Edge labeled extended duplicate graph of comb graph .

183 **Theorem : 3.3**

184 Extended duplicate graph of comb graph , $EDG(\text{comb})$, $m \geq 2$ is product E-cordial.

185 **Proof :**

186 From the construction of extended duplicate graph of comb graph, it is clear that

187 $EDG(\text{comb})$ has $4m + 4$ vertices and $4m + 4$ edges.

188 In order to label the edges, define a function $f : E \rightarrow \{0, 1\}$, using algorithm 2.

189 Clearly,

190 The number of edges labeled with '1' = $2m+2$.

191 The number of edges labeled with '0' = $2m+2$.

192 Thus the number of edges labeled with '1' and '0' differ by atmost one.

193 In order to get the label for vertices, define the induced map $f^* : V \rightarrow \mathbb{N}$ such that

$$194 f^*(v) = \left[\prod_{u \in N(v)} f(uv) \right]$$

195 Vertex labels are obtained as follows :

196 For $1 \leq i \leq m$

$$197 f^*(v_i) = f^*(v_{m+i+1}') = \begin{cases} 0 & \text{if } i \equiv 1 \pmod{2} \\ 1 & \text{otherwise} \end{cases}$$

$$199 f^*(v_i') = f^*(v_{m+i+1}) = \begin{cases} 1 & \text{if } i \equiv 1 \pmod{2} \\ 0 & \text{otherwise} \end{cases}$$

201

202 **If $m = 0 \pmod{2}$**

$$203 f^*(v_{m+1}) = f^*(v_{2m+2}') = 1$$

$$204 f^*(v_{m+1}') = f^*(v_{2m+2}) = 0$$

205 **If $m = 1 \pmod{2}$**

$$206 f^*(v_{m+1}) = f^*(v_{2m+2}') = 0$$

$$207 f^*(v_{m+1}') = f^*(v_{2m+2}) = 1$$

$$208 \text{ Thus the number of vertices labeled with '1' } = 2 + \frac{m}{2} + \frac{m}{2} + \frac{m}{2} + \frac{m}{2} = 2m + 2$$

$$209 \text{ Number of vertices labeled with '0' } = 2 + \frac{m}{2} + \frac{m}{2} + \frac{m}{2} + \frac{m}{2} = 2m + 2$$

210

211 Thus the number of vertices labeled with '1' and '0' is same as $2m + 2$

212 Hence the number of vertices labeled with '1' and '0' differ by atmost one.

213 Therefore **EDG (comb) graph** is product E-cordial.

214 **Theorem: 3.4**

215 Extended duplicate graph of comb graph , EDG (comb) , $m \geq 2$ is total product

216 E-cordial.

217 **Proof :**

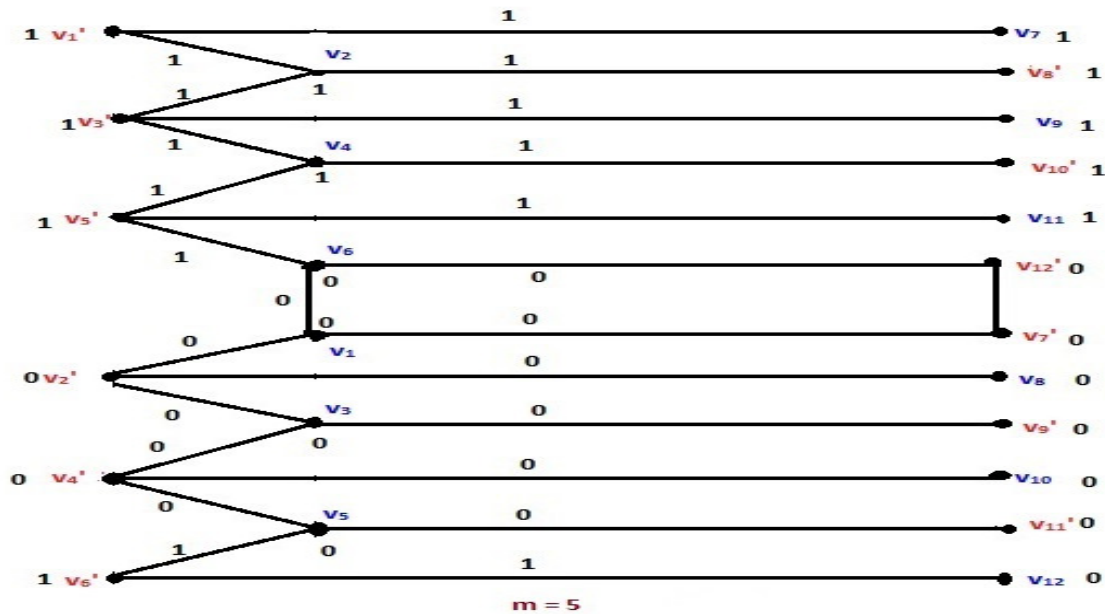
218 By theorem 3.3, we have the number of edges labeled with '1' and '0' are same as $2m + 2$.
 219 The number of vertices labeled with '1' and '0' are also same as $2m + 2$. Thus the total number
 220 of vertices and edges labeled with '1' is $4m+4$ and '0' is $4m+4$ respectively.

221 Hence the total number of vertices and edges labeled with '1' and the total number of
 222 vertices and edges labeled with '0' differ by atmost one.

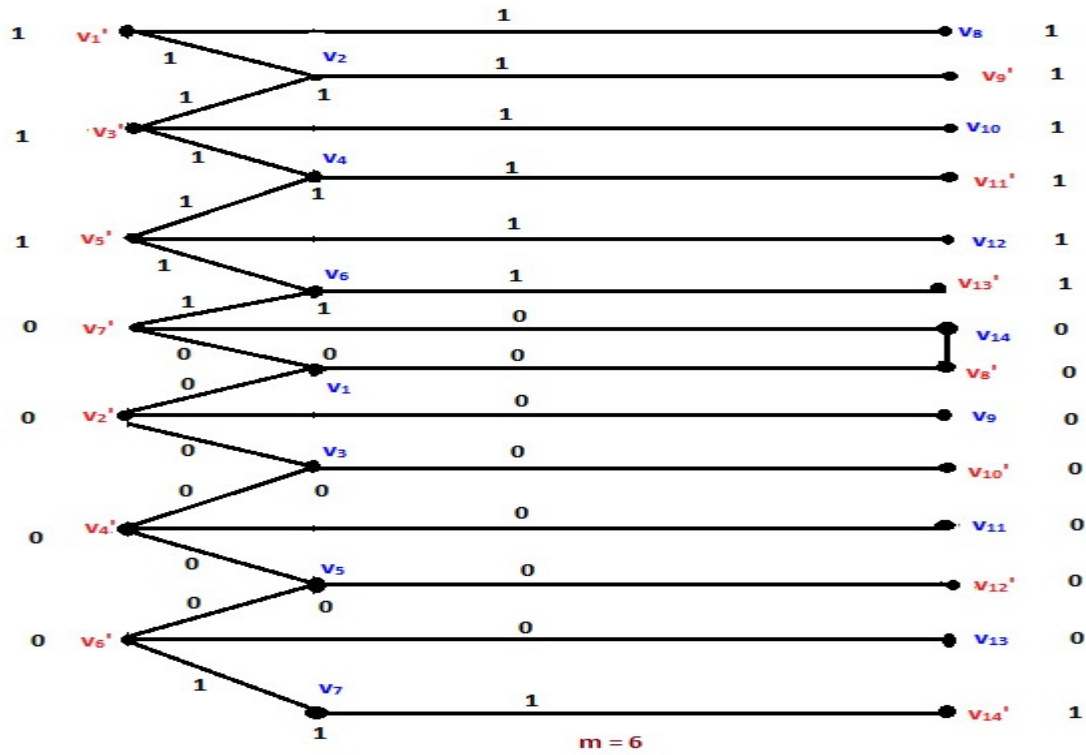
223 Therefore the extended duplicate graph of comb graph is total product E- cordial.

224 Example : 3.2

225 Product E- cordial labeling of EDG(comb) graph for $m=5$ and $m=6$ are given below.



226



227

228

229 **Algorithm : 3**

230 **Procedure :** (prime E-cordial labeling for extended duplicate graph of comb graph).

231

232 // assignment of labels to edges

233 $v_1v_2' \leftarrow 4$; $v_1v_{m+2}' \leftarrow 6$

234 for $i = 2$ to m do

235 $\{ v_i v_{i+1}' \leftarrow \begin{cases} 4i+2 & \text{if } i \equiv 1(\text{mod } 2) \\ 4i+3 & \text{otherwise} \end{cases}$

236

237 $v_i v_{m+i+1}' \leftarrow \begin{cases} 4i & \text{if } i \equiv 1(\text{mod } 2) \\ 4i+1 & \text{otherwise} \end{cases}$

238

239 }

240 end for

241 for $i = 1$ to m do

242 $\{ v_i' v_{i+1} \leftarrow \begin{cases} 4i+3 & \text{if } i \equiv 1(\text{mod } 2) \\ 4i+2 & \text{otherwise} \end{cases}$

243

244
$$v_i'v_{m+i+1} \leftarrow \begin{cases} 4i+1 & \text{if } i \equiv 1 \pmod{2} \\ 4i & \text{otherwise} \end{cases}$$

245

246
$$\}$$

247 end for

248 **if** $m \equiv 0 \pmod{2}$

249 $v_{m+1}'v_{2m+2} \leftarrow 1$, $v_1v_{m+1}' \leftarrow 2$, $v_{m+2}'v_{2m+2} \leftarrow 3$, $v_{m+1}v_{2m+2}' \leftarrow 4m+4$

250 **if** $m \equiv 1 \pmod{2}$

251 $v_{m+1}v_{2m+2}' \leftarrow 1$, $v_1v_{m+1} \leftarrow 2$; $v_{m+2}'v_{2m+2} \leftarrow 3$, $v_{m+1}'v_{2m+2} \leftarrow 4m+4$

252

253 end procedure

254 **output** : Edge labeled extended duplicate graph of comb graph.

255 **Theorem : 3.5**

256 Extended duplicate graph of comb graph admits prime E- cordial labeling

257 **Proof :**

258 EDG(comb) graph has $4m+4$ vertices and $4m+4$ edges

259 In order to label the edges , define a function $f : E \rightarrow \{ 1,2,3,\dots,4m+4 \}$ as

260 given in algorithm 3 .Clearly all the $4m+4$ edges are labeled.

261 Let $V_1 = \{ v_{m+2}, v_{m+3}, \dots, v_{2m+1}, v_{m+2}', v_{m+3}', \dots, v_{2m+1}' \}$ and $V_2 = V - V_1$.

262 The induced map $f^*: V \rightarrow N$ is defined such that ,

263 For all $v \in V_2$ $f^*(v) = \begin{cases} 1 & \text{if } \gcd(f(e)) = 1 \text{ where } e \in N(v) \\ 0 & \text{otherwise} \end{cases}$

264

265 For all $v \in V_1$, $f^*(v) = \begin{cases} 0 & \text{if } f(e) \equiv 0 \pmod{2} \text{ where } e \in N(v) \\ 1 & \text{otherwise} \end{cases}$

266 By the induced function , the vertex labels are obtained as follows:

267 For $1 \leq i \leq m+1$

268
$$f^*(v_i') = f^*(v_{m+i+1}) = \begin{cases} 1 & \text{if } i \equiv 1 \pmod{2} \\ 0 & \text{otherwise} \end{cases}$$

269

270

271
$$f^*(v_i) = f^*(v_{m+i+1}') = \begin{cases} 0 & \text{if } i \equiv 1 \pmod{2} \\ 1 & \text{otherwise} \end{cases}$$

272

273 Thus in case of $m \equiv 0(\text{mod } 2)$

274 The number of vertices labeled with '1' = $\frac{m}{2} + 1 + \frac{m}{2} + 1 + \frac{m}{2} + \frac{m}{2} = 2m+2$

275 The number of vertices labeled with '1' = $\frac{m}{2} + \frac{m}{2} + 1 + \frac{m}{2} + \frac{m}{2} + 1 = 2m+2$

276 In case of $m \equiv 1(\text{mod } 2)$

277 The number of vertices labeled with '1' = $\frac{m+1}{2} + \frac{m+1}{2} + \frac{m+1}{2} + \frac{m+1}{2} = 2m+2$

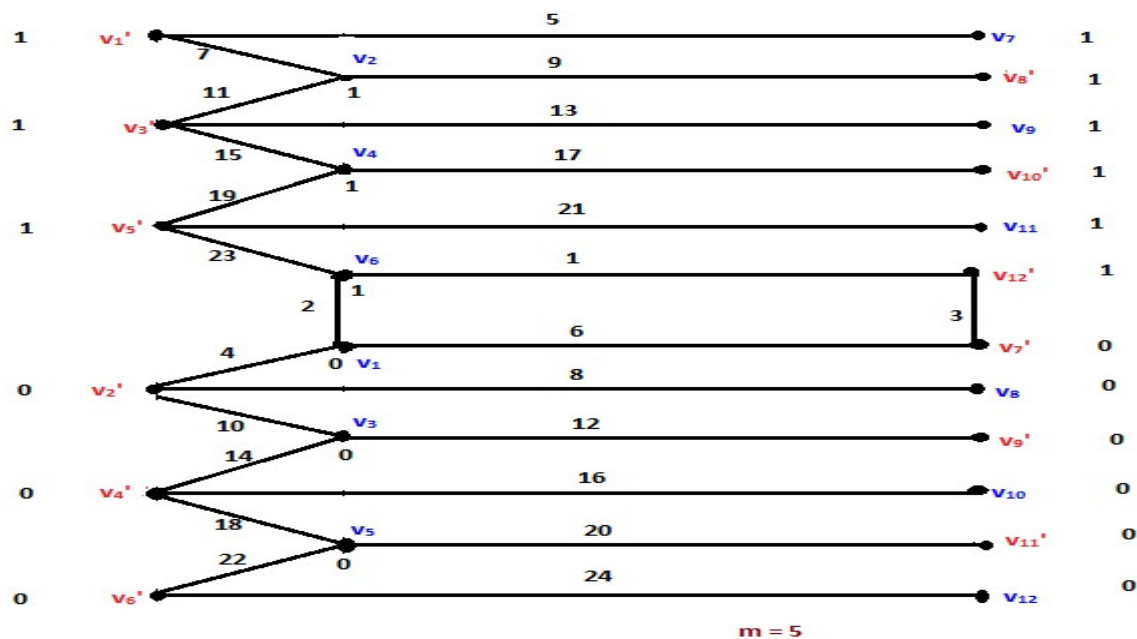
278 The number of vertices labeled with '1' = $\frac{m+1}{2} + \frac{m+1}{2} + \frac{m+1}{2} + \frac{m+1}{2} = 2m+2$

279 By the above two cases, the number of vertices labeled with '1' and '0' is same as
280 $2m+2$. Hence the number of vertices labeled with '1' and '0' differ by atmost '1'

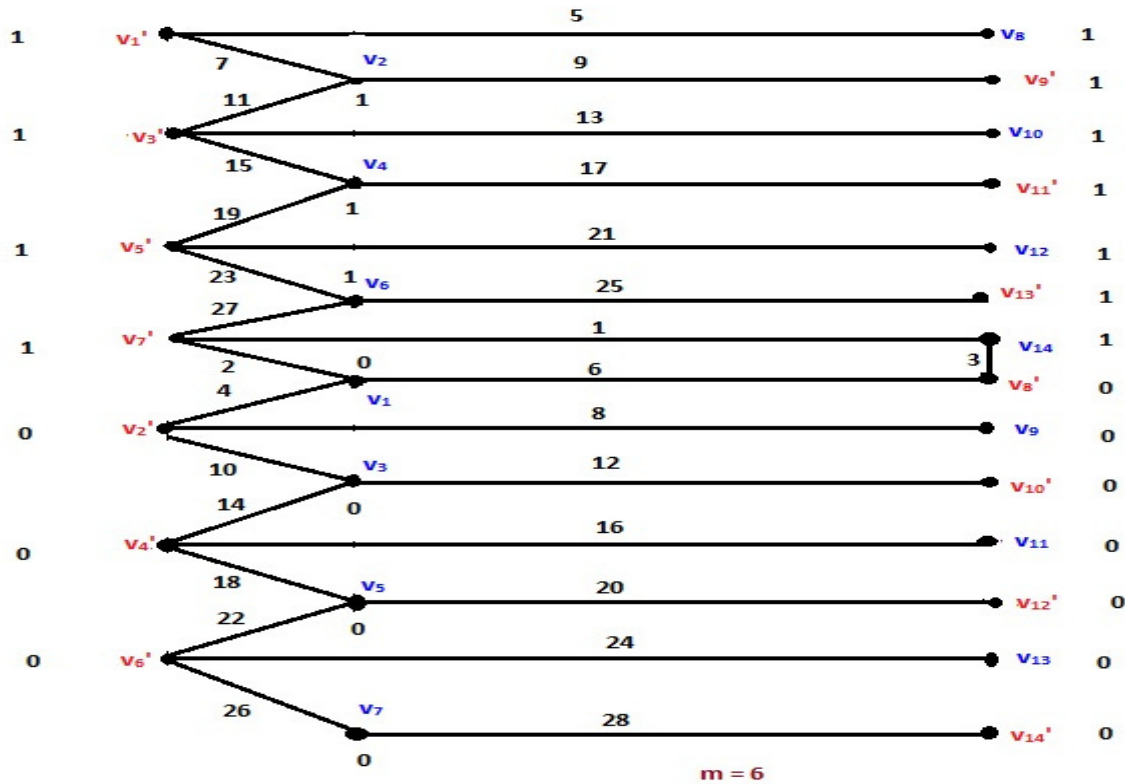
281 Therefore **EDG (comb) graph** is prime E – cordial.

282 Example: 3.3

283 Prime E- cordial labeling of EDG (comb) graph for $m=5$ and $m=6$ are given below.



285



286

287 **Algorithm : 4**

288 **Procedure :** (E_3 -cordial labeling for extended duplicate graph of comb graph).

289 // assignment of labels to edges

290 $v_1 v_{m+2}' \leftarrow 0$, $v_1 v_2'$, $v_1' v_{m+2} \leftarrow 2$.

291 **if** $m = 2n$

292 $v_1 v_{m+1}'$, $v_{m+1}' v_{2m+2}$, $v_{m+2}' v_{2m+2} \leftarrow 0$

293 $v_{m+1} v_{2m+2}' \leftarrow 2$

294 $v_1' v_2 \leftarrow \begin{cases} 0 & \text{if } m = 3n+1 \\ 1 & \text{otherwise} \end{cases}$

296 $v_m v_{m+1}' \leftarrow \begin{cases} 0 & \text{if } m = 3n \\ 1 & \text{if } m = 3n+1 \\ 2 & \text{if } m = 3n-1 \end{cases}$

299

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301

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302          $v_m' v_{m+1} \leftarrow \begin{cases} 0 & \text{if } m = 3n \\ 1 & \text{if } m = 3n-1 \\ 2 & \text{if } m = 3n+1 \end{cases}$ 
303
304
305     for  $i = 2$  to  $m-1$  do
306         {
307
308              $v_i v_{i+1}' \leftarrow \begin{cases} 0 & \text{if } i \in \{6n, 6n-1\} \\ 1 & \text{if } i \in \{6n-2, 6n-3\} \\ 2 & \text{if } i \in \{6n-4, 6n-5\} \end{cases}$ 
309
310
311              $v_i' v_{i+1} \leftarrow \begin{cases} 0 & \text{if } i \in \{6n, 6n-5\} \\ 1 & \text{if } i \in \{6n-4, 6n-3\} \\ 2 & \text{if } i \in \{6n-2, 6n-1\} \end{cases}$ 
312
313
314         }
315     end for
316
317     for  $i = 2$  to  $m$  do
318         {  $v_i v_{m+i+1}' \leftarrow \begin{cases} 0 & \text{if } i \in \{6n, 6n-5\} \\ 1 & \text{if } i \in \{6n-4, 6n-1\} \\ 2 & \text{if } i \in \{6n-2, 6n-3\} \end{cases}$ 
319
320
321
322          $v_i' v_{m+i+1} \leftarrow \begin{cases} 0 & \text{if } i \in \{6n-2, 6n-3\} \\ 1 & \text{if } i \in \{6n-4, 6n-1\} \\ 2 & \text{if } i \in \{6n, 6n-5\} \end{cases}$ 
323
324
325         }
326     end for
327     if  $m = 2n + 1$ 
328          $v_1 v_{m+1}, v_{m+1} v_{2m+2}', v_{m+2}' v_{2m+2}' \leftarrow 0$ 
329          $v_1' v_2 \leftarrow \begin{cases} 1 & \text{if } m = 3n+1 \\ 0 & \text{otherwise} \end{cases}$ 
330
331          $v_m' v_{m+1} \leftarrow \begin{cases} 1 & \text{if } m = 3n \\ 2 & \text{otherwise} \end{cases}$ 
332

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$$v_{m+1}'v_{2m+2} \leftarrow \begin{cases} 1 & \text{if } m = 3n+1 \\ 2 & \text{otherwise} \end{cases}$$

$$v_m v_{m+1}' \leftarrow \begin{cases} 0 & \text{if } m = 3n - 1 \\ 1 & \text{if } m = 3n \\ 2 & \text{if } m = 3n+2 \end{cases}$$

for $i = 2$ to $m-1$ do

$$\{ v_i v_{i+1}' \leftarrow \begin{cases} 0 & \text{if } i \in \{ 6n-1, 6n \} \\ 1 & \text{if } i \in \{ 6n-3, 6n-2 \} \\ 2 & \text{if } i \in \{ 6n-4, 6n-5 \} \end{cases}$$

$$v_i' v_{i+1} \leftarrow \begin{cases} 0 & \text{if } i \in \{ 6n, 6n-5 \} \\ 1 & \text{if } i \in \{ 6n-3, 6n-4 \} \\ 2 & \text{if } i \in \{ 6n-1, 6n-2 \} \end{cases}$$

$\}$

end for

for $i = 2$ to m do

$$\{ v_i v_{m+i+1}' \leftarrow \begin{cases} 0 & \text{if } i \in \{ 6n, 6n-5 \} \\ 1 & \text{if } i \in \{ 6n-1, 6n-4 \} \\ 2 & \text{if } i \in \{ 6n-3, 6n-2 \} \end{cases}$$

$$v_i' v_{m+i+1} \leftarrow \begin{cases} 0 & \text{if } i \in \{ 6n-3, 6n-2 \} \\ 1 & \text{if } i \in \{ 6n-1, 6n-4 \} \\ 2 & \text{if } i \in \{ 6n, 6n-5 \} \end{cases}$$

$\}$

end for

end procedure

output : Edge labeled extended duplicate graph of comb graph.

Theorem: 3.6

Extended duplicate graph of comb graph, EDG (comb) graph $m \geq 2$, admits E_3 -cordial labeling.

364 **Proof:**

365 EDG (comb) graph has $4m+4$ vertices and $4m+4$ edges.

366 To label the edges, define a function $f : E \rightarrow \{0,1,2\}$ as given in algorithm 4.

367 Denote $m_i(f) = \{e \in E / f(e)=i\}$.

368 Clearly the edge labels are as follows:

369 **If $m = 3n$**

370 Number of edges labeled with 0 is $4m+6 / 3$ and labeled with 1 and 2 are same as $4m+3/3$

371 **If $m = 3n - 1$**

372 Number of edges labeled with 0 ,1 and 2 are same as $4m+4/3$

373 **If $m = 3n + 1$**

374 Number of edges labeled with 1 is $4m+2 / 3$ and labeled with 0 and 2 are same as $4m+5/3$

375 Now all the $4m+4$ edges are labeled.

376 Thus for all $i,j \in \{0,1,2\}$; $|m_i(f) - m_j(f)| \leq 1$

377 The induced function $f^*: V \rightarrow \{0,1,2\}$ is defined such that

378 $f^*(v_i) = \{ \sum f(v_i v_j) / v_i v_j \in E \} \pmod{3}$ for all vertex $v \in V$

379 Denote $n_i(f) = \{ v \in V: f^*(v) = i \}$

380 By the induced function , the vertex labels are obtained as follows:

381 $f^*(v_1) = f^*(v_{m+2}) = 2$; $f^*(v_{m+2}') = 0$

382
$$f^*(v_2) = \begin{cases} 0 & \text{if } m = (6n-1) \text{ or } (6n-2) \text{ or } (6n-3) \\ 1 & \text{otherwise} \end{cases}$$

384 For $3 \leq i \leq m$

385
$$f^*(v_i) = \begin{cases} 0 & \text{if } i \in \{6n-1, 6n-4\} \\ 1 & \text{if } i \in \{6n-3, 6n-2\} \\ 2 & \text{if } i \in \{6n, 6n-5\} \end{cases}$$

388 For $2 \leq i \leq m - 1$

389
$$f^*(v_i') = \begin{cases} 0 & \text{if } i \in \{6n-3, 6n-2\} \\ 1 & \text{if } i \in \{6n-1, 6n-4\} \\ 2 & \text{if } i \in \{6n, 6n+1\} \end{cases}$$

392

393

394

395 For $2 \leq i \leq m$

$$396 \quad f^*(v_{m+i+1}) = \begin{cases} 0 & \text{if } i \in \{6n-3, 6n-2\} \\ 397 \quad 1 & \text{if } i \in \{6n-1, 6n-4\} \\ 398 \quad 2 & \text{if } i \in \{6n, 6n+1\} \end{cases}$$

399

$$400 \quad f^*(v_{m+i+1}') = \begin{cases} 0 & \text{if } i \in \{6n+1, 6n\} \\ 401 \quad 1 & \text{if } i \in \{6n-1, 6n-4\} \\ 402 \quad 2 & \text{if } i \in \{6n-3, 6n-2\} \end{cases}$$

403 **In case of $m \equiv 1 \pmod{2}$**

$$404 \quad f^*(v_{2m+2}') = 0$$

405 If $m = 3n$

$$406 \quad f^*(v_{m+1}) = 1, \quad f^*(v_1') = f^*(v_{2m+2}) = 2; \quad f^*(v_{m+1}') = f^*(v_m') = 0$$

407 If $m = 3n - 1$

$$408 \quad f^*(v_{m+1}) = f^*(v_1') = f^*(v_{2m+2}) = f^*(v_{m+1}') = 2, \quad f^*(v_m') = 1.$$

409 If $m = 3n + 1$

$$410 \quad f^*(v_{m+1}) = 2; \quad f^*(v_1') = f^*(v_{m+1}') = 0; \quad f^*(v_{2m+2}) = f^*(v_m') = 1$$

411 **In case of $m \equiv 0 \pmod{2}$**

$$412 \quad f^*(v_{2m+2}) = 0; \quad f^*(v_{2m+2}') = 2$$

413 If $m = 3n$

$$414 \quad f^*(v_m') = f^*(v_{m+1}) = 2 : f^*(v_1') = f^*(v_{m+1}') = 0$$

415 If $m = 3n - 1$

$$416 \quad f^*(v_m') = 1; \quad f^*(v_{m+1}) = f^*(v_1') = 0; \quad f^*(v_{m+1}') = 2$$

417 If $m = 3n + 1$

$$418 \quad f^*(v_m') = 0; \quad f^*(v_{m+1}) = f^*(v_{m+1}') = 1; \quad f^*(v_1') = 2$$

419 Thus we have ,

420 **If $m = 3n$**

421 Number of vertices labeled with 0 is $4m+6/3$ and labeled with 1 and 2 are same as $4m+3/3$

422 **If $m = 3n - 1$**

423 Number of vertices labeled with 0, 1 and 2 are same as $4m+4/3$

424 **If $m = 3n + 1$**

425 Number of vertices labeled with 1 is $4m+2/3$ and labeled with 0 and 2 are same as $4m+5/3$

Thus for all $i, j \in \{0, 1, 2\}$, $|n_i(f) - n_j(f)| \leq 1$

Hence **EDG (comb) graph** admits E_3 – cordial labeling

Theorem : 3.7

The extended duplicate graph of comb graph admits total E_3 – cordial labeling.

Proof :

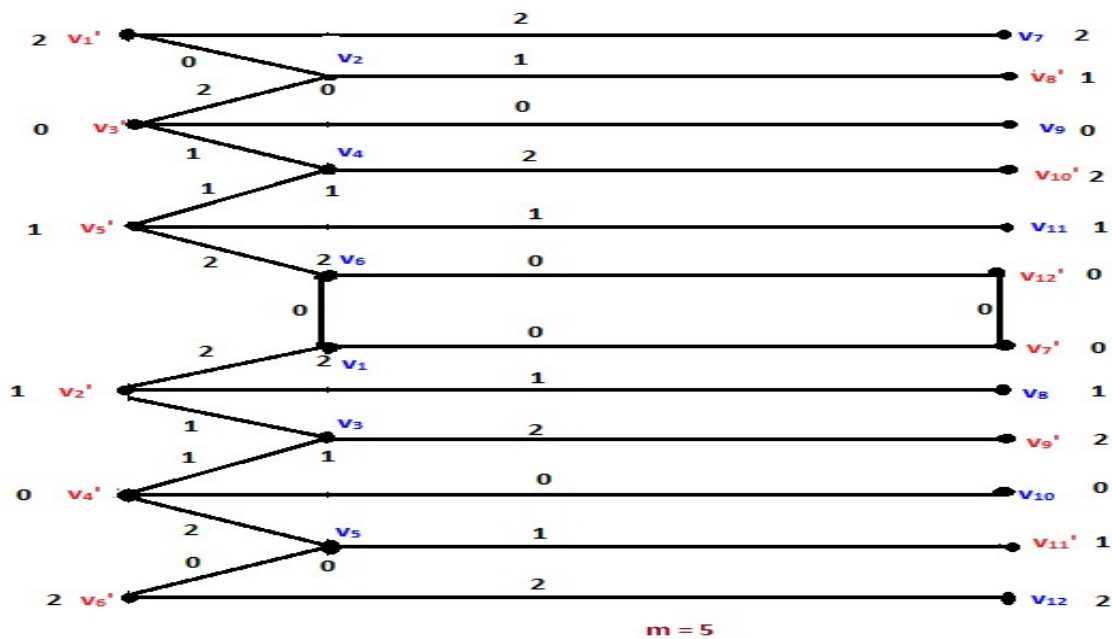
In theorem 3.6, the number of edges labeled with ‘0’ is $4m+6 / 3$ and labeled with ‘1’ and ‘2’ are same as $4m+3 / 3$ for the case $m = 3n$. The number of edges labeled with 0 , 1 , 2 are same as $4m+4/3$ for the case $m = 3n - 1$. The number of edges labeled with ‘1’ is $4m+2/3$ and ‘0’ and ‘2’ are same as $4m+5/3$. It was shown that the number of vertices labeled with ‘0’ is $4m+6 / 3$ and labeled with ‘1’ and ‘2’ are same as $4m+3 / 3$ for the case $m = 3n$. The number of vertices labeled with 0 , 1 , 2 are same as $4m+4/3$ for the case $m = 3n - 1$. The number of vertices labeled with ‘1’ is $4m+2/3$ and ‘0’ and ‘2’ are same as $4m+5/3$ and satisfies for all $i, j \in \mathbb{Z}_3$

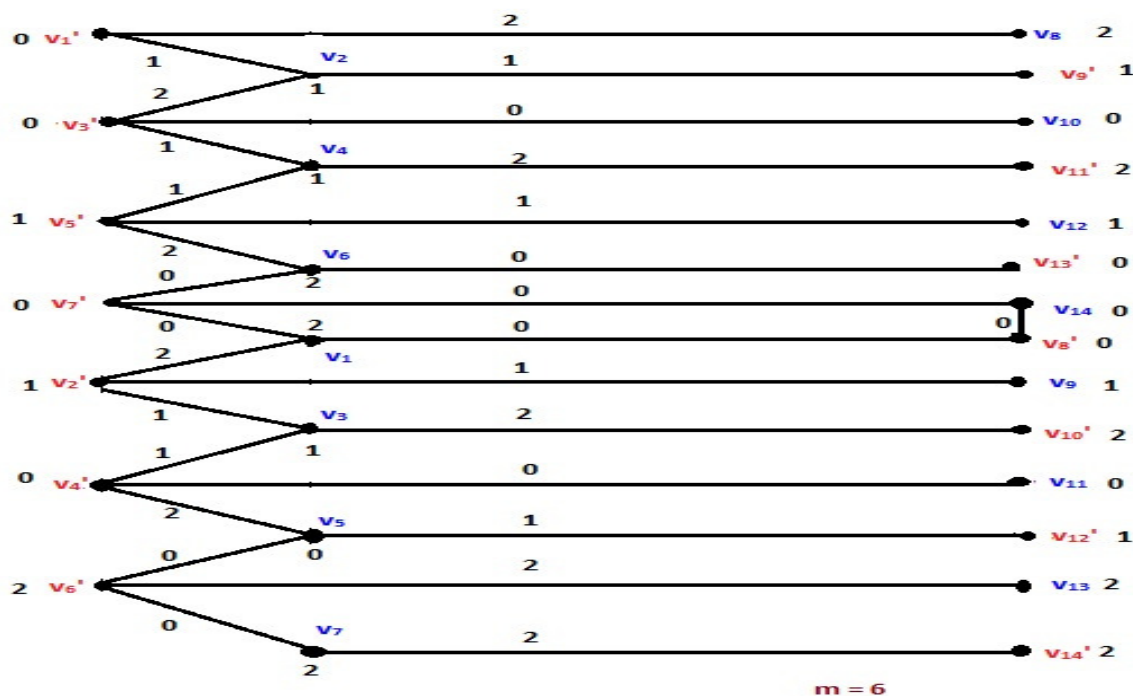
$|\{m_i(f) + n_i(f)\} - \{m_j(f) + n_j(f)\}| \leq 1$.

Hence the **extended duplicate graph** of comb graph is total E_3 – cordial.

Example: 3.4

E_3 -cordial labeling of EDG(comb) graph for $m=5$ and $m=6$ are given below.





Conclusion:

In this paper we have presented algorithms and proved that the extended duplicate graph of Comb graph admits E- cordial, total E- cordial,product E- cordial,total product E-cordial,prime E- cordial, E₃ cordial, and total E₃ cordial labeling.

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