

Assume data Pattern is

$$Y = AX_i + B \quad (1)$$

and  $Q = \sum_{i=1}^n (\bar{Y}_i - y_i)^2 = \min. \quad (2)$

$$\therefore \frac{\partial Q}{\partial A} = \sum_{i=1}^n \frac{\partial}{\partial A} (\bar{Y}_i - y_i)^2 = 0$$

$$\frac{\partial Q}{\partial B} = \sum_{i=1}^n \frac{\partial}{\partial B} (\bar{Y}_i - y_i)^2 = 0 \quad (3)$$

Substitute (1) into (3):

$$\sum_{i=1}^n \frac{\partial}{\partial A} (A\bar{X}_i + B - y_i)^2 = \sum_{i=1}^n 2(A\bar{X}_i + B - y_i) \cdot \bar{X}_i = 0$$

$$\sum_{i=1}^n \frac{\partial}{\partial B} (A\bar{X}_i + B - y_i)^2 = 2 \sum_{i=1}^n (A\bar{X}_i + B - y_i) \cdot 1 = 0 \quad (4)$$

Re-write (4) as:

$$A \sum_{i=1}^n \bar{X}_i^2 + B \sum_{i=1}^n \bar{X}_i = \sum_{i=1}^n y_i \bar{X}_i$$

$$A \sum_{i=1}^n \bar{X}_i + B \sum_{i=1}^n 1 = \sum_{i=1}^n y_i \quad (5)$$

Let:  $M_{11} = \sum_{i=1}^n \bar{X}_i^2$      $M_{12} = \sum_{i=1}^n \bar{X}_i$      $M_{13} = \sum_{i=1}^n y_i \bar{X}_i$

$$M_{21} = \sum_{i=1}^n \bar{X}_i \quad M_{22} = \sum_{i=1}^n 1 = n \quad M_{23} = \sum_{i=1}^n y_i \quad (6)$$

Thus, (5) becomes

$$M_{11}A + M_{12}B = M_{13} \quad (7)$$

$$M_{21}A + M_{22}B = M_{23}$$

Or 
$$\begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} \begin{pmatrix} A \\ B \end{pmatrix} = \begin{pmatrix} M_{13} \\ M_{23} \end{pmatrix} \quad (8)$$