

Covariant Almost Analytic Vector Field On Q -Quasi Umbilical Hypersurface of a Trans Sasakian Manifolds

Abstract. The present paper aims to study the axioms of covariant almost analytic vector field on Q -quasi umbilical hypersurface M of a trans-Sasakian manifold $\bar{M}$ with structure (ϕ, g, u, v, λ) and obtained the scalars a and b using 1-forms u, v covariant almost analytic for the hypersurface M to be totally umbilical and cylindrical.

1. Introduction

The study of hypersurfaces with almost contact structures are important parts of differentail geometry. It was 1967, M. Okumura [7] studied the totally umbilical hypersurfaces of a product Riemannian manifold and hypersurface of an almost r -paracontact Riemannian manifold studied by A. Bucki [2]. In 1970, Goldberg and Yano [3] studied the geometry of non-invariant hypersurfaces of almost contact manifolds. D. E. Blair [1] discussed the case of almost contact manifolds with Killing structure tensor. In 1989, D. Narain studied differential geometry of hypersurfaces with (f, g, u, v, λ) -structures [4]. Moreover, D. Narain and others also studied the non-invariant hypersurfaces of Sasakian, nearly Sasakian and para Sasakian manifold in [5,6,10] respectively. After that Prasad and Tripathi [9] studied the transversal hypersurfaces of Kenmotsu manifolds. In 1985, J. A. Oubina introduced a new class of almost

¹2010 Mathematical Subject Classification : 53C21, 53C25, 53C10

²Key words and phrases : Trans-Sasakian manifolds, Covariant almost analytic vector field, Q -quasi umbilical Hypersurface.

contact metric manifold known as trans-Sasakian manifold [8]. This class contains α -Sasakian and β -Kenmotsu manifold. Thus motivated by the studies referred above in the present paper, we have studied the covariant almost analytic vector field on Q -quasi umbilical hypersurface of a trans-Sasakian manifold.

The present paper is organized as follows. In section 2, we give the brief introduction about the trans-Sasakian manifold. In section 3, the hypersurfaces of a trans-Sasakian manifold with (ϕ, g, u, v, λ) -structure have been discussed. In section 4, expresses the Q -quasi umbilical hypersurface and in section 5, we obtained some results on covariant almost analytic vector field on Q -quasi umbilical hypersurface of trans-Sasakian manifold.

2. Preliminaries

Let $\bar{M}$ be an $(2n + 1)$ dimensional manifold with almost contact metric structure $(\bar{\phi}, \xi, \eta, \bar{g})$, where $\bar{\phi}$ is a $(1,1)$ tensor field, ξ is a vector field, η is a 1-form and $\bar{g}$ is a compatible Riemannian metric such that

$$(2.1) \quad \bar{\phi}^2 = -I + \eta \otimes \xi, \quad \phi\xi = 0, \quad \eta\phi = 0, \quad \eta(\xi) = 1,$$

$$(2.2) \quad g(\bar{\phi}X, \bar{\phi}Y) = \bar{g}(X, Y) - \eta(X)\eta(Y),$$

$$(2.3) \quad g(X, \phi Y) = -g(\phi X, Y), \quad \bar{g}(X, \xi) = \eta(X)$$

An almost contact metric structure $(\bar{\phi}, \xi, \eta, \bar{g})$, is called an trans Sasakian manifold if

$$(2.4) \quad (\bar{\nabla}_X \bar{\phi})(Y) = \alpha(\bar{g}(X, Y)\xi - \eta(Y)X) + \beta(\bar{g}(\phi X, Y)\xi - \eta(Y)\bar{\phi}X)$$

and it follows

$$(2.5) \quad \bar{\nabla}_X \xi = -\alpha\bar{\phi}X + \beta(X - \eta(X)\xi),$$

where $\bar{\nabla}$ denotes the Riemannian connection of the Riemannian metric g , then the structure $(\bar{\phi}, \xi, \eta, \bar{g})$ is called a trans-Sasakian manifold.

If we define $G(X, Y) = g(\phi X, Y)$, then we also find

$$(2.6) \quad G(X, Y) + G(Y, X) = 0,$$

$$(2.7) \quad G(X, \phi Y) = G(Y, \phi X),$$

$$(2.8) \quad G(\phi X, \phi Y) = G(X, Y).$$

3. Hypersurface of a trans-Sasakian manifold with (ϕ, g, u, v, λ) -structure.

Let us consider an $2n$ -dimensional manifold M embedded in $\bar{M}$ with embedding $i : M \rightarrow \bar{M}$. The map i induces a linear transformation map B , $B : T_p \rightarrow T_{ip}$. Let an affine unit normal N of M is in such way that $\bar{\phi}N$ is always tangent to the hypersurface and satisfying the linear transformations

$$(3.1) \quad \bar{\phi}BX = B\phi X + u(X)N,$$

$$(3.2) \quad \bar{\phi}N = -BU,$$

$$(3.3) \quad \xi = BV + \lambda N,$$

$$(3.4) \quad \eta(BX) = v(X),$$

where ϕ is $(1, 1)$ type tensor; U, V are vector fields; u, v are 1-form and λ is a C^∞ -function. If $u \neq 0$, M , is called a non-invariant hypersurface of $\bar{M}$.

Operating (3.1), (3.2), (3.3) and (3.4) by $\bar{\phi}$ and using (2.1), (2.2) and (2.3) and taking tangent and normal parts separately, we get the following induced structures on M .

$$(3.5) \quad \phi^2 X = -X + u(X)U + v(X)V,$$

$$(3.6) \quad u(\phi X) = \lambda v(X), \quad v(\phi X) = -\eta(N)u(X),$$

$$(3.7) \quad \phi U = -\eta(N)V, \quad \phi V = \lambda U,$$

$$(3.8) \quad u(U) = 1 - \lambda\eta(N), \quad u(V) = 0,$$

$$(3.9) \quad v(U) = 0, \quad v(V) = 1 - \lambda\eta(N)$$

and from (2.2) and (2.3), we get the induced metric g on M .

$$(3.10) \quad g(\phi X, \phi Y) = g(X, Y) - u(X)u(Y) - v(X)v(Y),$$

$$(3.11) \quad g(U, X) = u(X), \quad g(V, X) = v(X).$$

If we consider $\eta(N) = \lambda$, we get the following structures on M

$$(3.12) \quad \phi^2 = -I + u \otimes U + v \otimes V,$$

$$(3.13) \quad \phi U = \lambda V, \quad \phi V = \lambda U,$$

$$(3.14) \quad u \circ \phi = \lambda v, \quad v \circ \phi = -\lambda u,$$

$$(3.15) \quad u(U) = 1 - \lambda^2, \quad u(V) = 0,$$

$$(3.16) \quad v(U) = 0, \quad v(V) = 1 - \lambda^2.$$

A manifold M with a metric g satisfying (3.10), (3.11) and (3.12) is called manifold with (ϕ, g, u, v, λ) -structure. Let ∇ be the induced connection on the hypersurface M of the affine connection $\bar{\nabla}$ of $\bar{M}$.

Now the Gauss and Weingarten's equations are given by

$$(3.17) \quad \bar{\nabla}_{BX}BY = B\nabla_XY + h(X, Y)N,$$

$$(3.18) \quad \bar{\nabla}_{BX}N = BHX + w(X)N, \quad \text{where } g(HY, Z) = h(Y, Z).$$

Here h and H are the second fundamental tensor of type $(0, 2)$ and $(1, 1)$ and w is a 1-form.

Now differentiating (3.1), (3.2), (3.3) and (3.4) covariantly and using (3.17), (3.18) and (2.4) and again re-using (3.1), (3.2), (3.3) and (3.4), we get

$$(3.19) \quad (\nabla_Y\phi)(X) = \alpha \{v(X)Y - g(X, Y)V\} + \beta \{\lambda g(\phi X, Y)V - v(Y)\phi X\} \\ - h(X, Y)U - u(X)HY,$$

$$(3.20) \quad (\nabla_Yu)(X) = \alpha \lambda g(X, Y) + \beta \{\lambda g(\phi X, Y) - u(X)v(Y)\} \\ + g(X, \phi Y) - h(X, \phi Y),$$

$$(3.21) \quad (\nabla_Yv)(X) = \alpha g(\phi Y, X) - \beta \{\lambda g(Y, X) - u(X)u(Y) - v(X)v(Y)\} \\ + \lambda h(X, Y)$$

$$(3.22) \quad \nabla_YU = \phi HY + w(Y)U - \alpha \lambda Y,$$

$$(3.23) \quad \nabla_Y V = \alpha \phi Y + \lambda H Y,$$

$$(3.24) \quad h(Y, V) = \alpha u(Y) + \beta \lambda w(X),$$

$$(3.25) \quad h(Y, U) = u(HY) + w(X).$$

Since $h(X, Y) = g(HX, Y)$, then from (2.3) and (3.25), we get

$$(3.26) \quad h(Y, U) = 0 \Rightarrow HU = 0.$$

4. Q -Quasi Umbilical Hypersurface.

M is called Quasi-umbilical hypersurface if

$$(4.1) \quad h(X, Y) = ag(X, Y) + bq(X)q(Y),$$

where a, b are scalar functions, q is a 1-form and if $g(Q, X) = q(X)$, where Q is a vector field, then M is called Q -quasi umbilical hypersurface. If $a = 0, b \neq 0$, then Q -quasi umbilical hypersurface M is called *cylindrical hypersurface*. If $a \neq 0, b = 0$, then Q -quasi-umbilical hypersurface M is called *totally umbilical* and if $a = 0, b = 0$, then Q -quasi umbilical hypersurface is *totally geodesic*.

Using (4.1) in (3.19), (3.20), (3.21), (3.22), (3.23), (3.24) and (3.25) we get

$$(4.2) \quad (\nabla_Y \phi)(X) = \alpha \{v(X)Y - g(X, Y)V\} + \beta \{g(\phi X, Y)V - v(Y)\phi X\} \\ - \{ag(X, Y) + bq(X)q(Y)\}U - u(Y)\{aY + bq(Y)Q\}$$

$$(4.3) \quad (\nabla_Y u)(X) = -\{(1+a)g(\phi X, Y)\} + \beta \{\lambda g(\phi X, Y) - u(X)w(Y)\} \\ + \alpha \lambda g(X, Y) - bq(\phi X)q(Y)$$

$$(4.4) \quad (\nabla_Y v)(X) = \alpha g(\phi Y, X) - \beta \{\lambda g(Y, X) - u(X)u(Y) - v(X)v(Y)\} \\ \lambda + \{ag(X, Y) + bq(X)q(Y)\}$$

$$(4.5) \quad \nabla_Y U = \beta w(Y)U + \{a\phi Y + bq(Y)Q\} - \alpha \lambda Y$$

$$(4.6) \quad \nabla_Y V = \alpha \phi Y + \lambda \{aY + bq(Y)Q\}$$

$$(4.7) \quad h(Y, V) = ag(V, Y) + bq(V)q(Y)$$

$$(4.7) \quad |u(Q)|^2 = -\frac{a}{b}(1 - \lambda^2) + w(X).$$

Also from (4.1), (3.24) and (3.15), we get

$$(4.8) \quad w(U) = \frac{\alpha(1 - \lambda^2)}{\beta \lambda}$$

5. Covariant almost analytic vector field on Q -quasi umbilical hypersurface.

1-form u and v are said to be covariant almost analytic if

$$(5.1) \quad u \{(\nabla_X \phi)(Y) - (\nabla_Y \phi)(X)\} = (\nabla_{\phi X} u)(Y) - (\nabla_X u)(\phi Y)$$

and

$$(5.2) \quad v \{(\nabla_X \phi)(Y) - (\nabla_Y \phi)(X)\} = (\nabla_{\phi X} v)(Y) - (\nabla_X v)(\phi Y)$$

Theorem 5.1. On Q -quasi umbilical hypersurface M with (ϕ, g, u, v, λ) -structure of a trans-Sasakian manifold $\bar{M}$, if 1-form u be a covariant almost analytic, then we have

$$(5.3) \quad \alpha \{v(Y)u(X) - v(X)u(Y)\} = -(2 + 2a + 2\beta)g(\phi X, \phi Y) - 2\alpha\lambda g(\phi X, Y) \\ - bq(\phi Y)q(\phi X) + \beta \{u(\phi Y)w(X) - u(Y)w(\phi X)\} \\ + 2bq(X)u(Y)u(Q)$$

Proof. From equation (4.2), we have

$$(\nabla_Y \phi)(X) = \alpha \{v(X)Y - g(X, Y)V\} + \beta \{g(\phi X, Y)V - v(Y)\phi X\} \\ - \{ag(X, Y) + bq(X)q(Y)\}U - u(X) \{aY + bq(Y)Q\}$$

and

$$(\nabla_X \phi)(Y) = \alpha \{v(Y)X - g(Y, X)V\} + \beta \{g(\phi Y, X)V - v(X)\phi Y\} \\ - \{ag(Y, X) + bq(Y)q(X)\}U - u(Y) \{aX + bq(X)Q\} \\ (\nabla_X \phi)(Y) - (\nabla_Y \phi)(X) = \alpha \{v(Y)X - v(X)Y\} + \beta \{v(Y)\phi X - v(X)\phi Y\} \\ - u(Y) \{aX + bq(X)Q\} + u(X) \{aY + bq(Y)Q\}$$

$$(5.4) \quad u \{(\nabla_X \phi)(Y) - (\nabla_Y \phi)(X)\} = \alpha \{v(Y)X - v(X)Y\} \\ + b \{u(X)q(Y) - q(X)u(Y)\}u(Q).$$

Also from equation (4.3)

$$(\nabla_X u)(Y) = - \{(1 + a)g(\phi Y, X)\} + \beta \{\lambda g(\phi Y, X) - u(Y)w(X)\} \\ + \alpha \lambda g(Y, X) + bq(\phi Y)q(X)$$

replacing ϕ by ϕX in the above equation, we have

$$(\nabla_{\phi X} u)(Y) = - \{(1 + a)g(\phi Y, \phi X)\} + \beta \{\lambda g(\phi Y, \phi X) - u(Y)w(\phi X)\}$$

$$+ \alpha \lambda g(Y, \phi X) + bq(\phi Y)q(\phi X)$$

and

$$\begin{aligned} (\nabla_X u)(\phi Y) &= -\{(1+a)g(\phi^2 Y, X)\} + \beta \{\lambda g(\phi^2 Y, X) - u(\phi Y)w(X)\} \\ &\quad + \alpha \lambda g(\phi Y, X) + bq(\phi^2 Y)q(X) \\ (\nabla_X u)(\phi Y) &= -(1+a)g(Y, X) + (1+a)u(X)u(Y) + (1+a)v(X)v(Y) + bq(Y)q(X) \\ &\quad - bu(Y)q(U)q(X) - bv(Y)q(V)q(X) + \alpha \lambda g(\phi Y, X) - \beta \lambda g(Y, X) \\ &\quad + \beta \lambda u(X)u(Y) + \beta \lambda v(X)v(Y) - u(\phi Y)w(X) \end{aligned}$$

or

$$\begin{aligned} (\nabla_X u)(\phi Y) &= -(1+a)g(\phi X, \phi Y) + bq(Y)q(X) - bu(Y)q(U)q(X) \\ &\quad - \beta u(\phi Y)w(X) - \alpha \lambda g(X, \phi Y) \\ (5.5) \quad (\nabla_{\phi X} u)(Y) - (\nabla_X u)(\phi Y) &= -(2+2a+2\beta)g(\phi X, \phi Y) - bq(\phi Y)q(\phi X) \\ &\quad + bu(Y)q(U)q(X) + \beta \{u(\phi Y)w(X) - u(Y)w(\phi X)\} \\ &\quad - bq(Y)q(X) + \alpha \lambda g(X, \phi Y) - \alpha \lambda g(\phi X, Y) \end{aligned}$$

using equation (5.1) in (5.4) and (5.5), we get (5.3).

Corollary 5.1. On Q -quasi umbilical hypersurface M with (ϕ, g, u, v, λ) -structure of a trans-Sasakian manifold $\bar{M}$, if 1-form u be a covariant almost analytic, then we also have

$$(5.6) \quad 2\alpha\lambda^2 = 0, \quad \alpha \neq 0$$

where, λ is covariant constant along $\bar{M}$.

Proof. Taking $X = U$ and $Y = V$ in equation (5.3) and using equations (3.15) and (3.16), we get

$$\begin{aligned} \alpha(1-\lambda^2)^2 &= -(2+2a+2\beta).0 + 2\alpha\lambda^2(1-\lambda^2) + \beta\lambda(1-\lambda^2)w(U) \\ \alpha(1-\lambda^2) &= +2\alpha\lambda^2(1-\lambda^2) + \beta\lambda(1-\lambda^2)w(U) \\ \alpha(1-\lambda^2) &= +2\alpha\lambda^2(1-\lambda^2) + \beta\lambda \left\{ \frac{\alpha(1-\lambda^2)}{\beta\lambda} \right\} \\ \alpha(1-\lambda^2) &= 2\alpha\lambda^2 + \alpha(1-\lambda^2) \\ 2\alpha\lambda^2 &= 0, \quad \alpha \neq 0 \end{aligned}$$

i.e. λ is covariant constant along $\bar{M}$.

Theorem 5.2. On cylindrical hypersurface M with (ϕ, g, u, v, λ) -structure of a trans-Sasakian manifold M , with covariant analytic vector fields U and V , we have

$$(5.7) \quad \alpha \{v(Y)u(X) - v(X)u(Y)\} = -(2 + 2\beta)g(\phi X, \phi Y) - 2\alpha\lambda g(\phi X, Y) \\ - bq(\phi Y)q(\phi X) + \beta \{u(\phi Y)w(X) - u(Y)w(\phi X)\}$$

Proof. Putting $a = 0$ in equation (5.3) and using $u(Q) = 0$, we get equation (5.7).

Corollary 5.2. On cylindrical hypersurface M with (ϕ, g, u, v, λ) -structure of a trans-Sasakian manifold M , with covariant analytic vector fields U, V , we have

$$(5.8) \quad \alpha u(Y) + (1 + \beta)v(Y) = 0,$$

$$(5.9) \quad q(Q) = 0.$$

Proof. Putting $X = V$ in equation (5.7), we get

$$\begin{aligned} -\alpha(1 - \lambda^2)u(Y) &= -(2 + 2\beta)g(\phi V, \phi Y) - 2\alpha\lambda g(\phi V, Y) \\ &\quad - bq(\phi Y)q(\phi V) + \beta \{u(\phi Y)w(V) - u(Y)w(\phi V)\} \\ -\alpha(1 - \lambda^2)u(Y) &= -(2 + 2\beta)v(Y)\lambda^2 - 2\alpha\lambda^2 u(Y) + \beta \{u(\phi Y)w(V) - u(Y)w(\phi V)\} \\ -\alpha(1 - \lambda^2)u(Y) &= -(2 + 2\beta)v(Y)\lambda^2 - 2\alpha\lambda^2 u(Y) + \beta u(\phi Y)w(V) - \beta\lambda u(Y)w(U) \\ \alpha(3\lambda^2 - 1)u(Y) &= -\beta\lambda u(Y) \left\{ \frac{\alpha(1 - \lambda^2)}{\beta\lambda} \right\} - (2 + 2\beta)v(Y)\lambda^2 \\ 2\alpha\lambda^2 u(Y) &= -(2 + 2\beta)v(Y)\lambda^2 \\ \alpha u(Y) + (1 + \beta)v(Y) &= 0. \end{aligned}$$

Also from equation (5.7), we get

$$u(X)V - v(X)U = -(2 + 2\beta)\phi X - 2\alpha\lambda\phi X - b\phi Qq(\phi X) - bq(X)Q + \beta \{w(X)\phi U - w(\phi X)U\}$$

Contracting with respect to X , we get

$$0 = -bq(\phi^2 Q) - bq(Q) + \beta \{\lambda w(X) - \lambda w(V)\}$$

or

$$bq(Q) = (V\lambda) = 0$$

since $b \neq 0$, so $q(Q) = 0$.

Theorem 5.3. On Q -quasi umbilical hypersurface M with (ϕ, g, u, v, λ) -structure of a trans-Sasakian manifold $\bar{M}$, if 1-form u be a covariant almost analytic. If M is totally umbilical, then we have

$$(5.10) \quad \alpha \{v(Y)u(X) - v(X)u(Y)\} = -(2+2a+2\beta)g(\phi X, \phi Y) - 2\alpha\lambda g(\phi X, Y) \\ + \beta \{u(\phi Y)w(X) - u(Y)w(\phi X)\}$$

Proof. Putting $b = 0$ in equation (5.3), we get equation (5.10).

Corollary 5.3. On Q -quasi umbilical hypersurface M with (ϕ, g, u, v, λ) -structure of a trans-Sasakian manifold $\bar{M}$, if 1-form u is covariant almost analytic. If M is totally umbilical, then we have

$$(5.11) \quad a = \left\{ \frac{\alpha(1-3\lambda^2)v(Y)}{\lambda} \right\} - \left\{ \frac{\alpha(1-3\lambda^2)}{\lambda} + 2(1+\beta) \right\} u(Y)$$

Proof. Put $X = U$ in equation (5.10), we get

$$\alpha(1-\lambda^2)v(Y) = -(2+2a+2\beta)v(\phi Y) + 2\alpha\lambda^2v(Y) + \beta \{u(\phi U)w(X) - u(Y)w(\phi U)\}$$

$$\alpha(1-\lambda^2)v(Y) = -(2+2a+2\beta)u(\phi Y) + 2\alpha\lambda^2v(Y) + \beta \left\{ u(\phi Y) \frac{\alpha(1-\lambda^2)}{\beta\lambda} \right\}$$

$$\alpha(1-\lambda^2)v(Y) - 2\alpha\lambda^2v(Y) = -(2+2a+2\beta)u(Y) + u(Y) \left\{ u(\phi Y) \frac{\alpha(1-\lambda^2)}{\lambda} \right\}$$

$$\alpha(1-3\lambda^2)v(Y) = u(Y) \{-2\lambda - 2a\lambda - 2\beta + \alpha(1-\lambda^2)\}$$

$$\alpha(1-3\lambda^2)v(Y) - \alpha(1-\lambda^2)u(Y) = -2u(Y) \{\lambda + a\lambda + \beta\lambda\}$$

$$a\lambda = \alpha(1-3\lambda^2)V - \alpha(1-\lambda^2)U + 2U\lambda + 2\beta\lambda U$$

and further solving we get (5.11).

Theorem 5.4. On Q -quasi umbilical hypersurface M with (ϕ, g, u, v, λ) -structure of a trans-Sasakian manifold $\bar{M}$, if 1-form v is covariant almost analytic, then we have

$$(5.13) \quad \alpha \{u(X)v(Y) - u(Y)v(X)\} = -2\alpha g(\phi X, \phi Y) + 2(\lambda + \beta)ag(\phi X, Y) \\ + \lambda b \{q(\phi X)q(Y) - q(X)q(\phi Y)\}$$

Proof. From equation (4.2), we get

$$(\nabla_X \phi)(Y) - (\nabla_Y \phi)(X) = \alpha \{v(Y)X - v(X)Y\} + \beta \{v(Y)\phi X - v(X)\phi Y\}$$

$$\begin{aligned}
 & -u(Y)\{aX + bq(X)Q\} + u(X)\{aY + bq(Y)Q\} \\
 (5.14) \quad & v(\nabla_X \phi)(Y) - (\nabla_Y \phi)(X) = \beta \{v(Y)\phi X - v(X)\phi Y\} + a \{u(X)v(Y) - u(Y)v(X)\}.
 \end{aligned}$$

From (4.3), we have

$$(\nabla_{\phi X} v)(Y) = (\lambda a + \beta)g(\phi X, Y) + \lambda \{u(X)v(Y) - v(X)u(Y)\}$$

and

$$\begin{aligned}
 (\nabla_X v)(\phi Y) &= -\alpha \{g(X, Y) - u(X)u(Y) - v(X)v(Y)\} + (\lambda - \beta)g(X, \phi Y) \\
 &+ \lambda \{v(X)u(Y) - u(X)v(Y)\}.
 \end{aligned}$$

Therefore we have

$$\begin{aligned}
 (5.15) \quad & (\nabla_{\phi X} v)(Y) - (\nabla_X v)(\phi Y) = -2\alpha g(\phi X, \phi Y) + 2(\lambda + \beta)ag(\phi X, Y) \\
 & + \lambda b \{q(\phi X)q(Y) - q(X)q(\phi Y)\}.
 \end{aligned}$$

Using equation (5.2) in (5.14) and (5.15), we get (5.13).

Theorem 5.5. Let the 1-form v be a covariant almost analytic on Q -quasi umbilical hypersurface M with (ϕ, g, u, v, λ) -structure of a trans-Sasakian manifold $\bar{M}$ if it is also cylindrical, then we have

$$\begin{aligned}
 (5.16) \quad & 2g(\phi X, \phi Y) = \lambda b \{q(\phi X)q(Y) - q(X)q(\phi Y)\} \\
 & - \beta \{v(Y)v(\phi X) - v(X)v(\phi Y)\}
 \end{aligned}$$

with $X \neq U$.

Proof. Putting $\alpha = 0$ in equation (5.13), we obtained equation (5.16). Further $X = U$, then from (5.16), we have

$$\begin{aligned}
 2g(\phi U, \phi Y) &= \lambda b \{-\lambda q(V)q(Y) - q(U)q(\phi Y)\} \\
 &- \beta \{v(Y)v(\phi U)\} \\
 -2\lambda g(V, \phi Y) + \beta g(V, Y)g(V, \phi U) &= 0
 \end{aligned}$$

or

$$-2\lambda v(\phi Y) + \beta v(Y)V(\phi U) = 0$$

since $v(\phi Y) \neq 0$ and $\lambda \neq 0$, therefore $X \neq U$.

Theorem 5.6. Let the 1-form v be a covariant almost analytic on Q -quasi umbilical non-invariant hypersurface of M with (ϕ, g, u, v, λ) -structure of a

trans-Sasakian manifold $\bar{M}$, if it is also totally umbilical, then we have

$$(5.17) \quad \alpha \{u(X)v(Y) - v(Xu(Y))\} = -2\alpha g(\phi X, \phi Y) + 2(\lambda + \beta)ag(\phi X, Y) \\ - \beta \{(v(Y), v(\phi X) - v(X)v(\phi Y))\},$$

with $X \neq U$.

Proof. Putting $b = 0$ in equation(4.13), we get (4.17). If we take $X = U$ in (5.17), we get

$$a(1 - \lambda^2)v(Y) + 2(\lambda + \beta)av(Y) = 2\alpha\lambda v(\phi Y) - \beta v(Y)u(\phi U) \\ a \{1 - \lambda^2\} + 2(\lambda + \beta) + \beta u(\phi U) \} v(Y) = -2\alpha\lambda v(\phi Y) \\ a \{\lambda^2 - 2(\lambda + \beta) - \beta U - 1\} v(Y) = -2\alpha\lambda^2 U$$

i.e. V and U are linearly dependent, which is a contradiction. Thus $X \neq U$.

References

- [1] Blair, D. E., Almost contact manifolds with Killing structure tensor, Pacific J. Math. 39 (12), (1971), 285-292.
- [2] Bucki, A., Hypersurfaces of almost r -paracontact Riemannian manifold, Tensor N.S., 48, no. 3, (1989), 245-251.
- [3] Goldberg, S. I., Yano. K., Noninvariant hypersurfaces of almost contact manifolds, J. Math. Soc., Japan, 22 (1970), 25-34.
- [4] Narain, D., Hypersurfaces with (f, g, u, v, λ) -structure of an affinely cosymplectic manifold, Indian Jour. Pure and Appl. Math. 20 no.8 (1989), 799-803.
- [5] Narian, D., Noninvaraint hypersurfaces of Sasakian Space forms, Int. J. Contemp. Math. Sci. 4, no. 33 (2009), 1611-1617.
- [6] Narain, D., Srivastava, S. K. and Srivastava, K., A note of noninvariant hypersurfaces of para-Sasakian manifold, IOSR, J. Engg. 2, (2012), 363-368.

- [7] Okumura, M., Totally umbilical hypersurface of locally product Riemannian manifold, Kodai Math. Sem. Rep. 19 (1967), 35-42.
- [8] Oubina, J. A., New Class of almost contact metric structures, Publ. Math. Debrecen, 32(1985), 187-193.
- [9] Prasad, R. and Tripathi, M. M., Transversal hypersurfaces of Kenmotsu manifold, Indian Jour. Pure and Appl. Math. 34, no. 3, (2003), 433-452.
- [10] Sinha, B. B. and Narain, D., Hypersurface of nearly Sasakian manifold, Ann. Fac. Sci. Dekishasa Zaire, 3 (2), (1977), 267-280.