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Journal Name:	British Journal of Mathematics & Computer Science
Manuscript Number:	Ms_BJMCS_25884
Title of the Manuscript:	AN ITERATIVE ALGORITHM FOR ZEROS OF GENERALIZED PHI-STRONGLY MONOTONE AND BOUNDED MAPS IN UNIFORMLY CONVEX AND UNIFORMLY SMOOTH BANACH SPACES
Type of the Article	Original Research Article

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PART 1: Review Comments

	Reviewer's comment	Author's comment (if agreed with reviewer, correct the manuscript and highlight that part in the manuscript. It is mandatory that authors should write his/her feedback here)
Compulsory REVISION comments	The author proves interesting results, written in very good English. Please clarify the Plagiarism: However, the main results and their way of presentation are similar to those published by C. Diop, T.M.M. Sow, N. Djitte and C.E. Chidume in their paper "Constructive techniques for zeros of monotone mappings in certain Banach spaces", Springerplus, 2015, 4: 383, published online 2015, Jul 28, DOI 10.1186/s40064-015-1169-2 Such results are free available online. Therefore, the present work seems to be a review article, not an original research article. Moreover, main results remain unclear. For example, at p. 3, Theorem 1.1, a reference citation is required. The last line on page 3 is not clear: if (x_n) converges strongly to $\hat{x}$, assuming that $(Ax_n)_n$ is bounded, but not converges to zero, passing through the limit, one obtains $\hat{x} = \hat{x} - 0 \cdot \lim Ax_n$, that is a true equality: $0 = 0$, which does not imply	The following definitions will help clarify matters to this reviewer. Definition 0.1. (Classes of monotone-type mappings) Let E be a real normed space and E^* be its dual. A map $A : D(A) \subseteq E \rightarrow E^*$ is called <i>k-strongly monotone</i> if there exists $k \in (0, 1)$ such that $\forall x, y \in D(A)$, $\langle Ax - Ay, x - y \rangle \geq k\ x - y\ ^2.$ It is called <i>φ-strongly monotone</i> if there exists a strictly increasing function $\phi : [0, \infty) \rightarrow [0, \infty)$ with $\phi(0) = 0$, such that $\forall x, y \in D(A)$, $\langle Ax - Ay, x - y \rangle \geq \phi(\ x - y\)\ x - y\ .$ It is called <i>generalized Φ-strongly monotone</i> if there exists a strictly increasing function $\Phi : [0, \infty) \rightarrow [0, \infty)$ with $\Phi(0) = 0$, such that $\forall x, y \in D(A)$, $\langle Ax - Ay, x - y \rangle \geq \Phi(\ x - y\).$ The class of <i>generalized Φ-strongly monotone maps</i> is the largest class of monotone-type mappings for which, if $Au = 0$ has a solution, it is unique.



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	$A\hat{x} = 0$. P.5, line 2 from below: (ii): M used, but not defined. The relation (ii) should be explained or modified properly. The first two lines at page 6 need a serious explanation (on the modulus of continuity for J^{-1}). The proof of Theorem 3.1, (and consequently those of the Corollaries 3.2 and 3.3) should be rewritten in more detail and more clearly, such that the reader can understand easily all the arguments, without searching back. The author is kindly asked to give all the required explanations, line after line and step by step.	Comment: It is well known that the class of k-strongly monotone maps is a proper subclass of the class of ϕ-strongly monotone maps, (one takes $\phi(t) = kt, k \in (0, 1)$) and the later is a proper subclass of the class of generalized Φ-strongly monotone maps, (one takes $\Phi(t) = t\phi(t)$). Definition 0.2. Let E be a normed space with $\dim E \geq 2$. The modulus of smoothness of E is a function $\rho_E : [0, \infty) \rightarrow [0, \infty)$ defined by $\rho_E(\tau) := \sup \left\{ \frac{\ x+y\ + \ x-y\ }{2} - 1 : \ x\ = 1; \ y\ = \tau \right\}.$ Definition 0.3. (Classes of uniformly smooth Banach spaces) A normed space E is <i>uniformly smooth</i> if and only if $\lim_{t \rightarrow \infty} \frac{\rho_E(t)}{t} = 0.$ For $q > 1$, E is said to be <i>q-uniformly smooth</i> if there exists a constant $c > 0$ such that $\rho_E(t) \leq ct^q, t > 0$. L_p spaces for $2 \leq p < \infty$ are examples of <i>2-uniformly smooth</i> spaces whereas, L_p spaces for $1 < p < 2$ are not <i>2-uniformly smooth</i>. Comment: In theorem 3.1, which is the main theorem of the paper of Diop <i>et al.</i>, [1], the reviewer referred to, E is a <i>2-uniformly smooth spaces</i>. These spaces do not include L_p spaces for $1 < p < 2$. Comment: Our theorems are proved in real Banach spaces that are <i>uniformly smooth and uniformly convex</i>. These spaces include L_p spaces for all $1 < p < \infty$. In particular, they include L_p spaces for $1 < p < 2$. Furthermore, our theorems are proved for the much more general class of <i>generalized Φ-strongly monotone</i> maps.
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		Consequently, our theorems generalized, improve and extend the results in Diop <i>et al.</i>, [1], from the class of <i>k-strongly monotone</i> maps to the much more general class of <i>generalized Φ-strongly monotone maps</i>, and to the more general class of <i>uniformly convex and uniformly smooth real Banach spaces</i>. We have now included the above reference in our revised paper with these comments added. In iterative formulas involving parameters, limit taken in the formula does not reveal anything meaningful. For example, the Mann iterative formula which has been studied for over fifty years is defined by, $x_0 \in D(T)$, $x_{n+1} = (1 - \alpha_n)x_n + \alpha_n T x_n, \quad (*)$ where (i) $\sum_{n=1}^{\infty} \alpha_n = \infty$; (ii) $\lim \alpha_n = 0$. Observe that if $\{T x_n\}$ is bounded, by passing limit through in $(*)$ we have that $0=0$, which does not imply that $T x^* = x^*$ for some $x^* \in F(T) := \{x \in D(T) : T x = x\}$. But, for example, if T denotes the rotation of the unit ball in $\mathbb{R}^2$ about the origin of coordinates, the sequence given by $(*)$ converges strongly to $x^* \in \mathbb{R}^2$, where $T x^* = x^*$. It is now defined before the proof.
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<u>Minor</u> REVISION comments	P.4: "Unfortunately, these geometric..." (free space before 'these'). P.4: " In this paper, we introduce an iterative..." (delete "a"). P.5, lemma 2.5:"...,let $B_a(0)$...". P.6, line 3 from below: " w e have that $\ J_{u_{n+1}}\ $..." (never use capital letters in the middle of a phrase and delete the comma following "that"). P.6, line 2 from below: "... o f J^{-1} ... ". P.9, the proof of the Corollary 3.3: "...is bounded, w e show...".	The corrections have been effected. We have also acknowledged the useful comments of the referees.
<u>Optional/General</u> comments		