

Original research Articles

$L^p(\mu), 0 < p < 1$ as quasi 2-normed space

ABSTRACT

In [7], C. Park generalized the notion of the quasi-normed space, i.e. he introduced the notion of the quasi 2-normed space. He proved several properties of the quasi 2-norm. In [3], M. Kir and M. Acikgoz gave the procedure of completing the quasi 2-normed space. Several inequalities relating the quasi 2-normed spaces are given in [1], [4] and [5]. Later, in [6], some properties of the convergent sequences in quasi 2-normed spaces are proven. In this paper, we will introduce the quasi 2-norm of the space $L^p(\mu), 0 < p < 1$ and we will prove several inequalities relating the quasi 2-norm of this space.

Keywords: quasi 2-norm, $(2, p)$ -norm, modul of concavity

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1 INTRODUCTION

In 1965, S. Gähler introduced the 2-normed spaces ([2]). One of the axioms of the 2-norm is the inequality of parallelepiped which is fundamental in the theory of 2-normed space. As in the normed spaces, C. Park, in the 2-normed spaces, replaces this inequality with a new condition, which actually gives the following definition of the quasi 2-normed space.

Definition 1 ([7]). Let L be a real vector space with $\dim L \geq 2$. *Quasi 2-norm* is a real function $\|\cdot, \cdot\|: L \times L \rightarrow [0, \infty)$ which satisfies:

- a) $\|x, y\| \geq 0$, for all $x, y \in L$ and $\|x, y\| = 0$ if and only if the set $\{x, y\}$ is linearly dependent;
- b) $\|x, y\| = \|y, x\|$, for all $x, y \in L$;
- c) $\|\alpha x, y\| = |\alpha| \cdot \|x, y\|$, for all $x, y \in L$ and for each $\alpha \in \mathbf{R}$, and
- d) It exists a constant $K \geq 1$ such that $\|x + y, z\| \leq K(\|x, z\| + \|y, z\|)$, for all $x, y, z \in L$.

The pair $(L, \|\cdot, \cdot\|)$ is called *quasi 2-normed space*. The smallest number K that satisfies the condition d) is called the *modul of concavity* of the quasi 2-norm $\|\cdot, \cdot\|$.

In [3], M. Kir and M. Acikgoz give several examples of trivial quasi 2-normed spaces and they consider the question of completing the quasi 2-normed space, and in [7], C. Park gives a characterization of the quasi 2-normed space, i.e. he proved the following theorem.

Theorem 1 ([7]). Let $(L, \|\cdot, \cdot\|)$ be a quasi 2-normed space. Then there exists $p, 0 < p \leq 1$ and an equivalent quasi 2-norm $\|\cdot, \cdot\|$ on L such that

$$\|x + y, z\|^p \leq \|x, z\|^p + \|y, z\|^p, \quad (1)$$

40 for all $x, y, z \in L$. ■

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42 In connection with this theorem 1, C. Park, in [7], gives the following definition.

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44 **Definition 2 ([7]).** The quasi 2-norm of theorem 1 is called $(2, p)$ -norm and the quasi 2-normed space
45 L is called $(2, p)$ -normed space.

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47 **2 THE QUASI 2-NORM OF $L^p(\mu)$, $0 < p < 1$**

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49 In this part, we will introduce the quasi 2-norm of the space $L^p(\mu)$, $0 < p < 1$. For that purpose, we
50 will use the following two well known inequalities.

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52 **Lemma 1.** If $p \in (0, 1)$ and $a, b \geq 0$, then

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$$54 \quad (a+b)^p \leq a^p + b^p, \quad (2)$$

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56 and equality holds if and only if $a = 0$ or $b = 0$. ■

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58 **Lemma 2.** If $q > 1$ and $c, d > 0$, then

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$$60 \quad 2^{1-q}(c+d)^q \leq c^q + d^q, \quad (3)$$

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62 and equality holds if and only if $c = d$. ■

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64 **Theorem 1.** Let (Y, M) be a measurable space, μ be a positive measure on M and let $X = L^p(\mu)$,
65 $p \in (0, 1)$ be the space

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$$67 \quad X = \{f \mid f : Y \rightarrow \mathbf{R}, \int_Y |f|^p d\mu < +\infty\}.$$

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69 Then the function $\|\cdot, \cdot\| : L^p(\mu) \times L^p(\mu) \rightarrow \mathbf{R}$, defined by

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$$71 \quad \|f, g\| = \left\{ \int_{Y \times Y} \left| \frac{f(x) - f(y)}{g(x) - g(y)} \right|^p d(\mu \times \mu) \right\}^{\frac{1}{p}}, \quad (4)$$

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73 where $\mu \times \mu$ is the direct product of the measure μ , is the quasi 2-norm of the space $X = L^p(\mu)$.

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75 **Proof.** a) Because of the inequality (2), we have

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$$\begin{aligned} 77 \quad & \left| \frac{f(x) - f(y)}{g(x) - g(y)} \right|^p = |f(x)g(y) - g(x)f(y)|^p \\ & \leq (|f(x)g(y)| + |g(x)f(y)|)^p \\ 78 \quad & \leq |f(x)g(y)|^p + |g(x)f(y)|^p, \end{aligned}$$

79 for all $f, g \in L^p(\mu)$ and for each $(x, y) \in Y \times Y$. Now, using (4), the properties of the absolute value
 80 and the properties of integrating on a direct product of the measurable spaces $(Y \times Y, M \times M, \mu \times \mu)$,
 81 we obtain
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$$\begin{aligned} \|f, g\| &= \left\{ \int_{Y \times Y} \left| \frac{f(x)}{g(x)} - \frac{f(y)}{g(y)} \right|^p d(\mu \times \mu) \right\}^{\frac{1}{p}} \\ &\leq \left\{ \int_{Y \times Y} (|f(x)g(y)|^p + |g(x)f(y)|^p) d(\mu \times \mu) \right\}^{\frac{1}{p}} \\ 83 \quad &= \left\{ \int_{Y \times Y} |f(x)|^p |g(y)|^p d(\mu \times \mu) + \int_{Y \times Y} |g(x)|^p |f(y)|^p d(\mu \times \mu) \right\}^{\frac{1}{p}} \\ &= \left\{ \int_Y |f(x)|^p d\mu \int_Y |g(y)|^p d\mu + \int_Y |g(x)|^p d\mu \int_Y |f(y)|^p d\mu \right\}^{\frac{1}{p}} \\ &= 2^{\frac{1}{p}} \left\{ \int_Y |f(x)|^p d\mu \right\}^{\frac{1}{p}} \left\{ \int_Y |g(y)|^p d\mu \right\}^{\frac{1}{p}} < +\infty, \end{aligned}$$

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 85 which means that the function $\|\cdot, \cdot\|: L^p(\mu) \times L^p(\mu) \rightarrow \mathbf{R}$ is well defined.
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87 b) Let $f, g \in L^p(\mu)$. From the definition (4) directly follows that $\|f, g\| \geq 0$. Let $\|f, g\| = 0$, for
 88 some $f, g \in L^p(\mu)$, $f \neq 0, g \neq 0$ in respect to the measure μ , i.e.
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$$90 \quad \left\{ \int_{Y \times Y} \left| \frac{f(x)}{g(x)} - \frac{f(y)}{g(y)} \right|^p d(\mu \times \mu) \right\}^{\frac{1}{p}} = 0.$$

91 Then

$$92 \quad \left| \frac{f(x)}{g(x)} - \frac{f(y)}{g(y)} \right| = 0$$

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 94 almost everywhere in respect to the measure $\mu \times \mu$, which means that $f(x)g(y) - g(x)f(y) = 0$
 95 almost everywhere in respect to the measure $\mu \times \mu$. Therefore,
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$$97 \quad W_{f,g} = \{(x, y) \mid f(x)g(y) - g(x)f(y) \neq 0\}$$

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 99 is a set with measure 0, i.e. $(\mu \times \mu)(W_{f,g}) = 0$. The assumption $f \neq 0, g \neq 0$ in respect to the
 100 measure μ , implies that
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$$102 \quad U_f = \{x \mid x \in Y, f(x) \neq 0\} \text{ and } U_g = \{x \mid x \in Y, g(x) \neq 0\}$$

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 104 are sets with positive measure i.e. $\mu(U_f) > 0$ и $\mu(U_g) > 0$.

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 106 We will prove that
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$$108 \quad \mu(U_f \cap U_g) > 0, \mu(U_f \setminus U_g) = 0 \text{ and } \mu(U_g \setminus U_f) = 0.$$

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110 Let us suppose that $\mu(U_f \cap U_g) = 0$. Then, without loss of the generality, we can assume that
 111 $U_f \cap U_g = \emptyset$ in respect to the measure μ . For every $(x, y) \in U_f \times U_g$ we have that
 112 $f(x) \neq 0, g(y) \neq 0, g(x) = 0$ and $f(y) = 0$, so

$$113 \quad f(x)g(y) - g(x)f(y) = f(x)g(y) \neq 0.$$

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 115 Therefore $(x, y) \in W_{f,g}$, i.e. $U_f \times U_g \subseteq W_{f,g}$. Using the properties of the measure $\mu \times \mu$, i.e. from
 116 its definition we get that
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$$119 \quad \begin{aligned} (\mu \times \mu)(W_{f,g}) &\geq (\mu \times \mu)(U_f \times U_g) \\ &= \mu(U_f)\mu(U_g) > 0, \end{aligned}$$

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 121 which contradicts to $(\mu \times \mu)(W_{f,g}) = 0$. So, we conclude that $\mu(U_f \cap U_g) > 0$.
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123 Let us now suppose that $\mu(U_f \setminus U_g) > 0$. If $(x, y) \in (U_f \setminus U_g) \times (U_f \cap U_g)$, then $x \in U_f, x \notin U_g,$
 124 $y \in U_f$ and $y \in U_g$, so $f(x) \neq 0, g(x) = 0, f(y) \neq 0$ and $g(y) \neq 0$, which implies that

$$125 \quad f(x)g(y) - g(x)f(y) = f(x)g(y) \neq 0.$$

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 127 Therefore $(U_f \setminus U_g) \times (U_f \cap U_g) \subseteq W_{f,g}$. Using this and the fact that $\mu(U_f \cap U_g) > 0$, we get that
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$$130 \quad \begin{aligned} (\mu \times \mu)(W_{f,g}) &\geq (\mu \times \mu)[(U_f \setminus U_g) \times (U_f \cap U_g)] \\ &= \mu(U_f \setminus U_g)\mu(U_f \cap U_g) > 0, \end{aligned}$$

131
 132 which, again, contradicts to $(\mu \times \mu)(W_{f,g}) = 0$. We conclude that $\mu(U_f \setminus U_g) = 0$. Analogously, it
 133 can be proved that $\mu(U_g \setminus U_f) = 0$.
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135 The equalities
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$$137 \quad \begin{aligned} U_f &= (U_f \setminus U_g) \cup (U_f \cap U_g), \quad U_g = (U_g \setminus U_f) \cup (U_f \cap U_g), \\ 138 \quad \mu(U_f \setminus U_g) &= 0 \text{ and } \mu(U_g \setminus U_f) = 0, \end{aligned}$$

139
 140 imply that $U_f = U_g = U$, in respect to the measure μ .
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142 Clearly, $\mu(U) > 0$. For each $x \in U$, we define the set
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$$144 \quad U_x = \{y \mid y \in U, f(x)g(y) - f(y)g(x) \neq 0\}.$$

145 We will prove that there exists $x_0 \in U$ such that $\mu(U_{x_0}) = 0$. Let us suppose the contrary, i.e. that
 146 $\mu(U_x) > 0$, for each $x \in U$. $f = g = 0$ almost everywhere on $Y \setminus U$ in respect to the measure μ ,
 147 implies that $f = g = 0$ almost everywhere on $Y \setminus U_x$ in respect to the measure μ , and
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$$149 \quad |f(x)g(y) - g(x)f(y)|^p > 0 \text{ on } U_x.$$

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151 Then

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$$\begin{aligned} h(x) &= \int_Y |f(x)g(y) - g(x)f(y)|^p d\mu_y \\ &= \int_{U_x} |f(x)g(y) - g(x)f(y)|^p d\mu_y > 0, \end{aligned}$$

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154 which implies that

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$$\int_{Y \times Y} \left| \frac{f(x)}{g(x)} - \frac{f(y)}{g(y)} \right|^p d(\mu \times \mu) = \int_Y h(x) d\mu = \int_U h(x) d\mu > 0,$$

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158 which contradicts the assumption $\|f, g\| = 0$. Therefore, there must exist $x_0 \in U$ such that

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$$f(x_0)g(y) - g(x_0)f(y) = 0,$$

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162 almost everywhere on U . Since $f(x_0) \neq 0, g(x_0) \neq 0$ we get that $g(y) = \frac{g(x_0)}{f(x_0)} f(y)$, for each $y \in U$,

163 which together with the fact $f \equiv 0, g \equiv 0$ on $Y \setminus U$, gives that $g(y) = \frac{g(x_0)}{f(x_0)} f(y)$ на Y , i.e.

164 $g(y) = \alpha f(y)$, where $\alpha = \frac{g(x_0)}{f(x_0)}$. So, we proved the condition $i)$ from the definition 1.

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166 c) For all $f, g \in L^p(\mu)$, it holds

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$$\begin{aligned} \|f, g\| &= \left\{ \int_{Y \times Y} \left| \frac{f(x)}{g(x)} - \frac{f(y)}{g(y)} \right|^p d(\mu \times \mu) \right\}^{\frac{1}{p}} \\ &= \left\{ \int_{Y \times Y} \left| \frac{g(x)}{f(x)} - \frac{g(y)}{f(y)} \right|^p d(\mu \times \mu) \right\}^{\frac{1}{p}} = \|g, f\| \end{aligned}$$

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170 i.e. the condition $ii)$ from the definition 1 is true.

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172 d) Let $f, g \in L^p(\mu)$ and $\alpha \in \mathbf{R}$. Then

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$$\begin{aligned} \|\alpha f, g\| &= \left\{ \int_{Y \times Y} \left| \frac{\alpha f(x)}{g(x)} - \frac{\alpha f(y)}{g(y)} \right|^p d(\mu \times \mu) \right\}^{\frac{1}{p}} \\ &= |\alpha| \left\{ \int_{Y \times Y} \left| \frac{f(x)}{g(x)} - \frac{f(y)}{g(y)} \right|^p d(\mu \times \mu) \right\}^{\frac{1}{p}} = |\alpha| \cdot \|f, g\| \end{aligned}$$

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176 i.e. the condition $iii)$ from the definition 1 holds.

177 e) Using the inequality (2), and then the inequality (3), with $q = \frac{1}{p}$, we get that, for all

178 $f, g, h \in L^p(\mu)$, it holds

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$$\begin{aligned}
 \|f + g, h\| &= \left\{ \int_{Y \times Y} \left| \frac{f(x) + g(x)}{h(x)} - \frac{f(y) + g(y)}{h(y)} \right|^p d(\mu \times \mu) \right\}^{\frac{1}{p}} \\
 &\leq \left\{ \int_{Y \times Y} \left(\left| \frac{f(x)}{h(x)} - \frac{f(y)}{h(y)} \right| + \left| \frac{g(x)}{h(x)} - \frac{g(y)}{h(y)} \right| \right)^p d(\mu \times \mu) \right\}^{\frac{1}{p}} \\
 &\leq \left\{ \int_{Y \times Y} \left| \frac{f(x)}{h(x)} - \frac{f(y)}{h(y)} \right|^p d(\mu \times \mu) + \int_{Y \times Y} \left| \frac{g(x)}{h(x)} - \frac{g(y)}{h(y)} \right|^p d(\mu \times \mu) \right\}^{\frac{1}{p}} \\
 &= \left\{ \int_{Y \times Y} \left| \frac{f(x)}{h(x)} - \frac{f(y)}{h(y)} \right|^p d(\mu \times \mu) + \int_{Y \times Y} \left| \frac{g(x)}{h(x)} - \frac{g(y)}{h(y)} \right|^p d(\mu \times \mu) \right\}^{\frac{1}{p}} \\
 &\leq 2^{\frac{1}{p}-1} \left\{ \left(\int_{Y \times Y} \left| \frac{f(x)}{h(x)} - \frac{f(y)}{h(y)} \right|^p d(\mu \times \mu) \right)^{\frac{1}{p}} + \left(\int_{Y \times Y} \left| \frac{g(x)}{h(x)} - \frac{g(y)}{h(y)} \right|^p d(\mu \times \mu) \right)^{\frac{1}{p}} \right\} \\
 &= 2^{\frac{1}{p}-1} (\|f, h\| + \|g, h\|),
 \end{aligned}$$

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182 which means that, for $K = 2^{\frac{1}{p}-1} > 1$, the condition iv) from the definition 1 holds. ■

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184 In [7], C. Park defines Cauchy sequence, convergent sequence and complete quasi 2-normed space i.e. he gives the following definition.

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Definition 3. Let L be a quasi 2-normed space.

a) The sequence $\{x_n\}_{n=1}^{\infty}$ in L is a *Cauchy* if

$$\lim_{n \rightarrow \infty} \|x_m - x_n, z\| = 0, \text{ for each } z \in L.$$

b) The sequence $\{x_n\}_{n=1}^{\infty}$ in L is *convergent* if there exists $x \in L$ such that

$$\lim_{n \rightarrow \infty} \|x_m - x, z\| = 0, \text{ for each } z \in L.$$

The vector $x \in L$ is called the *boundary* of the sequence $\{x_n\}_{n=1}^{\infty}$.

c) A quasi 2-normed space L in which every Cauchy sequence is convergent is called a *quasi 2-complete* (a *quasi 2-Banach*) *space*.

Problem. Whether the quasi 2-normed space $L^p(\mu)$, $0 < p < 1$ with the quasi norm, introduced by (4), is complete?

3. TWO INEQUALITIES IN $L^p(\mu)$, $0 < p < 1$ AS QUASI 2-NORMED SPACE

For the quasi 2-normed spaces in [5], it is proven the following Lemma.

Lemma 3. If L is quasi 2-normed space with modul of concavity $K \geq 1$, then, for each $n > 1$ and for all $z, x_1, x_2, \dots, x_n \in L$, it holds

$$\left\| \sum_{i=1}^n x_i, z \right\| \leq K^{1+\lceil \log_2(n-1) \rceil} \sum_{i=1}^n \|x_i, z\|. \quad \blacksquare \quad (5)$$

From Lemma 3 and the proof of Theorem 1, directly follows the consequence.

211 **Corollary 1.** Let $L^p(\mu)$, $0 < p < 1$ be a quasi 2-normed space in which the quasi 2-norm is given by

212 (4). Then for all $f_1, f_2, \dots, f_n, g \in L^p(\mu)$, $n \geq 2$, it holds

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$$214 \quad \left\| \sum_{i=1}^n f_i, g \right\| \leq 2^{(1+\lceil \log_2(n-1) \rceil)(\frac{1}{p}-1)} \sum_{i=1}^n \|f_i, g\|. \quad (6)$$

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216 **Proof.** It is enough to take $K = 2^{\frac{1}{p}-1} > 1$ in the inequality (5). ■

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218 In the following theorem, we will prove stricter inequality than the inequality (6). For that purpose,
219 we will use the following well known inequality.

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221 **Lemma 4.** Let $t \in [1, \infty)$. Then for all $a_1, a_2, \dots, a_n \in [0, \infty)$, it holds

$$222 \quad \left(\sum_{i=1}^n a_i \right)^t \leq n^{t-1} \sum_{i=1}^n a_i^t. \quad \blacksquare$$

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224 **Theorem 2.** Let $L^p(\mu)$, $0 < p < 1$ be a quasi 2-normed space in which the quasi 2-norm is given by

225 (4). Then for all $f_1, f_2, \dots, f_n, g \in L^p(\mu)$, it holds

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$$227 \quad \left\| \sum_{i=1}^n f_i, g \right\| \leq n^{\frac{1}{p}-1} \sum_{i=1}^n \|f_i, g\|. \quad (7)$$

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229 **Proof.** Since $t = \frac{1}{p} > 1$, the inequality (4), the properties of the integral, Lemma 1 and Lemma 4 imply
230 that

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$$\begin{aligned} \left\| \sum_{i=1}^n f_i, g \right\| &= \left\{ \int_{Y \times Y} \left| \sum_{i=1}^n \frac{f_i(x)}{g(x)} \frac{f_i(y)}{g(y)} \right|^p d(\mu \times \mu) \right\}^{\frac{1}{p}} \\ &\leq \left\{ \int_{Y \times Y} \left(\sum_{i=1}^n \left| \frac{f_i(x)}{g(x)} \frac{f_i(y)}{g(y)} \right| \right)^p d(\mu \times \mu) \right\}^{\frac{1}{p}} \\ &\leq \left\{ \int_{Y \times Y} \sum_{i=1}^n \left| \frac{f_i(x)}{g(x)} \frac{f_i(y)}{g(y)} \right|^p d(\mu \times \mu) \right\}^{\frac{1}{p}} \\ 232 \quad &= \left\{ \sum_{i=1}^n \int_{Y \times Y} \left| \frac{f_i(x)}{g(x)} \frac{f_i(y)}{g(y)} \right|^p d(\mu \times \mu) \right\}^{\frac{1}{p}} \\ &\leq n^{\frac{1}{p}-1} \sum_{i=1}^n \left\{ \int_{Y \times Y} \left| \frac{f_i(x)}{g(x)} \frac{f_i(y)}{g(y)} \right|^p d(\mu \times \mu) \right\}^{\frac{1}{p}} \\ &= n^{\frac{1}{p}-1} \sum_{i=1}^n \|f_i, g\|. \end{aligned}$$

233 Clearly, the inequality (7) is stricter than the inequality (6), since for each $n \geq 2$, it holds

234 $n \leq 2^{1+\lceil \log_2(n-1) \rceil}$, and the equality holds for $n = 2^k, k = 1, 2, 3, \dots$.

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236 To prove that $n^{\frac{1}{p}-1}$ is the best possible constant, it is enough to take that E and F are measurable
237 sets such that $\mu(E) = \alpha < \infty, \mu(F) = \beta < \infty$, $g = 1_F$ and $E_i = E$ and $f_i = 1_{E_i}$, for $i = 1, 2, \dots, n$. Then

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$$\begin{aligned} \|\sum_{i=1}^n f_i, g\| &= \left\{ \sum_{i=1}^n (\mu \times \mu)(E_i \times F) \right\}^{\frac{1}{p}} = \left\{ \sum_{i=1}^n \mu(E_i) \mu(F) \right\}^{\frac{1}{p}} = (n\alpha\beta)^{\frac{1}{p}} \\ &= n^{\frac{1}{p}-1} \{n(\alpha\beta)^{\frac{1}{p}}\} = n^{\frac{1}{p}-1} \sum_{i=1}^n \{\mu(E_i) \mu(F)\}^{\frac{1}{p}} \\ &= n^{\frac{1}{p}-1} \sum_{i=1}^n (\mu \times \mu)(E_i \times F)^{\frac{1}{p}} = n^{\frac{1}{p}-1} \sum_{i=1}^n \|f_i, g\|. \blacksquare \end{aligned}$$

4. CONCLUSION

In this paper, it's introduced the quasi 2-norm of the space $L^p(\mu)$, $0 < p < 1$ and there are proven several inequalities relating the quasi 2-norm of this space. Also is given the following open problem: Whether the quasi 2-normed space $L^p(\mu)$, $0 < p < 1$ with the quasi norm, introduced by (4), is complete?

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