

On V_4 -cordial Labeling of Graphs

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Abstract

In this Research article we prove the following results:

1. The crown graph $C_n \odot K_1$ is V_4 -cordial for all n .
2. The armed crown AC_n is V_4 -cordial for all n .
3. The pan graph C_n^{+1} is V_4 -cordial for all n .
4. The corona graph $C_n \odot mK_1$ is V_4 -cordial for all n and m .

Keywords: Klein-four Group; V_4 -cordial Labeling; Crown; Armed Crown; Pan Graph.

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1 Introduction

In this research article, by a graph we mean finite, connected, undirected and simple graph $G = (V(G), E(G))$ of order $|V(G)|$ and size $|E(G)|$. Here we discuss V_4 -cordial labeling of standard graphs which are obtained by using some graph operations.

In this paper we consider the following definitions.

Definition 1.1. The Corona $G_1 \odot G_2$ of two graphs G_1 and G_2 is defined as a graph obtained by taking one copy of G_1 (which has p_1 vertices) and p_1 copies of G_2 and joining the i^{th} vertex of G_1 with an edge to every vertex in the i^{th} copy of G_2 .

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Definition 1.2. The *Crown* $C_n \odot K_1$ is obtained by joining a pendant edge to each vertex of cycle C_n .

Definition 1.3. An *Armed Crown* is a graph in which path P_2 is attached at each vertex of cycle C_n by an edge. It is denoted by AC_n where n is the number of vertices in cycle C_n .

Definition 1.4. The *n-pan* graph C_n^{+1} is the graph obtained by joining a cycle graph C_n to a complete graph K_1 with a bridge. In other words the pan graph is obtained by adding a pendant edge to any vertex of cycle C_n .

Definition 1.5. A *graph labeling* is an assignment of integers to the vertices or edges or both subject to certain condition(s).

The latest updates of various graph labeling techniques can be found in [Gallian (2014)].

Definition 1.6. Let $\langle A, * \rangle$ be any Abelian group. A graph $G = (V(G), E(G))$ is said to be *A-cordial* if there is a mapping $f : V(G) \rightarrow A$ which satisfies the following two conditions when the edge $e = uv$ is labeled as $f(u) * f(v)$

(i) $|v_f(a) - v_f(b)| \leq 1$; for all $a, b \in A$,

(ii) $|e_f(a) - e_f(b)| \leq 1$; for all $a, b \in A$.

Where

$v_f(a)$ =the number of vertices with label a ;

$v_f(b)$ =the number of vertices with label b ;

$e_f(a)$ =the number of edges with label a ;

$e_f(b)$ =the number of edges with label b .

We note that if $A = \langle Z_k, +_k \rangle$, that is additive group of modulo k then the labeling is known as *k-cordial* labeling.

Definition 1.7. Let $V_4 = Z_2 \times Z_2 = \{0 = \langle 0, 0 \rangle, a = \langle 1, 0 \rangle, b = \langle 0, 1 \rangle, c = \langle 1, 1 \rangle\}$ be the Klein-four group in which the operation $*$ is defined as follows:

*	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

The graph $G = (V(G), E(G))$, with vertex set $V(G)$ and edge set $E(G)$, is said to be *V₄-cordial* if there exist a mapping $f : V(G) \rightarrow V_4$ which satisfies the following two conditions when the edge $e = uv$ is labeled as $f(u) * f(v)$

(i) $|v_f(p) - v_f(q)| \leq 1$; for all $p, q \in V_4$,

(ii) $|e_f(p) - e_f(q)| \leq 1$; for all $p, q \in V_4$.

Where

$v_f(p)$ =the number of vertices with label p ;

$v_f(q)$ =the number of vertices with label q ;

$e_f(p)$ =the number of edges with label p ;

$e_f(q)$ =the number of edges with label q .

For any undefined term in graph theory we rely upon [Gross and Yellen (2004)].

The concept of V_4 -cordiality was introduced by [A. Riskin (2013)] and proved the following result.

1. Complete graph K_n is V_4 -cordial if and only if $n < 4$.
2. The Star graph $K_{1,n}$ is V_4 -cordial.

[M. Seenivasan and A. Lourdusamy (2010)] proved the following results.

1. If G is an Eulerian graph with q edges, where $q \equiv 2(mod 4)$, then G has no V_4 -cordial labeling.
2. If f be a V_4 -cordial labeling of a graph G with $p \geq 4$ and uv be an edge of G such that $f(u) = 0$ and $f(u) \neq f(v)$. Then the graph G' obtained from G by replacing the edge uv by a path of length five is V_4 -cordial.
3. The Path P_4 and P_5 are not V_4 -cordial.
4. All Trees except P_4 and P_5 are V_4 -cordial.
5. The Cycle C_n is V_4 -cordial if and only if $n \neq 4$ or 5 or $n \not\equiv 2(mod 4)$.

[O. Pechenik and J. Wise (2012)] proved the following results.

1. The Complete Bipartite graph $K_{m,n}$ is V_4 -cordial if and only if m and n are not both congruent to $2(mod 4)$.
2. The Path P_n is V_4 -cordial unless $n \in \{4, 5\}$.
3. The Cycle C_n is V_4 -cordial if and only if $n \notin \{4, 5\}$ and $n \not\equiv 2(mod 4)$.
4. All Ladders $P_2 \times P_k$ are V_4 -cordial, except $P_2 \times P_2$.
5. The Prism $P_2 \times C_k$ is V_4 -cordial if and only if $k \not\equiv 2(mod 4)$.
6. The d -dimensional hypercube Q_d is V_4 -cordial, unless $d = 2$.

In the present work there are new results corresponding to V_4 -cordial labeling of graphs are investigated.

2 Main Results

Theorem 2.1. The Crown $C_n \odot K_1$ is V_4 -cordial for all n .

Proof. Let $G = C_n \odot K_1$ be the crown graph. Let $v_1, v_2, \dots, v_n$ be the vertices of cycle C_n and $v'_1, v'_2, \dots, v'_n$ are the pendant vertices of crown $C_n \odot K_1$. We note that $|V(G)| = 2n$ and $|E(G)| = 2n$.

To define V_4 -cordial labeling $f : V(G) \rightarrow V_4$ we consider the following cases.

Case 1: $n \equiv 0, 3(mod 4)$

$$\begin{aligned} f(v_i) &= 0; & i &\equiv 0(mod 4); \\ f(v_i) &= a; & i &\equiv 1(mod 4); \\ f(v_i) &= b; & i &\equiv 2(mod 4); \\ f(v_i) &= c; & i &\equiv 3(mod 4); & 1 \leq i \leq n, \\ f(v'_i) &= 0; & i &\equiv 2(mod 4); \\ f(v'_i) &= a; & i &\equiv 1(mod 4); \\ f(v'_i) &= b; & i &\equiv 0(mod 4); \\ f(v'_i) &= c; & i &\equiv 3(mod 4); & 1 \leq i \leq n. \end{aligned}$$

Case 2: $n \equiv 1(mod 4)$

$$f(v_i) = 0; \quad i \equiv 0(mod 4);$$

$$\begin{aligned}
 f(v_i) &= a; & i \equiv 1(\text{mod}4); \\
 f(v_i) &= b; & i \equiv 2(\text{mod}4); \\
 f(v_i) &= c; & i \equiv 3(\text{mod}4); & 1 \leq i \leq n-1, \\
 f(v_n) &= 0; \\
 f(v'_i) &= 0; & i \equiv 2(\text{mod}4); \\
 f(v'_i) &= a; & i \equiv 1(\text{mod}4); \\
 f(v'_i) &= b; & i \equiv 0(\text{mod}4); \\
 f(v'_i) &= c; & i \equiv 3(\text{mod}4); & 1 \leq i \leq n-1, \\
 f(v'_n) &= b.
 \end{aligned}$$

Case 3: $n \equiv 2(\text{mod}4)$

$$\begin{aligned}
 f(v_i) &= 0; & i \equiv 0(\text{mod}4); \\
 f(v_i) &= a; & i \equiv 1(\text{mod}4); \\
 f(v_i) &= b; & i \equiv 2(\text{mod}4); \\
 f(v_i) &= c; & i \equiv 3(\text{mod}4); & 1 \leq i \leq n-2, \\
 f(v_{n-1}) &= 0; \\
 f(v_n) &= b; \\
 f(v'_i) &= 0; & i \equiv 2(\text{mod}4); \\
 f(v'_i) &= a; & i \equiv 1(\text{mod}4); \\
 f(v'_i) &= b; & i \equiv 0(\text{mod}4); \\
 f(v'_i) &= c; & i \equiv 3(\text{mod}4); & 1 \leq i \leq n-1, \\
 f(v'_n) &= c.
 \end{aligned}$$

The Table 1 shows that the labeling pattern defined above covers all possible arrangement of vertices. In each possibility the graph under consideration satisfies the vertex conditions and edge conditions for V_4 -cordial labeling. Hence, the crown $C_n \odot K_1$ is V_4 -cordial for all n . Let $n = 4p + q$, where $p, q \in N \cup \{0\}$.

q	Vertex conditions	Edge conditions
0,2	$v_f(0) = v_f(a) = v_f(b) = v_f(c)$	$e_f(0) = e_f(a) = e_f(b) = e_f(c)$
1	$v_f(0) = v_f(a) + 1 = v_f(b) = v_f(c) + 1$	$e_f(0) = e_f(a) + 1 = e_f(b) = e_f(c) + 1$
3	$v_f(0) + 1 = v_f(a) = v_f(b) + 1 = v_f(c)$	$e_f(0) = e_f(a) + 1 = e_f(b) = e_f(c) + 1$

Table 1

Illustration 2.2. The crown $C_5 \odot K_1$ and its V_4 -cordial labeling is shown in Figure 1.

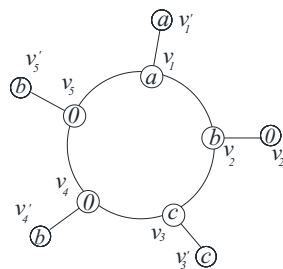


Figure 1 V_4 -cordial labeling of crown $C_5 \odot K_1$.

Theorem 2.3. The Armed Crown AC_n is V_4 -cordial for all n .

Proof. Let $G = AC_n$ be the armed crown. Let $v_1, v_2, \dots, v_n$ be the vertices of cycle C_n . Let v_i, v'_i and v''_i be the vertices of path where $1 \leq i \leq n$. Here, each v'_i is adjacent to both v_i and v''_i for all i . We note that $|V(G)| = 3n$ and $|E(G)| = 3n$.

To define V_4 -cordial labeling $f : V(G) \rightarrow V_4$ we consider the following cases.

Case 1: $n \equiv 0(mod 4)$

$$\begin{aligned} f(v_i) &= 0; & i &\equiv 0(mod 4); \\ f(v_i) &= a; & i &\equiv 1(mod 4); \\ f(v_i) &= b; & i &\equiv 2(mod 4); \\ f(v_i) &= c; & i &\equiv 3(mod 4); & 1 \leq i \leq n, \\ f(v'_i) &= 0; & i &\equiv 2(mod 4); \\ f(v'_i) &= a; & i &\equiv 1(mod 4); \\ f(v'_i) &= b; & i &\equiv 0(mod 4); \\ f(v'_i) &= c; & i &\equiv 3(mod 4); & 1 \leq i \leq n, \\ f(v''_i) &= 0; & i &\equiv 3(mod 4); \\ f(v''_i) &= a; & i &\equiv 1(mod 4); \\ f(v''_i) &= b; & i &\equiv 2(mod 4); \\ f(v''_i) &= c; & i &\equiv 0(mod 4); & 1 \leq i \leq n. \end{aligned}$$

Case 2: $n \equiv 1(mod 4)$

$$\begin{aligned} f(v_i) &= 0; & i &\equiv 0(mod 4); \\ f(v_i) &= a; & i &\equiv 1(mod 4); \\ f(v_i) &= b; & i &\equiv 2(mod 4); \\ f(v_i) &= c; & i &\equiv 3(mod 4); & 1 \leq i \leq n, \\ f(v'_i) &= 0; & i &\equiv 2(mod 4); \\ f(v'_i) &= a; & i &\equiv 1(mod 4); \\ f(v'_i) &= b; & i &\equiv 0(mod 4); \\ f(v'_i) &= c; & i &\equiv 3(mod 4); & 1 \leq i \leq n-1, \\ f(v'_n) &= b; \\ f(v''_i) &= 0; & i &\equiv 3(mod 4); \\ f(v''_i) &= a; & i &\equiv 1(mod 4); \\ f(v''_i) &= b; & i &\equiv 2(mod 4); \\ f(v''_i) &= c; & i &\equiv 0(mod 4); & 1 \leq i \leq n-1, \\ f(v''_n) &= c. \end{aligned}$$

Case 3: $n \equiv 2(mod 4)$

$$\begin{aligned} f(v_i) &= 0; & i &\equiv 0(mod 4); \\ f(v_i) &= a; & i &\equiv 1(mod 4); \\ f(v_i) &= b; & i &\equiv 2(mod 4); \\ f(v_i) &= c; & i &\equiv 3(mod 4); & 1 \leq i \leq n, \\ f(v'_i) &= 0; & i &\equiv 2(mod 4); \\ f(v'_i) &= a; & i &\equiv 1(mod 4); \\ f(v'_i) &= b; & i &\equiv 0(mod 4); \\ f(v'_i) &= c; & i &\equiv 3(mod 4); & 1 \leq i \leq n-2, \\ f(v'_{n-1}) &= 0; \\ f(v'_n) &= c; \\ f(v''_i) &= 0; & i &\equiv 3(mod 4); \\ f(v''_i) &= a; & i &\equiv 1(mod 4); \\ f(v''_i) &= b; & i &\equiv 2(mod 4); \end{aligned}$$

$$\begin{aligned} f(v_i'') &= c; & i \equiv 0(\text{mod } 4); & \quad 1 \leq i \leq n-2, \\ f(v_{n-1}'') &= b; \\ f(v_n'') &= c. \end{aligned}$$

Case 4: $n \equiv 3(\text{mod } 4)$

$$\begin{aligned} f(v_i) &= 0; & i \equiv 0(\text{mod } 4); \\ f(v_i) &= a; & i \equiv 1(\text{mod } 4); \\ f(v_i) &= b; & i \equiv 2(\text{mod } 4); \\ f(v_i) &= c; & i \equiv 3(\text{mod } 4); & \quad 1 \leq i \leq n, \\ f(v_i') &= 0; & i \equiv 2(\text{mod } 4); \\ f(v_i') &= a; & i \equiv 1(\text{mod } 4); \\ f(v_i') &= b; & i \equiv 0(\text{mod } 4); \\ f(v_i') &= c; & i \equiv 3(\text{mod } 4); & \quad 1 \leq i \leq n, \\ f(v_i'') &= 0; & i \equiv 3(\text{mod } 4); \\ f(v_i'') &= a; & i \equiv 1(\text{mod } 4); \\ f(v_i'') &= b; & i \equiv 2(\text{mod } 4); \\ f(v_i'') &= c; & i \equiv 0(\text{mod } 4); & \quad 1 \leq i \leq n-3, \\ f(v_{n-2}'') &= 0; \\ f(v_{n-1}'') &= c; \\ f(v_n'') &= b. \end{aligned}$$

The Table 2 shows that the labeling pattern defined above covers all possible arrangement of vertices. In each possibility the graph under consideration satisfies the vertex conditions and edge conditions for V_4 -cordial labeling. Hence, the armed crown AC_n is V_4 -cordial for all n . Let $n = 4p + q$, where $p, q \in N \cup \{0\}$.

q	Vertex conditions	Edge conditions
0	$v_f(0) = v_f(a) = v_f(b) = v_f(c)$	$e_f(0) = e_f(a) = e_f(b) = e_f(c)$
1	$v_f(0) + 1 = v_f(a) = v_f(b) = v_f(c)$	$e_f(0) = e_f(a) = e_f(b) + 1 = e_f(c)$
2	$v_f(0) + 1 = v_f(a) + 1 = v_f(b) = v_f(c)$	$e_f(0) + 1 = e_f(a) = e_f(b) + 1 = e_f(c)$
3	$v_f(0) + 1 = v_f(a) + 1 = v_f(b) + 1 = v_f(c)$	$e_f(0) + 1 = e_f(a) = e_f(b) + 1 = e_f(c) + 1$

Table 2

Illustration 2.4 The armed crown AC_5 and its V_4 -cordial labeling is shown in Figure 2.

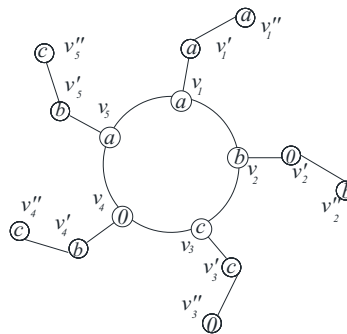


Figure 2 V_4 -cordial labeling of an armed crown AC_5 .

Theorem 2.5. The Pan Graph C_n^{+1} is V_4 -cordial for all n .

Proof. Let $G = C_n^{+1}$ be the cycle with one pendant edge known as Pan graph. Let $v_1, v_2, \dots, v_n$ be the cycle vertices and v' be the pendant vertex. We note that $|V(G)| = n + 1$ and $|E(G)| = n + 1$.

To define V_4 -cordial labeling $f : V(G) \rightarrow V_4$ we consider the following cases.

Case 1: $n \equiv 0(mod 8)$.

$$\begin{aligned} f(v_i) &= 0; & i &\equiv 0, 7(mod 8); \\ f(v_i) &= a; & i &\equiv 1, 5(mod 8); \\ f(v_i) &= b; & i &\equiv 4, 6(mod 8); \\ f(v_i) &= c; & i &\equiv 2, 3(mod 8); & 1 \leq i \leq n, \\ f(v') &= a. \end{aligned}$$

Case 2: $n \equiv 1(mod 8)$.

$$\begin{aligned} f(v_i) &= 0; & i &\equiv 0, 7(mod 8); \\ f(v_i) &= a; & i &\equiv 1, 5(mod 8); \\ f(v_i) &= b; & i &\equiv 4, 6(mod 8); \\ f(v_i) &= c; & i &\equiv 2, 3(mod 8); & 1 \leq i \leq n, \\ f(v') &= c. \end{aligned}$$

Case 3: $n \equiv 2(mod 8)$.

$$\begin{aligned} f(v_i) &= 0; & i &\equiv 0, 7(mod 8); \\ f(v_i) &= a; & i &\equiv 1, 5(mod 8); \\ f(v_i) &= b; & i &\equiv 4, 6(mod 8); \\ f(v_i) &= c; & i &\equiv 2, 3(mod 8); & 1 \leq i \leq n - 3, \\ f(v_{n-2}) &= a; \\ f(v_{n-1}) &= b; \\ f(v_n) &= 0; \\ f(v') &= 0. \end{aligned}$$

Case 4: $n \equiv 3(mod 8)$.

$$\begin{aligned} f(v_i) &= 0; & i &\equiv 0, 7(mod 8); \\ f(v_i) &= a; & i &\equiv 1, 5(mod 8); \\ f(v_i) &= b; & i &\equiv 4, 6(mod 8); \\ f(v_i) &= c; & i &\equiv 2, 3(mod 8); & 1 \leq i \leq n - 5, \\ f(v_{n-4}) &= a; \\ f(v_{n-3}) &= c; \\ f(v_{n-2}) &= b; \\ f(v_{n-1}) &= 0; \\ f(v_n) &= 0; \\ f(v') &= 0. \end{aligned}$$

Case 5: $n \equiv 4, 5, 7(mod 8)$.

$$\begin{aligned} f(v_i) &= 0; & i &\equiv 0, 7(mod 8); \\ f(v_i) &= a; & i &\equiv 1, 5(mod 8); \\ f(v_i) &= b; & i &\equiv 4, 6(mod 8); \\ f(v_i) &= c; & i &\equiv 2, 3(mod 8); & 1 \leq i \leq n, \\ f(v') &= 0. \end{aligned}$$

Case 6: $n \equiv 6(mod 8)$.

$$f(v_i) = 0; \quad i \equiv 0, 7(mod 8);$$

$$\begin{aligned}
 f(v_i) &= a; & i \equiv 1, 5(\text{mod } 8); \\
 f(v_i) &= b; & i \equiv 4, 6(\text{mod } 8); \\
 f(v_i) &= c; & i \equiv 2, 3(\text{mod } 8); \quad 1 \leq i \leq n-3, \\
 f(v_{n-2}) &= 0; \\
 f(v_{n-1}) &= b; \\
 f(v_n) &= a; \\
 f(v') &= 0.
 \end{aligned}$$

The Table 3 shows that the labeling pattern defined above covers all possible arrangement of vertices. In each possibility the graph under consideration satisfies the vertex conditions and edge conditions for V_4 -cordial labeling. Hence, the pan graph C_n^{+1} is V_4 -cordial for all n . Let $n = 8p + q$, where $p, q \in N \cup \{0\}$.

q	Vertex conditions	Edge conditions
0	$v_f(0) + 1 = v_f(a) = v_f(b) + 1 = v_f(c) + 1$	$e_f(0) + 1 = e_f(a) = e_f(b) + 1 = e_f(c) + 1$
1	$v_f(0) + 1 = v_f(a) = v_f(b) + 1 = v_f(c)$	$e_f(0) = e_f(a) + 1 = e_f(b) = e_f(c) + 1$
2	$v_f(0) = v_f(a) = v_f(b) = v_f(c) + 1$	$e_f(0) + 1 = e_f(a) = e_f(b) = e_f(c)$
3,7	$v_f(0) = v_f(a) = v_f(b) = v_f(c)$	$e_f(0) = e_f(a) = e_f(b) = e_f(c)$
4	$v_f(0) + 1 = v_f(a) + 1 = v_f(b) + 1 = v_f(c)$	$e_f(0) + 1 = e_f(a) + 1 = e_f(b) = e_f(c) + 1$
5	$v_f(0) + 1 = v_f(a) = v_f(b) + 1 = v_f(c)$	$e_f(0) = e_f(a) = e_f(b) + 1 = e_f(c) + 1$
6	$v_f(0) = v_f(a) = v_f(b) + 1 = v_f(c)$	$e_f(0) = e_f(a) + 1 = e_f(b) = e_f(c)$

Table 3

Illustration 2.6 The pan graph C_5^{+1} and its V_4 -cordial labeling is shown in Figure 3.

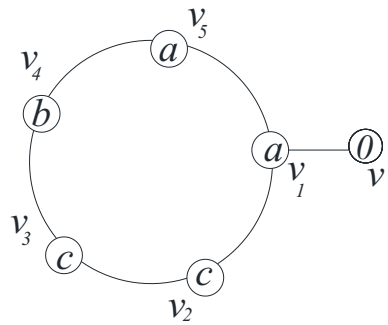


Figure 3 V_4 -cordial labeling of Pan graph C_5^{+1} .

Theorem 2.7. The Corona $C_n \odot mK_1$ of cycle C_n and m complete graph K_1 is V_4 -cordial for all n and m .

Proof. Let $G = C_n \odot mK_1$ be the corona of cycle C_n and m complete graph K_1 . Let $v_1, v_2, \dots, v_n$ be the cycle vertices and $v_{11}, v_{12}, \dots, v_{1m}, v_{21}, v_{22}, \dots, v_{2m}, v_{31}, v_{32}, \dots, v_{3m}, \dots,$

$v_{i1}, v_{i2}, \dots, v_{ij}, \dots, v_{im}, \dots, v_{n1}, v_{n2}, \dots, v_{nj}, \dots, v_{nm}$

be the vertices of m complete graph K_1 , where $1 \leq i \leq n$ and $1 \leq j \leq m$. We note that $|V(G)| = (m+1)n$ and $|E(G)| = (m+1)n$.

To define V_4 -cordial labeling $f : V(G) \rightarrow V_4$ we consider the following cases.

Case 1: $n \equiv 0(mod 4)$.

Label the vertices $v_1, v_2, \dots, v_n$ of cycle as follows.

$$\begin{aligned} f(v_i) &= 0; & i &\equiv 0(mod 4); \\ f(v_i) &= a; & i &\equiv 1(mod 4); \\ f(v_i) &= b; & i &\equiv 2(mod 4); \\ f(v_i) &= c; & i &\equiv 3(mod 4); \quad 1 \leq i \leq n. \end{aligned}$$

The labeling pattern of the vertices $v_{1j}, v_{2j}, \dots, v_{nj}$, where $1 \leq j \leq m$ is divided into following two subcases.

Subcase 1: $m = 1$.

$$\begin{aligned} f(v_{i1}) &= 0; & i &\equiv 2(mod 4); \\ f(v_{i1}) &= a; & i &\equiv 1(mod 4); \\ f(v_{i1}) &= b; & i &\equiv 0(mod 4); \\ f(v_{i1}) &= c; & i &\equiv 3(mod 4); \quad 1 \leq i \leq n. \end{aligned}$$

Subcase 2: $m > 1$.

$$\begin{aligned} f(v_{ij}) &= 0; & i &\equiv 2(mod 4); \\ f(v_{ij}) &= a; & i &\equiv 1(mod 4); \\ f(v_{ij}) &= b; & i &\equiv 3(mod 4); \\ f(v_{ij}) &= c; & i &\equiv 0(mod 4); \quad 1 \leq i \leq n \text{ and } 2 \leq j \leq m. \end{aligned}$$

Case 2: $n \equiv 1(mod 4)$.

Label the vertices $v_1, v_2, \dots, v_n$ of cycle as follows.

$$\begin{aligned} f(v_i) &= 0; & i &\equiv 0(mod 4); \\ f(v_i) &= a; & i &\equiv 1(mod 4); \\ f(v_i) &= b; & i &\equiv 2(mod 4); \\ f(v_i) &= c; & i &\equiv 3(mod 4); \quad 1 \leq i \leq n-1, \\ f(v_n) &= 0. \end{aligned}$$

The labeling pattern of the vertices $v_{1j}, v_{2j}, \dots, v_{nj}$, where $1 \leq j \leq m$ is divided into following subcases.

Subcase 1: $m = 1$.

$$\begin{aligned} f(v_{i1}) &= 0; & i &\equiv 2(mod 4); \\ f(v_{i1}) &= a; & i &\equiv 1(mod 4); \\ f(v_{i1}) &= b; & i &\equiv 0(mod 4); \\ f(v_{i1}) &= c; & i &\equiv 3(mod 4); \quad 1 \leq i \leq n-1, \\ f(v_{n1}) &= a. \end{aligned}$$

Subcase 2: $m \equiv 0(mod 4)$.

$$\begin{aligned} f(v_{ij}) &= 0; & i &\equiv 2(mod 4); \\ f(v_{ij}) &= a; & i &\equiv 1(mod 4); \\ f(v_{ij}) &= b; & i &\equiv 3(mod 4); \\ f(v_{ij}) &= c; & i &\equiv 0(mod 4); \quad 1 \leq i \leq n-1 \text{ and } 2 \leq j \leq m, \end{aligned}$$

$$f(v_{nj}) = 0.$$

Subcase 3: $m \equiv 1(mod4)$.

$$\begin{aligned} f(v_{ij}) &= 0; & i &\equiv 2(mod4); \\ f(v_{ij}) &= a; & i &\equiv 1(mod4); \\ f(v_{ij}) &= b; & i &\equiv 3(mod4); \\ f(v_{ij}) &= c; & i &\equiv 0(mod4); & 1 \leq i \leq n-1 \text{ and } 2 \leq j \leq m, \\ f(v_{nj}) &= a. \end{aligned}$$

Subcase 4: $m \equiv 2(mod4)$.

$$\begin{aligned} f(v_{ij}) &= 0; & i &\equiv 2(mod4); \\ f(v_{ij}) &= a; & i &\equiv 1(mod4); \\ f(v_{ij}) &= b; & i &\equiv 3(mod4); \\ f(v_{ij}) &= c; & i &\equiv 0(mod4); & 1 \leq i \leq n-1 \text{ and } 2 \leq j \leq m, \\ f(v_{nj}) &= b. \end{aligned}$$

Subcase 5: $m \equiv 3(mod4)$.

$$\begin{aligned} f(v_{ij}) &= 0; & i &\equiv 2(mod4); \\ f(v_{ij}) &= a; & i &\equiv 1(mod4); \\ f(v_{ij}) &= b; & i &\equiv 3(mod4); \\ f(v_{ij}) &= c; & i &\equiv 0(mod4); & 1 \leq i \leq n-1 \text{ and } 2 \leq j \leq m, \\ f(v_{nj}) &= c. \end{aligned}$$

Case 3: $n \equiv 2(mod4)$.

Label the vertices $v_1, v_2, \dots, v_n$ of cycle as follows.

$$\begin{aligned} f(v_i) &= 0; & i &\equiv 0(mod4); \\ f(v_i) &= a; & i &\equiv 1(mod4); \\ f(v_i) &= b; & i &\equiv 2(mod4); \\ f(v_i) &= c; & i &\equiv 3(mod4); & 1 \leq i \leq n-2, \\ f(v_{n-1}) &= 0; \\ f(v_n) &= c. \end{aligned}$$

The labeling pattern of the vertices $v_{1j}, v_{2j}, \dots, v_{nj}$, where $1 \leq j \leq m$ is divided into following subcases.

Subcase 1: $m = 1$.

$$\begin{aligned} f(v_{i1}) &= 0; & i &\equiv 2(mod4); \\ f(v_{i1}) &= a; & i &\equiv 1(mod4); \\ f(v_{i1}) &= b; & i &\equiv 0(mod4); \\ f(v_{i1}) &= c; & i &\equiv 3(mod4); & 1 \leq i \leq n-1, \\ f(v_{n1}) &= b. \end{aligned}$$

Subcase 2: $m \equiv 0(mod2)$.

$$\begin{aligned} f(v_{ij}) &= 0; & i &\equiv 2(mod4); \\ f(v_{ij}) &= a; & i &\equiv 1(mod4); \\ f(v_{ij}) &= b; & i &\equiv 3(mod4); \\ f(v_{ij}) &= c; & i &\equiv 0(mod4); & 1 \leq i \leq n \text{ and } 2 \leq j \leq m. \end{aligned}$$

Subcase 3: $m \equiv 1(mod2)$.

$$\begin{aligned} f(v_{ij}) &= 0; & i &\equiv 2(mod4); \\ f(v_{ij}) &= a; & i &\equiv 1(mod4); \\ f(v_{ij}) &= b; & i &\equiv 3(mod4); \\ f(v_{ij}) &= c; & i &\equiv 0(mod4); & 1 \leq i \leq n-2 \text{ and } 2 \leq j \leq m, \\ f(v_{(n-1)j}) &= b; \\ f(v_{nj}) &= c. \end{aligned}$$

Case 4: $n \equiv 3(mod4)$.

Label the vertices $v_1, v_2, \dots, v_n$ of cycle as follows.

$$\begin{aligned} f(v_i) &= 0; & i &\equiv 0(mod4); \\ f(v_i) &= a; & i &\equiv 1(mod4); \\ f(v_i) &= b; & i &\equiv 2(mod4); \\ f(v_i) &= c; & i &\equiv 3(mod4); & 1 \leq i \leq n. \end{aligned}$$

The labeling pattern of the vertices $v_{1j}, v_{2j}, \dots, v_{nj}$, where $1 \leq j \leq m$ is divided into following subcases.

Subcase 1: $m = 1$.

$$\begin{aligned} f(v_{ij}) &= 0; & i &\equiv 2(mod4); \\ f(v_{ij}) &= a; & i &\equiv 1(mod4); \\ f(v_{ij}) &= b; & i &\equiv 0(mod4); \\ f(v_{ij}) &= c; & i &\equiv 3(mod4); & 1 \leq i \leq n. \end{aligned}$$

Subcase 2: $m \equiv 0(mod8)$.

$$\begin{aligned} f(v_{ij}) &= 0; & i &\equiv 2(mod4); \\ f(v_{ij}) &= a; & i &\equiv 1(mod4); \\ f(v_{ij}) &= b; & i &\equiv 3(mod4); \\ f(v_{ij}) &= c; & i &\equiv 0(mod4); & 1 \leq i \leq n-3 \text{ and } 2 \leq j \leq m, \\ f(v_{(n-2)j}) &= b; \\ f(v_{(n-1)j}) &= c; \\ f(v_{nj}) &= a. \end{aligned}$$

Subcase 3: $m \equiv 1(mod8)$.

$$\begin{aligned} f(v_{ij}) &= 0; & i &\equiv 2(mod4); \\ f(v_{ij}) &= a; & i &\equiv 1(mod4); \\ f(v_{ij}) &= b; & i &\equiv 3(mod4); \\ f(v_{ij}) &= c; & i &\equiv 0(mod4); & 1 \leq i \leq n-1 \text{ and } 2 \leq j \leq m, \\ f(v_{nj}) &= c. \end{aligned}$$

Subcase 4: $m \equiv 2(mod8)$.

$$\begin{aligned} f(v_{ij}) &= 0; & i &\equiv 2(mod4); \\ f(v_{ij}) &= a; & i &\equiv 1(mod4); \\ f(v_{ij}) &= b; & i &\equiv 3(mod4); \\ f(v_{ij}) &= c; & i &\equiv 0(mod4); & 1 \leq i \leq n-3 \text{ and } 2 \leq j \leq m, \\ f(v_{(n-2)j}) &= 0; \\ f(v_{(n-1)j}) &= a; \\ f(v_{nj}) &= b. \end{aligned}$$

Subcase 5: $m \equiv 3(mod8)$.

$$\begin{aligned}
 f(v_{ij}) &= 0; & i &\equiv 2(\text{mod}4); \\
 f(v_{ij}) &= a; & i &\equiv 1(\text{mod}4); \\
 f(v_{ij}) &= b; & i &\equiv 3(\text{mod}4); \\
 f(v_{ij}) &= c; & i &\equiv 0(\text{mod}4); & 1 \leq i \leq n-3 \text{ and } 2 \leq j \leq m, \\
 f(v_{(n-2)j}) &= b; \\
 f(v_{(n-1)j}) &= 0; \\
 f(v_{nj}) &= c.
 \end{aligned}$$

Subcase 6: $m \equiv 4(\text{mod}8)$.

$$\begin{aligned}
 f(v_{ij}) &= 0; & i &\equiv 2(\text{mod}4); \\
 f(v_{ij}) &= a; & i &\equiv 1(\text{mod}4); \\
 f(v_{ij}) &= b; & i &\equiv 3(\text{mod}4); \\
 f(v_{ij}) &= c; & i &\equiv 0(\text{mod}4); & 1 \leq i \leq n-3 \text{ and } 2 \leq j \leq m, \\
 f(v_{(n-2)j}) &= a; \\
 f(v_{(n-1)j}) &= c; \\
 f(v_{nj}) &= 0.
 \end{aligned}$$

Subcase 7: $m \equiv 5(\text{mod}8)$.

$$\begin{aligned}
 f(v_{ij}) &= 0; & i &\equiv 2(\text{mod}4); \\
 f(v_{ij}) &= a; & i &\equiv 1(\text{mod}4); \\
 f(v_{ij}) &= b; & i &\equiv 3(\text{mod}4); \\
 f(v_{ij}) &= c; & i &\equiv 0(\text{mod}4); & 1 \leq i \leq n-3 \text{ and } 2 \leq j \leq m, \\
 f(v_{(n-2)j}) &= b; \\
 f(v_{(n-1)j}) &= 0; \\
 f(v_{nj}) &= c.
 \end{aligned}$$

Subcase 8: $m \equiv 6(\text{mod}8)$.

$$\begin{aligned}
 f(v_{ij}) &= 0; & i &\equiv 2(\text{mod}4); \\
 f(v_{ij}) &= a; & i &\equiv 1(\text{mod}4); \\
 f(v_{ij}) &= b; & i &\equiv 3(\text{mod}4); \\
 f(v_{ij}) &= c; & i &\equiv 0(\text{mod}4); & 1 \leq i \leq n-3 \text{ and } 2 \leq j \leq m, \\
 f(v_{(n-2)j}) &= c; \\
 f(v_{(n-1)j}) &= a; \\
 f(v_{nj}) &= b.
 \end{aligned}$$

Subcase 9: $m \equiv 7(\text{mod}8)$.

$$\begin{aligned}
 f(v_{ij}) &= 0; & i &\equiv 2(\text{mod}4); \\
 f(v_{ij}) &= a; & i &\equiv 1(\text{mod}4); \\
 f(v_{ij}) &= b; & i &\equiv 3(\text{mod}4); \\
 f(v_{ij}) &= c; & i &\equiv 0(\text{mod}4); & 1 \leq i \leq n-3 \text{ and } 2 \leq j \leq m, \\
 f(v_{(n-2)j}) &= 0; \\
 f(v_{(n-1)j}) &= b; \\
 f(v_{nj}) &= a.
 \end{aligned}$$

The *Table 4* shows that the labeling pattern defined above covers all possible arrangement of vertices. In each possibility the graph under consideration satisfies the vertex conditions and edge conditions for V_4 -cordial labeling. Hence, the corona $C_n \odot mK_1$ is V_4 -cordial for all n and m .

Let $n = 4p + q$ and $(j = 1, 2, 3, \dots, m)$, where $m, p, q \in \mathbb{N} \cup \{0\}$.

q	m	Vertex conditions	Edge conditions
0	$m \equiv 0, 1(\text{mod}2)$	$v_f(0) = v_f(a)$ $v_f(b) = v_f(c)$	$e_f(b) = e_f(c)$ $e_f(b) = e_f(c)$
1	$m \equiv 0(\text{mod}4)$	$v_f(0) = v_f(a) + 1$ $v_f(b) + 1 = v_f(c) + 1$	$e_f(0) = e_f(a) + 1$ $e_f(b) + 1 = e_f(c) + 1$
	$m \equiv 1(\text{mod}4)$	$v_f(0) = v_f(a)$ $v_f(b) + 1 = v_f(c) + 1$	$e_f(0) = e_f(a)$ $e_f(b) + 1 = e_f(c) + 1$
	$m \equiv 2(\text{mod}4)$	$v_f(0) = v_f(a)$ $v_f(b) = v_f(c) + 1$	$e_f(0) = e_f(a)$ $e_f(b) = e_f(c) + 1$
	$m \equiv 3(\text{mod}4)$	$v_f(0) = v_f(a)$ $v_f(b) = v_f(c)$	$e_f(0) = e_f(a)$ $e_f(b) = e_f(c)$
2	$m \equiv 0(\text{mod}2)$	$v_f(0) = v_f(a)$ $v_f(b) + 1 = v_f(c) + 1$	$e_f(0) + 1 = e_f(a)$ $e_f(b) + 1 = e_f(c)$
	$m \equiv 1(\text{mod}2)$	$v_f(0) = v_f(a)$ $v_f(b) = v_f(c)$	$e_f(0) = e_f(a)$ $e_f(b) = e_f(c)$
3	$m \equiv 0(\text{mod}8)$	$v_f(0) + 1 = v_f(a)$ $v_f(b) = v_f(c)$	$e_f(0) + 1 = e_f(a)$ $e_f(b) = e_f(c)$
	$m \equiv 1(\text{mod}8)$	$v_f(0) + 1 = v_f(a)$ $v_f(b) + 1 = v_f(c)$	$e_f(0) = e_f(a) + 1$ $e_f(b) = e_f(c) + 1$
	$m \equiv 2(\text{mod}8)$	$v_f(0) + 1 = v_f(a)$ $v_f(b) + 1 = v_f(c) + 1$	$e_f(0) + 1 = e_f(a)$ $e_f(b) + 1 = e_f(c) + 1$
	$m \equiv 3, 7(\text{mod}8)$	$v_f(0) = v_f(a)$ $v_f(b) = v_f(c)$	$e_f(0) = e_f(a)$ $e_f(b) = e_f(c)$
	$m \equiv 4(\text{mod}8)$	$v_f(0) = v_f(a)$ $v_f(b) + 1 = v_f(c)$	$e_f(0) = e_f(a)$ $e_f(b) + 1 = e_f(c)$
	$m \equiv 5(\text{mod}8)$	$v_f(0) = v_f(a) + 1$ $v_f(b) + 1 = v_f(c)$	$e_f(0) = e_f(a) + 1$ $e_f(b) + 1 = e_f(c)$
	$m \equiv 6(\text{mod}8)$	$v_f(0) + 1 = v_f(a) + 1$ $v_f(b) + 1 = v_f(c)$	$e_f(0) + 1 = e_f(a) + 1$ $e_f(b) + 1 = e_f(c)$

Table 4

Illustration 2.8 The corona graph $C_4 \odot 5K_1$ and its V_4 -cordial labeling is shown in Figure 4.

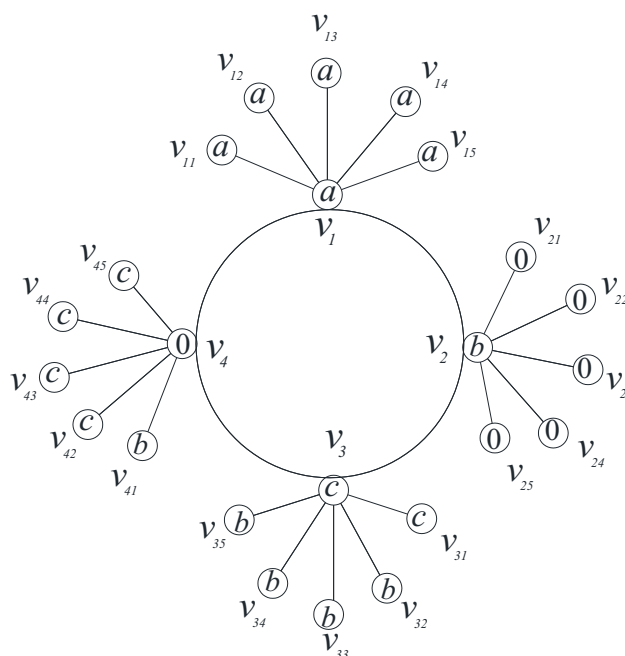


Figure 4 V_4 -cordial labeling of corona $C_4 \odot 5K_1$.

Concluding Remarks

From the above study it can be concluded that a Graph can be labeled by either of the two techniques (4-cordial and V_4 -cordial) in some cases may be by both of them and in certain cases may be by neither of them. Thus 4-cordial and V_4 -cordial labeling techniques do not show any inter-dependence and can be considered to be two distinct graph labeling techniques. Following are illustrative examples of the mentioned facts.

1. The Path P_4 is 4-cordial proved by [M. Hovey (1991)] but not V_4 -cordial proved by [M. Seenivasan and A. Lourdasamy (2010)].
2. The Star $K_{1,n}$ are 4-cordial proved by [M. Hovey (1991)] and V_4 -cordial proved by [A. Riskin (2013)].
3. The Cycle C_4 is neither 4-cordial proved by [M. Hovey (1991)] nor V_4 -cordial proved by [M. Seenivasan and A. Lourdasamy (2010)].

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