

On k-cordial Labeling of Some Graphs

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Abstract

In this paper we present k -cordiality of one point union of some path, cycle and star related graphs. We prove that bistar graph $B_{m,n}$ is k -cordial for all k , restricted square graph $B_{n,n}^2$ of Bistar is k -cordial for all k . We also prove that one point union of cycle C_3 with star graph $K_{1,n}$ and one point union of path P_n with K_1 are k -cordial for all k .

Keywords: Abelian Group; k -cordial Labeling; Bistar, Comb, One Point Union.

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1 Introduction

Graph Theory is one of the Expanding Branches of Discrete Mathematics with applications in almost all the disciplines of Science and Technology. There are many potential fields of research in Graph Theory. Some of them are Enumeration of Graphs, Domination in Graphs, Algorithmic Graph Theory, Topological Graph Theory, Fuzzy Graph Theory, Labeling of Graph etc. Graph Labeling is an Integral part of Graph Theory which assigns numeral values to the vertices or edges or both, subject to certain conditions. The concept of Graph Labeling was introduced by A. Rosa in 1967. The present authors are motivated by the Research Article "A-cordial graphs" by Hovey [Hovey (1991)].

Throughout this work, by a graph we mean finite, connected, undirected, simple graph $G = (V(G), E(G))$ of order $|V(G)|$ and size $|E(G)|$.

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Definition 1.1. A *graph labeling* is an assignment of integers to the vertices or edges or both subject to certain condition(s).

The latest updates of various graph labeling techniques can be found in Gallian[Gallian (2014)].

Definition 1.2. Let $\langle A, * \rangle$ be any Abelian group. A graph $G = (V(G), E(G))$ is said to be *A-cordial* if there is a mapping $f : V(G) \rightarrow A$ which satisfies the following two conditions when the edge $e = uv$ is labeled as $f(u) * f(v)$

(i) $|v_f(a) - v_f(b)| \leq 1$; for all $a, b \in A$,

(ii) $|e_f(a) - e_f(b)| \leq 1$; for all $a, b \in A$.

Where

$v_f(a)$ =the number of vertices with label a ;

$v_f(b)$ =the number of vertices with label b ;

$e_f(a)$ =the number of edges with label a ;

$e_f(b)$ =the number of edges with label b .

We note that if $A = \langle \mathbb{Z}_k, +_k \rangle$, that is additive group of modulo k then the labeling is known as *k-cordial labeling*.

The concept of *A-cordial labeling* was introduced by Hovey [Hovey (1991)].

Definition 1.3. If G is graph of order n , the corona of G with another graph H , $G \odot H$ is the graph obtained by taking one copy of G and n copies of H and joining the i^{th} vertex of G with an edge to every vertex in the i^{th} copy of H .

Definition 1.4. The *Bistar* $B_{m,n}$ is the graph obtained by joining the center(apex) vertices of $K_{1,m}$ and $K_{1,n}$ by an edge.

Definition 1.5. Let G be a simple connected graph. The *square of graph* G denoted by G^2 is defined to be the graph with the same vertex set as G and in which two vertices u and v are joined by an edge $\Leftrightarrow$ in G we have $1 \leq d(u, v) \leq 2$.

Definition 1.6. The graph obtained by joining a pendent edge at each vertex of a path P_n is called a *Comb* and is denoted by $P_n \odot K_1$.

Definition 1.7. The graph obtained by joining n pendent edge at one vertex of the cycle C_3 is denoted by $C_3 \odot K_{1,n}$.

A graph may be labeled by both the labeling techniques cordial labeling as well as *k-cordial labeling* at a time, may be labeled by any one of them, or may not at all be labeled by any of them. In other words these two graph labeling techniques are independent. Following are some examples to emphasize the fact.

- Fans are cordial[Cahit (1990)] as well as *k-cordial* for all k [Modha and Kanani (2015)].
- Wheels W_n are cordial if and only if $n \not\equiv 3(mod 4)$ [Cahit (1990)] where as wheels are *k-cordial* for all odd k if and only if $n \not\equiv \frac{k-1}{2}(mod k)$.
- Cycles C_n are cordial unless C_{4n+2} [Ho, Lee and Shee (1989)] while cycles C_n are *k-cordial* for all odd k [Hovey (1991)].

2 Main Results

Theorem 2.1. The Bistar $B_{m,n}$ is k -cordial graph for all k .

Proof. Let $G = B_{m,n}$ be the bistar with vertex set $\{u; v; u_i; v_j; 1 \leq i \leq m, 1 \leq j \leq n\}$ where u_i, v_j are pendant vertices. We note that $|V(G)| = m + n + 2$ and $|E(G)| = m + n + 1$.

Define k -cordial labeling $f : V(G) \rightarrow Z_k$ as follows.

$$\begin{aligned} f(u) &= k - 1; \\ f(v) &= 1; \\ f(u_i) &= p_i; \quad (2i + 1) \equiv p_i \pmod{k}; \quad 1 \leq i \leq m. \\ f(v_j) &= p_j; \quad 2(j - 1) \equiv p_j \pmod{k}; \quad 1 \leq j \leq n. \end{aligned}$$

The labeling pattern defined above covers all possible arrangement of vertices. In all possibilities the graph under consideration satisfies the vertex conditions and edge conditions for k -cordial labeling. Hence the Bistar $B_{m,n}$ is k -cordial for all k .

Illustration 2.2. The Bistar graph $B_{4,6}$ and its 15-cordial labeling is shown in Figure 1.

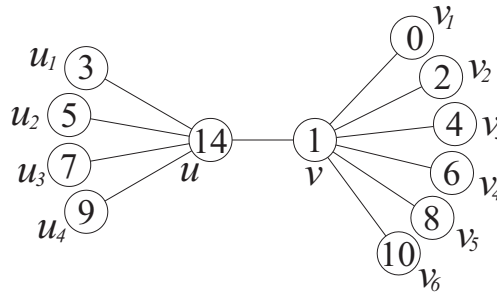


Figure 1 15-Cordial labeling of bistar graph $B_{4,6}$.

Theorem 2.3. The restricted square graph $B_{n,n}^2$ of Bistar $B_{n,n}$ is k -cordial for all k .

Proof. Let $B_{n,n}$ be the bistar with vertex set $\{u; v; u_i; v_i; 1 \leq i \leq n\}$, where u_i, v_i are pendant vertices. Let G be the square graph $B_{n,n}^2$ with $V(G) = V(B_{n,n})$ and $E(G) = \{E(B_{n,n}) \cup uv_i, vu_i; 1 \leq i \leq n\}$. We note that $|V(G)| = 2n + 2$ and $|E(G)| = 4n + 1$.

To define k -cordial labeling $f : V(G) \rightarrow Z_k$ we consider the following two cases.

Case 1: k is odd.

$$\begin{aligned} f(u) &= 0; \\ f(v) &= k - 1; \\ f(u_i) &= 4i - 1; \quad 1 \leq i \leq \lfloor \frac{k-1}{4} \rfloor. \\ &= p_i; \quad (4i + 1) \equiv p_i \pmod{k}; \quad \lfloor \frac{k-1}{4} \rfloor + 1 \leq i \leq n. \\ f(v_i) &= 4i - 3; \quad 1 \leq i \leq \lceil \frac{k-1}{4} \rceil. \\ &= p_i; \quad (4i - 1) \equiv p_i \pmod{k}; \quad \lceil \frac{k-1}{4} \rceil + 1 \leq i \leq n. \end{aligned}$$

Case 2: k is even.

$$\begin{aligned} f(u) &= 0; \\ f(v) &= k - 1; \end{aligned}$$

$$f(u_i) = p_i; \quad (2i - 1) \equiv p_i(\text{mod } k); 1 \leq i \leq n.$$

$$f(v_i) = k - p_i; \quad (2i) \equiv p_i(\text{mod } k); \text{ if } p_i \neq 0, \\ = 0; \quad (2i) \equiv p_i(\text{mod } k), \text{ if } p_i = 0, 1 \leq i \leq n.$$

The labeling pattern defined above covers all possible arrangement of vertices. In all possibilities the graph under consideration satisfies the vertex conditions and edge conditions for k -cordial labeling. Hence $B_{n,n}^2$ are k -cordial for all k .

Illustration 2.4 The square graph $B_{5,5}^2$ and its 15-cordial labeling is shown in Figure 2.

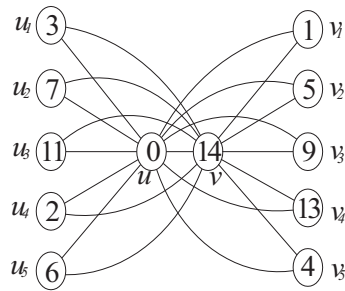


Figure 2 15-Cordial labeling of square graph $B_{5,5}^2$.

Theorem 2.5. The Graph $C_3 \odot K_{1,n}$ is k -cordial for all k .

Proof. Let G be the graph obtained by one point union of cycle C_3 and star $K_{1,n}$. Let $\{u, v, w\}$ be the vertices of the cycle C_3 and $\{v_1, v_2, \dots, v_n\}$ be the vertices of n pendent edges. We note that $|V(G)| = |E(G)| = n + 3$.

To define k -cordial labeling $f : V(G) \rightarrow Z_k$ we consider the following two cases.

Case 1: k is odd.

$$f(u) = \frac{k-1}{2}; \\ f(v) = 0; \\ f(w) = \frac{k+1}{2}; \\ f(v_i) = i; \quad 1 \leq i \leq \frac{k-3}{2}$$

$$f(v_i) = p_i; \quad (i + 2) \equiv p_i(\text{mod } k); \quad \frac{k-1}{2} \leq i \leq n.$$

Case 2: k is even.

$$f(u) = \frac{k-2}{2}; \\ f(v) = 0; \\ f(w) = \frac{k+2}{2}; \\ f(v_i) = i; \quad 1 \leq i \leq \frac{k-4}{2}$$

$$f(v_i) = i + 1; \quad i = \frac{k-2}{2}$$

$$f(v_i) = p_i; \quad (i + 2) \equiv p_i(\text{mod } k); \quad \frac{k}{2} \leq i \leq n.$$

The labeling pattern defined above covers all possible arrangement of vertices. In all possibilities the graph under consideration satisfies the vertex conditions and edge conditions for k -cordial labeling. Hence the Graph $C_3 \odot K_{1,n}$ is k -cordial for all odd k .

Illustration 2.6 The one point union of C_3 and $K_{1,8}$ and its 11-cordial labeling is shown in Figure 3.

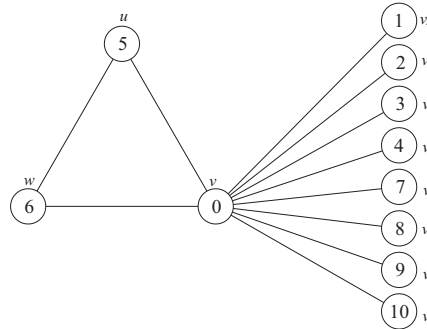


Figure 3 11-Cordial labeling of the one point union of C_3 and $K_{1,8}$ graph.

Theorem 2.7. The Comb Graph $P_n \odot K_1$ is k -cordial for all k .

Proof. Let $G = P_n \odot K_1$ be the comb graph. Let $\{v_1, v_2, v_3, \dots, v_n\}$ be the vertices of the path P_n and $\{u_1, u_2, u_3, \dots, u_n\}$ be the vertices adjacent to each vertex of the path P_n . We note that $|V(G)| = 2n$ and $|E(G)| = 2n - 1$.

To define k -cordial labeling $f : V(G) \rightarrow Z_k$ we consider the following two cases.

Case 1: k is odd.

$$\begin{aligned} f(v_i) &= p_i; & 2(i-1) &\equiv p_i \pmod{k}; i \text{ is odd,} \\ &= p_i; & (2i-1) &\equiv p_i \pmod{k}; i \text{ is even, } 1 \leq i \leq n. \\ f(u_i) &= p_i; & (2i-1) &\equiv p_i \pmod{k}; i \text{ is odd, } i \text{ is odd,} \\ &= p_i; & 2(i-1) &\equiv p_i \pmod{k}; i \text{ is even, } 1 \leq i \leq n. \end{aligned}$$

Case 2: k is even.

Subcase I: $\frac{k}{2}$ is odd.

$$\begin{aligned} f(v_i) &= p_i; & 2(i-1) &\equiv p_i \pmod{k}; 1 \leq i \leq n. \\ f(u_i) &= p_i; & (2i-1) &\equiv p_i \pmod{k}; 1 \leq i \leq n. \end{aligned}$$

Subcase II: $\frac{k}{2}$ is even.

$$\begin{aligned} f(v_i) &= p_i; & (i-1) &\equiv p_i \pmod{k}; 1 \leq i \leq n. \\ f(u_i) &= p_i; & (\frac{k}{2} + (i-i)) &\equiv p_i \pmod{k}; 1 \leq i \leq n \end{aligned}$$

The labeling pattern defined above covers all possible arrangement of vertices. In all possibilities the graph under consideration satisfies the vertex conditions and edge conditions for k -cordial labeling. Hence the Comb Graph $P_n \odot K_1$ is k -cordial for all k .

Illustration 2.8 The comb graph $P_{11} \odot K_1$ and its 12-cordial labeling is shown in Figure 4.

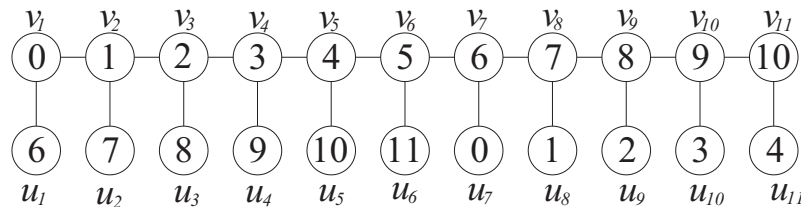


Figure 4 12-Cordial labeling of the comb graph $P_{11} \odot K_1$.

Concluding Remarks

Labeled graph is the topic of current interest for many researchers as it has several diversified applications. We discuss here k -cordial labeling in the context of graph operation namely one point union. This approach is novel and contributes four new graphs to the theory of k -cordial graphs. The derived labeling pattern is demonstrated by means of sufficient illustrations which provide better understanding of the derived results. The results reported here are new and will add new dimension to the theory of k -cordial graphs.

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