

Original Research Articles

Oscillatory Blood Flow in Convergent and Divergent Channels, Part 1: Effects of Pulse Amplitude and Local Constriction Height

ABSTRACT Oscillatory blood flow in convergent and divergent channels is investigated. The problem which involves a set of non-linear differential equations is handled analytically using the method of regular perturbation series solutions. Solutions are obtained for the velocities, pressure and wall shear stress, and are analyzed graphically. It is found that the variations in the pulse amplitude and height of constriction reduce the axial velocity and pressure but increase the radial velocity and wall shear stress. More so, it is observed that flow separation occurs in the radial velocity and pressure structures in the convergent and divergent regions respectively, when the height of the constriction are varied.

Keywords: oscillatory flow, pulse amplitude, constriction height, convergent and divergent channels

1. INTRODUCTION

The human cardiovascular system is infested by a number of diseases. Some of which are due to the nature of the food we eat. An example of such is the atherosclerosis of the artery. Atherosclerosis involves the lesion or hardening of the arteries as a result of the deposition of lipids in its internal walls. The local disorder causes blood platelets adherence to the walls, and the insoluble proteins getting calcified such that the arterial walls at such points of the plaque loses its distensibility and becomes rigid.

The plaque also called arterial stenosis are found predominantly in the internal carotid artery which supplies blood to the brain; the coronary artery which supplies blood to the cardiac muscles, and the femoral artery which supplies blood to the lower limbs [1]. The constrictions due to the plaque tend to block the arterial blood flow. Moderate and severe stenosis leads to head losses which reduces blood supply to other parts of the body, and thus imposes extra loads on the heart musculature ([1], [2]). To maintain the peripheral flow, the heart enlarges itself, a condition that leads to increase in the amplitude of the pulse or pressure wave, which subsequently leads to abrupt rise in the flow variables ([2], [3]).

Furthermore, due to the progressive encroachment of the plaque on the lumen the arterial channel loses its taperedness to other geometrical forms which invariably changes the flow pattern. The evolving geometries which are slowly varying in cross-sections have been approximated by many researchers as locally constricted (see ([4], [5]) when only one point is affected, and peristaltic when many points are affected (see [6], [7]).

There are some reports on the previous work done on oscillatory flow in channels. For example, [8] examined the characteristics of pulsating flow in a confined channel, and observed that a given rate of mass flow is obtained in oscillating motion under an average pressure gradient as in steady flow. [9] investigated the pulsating flow of an incompressible viscous fluid in tubes of varying cross-sections using the perturbation series expansion solution which they developed, and observed that flow separation occurs in the wall shear stress structure in all geometries.

The purpose of this paper is to address the effects of the amplitude of the pressure wave and height of constriction on the flow velocity, pressure and wall shear stress structure in the regions before and after the peak of the constriction using the perturbation series solution of the form developed by [9].

This paper is organized in the following manner: section 2 is the methodology; section 3 gives the results and discussion, while section 4 holds the conclusions.

2 METHODOLOGY

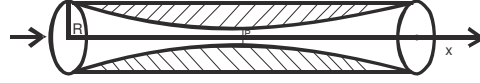


Fig.1 The convergent and divergent models of the sclerotic artery.

We consider the flow of blood in axisymmetric convergent and divergent channels whose radii vary slowly with the cross-sections. The fluid viscosity is a function of temperature. With the loss of distensibility, the channels now have rigid walls. At the entrance of the tube, we assume the flow to be fully developed and pulsating with a prescribed periodic frequency β and time average volume flux Q . If (R, θ, X) and (u, v, w) are the polar cylindrical orthogonal coordinates and vector components, respectively, assuming the flow is symmetrical about the x -axis, then for a two-dimensional flow situation the continuity, momentum and energy equations describing the flow are:

$$\frac{1}{R} \frac{\partial}{\partial R} (Ru) + \frac{\partial u}{\partial X} = 0 \quad (1)$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial R} + w \frac{\partial u}{\partial X} = -\frac{1}{\rho} \frac{\partial p}{\partial R} + \nu \left(\frac{\partial^2 u}{\partial R^2} + \frac{1}{R} \frac{\partial u}{\partial R} + \frac{\partial^2 u}{\partial X^2} \right) \quad (2)$$

$$\frac{\partial w}{\partial t} + u \frac{\partial w}{\partial R} + w \frac{\partial w}{\partial X} = -\frac{1}{\rho} \frac{\partial p}{\partial X} + \nu \left(\frac{\partial^2 w}{\partial R^2} + \frac{1}{R} \frac{\partial w}{\partial R} + \frac{\partial^2 w}{\partial X^2} \right) \quad (3)$$

where ρ the density, p the pressure, ν the kinematic viscosity of the fluid, and t is the time. $R=0$ is the centre or symmetric axis of the tube; $R = a_o(X, t)$, which is an arbitrary function of X and t is the cross-sectional radius of the tube; a_o is the characteristic radius of the tube, and t is the time.

The boundary conditions are:

$$(i) \quad w = 0, \quad \frac{\partial u}{\partial R} = 0 \quad \text{at } R=0 \quad (4)$$

(ii) there is no tangential motion at the wall of the tube ie:

$$u(a_o, t) = 0 \text{ at } R = a_o(X, t) \quad (5)$$

(iii) the flux across a cross-section of the tube is prescribed as:

$$a_o(X, t) \int_0^{2\pi} R u d\theta = 2 \pi \psi_o (1 + k e^{i\beta t}) \quad (6)$$

where ψ_o is a constant, k is the amplitude of the pulse which is assumed to be small, and β is the frequency of oscillation.

We eliminate the pressure terms in equations (2) and (3) by taking the $\frac{\partial}{\partial R}$ of equation (2) and $\frac{\partial}{\partial X}$ of equation (3), then subtracting the first result from the second one, we have

$$\begin{aligned} & \frac{\partial}{\partial t} \left(\frac{\partial u}{\partial R} - \frac{\partial w}{\partial X} \right) + \frac{\partial u}{\partial X} \left(\frac{\partial u}{\partial R} - \frac{\partial w}{\partial X} \right) + \frac{\partial w}{\partial R} \left(\frac{\partial u}{\partial R} - \frac{\partial w}{\partial X} \right) = \\ & \nu \left[\frac{\partial^2}{\partial X^2} \left(\frac{\partial u}{\partial R} - \frac{\partial w}{\partial X} \right) + \frac{\partial^2}{\partial R^2} \left(\frac{\partial u}{\partial R} - \frac{\partial w}{\partial X} \right) + \frac{1}{R} \frac{\partial}{\partial R} \left(\frac{\partial u}{\partial R} - \frac{\partial w}{\partial X} \right) - \frac{1}{R^2} \left(\frac{\partial u}{\partial R} - \frac{\partial w}{\partial X} \right) \right] \quad (7) \end{aligned}$$

Also, the pressure expression in the axial direction can be obtained from equation (3), as

$$\frac{\partial p}{\partial X} = \nu \left(\frac{\partial^2 w}{\partial R^2} + \frac{1}{R} \frac{\partial w}{\partial R} + \frac{\partial^2 w}{\partial X^2} \right) - \left(\frac{\partial w}{\partial t} + u \frac{\partial w}{\partial R} + w \frac{\partial w}{\partial X} \right) \quad (8)$$

Introducing the stream function ψ and vortices Ω , respectively i.e.

$$u = -\frac{1}{R} \frac{\partial \psi}{\partial R}, \quad w = \frac{1}{R} \frac{\partial \psi}{\partial X} \quad (9)$$

$$\Omega = \frac{\partial u}{\partial R} - \frac{\partial w}{\partial X} = \frac{1}{R} \frac{\partial^2 \psi}{\partial R^2} - \frac{1}{R^2} \frac{\partial \psi}{\partial R} + \frac{1}{R} \frac{\partial^2 \psi}{\partial X^2} \quad (10)$$

into equations (7) and (8), we have

$$\frac{\partial \Omega}{\partial t} - \frac{1}{R} \frac{\partial \Omega}{\partial X} \frac{\partial \psi}{\partial R} + \frac{1}{R} \frac{\partial \Omega}{\partial R} \frac{\partial \psi}{\partial X} + \frac{\Omega}{R^2} \frac{\partial \psi}{\partial X} = v \left[\frac{\partial^2 \Omega}{\partial X^2} + \frac{\partial^2 \Omega}{\partial R^2} + \frac{1}{R} \frac{\partial \Omega}{\partial R} - \frac{\Omega}{R^2} \right] \quad (11)$$

$$\frac{\partial p}{\partial X} = v \left(-\frac{1}{R} \frac{\partial^3 \psi}{\partial X^2 \partial R} - \frac{1}{R} \frac{\partial^3 \psi}{\partial R^3} - \frac{1}{R^2} \frac{\partial^2 \psi}{\partial R^2} \right) - \frac{1}{R^2} \frac{\partial \psi}{\partial R} \frac{\partial^2 \psi}{\partial X \partial R} + \frac{1}{R^2} \frac{\partial \psi}{\partial X} \frac{\partial^2 \psi}{\partial R^2} + \frac{1}{R} \frac{\partial^2 \psi}{\partial t \partial R} \quad (12)$$

with the boundary conditions as

$$\psi = 0, \quad \frac{1}{R} \frac{\partial \psi}{\partial X} = 0, \quad \frac{1}{R} \frac{\partial^2 \psi}{\partial R^2} - \frac{1}{R^2} \frac{\partial \psi}{\partial R} = 0 \quad \text{at } R = 0 \quad (13)$$

$$\frac{\partial \psi}{\partial R} = 0, \quad \psi = \psi_o (1 + k e^{i\beta t}) \quad \text{at } R = a(X, t) \quad (14)$$

Also, we assume that the cross-section of the tube in the model vary in the axial direction, and for which we take $a(X, t) = a_o s(\varepsilon X/a_o, t)$, $r = R_o(1 + \varepsilon f(x, t))$, $0 \leq r \leq 1$, $0 \leq x \leq 1$ where s is an

arbitrary function of X ; a is the variation in the r along the axial direction; $0 < \varepsilon = \frac{a_o}{L} \ll 1$ is a small dimensionless parameter that characterizes the slow variation in the channel radius; L defines the channel characteristic length. $\varepsilon = 0$ corresponds to a tube with constant radius. As ε increases from zero the variation of ψ in the axial direction depends upon εX instead of X .

Similarly, introducing the following non-dimensionalized variables

$$r = \frac{R}{a_o}, \quad x = \varepsilon \frac{X}{a_o}, \quad T = \beta t, \quad \phi(r, x, T) = \frac{\psi}{\psi_o},$$

$$\omega(r, x, T) = \frac{\Omega a_o^3}{\psi_o}, \quad \text{Re} = \frac{\psi_o}{a_o v}, \quad \eta = \frac{\beta}{a_o^2 v}, \quad a_o = \frac{a}{s}$$

where Re is the Reynolds number of the flow, η is a dimensionless number for the frequency of oscillation, T is the dimensionless time, ε is the height of constriction, ϕ, ω are the dimensionless stream function and vortices respectively; into equations (8) - (12) respectively, we have

$$u = -\frac{1}{r} \frac{\partial \phi}{\partial r} \quad (15)$$

$$w = -\frac{1}{r} \frac{\partial \phi}{\partial x} \quad (16)$$

$$\omega = \frac{1}{r} \frac{\partial^2 \phi}{\partial r^2} - \frac{1}{r^2} \frac{\partial \phi}{\partial r} \quad (17)$$

$$\eta \frac{\partial \omega}{\partial T} + \frac{\text{Re}}{r} \varepsilon \left(\frac{\partial \omega}{\partial r} \frac{\partial \phi}{\partial x} + \frac{\partial \omega}{\partial x} \frac{\partial \phi}{\partial r} + \frac{\omega}{r} \frac{\partial \phi}{\partial x} \right) = \frac{1}{r^2} \frac{\partial^2 \omega}{\partial r^2} + \frac{1}{r} \frac{\partial \omega}{\partial r} - \frac{1}{r} \omega \quad (18)$$

$$\frac{\partial p}{\partial X} = -\frac{\text{Re}}{\varepsilon} \left(-\frac{1}{r} \frac{\partial^3 \phi}{\partial r^3} + \frac{1}{r^2} \frac{\partial^2 \phi}{\partial r^2} \right) + \frac{\eta}{\varepsilon r} \frac{\partial^2 \phi}{\partial r \partial T} \quad (19)$$

Equation (19) tends to suggest that the pressure field is of order $\frac{1}{\varepsilon}$. Also, the $O(\varepsilon)^2$ terms are assumed to be very small, therefore, they are neglected.

Similarly, the wall shear stress is given as

$$\tau_w = \rho \nu \Omega \quad (\text{see [1]}) \text{ yielding}$$

$$\tau_w = \frac{1}{r} \frac{\partial^2 \phi}{\partial r^2} + \frac{1}{r^2} \frac{\partial \phi}{\partial r} \quad (20)$$

The boundary conditions become:

$$\phi = 0, \quad \frac{\partial \phi}{\partial x} = 0, \quad \frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial \phi}{\partial r} \right) = 0 \quad \text{at } r = 0 \quad (21)$$

$$\phi = 1 + k e^{iT}, \quad \frac{\partial \phi}{\partial r} = 0 \quad \text{at } r = s(x, T) \quad (22)$$

Now, we shall seek for perturbation series solutions about the small parameter ε . In particular, we shall use the form developed by [9], and which is of the form:

$$f = f^{(0)} + k e^{iT} \bar{f}^{(0)} + \varepsilon \left(f^{(1)} + k e^{iT} \bar{f}^{(1)} \right) + \dots \quad (23)$$

where f_n represents ω and ϕ .

Substituting equation (23) into equations (17), (18), (21) and (22), we have: the zeroth order terms as

$$\omega^{(0)} = \frac{1}{r} \left(\frac{\partial^2 \phi^{(0)}}{\partial r^2} - \frac{1}{r} \frac{\partial \phi^{(0)}}{\partial r} \right) \quad (24)$$

$$\omega^{(0)} = \frac{1}{r} \left(\frac{\partial^2 \bar{\phi}^{(0)}}{\partial r^2} + \frac{1}{r} \frac{\partial \bar{\phi}^{(0)}}{\partial r} \right) \quad (25)$$

$$\frac{\partial^2 \omega^{(0)}}{\partial r^2} + \frac{1}{r} \frac{\partial \omega^{(0)}}{\partial r} - \frac{1}{r^2} \omega^{(0)} = 0 \quad (26)$$

$$\frac{\partial^2 \varpi^{(0)}}{\partial r^2} + \frac{1}{r} \frac{\partial \varpi^{(0)}}{\partial r} - \left(\lambda^2 + \frac{1}{r} \right) \varpi^{(0)} = 0 \quad (27)$$

where $\lambda^2 = i\eta$
with the boundary conditions are:

$$\left. \begin{aligned} \phi^{(0)} = \bar{\phi}^{(0)} = 0 \\ \frac{1}{r} \frac{\partial \phi^{(0)}}{\partial x} = \frac{1}{r} \frac{\partial \bar{\phi}^{(0)}}{\partial x} = 0 \\ \frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial \phi^{(0)}}{\partial r} \right) = \frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial \bar{\phi}^{(0)}}{\partial r} \right) = 0 \end{aligned} \right\} \text{ at } r = 0 \quad (28)$$

$$\left. \begin{aligned} \frac{\partial \phi^{(0)}}{\partial r} = \frac{\partial \bar{\phi}^{(0)}}{\partial r} = 0 \\ \phi^{(0)} = \bar{\phi}^{(0)} = 1 \end{aligned} \right\} \text{ at } r = s \quad (29)$$

while the first order terms are

$$\omega^{(1)} = \frac{1}{r} \left(\frac{\partial^2 \phi^{(1)}}{\partial r^2} - \frac{1}{r} \frac{\partial \phi^{(1)}}{\partial r} \right) \quad (30)$$

$$\varpi^{(1)} = \frac{1}{r} \left(\frac{\partial^2 \bar{\phi}^{(1)}}{\partial r^2} - \frac{1}{r} \frac{\partial \bar{\phi}^{(1)}}{\partial r} \right) \quad (31)$$

$$\frac{\partial^2 \omega^{(1)}}{\partial r^2} + \frac{1}{r} \frac{\partial \omega^{(1)}}{\partial r} - \frac{1}{r} \omega^{(1)} = \frac{\text{Re}}{r} \left[\frac{\partial \phi^{(0)}}{\partial r} \frac{\partial \omega^{(0)}}{\partial x} + \frac{\partial \phi^{(0)}}{\partial x} \left(\frac{\omega^{(0)}}{r} - \frac{\partial \omega^{(0)}}{\partial r} \right) \right] \quad (32)$$

$$\begin{aligned} \frac{\partial^2 \varpi^{(1)}}{\partial r^2} + \frac{1}{r} \frac{\partial \varpi^{(1)}}{\partial r} - \left(\lambda^2 + \frac{1}{r^2} \right) \varpi^{(1)} = \frac{\text{Re}}{r} \left[\frac{\partial \phi^{(0)}}{\partial r} \frac{\partial \omega^{(0)}}{\partial x} + \frac{\partial \bar{\phi}^{(0)}}{\partial r} \frac{\partial \omega^{(0)}}{\partial x} \right. \\ \left. - \frac{\partial \varpi^{(0)}}{\partial x} \left(\frac{\partial \omega^{(0)}}{\partial r} - \frac{\varpi^{(0)}}{r} \right) - \frac{\partial \bar{\phi}^{(0)}}{\partial x} \left(\frac{\partial \omega^{(0)}}{\partial r} - \frac{\omega^{(0)}}{r} \right) \right] \quad (33) \end{aligned}$$

with boundary conditions

$$\left. \begin{aligned} \phi^{(1)} = \bar{\phi}^{(1)} = 0 \\ \frac{1}{r} \frac{\partial \phi^{(1)}}{\partial x} = \frac{1}{r} \frac{\partial \bar{\phi}^{(1)}}{\partial x} = 0 \\ \frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial \phi^{(1)}}{\partial r} \right) = \frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial \bar{\phi}^{(1)}}{\partial r} \right) = 0 \end{aligned} \right\} \text{ at } r = 0 \quad (34)$$

$$\left. \begin{aligned} \frac{\partial \phi^{(1)}}{\partial r} &= \frac{\partial \bar{\phi}^{(1)}}{\partial r} = 0 \\ \phi^{(1)} &= \bar{\phi}^{(1)} = 0 \end{aligned} \right\} \quad \text{at } r = s \quad (35)$$

Moreso, by equation (22), equations (15), (16), (19) and (20) become

$$u = -\frac{1}{r} \frac{\partial \phi}{\partial r} = -\frac{1}{r} \frac{\partial}{\partial r} (\phi^{(o)} + ke^{iT} \phi^{(o)} + \varepsilon \phi^{(1)} + \varepsilon ke^{It} \phi^{(1)}) \quad (36)$$

$$w = \frac{1}{r} \frac{\partial \phi}{\partial x} = \frac{1}{r} \frac{\partial s}{\partial x} \cdot \frac{\partial \phi}{\partial s} = \frac{1}{r} \frac{\partial s}{\partial x} \cdot \frac{\partial}{\partial s} (\phi^{(o)} + ke^{iT} \phi^{(o)} + \varepsilon \phi^{(1)} + \varepsilon ke^{It} \phi^{(1)}) \quad (37)$$

$$\frac{\partial p}{\partial x} = -\frac{\text{Re}}{\varepsilon} \left[-\frac{1}{r} \frac{\partial^3 \phi^{(o)}}{\partial r^3} + \frac{1}{r^2} \frac{\partial^2 \phi^{(o)}}{\partial r^2} + ke^{iT} \left(-\frac{1}{r} \frac{\partial^3 \bar{\phi}^{(o)}}{\partial r^3} + \frac{1}{r^2} \frac{\partial^2 \bar{\phi}^{(o)}}{\partial r^2} \right) \right] \quad (38)$$

$$\begin{aligned} \tau_w = & -\left(\frac{1}{r} \frac{\partial^2 \phi^{(o)}}{\partial r^2} + \frac{1}{r^2} \frac{\partial \phi^{(o)}}{\partial r} \right) - ke^{iT} \left(\frac{1}{r} \frac{\partial^2 \bar{\phi}^{(o)}}{\partial r^2} + \frac{1}{r^2} \frac{\partial \bar{\phi}^{(o)}}{\partial r} \right) - \varepsilon \left(\frac{1}{r} \frac{\partial^2 \phi^{(1)}}{\partial r^2} + \frac{1}{r^2} \frac{\partial \phi^{(1)}}{\partial r} \right) \\ & - \varepsilon ke^{iT} \left(\frac{1}{r} \frac{\partial^2 \bar{\phi}^{(1)}}{\partial r^2} + \frac{1}{r^2} \frac{\partial \bar{\phi}^{(1)}}{\partial r} \right) \end{aligned} \quad (39)$$

The analyses of the solutions show that the non-steady part is very insignificant.

By exhaustive algebraic operations, the solutions of the zeroth order equations (23) - (26), (29) - (32) and (35) - (38), are:

$$\phi^{(o)} = 2 \left(\frac{r}{s} \right)^2 - \left(\frac{r}{s} \right)^4 \quad (40)$$

$$\bar{\phi}^{(o)} = \frac{r^2}{\lambda s^2 I_2(\lambda s)} [I_o(\lambda s) - I_o(\lambda r) + I_2(\lambda r)] \quad (41)$$

$$\omega^o = -\frac{8}{s^3} \left(\frac{r}{s} \right) \quad (42)$$

$$\bar{\omega}^{(o)} = -2 \frac{\lambda I_1(\lambda r)}{s^2 I_2(\lambda s)} \quad (43)$$

$$\phi^{(1)} = -\frac{\text{Re}}{s} \frac{\partial s \varepsilon}{\partial x} \left[\frac{1}{9} \left(\frac{r}{s} \right)^8 - \frac{2}{3} \left(\frac{r}{s} \right)^6 + \left(\frac{r}{s} \right)^4 - \frac{4}{9} \left(\frac{r}{s} \right)^2 \right] \quad (44)$$

$$\begin{aligned} \bar{\phi}^{(1)} = & \frac{4 \text{Re}}{\lambda^4 s^4} \frac{\partial s}{\partial x} \left\{ \frac{I_1(\lambda r)}{s \lambda^2 I_2(\lambda s)} \left[\left(\frac{2}{3} s^4 \lambda^4 - 5 s^2 \lambda^2 - 32 \right) \lambda I_1^2(\lambda s) \right. \right. \\ & + \left(\frac{1}{2} s^2 \lambda^2 + 16 \right) \lambda^2 s I_o(\lambda s) I_1(\lambda s) + \left(\frac{2}{3} \lambda^4 s^4 + 12 \lambda^2 s^2 - 128 \right) \frac{I_1(\lambda s)}{s} \\ & \left. \left. + \left(64 - 6 \lambda^2 s^2 - \frac{1}{4} \lambda^4 s^4 \right) \lambda I_o(\lambda s) \right] \right\} \end{aligned} \quad (45)$$

$$\omega^{(1)} = \frac{8\text{Re}}{s^4} \frac{\partial s}{\partial x} \left[\frac{2}{3} \left(\frac{r}{s} \right)^5 - 2 \left(\frac{r}{s} \right)^3 + \left(\frac{r}{s} \right) \right] \quad (46)$$

$$\begin{aligned} \bar{\omega}^{(1)} = & 4 \frac{\text{Re}}{\lambda^4 s^4} \frac{\partial s}{\partial x} \left\{ \frac{1}{\lambda^2 I^2(\lambda s)} \left[\left(\frac{2s^4 \lambda^4}{3} - 5s^2 \lambda^2 - 32 \right) \lambda I_1^2(\lambda s) \right] \right. \\ & \left. + \left[I_o(\lambda r) \left(\lambda s^2 r - \frac{\lambda r^2}{3} \right) I_1(\lambda s) - \frac{r^3}{s} + 2sr \right] \right\} \quad (47) \end{aligned}$$

$$u = -\frac{4}{s^2} + \frac{4r^2}{s^4} - \frac{ke^{iT}}{\lambda s^2 I_2(\lambda s)} 2(I_o(\lambda s) - I_o(\lambda r) + \lambda r I_1(\lambda r))$$

$$+ \frac{\varepsilon \text{Re}}{s^3} \frac{\partial s}{\partial x} \left[\frac{8}{9} \left(\frac{r}{s} \right)^6 - 4 \left(\frac{r}{s} \right)^4 + 4 \left(\frac{r}{s} \right)^2 - 18 \right]$$

$$\begin{aligned} & + \frac{\varepsilon ke^{iT}}{r \lambda^4 s^6 I_2(\lambda s)} 4 \text{Re} \frac{\partial s}{\partial x} \left\{ \frac{I_o(\lambda r)}{s \lambda I_2(\lambda s)} \left[\begin{aligned} & \left(\frac{2}{3} s^4 \lambda^4 - s^5 \lambda^2 - 32 \right) \lambda I_1^2(\lambda s) \\ & + \left(\frac{1}{2} s^2 \lambda^2 + 16 \right) \lambda^2 s I_o(\lambda s) \\ & + \left(\frac{3}{2} s^4 \lambda^4 - 12 s^2 \lambda^2 - 128 \right) \frac{I_1(\lambda s)}{s} \\ & + \left(64 - 6 s^2 \lambda^2 - \frac{1}{4} s^4 \lambda^4 \right) I_o(\lambda s) \end{aligned} \right] \right. \\ & \left. + \lambda^4 r^2 \left(\frac{3}{2} s^2 - \frac{8}{\lambda^2} \right) I_1(\lambda r) I_1(\lambda s) - \frac{4}{3} \lambda^3 r^2 I_o(\lambda r) I_1(\lambda s) + \dots \right\} \quad (48) \end{aligned}$$

$$w = \left(4 \frac{r^3}{s^5} - 2 \frac{r}{s^3} \right) \frac{\partial s}{\partial x} - \frac{ke^{iT} r}{s I_2^2(\lambda s)} \frac{\partial s}{\partial x} [I_o(\lambda s) I_1(\lambda s) - I_1(\lambda s) I_o(\lambda r) + I_1(\lambda s) I_2(\lambda r)]$$

$$+ \varepsilon \frac{\text{Re}}{s^3} \left(\frac{\partial s}{\partial x} \right)^2 \left[\left(\frac{r}{s} \right)^7 - \frac{14}{3} \left(\frac{r}{s} \right)^5 + 5 \left(\frac{r}{s} \right)^3 - 27 \left(\frac{r}{s} \right) \right]$$

$$\begin{aligned}
 & + \epsilon k e^{iT} \left\{ \begin{aligned} & \left[\frac{2}{3} \lambda^6 \left(2s^3 I_o(\lambda s) I_1(\lambda s) I_2^2(\lambda s) + \frac{s^2}{\lambda} (I_1^2(\lambda s) I_2^2(\lambda s)) \right) \right. \\ & \left. \frac{4 \operatorname{Re}}{r \lambda^6} I_2(\lambda r) \frac{\partial s}{\partial x} - 5 \lambda^4 \left(2s^5 I_o(\lambda s) I_1(\lambda s) I_2^2(\lambda s) - \frac{s^4}{\lambda} I_1^2(\lambda s) I_2^2(\lambda s) \right) \right. \\ & \left. - 32 \lambda^2 \left(2s^7 I_o(\lambda s) I_1(\lambda s) I_2^2(\lambda s) - 3 \frac{s^6}{\lambda} I_1^2(\lambda s) I_2^2(\lambda s) \right) \right] \\ & + \frac{4 \operatorname{Re} r}{s 4 I_2^2(\lambda s)} I_2(\lambda r) \frac{\partial s}{\partial x} \left[\begin{aligned} & I_o(\lambda s) - \frac{4}{3 s^2} \left(I_o(\lambda s) - \frac{2 I_1(\lambda s)}{\lambda s} \right) \\ & - \frac{8}{\lambda^2 s^2} \left(I_o(\lambda s) - \frac{2 I_1(\lambda s)}{\lambda s} \right) \end{aligned} \right] \end{aligned} \right\} \\
 & \quad \quad \quad (49)
 \end{aligned}$$

$$p = \frac{1}{\epsilon} \left[\frac{36x}{s^4} - \frac{k e^{iT} x}{s^2 I_2(\lambda r)} \left[\frac{10}{r} I_1(\lambda r) + 7 \lambda r I_o(\lambda r) - (\lambda^2 r + \lambda^2 r^2) I_1(\lambda r) \right] \right] \quad 0 \leq x \leq 1 \quad (50)$$

$$\begin{aligned}
 & \tau_w = \frac{8}{s^2} - \frac{k e^{iT} x}{s^2 I_2(\lambda s)} (\lambda s^2 I_o(\lambda s) + 4 s I_1(\lambda s)) + \epsilon \frac{8}{3 s^3} \operatorname{Re} \frac{\partial s}{\partial x} + \\
 & \frac{\epsilon k e^{iT}}{\lambda^4 s^6 I_2(\lambda r)} \operatorname{Re} \frac{\partial s}{\partial x} \left\{ \begin{aligned} & \left[\begin{aligned} & \left(\frac{2}{3} s^4 \lambda^4 - s^2 \lambda^2 - 32 \right) I_1^2(\lambda s) \\ & + \left(\frac{1}{2} s^2 \lambda^2 + 16 \right) \lambda^2 s I_o(\lambda s) I_1(\lambda s) \end{aligned} \right] \\ & \frac{I_1(\lambda s)}{s^2 I_2(\lambda s)} \left(\frac{3}{2} s^4 \lambda^4 - 12 s^2 \lambda^2 - 128 \right) \frac{I_1(\lambda s)}{s} \\ & + \left(64 - 6 s^2 \lambda^2 - \frac{1}{4} s^4 \lambda^4 \right) I_o(\lambda s) \end{aligned} \right\} \\
 & \quad \quad \quad + \lambda^4 \left(\frac{4s}{3} - 6s^3 + \frac{32s}{\lambda^2} \right) I_1^2(\lambda s) \quad (51)
 \end{aligned}$$

where $I_o(\lambda r)$, $I_1(\lambda r)$, $I_2(\lambda r)$, $I_o(\lambda s)$, $I_1(\lambda s)$ and $I_2(\lambda s)$ are the modified Bessel function of order zero, one, two, respectively. It is worthy to note that the solutions to equations (31) and (33) as expressed in equations (45) and (47) are culled and developed from ([9], p. 206).

Furthermore, the geometries under consideration are: the convergent channel $s = e^{-x/2}$, and divergent channel $s = e^{x/2}$

3 RESULTS AND DISCUSSION

We considered the flow of blood through a stenosis artery with emphasis on the effects of pressure wave amplitude and height of constriction on the overall flow structure in the regions before and after the peak of the constriction. These two regions under consideration approximate the convergent and divergent channels, as shown in figure (1). The analyses of the results show that variations in the amplitude and

height of the constriction have tremendous effects on the flow structure. To this end, using Maple 12 computational software and the following physically realistic parameters: $T=\pi/4$; $\lambda=2$; $k = 0.01, 0.03, 0.05, 0.07$; $\varepsilon = 0.01, 0.03, 0.05, 0.07$; $Re= 10, 100, 500, 1000$ we obtained the results shown in Fig. 2 – Fig. 17. These figures show the profiles of the computational results for the velocities pressure and wall shear stress. The profiles show that increase in the amplitude decreases the axial velocity and axial pressure (see Fig. 2, Fig. 3, Fig. 6 and Fig. 7), but increases the radial velocity and wall shear stress (see Fig. 4, Fig. 5, Fig. 8 and Fig. 9. Blood flow in a normal artery is laminar and Poiseuille. But due to the stenosis, the arterial channel is blocked moderately or severely. For this, the heart enlarges itself to maintain the peripheral flow. The enlargement of the heart leads to increase in the amplitude of the pulse or pressure wave. And, the increase in the amplitude energizes the flow variables in the direction of the increase, which in this case, the radial velocity and wall shear stress. This accounts for what is seen in Fig. 4, Fig. 5, Fig. 8 and Fig. 9. On the other hand, the increase in the amplitude reduces the axial velocity and axial pressure as seen in Fig. 2, Fig. 3, Fig. 6 and Fig. 7). This could be due to the increase in the radial velocity.

Furthermore, the profiles show that the height of the constriction decreases the axial velocity (see Fig. 10 and Fig. 11), while it increases the axial pressure and wall shear stress (see Fig. 14, Fig. 16 and Fig. 17). As said above, blood flow in the artery is laminar, Poiseuille and fully developed. It moves with the velocity with which it is released from the heart. As it gets to the region of constriction, the flow pattern changes. The blockage caused by the stenosis reduces the axial pressure as shown in Fig. 11 and Fig. 11.

Moreso, the changes in the geometrical configuration affects the flow. In particular, for the convergent channel, as the geometry thins down exponentially, the pressure factor increases, and subsequently, the velocities and wall shear stress increase. It is also seen that in the divergent channel whose geometry increases exponentially, the radial velocity, axial pressure and wall shear stress are increased. The increase could be due to the influence of the flow situations in the convergent channel. All these account for the results in Fig. 13, Fig. 14, Fig. 16 and Fig. 17.

Similarly, a special feature called the flow separation occurs in the radial and pressure structures at $r \leq 0.3$ and $r \leq 0.46$, respectively (see Fig. 12 and Fig. 15). Initially, the radial velocity and pressure increase with the height of constriction drop at these points, and then the flow pattern changes such that these variables decrease as the height of constriction increases. The occurrence of the flow separation (which is in agreement with [9]) is due to the adverse flow conditions at such points. [9] adduced that it is due to viscous effects.

The overall analyses show that the axial velocity drops as the pulse amplitude and height of constriction increase. This has some health implications on the stenotic patient. The drop in the axial velocity leads subsequent to a drop in the rate at which the rejuvenated blood which bears oxygen and food nutrients is transported to other parts of the body, thus causing cells starvation.

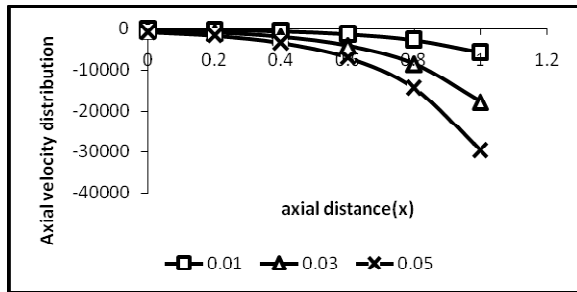


Fig. 2 Axial velocity-amplitude (k) profiles in a convergent channel

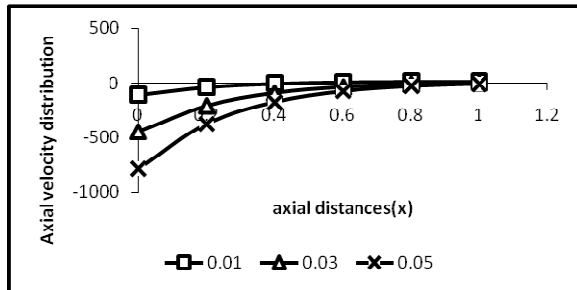


Fig. 3 Axial velocity-amplitude (k) profiles in a divergent channel

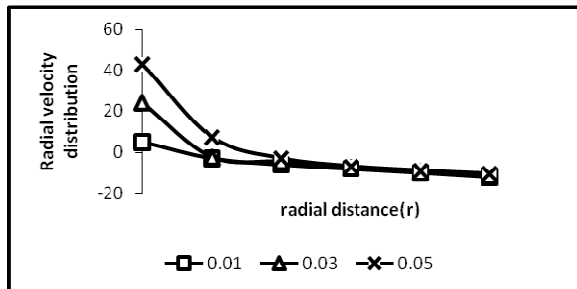


Fig. 4 Radial velocity-amplitude (k) profiles in a convergent channel

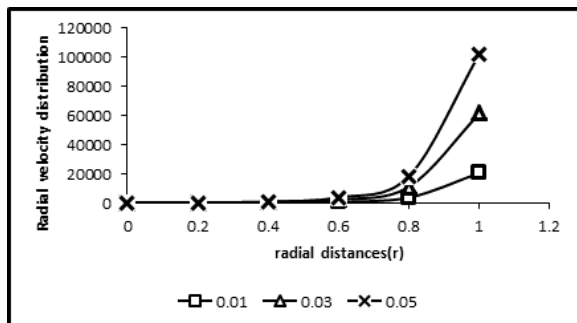


Fig. 5 Radial velocity-amplitude (k) profiles in a divergent channel

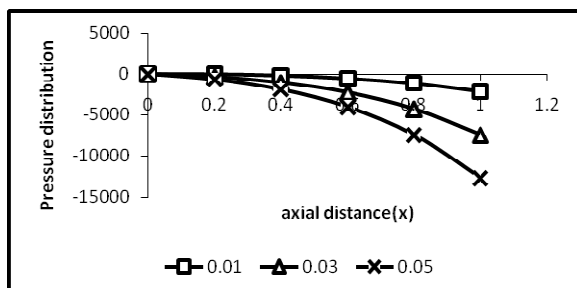


Fig. 6 Pressure-amplitude (k) profiles in a convergent channel

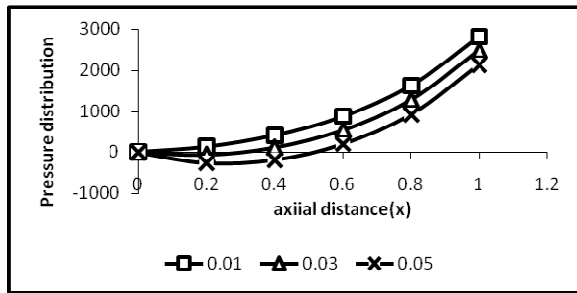


Fig. 7 Pressure-amplitude (k) profiles in a divergent channel

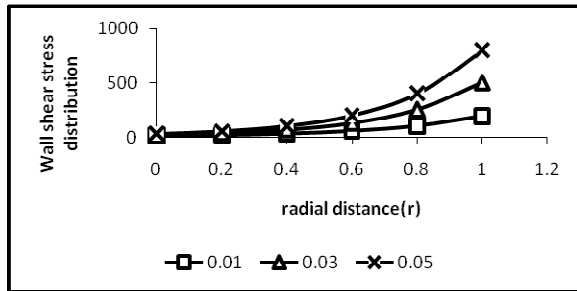


Fig. 8 Wall shear stress-amplitude (k) profiles in a convergent channel

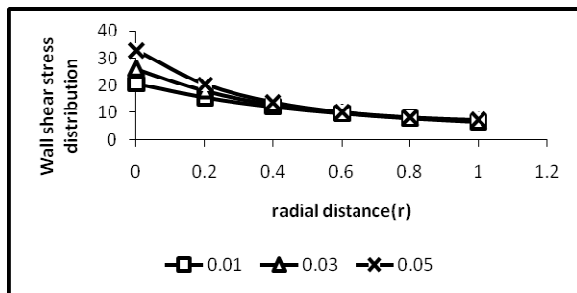


Fig. 9 Wall shear stress-amplitude (k) profiles in a divergent channel

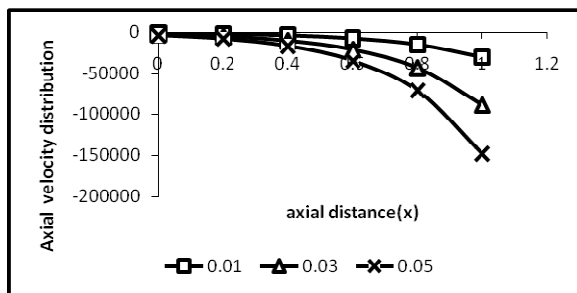


Fig. 10 Axial velocity-height of constriction (ϵ) profiles in a convergent channel

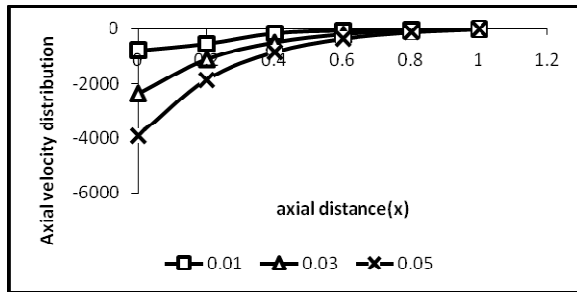


Fig. 11 Axial velocity-height of constriction (ϵ) profiles in a divergent channel

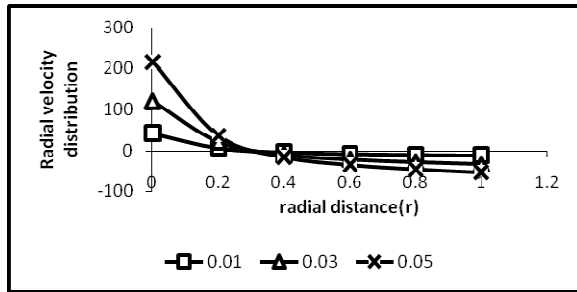


Fig. 12 Radial velocity-height of constriction (ϵ) profiles in a convergent channel

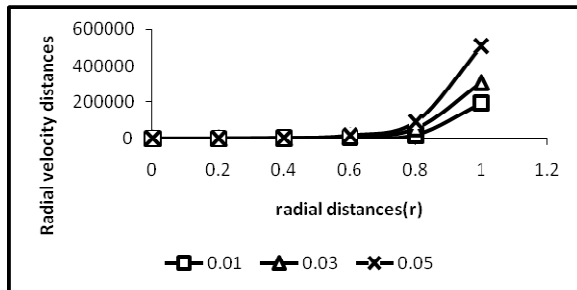


Fig. 13 Radial velocity-height of constriction (ϵ) profile in a divergent channel

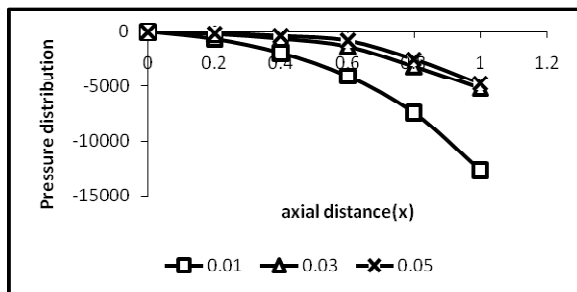


Fig. 14 Pressure-height of constriction (ϵ) profiles in a convergent channel

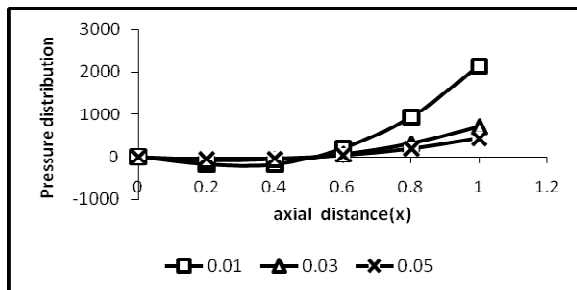


Fig. 15 Pressure-height of constriction (ϵ) profiles in a divergent channel

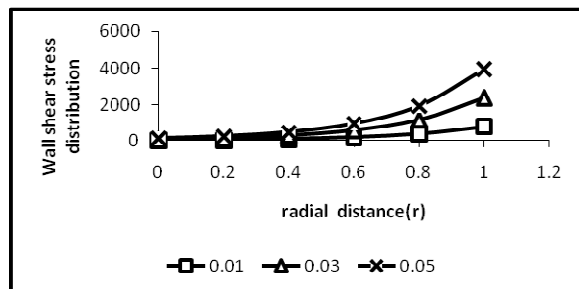


Fig. 16 Wall shear stress-height of constriction (ϵ) profiles in a convergent channel

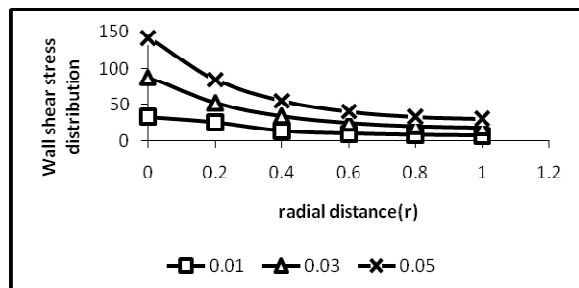


Fig. 17 Wall shear stress-height of constriction (ϵ) profiles in a divergent channel

4 CONCLUSIONS

The analyses of the results indicate that the pulse amplitude and height of constriction reduce the axial velocity and pressure but increase the radial velocity and wall shear stress. The occurrence of flow separation in the radial velocity and pressure structures are due to some adverse flow situations at such points. Furthermore, the drop in the axial velocity has some physiological implications on the stenotic patient.

REFERENCES

- [1] Mittal R, Simmons SP, Najjar F. Numerical study of pulsatile flow in a constricted channel. *Journal of Fluid Mechanics*. 2003; 485:337-378,
- [2] Anliker M, Rockwell RL, Odgen, E. Non-linear analysis of flow pulses and shock waves in arteries, 1 and 2. *Z. angew. Maths Physics*. 1971; 22:217-246, 563-581.
- [3] Ayeni RO, Akinrelere AE. On temperature factor and effect of viscosity on blood temperature in arteries. *Journal of Theoretical Biology*. 1984; 109:479-487.
- [4] Burns JC, Parkes T. Flow of incompressible fluid in peristaltic tubes. *Journal of Fluid Mechanics*. 1967; 29:731
- [5] Shapiro AH, Jaffins MY, Weinburg SL. Flow of incompressible fluid in peristaltic tubes, *Journal of Fluid Mechanics*. 1969; 37:409.
- [6] Lee JS, Fung YC. Flow of incompressible fluid in locally constricted tubes. *Journal of Applied Mechanics*. 1970; 37:9.
- [7] Yin FC, Fung YC. Flow of incompressible fluid in peristaltic tubes. *Journal of Applied Mechanics*. 1971; 47:93.

- 308 [8] Uchida S. Pulsating viscous flow superimposed on the steady-laminar motion of incompressible fluid in
309 a circular pipe. Z. angew. Maths Physics. 1956; 2:403-421.
- 310 [9] Rao AR, Devanathan R. Pulsatile flow in tubes of varying cross-sections, Z. angew Maths Physics.
311 1973; 24:203-213.