



SDI FINAL EVALUATION FORM 1.1

PART 1:

Journal Name:	British Journal of Mathematics & Computer Science
Manuscript Number:	Ms_BJMCS_23501
Title of the Manuscript:	A Refinement of the Asymptotic growth bounds for the Vlasov Poisson system with a point charge
Type of the Article	Short Research Article

PART 2:

FINAL EVALUATOR'S comments on revised paper (if any)	Authors' response to final evaluator's comments
on the article A refinement of the asymptotic growth bounds for the Vlasov Poisson system with a point charge The revision has not improved the article. My comment concerning the unexplained symbols has been ignored, so I repeat it in more details. The unknown functions in the characteristic system (2.1) are denoted by $X(s; 0)$ and $V(s; 0)$ or, minutely, $X(s; 0, x, v)$ and $V(s; 0, x, v)$. But further on this page there is the formula $f(t, x, v) = f_0(X(0; t), V(0; t)).$ Is $X(0; t)$ the same as $X(t; 0)$ or something other? Then on p. 5 there appear $X(s; t - \delta(t), x, v)$ and $V(s; t - \delta(t), x, v)$. But the authors defined earlier only $X(s; 0, x, v)$ and $V(s; 0, x, v)$ (it is not even clear from this notation whether 0 is spatial or temporal), not $X(s; t, x, v)$ and $V(s; t, x, v)$. It is not explained why $H(0) \leq h(t, x, v)$, which is the second inequality in formula (2.3). Also, I cannot grasp the logic why out of the two conservation laws $H(t) = H(0)$, $h(t, x, v) = h(0, x, v)$ the authors use in (2.3) only the former. Proposition 2.2 reads :“For any $(y, w) \dots$ we have. . .”. But the asserted formula (2.8) does not contain y and w. The phrase “Using Lemma 2.1 and definition of $P(t)$” on p. 5 does not lead straightforwardly to the subsequent formula. More rationale is required. The left-hand side of inequality (2.15) (the conclusion of Lemma 2.4) contains the indicator of the set S_3 whose definition contains, in turn, parameter L which, however, does not enter the right-hand side of the (2.15). Thus it is implicitly stated that the inequality is uniform in L. But the inequality in line 2 of p. 9 (in the proof of that lemma) shows that it is not the case. And what is k in that inequality? In general, the derivations are not verifiable and therefore the result is not reliable.	Authors' response to final evaluator's comments We write revisions in more detail carefully and hope that satisfy all requirements : 1- Re- formula function $f(t,x,v)$ in page 3 by $f(t,x,v)=f_0(x,v)$ and the function $X(0;t)=X(0;t,x,v)$ but the function $X(t,0)=X(t;0,x,v)$, therefore the function f must write $f(t,x,v)=f_0(X(0;0),V(0;0))$ or by the above with thanks for this important notice. 2- The first appear of $(X(s;t-\delta),V(s;t-\delta))$ in page 4 in definition of $Q(t,\delta)$ which explain $(x,v) \in \text{supp } f(t-\delta)$ and details follow from this case. Some amendments and arrangements make according for suggestions in page 5. 3- We explaine that $H(0)\leq h(t,x,v)$ in line 3 of page 4(because the choice of $K\geq \max\{H(0),1\}$ and definition of $h(t,x,v)$in (1.2) with $K=G$ therefore $H(0)\leq h(t,x,v)$) 4- We use the conservation law of $H(t)=H(0)$ to prove two important inequalities (2.3) and (2.4) without use $h(t,x,v)=h(o,x,v)$ in paper. We hope clear this point. 5- Re-write operative of proposition(2.2) by(Let (f,ζ) be a classical solution of (1.1) on $[0,T]$. we have) 6- The phrase “Using lemma 2.1 and definition of $P(t)$ to get inequality...”, in more than one references give details like[1],[2] and [15], so we skip it and add indicator in this phrase in page 5 of the paper. 7- In the set S_3 there exist two cases: The first case if $V(t;t-\delta(t))-V^*(t) \geq 4p$ which is uniform in L. The second case if $V(t;t-\delta(t))-V^*(t) \leq 4p$ we obtain steps in paper in the first of page 9. The constant k appears from use the lemma 2.1 in the concerned inequality.