

OPTIMAL PORTFOLIO FOR N-RISKY AND N-RISKLESS IRREVERSIBLE INVESTMENTS

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Abstract

When a firm decides to make irreversible investment expenditure, it neglects its right without obligation to invest but wait for new desirable information that might affect investment decision. Studies have shown irreversible investments with the underlying value process driven by stochastic differential equations. However, this study had no consideration for a portfolio of N-risky and N-riskless investments ($N = 2, 3, , T < \infty$) in relation to the set of: Variable Production Costs (VPC), Periodic Loan Repayment (PLR) with penalty for default, Investment Reserve (IR) and Equipment Value (EV) equations. This study was designed to consider a portfolio of N-risky and N-riskless investments including a set of processes describing VPC for the portfolio, interval PLR for the same investments with penalty processes for default in PLR.

A set of instantaneous Expected Revenue (ER) from the risky investments were formulated using Geometric Brownian Motion (GBM) with drift term restricted to zero and volatility strictly positive (Martingale condition). The resulting equation was solved using Ito's formula. The IR and EV equations were formulated with predictive adaptive processes to examine liquidity of business plan and Equipment Depreciation (ED) value respectively. Maintenance Process (MP) was introduced into the EV equation. The equation describing portfolio option value was formulated using Bellman utility function and solved. Risks associated with the risky investments were measured using Arrow-Pratt measure. Linear Dynamic Optimization (LDO) was adopted to maximise the value of the N-risky and N-riskless investments; and also used for Capital Equipment (CE) value equation with MP to examine deprecia-

tion values.

Application of this model to a Nigeria maritime industry showed that the set of ER obtained were fluctuating and met the Martingale condition. The equations obtained for the N-Risky and N-Riskless investments gave values as $VPC \cong N29.2M/month$, $PLR \cong N37.9M/month$, $IR=N234.4M$, $ED \in (0.3, 0.4)$. A set of threshold values for the N-irreversible investments were obtained. The equations obtained from Arrow Pratt yielded 18 to 20 % as investors' attitude towards risk management.

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1 Introduction

In any business venture, the optimal objective for setting aside money or acquired property for future gains is to maximize such gains while minimizing loses. An investor may have an incentive to postpone commitment and wait for new information in order to avoid expensive mistakes. The value of waiting can be more rewarding especially in highly uncertain environments. Recent literature has shown that the option value of waiting can be considerably high in uncertain environment.

In this paper, we consider the real option problem with a portfolio of N- risky irreversible investments and N-riskless investments such that the portfolio is self-financing. This model is applicable to any capital intensive proposition that is irreversible in nature in a market with default risk, friction and the investor is a price taker.

An investment is irreversible if the major equipment used for the production of goods or rendering services can only be used for the purpose for which it was purchased. In the aviation industry, a passenger plane is meant for conveying only human beings but not goods; in maritime industry, an oil

tanker is meant to transport only crude or its product; in a cement factory, cement mill can only be used solely for cement production. In all these, these equipments can be used only for the purposes for which they were purchased but not for other purposes should the proposition turns unfavourable. More detailed information on irreversible investment can found in [5,6] and some of the references therein.

In real world situations, the market is usually incomplete. An investor has to decide which investments to consider, when to consider such investments and how to manage the investments in order to guarantee maximum utility (satisfaction). This satisfaction derived depends on individual investor since the approaches adopted by these investors differs. This paper considers an investment strategy in which a major part of the limited but scare resources is committed to risky assets with high return and the remaining part to riskless assets with low return. This enables the investor to hedge the risk in the market. The expected revenues from the investment are designed to accommodate the noise in the market. The problem considers sets of; loans, interest rates, moratorium periods, and the process for servicing the loans. This enables an investors to manage their limited but scare resources to their upmost advantages. Considering periodic measurements of the value of the capital investment equipments enables investors to balance the cost of replacement with the cost of maintenances .

Most profitable investments fail because of lack of liquidity. Thus, the need for the consideration of the liquidity of the investment proposition, the effect of exchange rate and pattern of expenditures.

2 Mathematical Model

We consider a portfolio of N-risky irreversible investments and N-riskless investments opportunities with total Variable Production Costs given by

(VPC),

$$\mathfrak{K} = \sum_{\Omega=1}^N (k_{\Omega_j,t}) = \sum_{\Omega=1}^N \sum_{n=1}^y X_{\Omega_t,n} \quad (1)$$

for the j th year and t^{th} month, and $X_{t,n}$ is the n th variable cost for the Ω -investment at month t . That is,

- $X_{1_{t,1}}$ is employees' wage at a month t for the $\Omega = 1$ corresponding to the index $n = 1$.
- $X_{2_{t,1}}$ is employees' wage at a month t for the $\Omega = 2$ corresponding to the index $n = 1$.
- $X_{3_{t,1}}$ is employees' wage at a month t for the $\Omega = 3$ corresponding to the index $n = 1$.
- $\vdots$
- $X_{N_{t,1}}$ is employees' wage at a month t for the $\Omega = N$ corresponding to the index $n = 1$.

Thus, $X_{\Omega_{t,1}}$ is a vector of employees' wages at month t for the $\Omega = 1, 2, 3, \dots, N$ investment. Similarly, $X_{\Omega_{t,2}}$ is a vector of the value of light bills at month t for $\Omega = 1, 2, 3, \dots, N$. This continues up till $X_{\Omega_{t,y}}$, a vector of the value of the y^{th} expenditure at month t for $\Omega = 1, 2, 3, \dots, N$ investment. We take $E(X_{\Omega_{t,n}}) < \infty$ for each Ω -irreversible investment. Therefore, (1) is the sum of the vectors of the expenditures with respect to a month t in a year j ($1 \leq j \leq T$).

The Ω -instantaneous expected fluctuated revenue, $p_{\Omega_j,t}$, for the Ω -investment follows a geometric brownian motion given by ;

$$dp_{\Omega_j,t} = \alpha_{\Omega} p_{\Omega_j,t} dt + \sigma_{\Omega} p_{\Omega_j,t} dz_{\Omega_t} \quad (2)$$

$$p_{\Omega_j,t_0} = p_{\Omega} \geq 0$$

where $\sigma_{\Omega} > 0$ is the diffusion term which is assumed to be constant, z_{Ω_t} is a Wiener standard process for Ω -instantaneous revenue generated by the irreversible investment, [3, 4, 6]. Let $p_{\Omega_0} = p_{\Omega} > 0$ denotes the current revenue for each Ω . For $\alpha_{\Omega} = 0$ in equation (2), we obtain a fair value for the expected

fluctuated revenue in terms of today and future value for the generated revenue.

We denote the set of instantaneous Expected Revenue (ER) for Ω -irreversible investment with respect to the filtered probability space, $\mathfrak{G} = (\omega, H, \mu, F(H))$ by $\mathfrak{P}$. i.e.

$$\mathfrak{P} = \{\{p_{\Omega,j,t}\} : p_{\Omega,j,t} \in \mathfrak{G} \text{ for each } j \in (0, \infty), t \in [0, 12] \text{ and for each } \Omega\}$$

Let the set of loan, I_Ω , taken for each Ω -irreversible investment with instantaneous constant riskless interest rate, r_Ω be denoted by L.

i.e $L = \{\mathbf{I}_\Omega : \Omega = 1, \dots, N\}$. Each I_Ω has a moratorium of H_Ω months with life span of T- years. The life span of the loan in months denoted by $\hat{m}$ is given by $\hat{m} = 12T$ months. We set $r_\Omega > \alpha$ in order to avoid arbitrage opportunity. The future value of the loan I_Ω for each Ω -irreversible investment denoted by Fv_Ω and given as,

$$Fv_\Omega = \mathbf{I}_\Omega(1 + b_\Omega)^{\hat{m}} \quad (3)$$

where $b_\Omega = \frac{r_\Omega}{12}$. But with moratorium,

$$Fv_\Omega = \mathbf{I}_\Omega(1 + b_\Omega)^{\hat{m} - H_\Omega} = \mathbf{I}_\Omega(1 + b_\Omega)^d \quad (4)$$

where $d = \hat{m} - H_\Omega$. The Total future value of the loans for the N-irreversible investments denoted by $\widehat{Fv}_\Omega$ is given by

$$\widehat{Fv}_\Omega = \sum_{\Omega=1}^N Fv_{\Omega*} \quad (5)$$

The amount repaid monthly with respect to the loan for the Ω irreversible investment denoted by $a_{\Omega,j,t}$ is expressed by,

$$a_{\Omega,j,t} = \frac{Fv_\Omega}{\hat{m} - H_\Omega}, \quad (6)$$

Thus, the amount repaid monthly with respect to the loan at month t in year j for the Ω -Irreversible Investment with penalty for default in payment denoted by A_Ω is given as

$$A_\Omega = (a_{\Omega,j,t})_{T,s} + \epsilon(a_{\Omega,j,t})_{T,s} \quad (7)$$

where $\epsilon > 0$ is a real scalar, measuring default w.r.t. time of payment . In the case where $\epsilon = 0$, it means payment is made within given time frame, and

$$A_{\Omega} = (a_{\Omega_{j,t}}) \quad (8)$$

A_{Ω} is the sum of two matrices $(a_{\Omega_{j,t}})_{T,s}$ and $\epsilon(a_{\Omega_{j,t}})_{T,s}$.

The total amount repaid with respect to the loans for the N-irreversible investments is given by

$$\mathfrak{A} = \sum_{\Omega=1}^N A_{\Omega} \quad (9)$$

If the N-irreversible investments are considered at the current month, (i.e. $t=0$) in the current year, then instantaneous net cashflow from the project is:

$$f_{\Omega_{j,t}} = p_{\Omega_{j,t}} - m_{\Omega_{j,t}} \quad (10)$$

where

$$m_{\Omega_{j,t}} = k_{\Omega_{j,t}} + a_{\Omega_{j,t}} + \epsilon(a_{\Omega_{j,t}}) \quad (11)$$

is the expenditure made at the j th year, t month with respect to the Ω -irreversible investment.

The N-investments net cashflow denoted by $\mathfrak{N}$ is expressed as

$$\mathfrak{N} = \mathfrak{P} - \mathfrak{M} \quad (12)$$

$\mathfrak{P}$ is a collection of the N revenue matrices, each of order $T \times s$. Similarly, $\mathfrak{M}$ is a collection the N expenditure matrices of order $T \times s$

Therefore, $\mathfrak{N}$ is the sum of all the differences between a revenue matrix and its corresponding expenditure matrix and the possible entries of $\mathfrak{N}$ are:

- $\sum_{\Omega=1}^N -\mu_{\Omega}$ if $\sum_{\Omega=1}^N p_{\Omega_{j,t}} < \sum_{\Omega=1}^N m_{\Omega_{j,t}}$
- 0 if $\sum_{\Omega=1}^N p_{\Omega_{j,t}} = \sum_{\Omega=1}^N m_{\Omega_{j,t}}$
- $\sum_{\Omega=1}^N \mu_{\Omega}$ if $\sum_{\Omega=1}^N p_{\Omega_{j,t}} \geq \sum_{\Omega=1}^N m_{\Omega_{j,t}}$

where $\sum_{\Omega=1}^N m_{\Omega_{j,t}} = \sum_{\Omega=1}^N \left(k_{\Omega_{j,t}} + a_{\Omega_{j,t}} + \epsilon(a_{\Omega_{j,t}}) \right)$.

Let $\Psi := \sum_{\Omega=1}^N \mu_{\Omega}$, the range for ncf is $[-\Psi, \infty)$.

2.1 Valuation of N-Risky and N-Riskless Investments Opportunities

To value the N-Risky Investments, we use the von Neumann Morgenstern utility function

$$u_\Omega : [-\Psi, \infty) \longrightarrow \mathbb{R}$$

with the following properties:

- i Additive
- ii Monotone increasing
- iii $u_\Omega \in C^2(-\Psi, \infty)$

The expected utility u_Ω of the ncf obtained from the Ω -irreversible investment is

$$\widehat{\mathbf{V}}_\Omega = E\left(\sum_{j=1}^T \left(\int_{t=0}^{\infty} (u_{\Omega_{j,t}}(f_{j,t}))dt\right)\right) \quad (13)$$

for the j th year and t^{th} month. The expected utility from the Ω -irreversible investment for first (current) year and first (current) month of investment is

$$\mathbf{V}_\Omega = E\left(\frac{1}{(1+r_\Omega)^T} \sum_{j=1}^T \left(\int_{t=0}^{\infty} e^{-r_\Omega t} (u_{\Omega_{j,t}}(f_{j,t}))dt\right)\right) \quad (14)$$

The satisfaction the Investor derives with respect to the t^{th} month in the j^{th} year is given by:

$$u_{\Omega_{j,t}}(p_{j,t}) = E(e^{-r_\Omega t} [(1 - \eta_\Omega)(p_{\Omega_{j,t}} - m_{\Omega_{j,t}}) + \eta_\Omega(1 - (p_{\Omega_{j,t}} - m_{\Omega_{j,t}})^2)]) \quad (15)$$

When $t > 0$, $p_{\Omega_{j,t}}$ is equal to the solution of the GBM, [2].

i.e. $p_{\Omega_{j,t}} := p_0^\Omega e^{(\alpha_\Omega - 0.5\sigma_\Omega^2)\Delta t + \sigma_\Omega N(0,1)\sqrt{\Delta t}}$. The parameter η_Ω satisfy the condition $\eta_\Omega \leq 1$ and η_Ω measures Ω -investor's preference for risk. When $\eta_\Omega = 0$, equation (15) reduces to

$$u_{\Omega_{j,t}}(p_{j,t}) = E(e^{-r_\Omega t} (p_{\Omega_{j,t}} - m_{\Omega_{j,t}})) \quad (16)$$

When $\eta_\Omega = 1$, equation (15) reduces to

$$u_{\Omega,j,t}(p_{j,t}) = E(e^{-r_\Omega t}[1 - (p_{\Omega,j,t} - m_{\Omega,j,t})^2]) \quad (17)$$

Therefore, for all the risky and riskless N-Investments, the expected utility is $\mathfrak{V} = \sum_{\Omega=1}^N u_{\Omega,j,t}(p_{j,t})$. For $t \geq 0$, we observed that the N-Risky investments would be worthwhile since

$$\max\{u_{\Omega,j,t}(p_{j,t}) : \Omega = 1, 2, \dots, N\} > \min\{u_{\Omega,j,t}(p_{j,t}) : \Omega = 1, 2, \dots, N\} \quad (18)$$

will always hold due to the property of u_Ω for each Ω . This is not a sufficient condition to consider the Investments. Let the linear, twice differentiable function with respect to $p_{\Omega,j,t}$, $c_\Omega(p_{j,t})$ denote the option value of postponing the Ω -risky irreversible investment at month $t \geq 0$ with respect to the j^{th} year. Let $u_\Omega(0)$ denote the utility of waiting during a time interval $[0, dt]$ in which no cash flow occur, and $c(p_{\Omega,j,t} + dp_{\Omega,j,t})$ represent future utility of waiting. We use the Bellman equation:

$$c_\Omega(p_{j,t}) = \underbrace{u_\Omega(0)dt}_{\text{No cash flow}} + \underbrace{e^{-r_\Omega dt} E\{c_\Omega(p_{\Omega,j,t} + dp_{\Omega,j,t})\}}_{\text{Discounted expected utility of waiting}} \quad (19)$$

where r is the constant riskless discount factor for each Ω -risky irreversible investment to value the option embedded in the investment

Without loss of generality, $u_\Omega(0) = 0$ since the cash inflow with positive utility equals the cash outflow with negative utility. Using Ito lemma, we re-write RHS of (19) as:

$c_\Omega(p_{j,t}) + [\frac{1}{2}\sigma_\Omega^2 p_\Omega^2 c''(p) + \alpha_\Omega p c'(p) - r_\Omega c(p)]dt + o(dt)$, and substituting in (19), dividing by dt and letting $dt \rightarrow 0$ gives a differential equation

$$\frac{1}{2}\sigma_\Omega^2 p_\Omega^2 c''(p) + \alpha_\Omega p c'(p) - r_\Omega c(p) = 0$$

whose solution is

$$c_\Omega(p_{j,t}) = a_{\Omega_1} p^{\beta_{\Omega_1}} + a_{\Omega_2} p^{\beta_{\Omega_2}}$$

, where

$$\beta_{\Omega_1} = \frac{\sigma_\Omega^2 - 2\alpha_\Omega + \sqrt{4\alpha_\Omega^2 - 4\alpha_\Omega\sigma_\Omega^2 + \sigma_\Omega^4 + 8\sigma_\Omega^2 r_\Omega}}{2\sigma_\Omega^2} > 1, \text{ and } \beta_{\Omega_2} < 0.$$

Thus,

$$c_\Omega(p_{j,t}) = a_{\Omega_1} p_{j,t}^{\beta_{\Omega_1}} \quad (20)$$

express the option value for $\Omega = 1, 2, 3, \dots, N < \infty$ since $c_\Omega(p_{j,t}) \leq 0$ whenever $\beta_{\Omega_2} < 0$.

2.2 Determination of Critical Revenue for Investment Consideration

Let p_Ω denotes the revenue for each Ω -risky irreversible investment. If p_Ω satisfies the conditions ,

$$V_\Omega(p) = c_\Omega(p) \quad (21)$$

$$V'_\Omega(p) = c'_\Omega(p) \quad (22)$$

then p_Ω is called a threshold revenue. i.e revenue at which an investor is indifferent between investment and delay, and it is denoted by p_Ω^* .

Theorem 2.1

Let $\mathfrak{P}^* = \{(p_\Omega^*)_{T,s}\}$ denote the collection of matrices of expected threshold revenues and $\mathfrak{M}$ denote the corresponding matrices of expenditures. Consider a risk neutral or risk averse investor at the j^{th} year, t^{th} month contemplating N -tuple irreversible investments. Then $p_\Omega^* \in (p_\Omega^*)_{T,s}$ for each Ω , satisfying the conditions

$$\sum_{\Omega=1}^N V_\Omega(p^*) = \sum_{\Omega=1}^N C_\Omega(p^*) \quad (23)$$

$$\sum_{\Omega=1}^N V'_\Omega(p_\Omega^*) = \sum_{\Omega=1}^N C'_\Omega(p_\Omega^*) \quad (24)$$

is unique for that Ω -irreversible investment provided the utility function is unbounded.

2.3 Remark

The investor should invest if for the set of current market revenue $\{p_\Omega\}$ and set of critical (threshold) revenue $\{p_\Omega^*\}$, we have that $p_\Omega > p_\Omega^*$ for each Ω -irreversible investments otherwise don't invest .

2.4 Behavioural Attitude in the Investor's Preferences

We consider the following utility function for the Ω -irreversible investment.

$$u_\Omega(x) = (1 - \eta_\Omega)x + \eta_\Omega(1 - x^2) \quad (25)$$

where $0 \leq \eta_\Omega \leq 1$. For $0 < \eta_\Omega \leq 1$, the Arrow-Pratt measure of absolute risk aversion is expressed as

$$R_{\varphi_A}(x) = -\frac{u''_\Omega(x)}{u'_\Omega(x)} = \left[\frac{1}{2} \left(\frac{1 - \eta_\Omega + 2\eta_\Omega x}{\eta_\Omega} \right) \right]^{-1} \quad (26)$$

where η_Ω measures the degree of risk aversion of an investor.

The value of each investment in the Portfolio denoted by V_Ω is expressed as :

$$V_\Omega = \sum_{j=1}^T \left(E \int_{t=0}^{\infty} e^{-r_\Omega t} [(1 - \eta_\Omega)(P_\Omega - M_\Omega) + \eta_\Omega(1 - (P_\Omega - M_\Omega)^2)] dt \right) \\ \text{for } \Omega = 1, 2, \dots, N \\ V_\Omega(P) = (\Upsilon_\Omega) \left[\frac{P_\varphi}{r_\Omega - \alpha_\Omega} \right] + \eta_\Omega \left[\frac{1}{r_\Omega} - H(P) \right]$$

where

$$\Upsilon_\Omega = 1 - \eta_\Omega \quad (27)$$

$$H(P) \equiv E \int_{t=0}^{\infty} e^{-r_\Omega t} (P_\Omega - M_\Omega)^2 dt \quad (28)$$

Proof of Theorem 2.1

Consider the set of threshold revenues for the N-irreversible investments

$\{p_\Omega^* : p_\Omega^* \in [0, \infty) \text{ for } \Omega = 1, 2, \dots, N\}$ satisfying (21) and (22)

Let $F_\Omega : [0, \infty) \rightarrow \mathbb{R}$ be a twice differentiable function for each of the Ω -irreversible investment satisfying

$$F_\Omega(p) = \beta_{\Omega_1} V_\Omega(p) - p_{\Omega_{j,t}} V'_\Omega(p) \quad (29)$$

Now $u''_\Omega(.) < 0$ for risk aversion, and $u''_\Omega(.) = 0$ for risk neutrality. In either case, $u'_\Omega(.) > 0$. Since an investor can either be risk neutral or risk averse, then $u''_\Omega(.) \leq 0$.

From (21) and (22), we have that $F_\Omega(p) = 0$ for each Ω . Differentiating equation (29) with respect to p, we have

$$F'_\Omega(p) = (\beta_{\Omega_1} - 1) V'_\Omega(p) - p_{\Omega_{j,t}} V''_\Omega(p) \quad (30)$$

which is positive since $\beta_{\Omega_1} > 1$, $u'_\Omega > 0$ and $u''_\Omega \leq 0$. Also

$$F_\Omega(p) = \underbrace{\frac{\beta_{\Omega_1} u_\Omega(-\mu_\Omega)}{r_\Omega}}_{\text{revenue} < \text{expenditure}} < \underbrace{\frac{\beta_{\Omega_1} u_\Omega(0)}{r_\Omega}}_{\text{revenue} = \text{expenditure}} = 0 < \underbrace{\frac{\beta_{\Omega_1} u_\Omega(\mu^\Omega)}{r_\Omega}}_{\text{revenue} > \text{expenditure}} \quad \text{since } u'_\Omega > 0$$

By continuity of the function, F_Ω , $p_\Omega^* > 0$ and unique for some Ω .

A sufficient condition for existence of p_Ω^* is unboundedness of the utility function.

Substituting equation (14) in equation (29), we observe that:

$$\frac{F_\Omega(p)}{p_\Omega} = \left(\frac{1}{(1+r_\Omega)^T} \sum_{j=1}^T \left(E \int_{t=0}^{\infty} e^{-r_\Omega t} \left[\beta_{\Omega_1} \frac{u_\Omega(f_{\Omega_{j,t}})}{p_{\Omega_{j,t}}} - u'_\Omega(f_{\Omega_{j,t}}) \right] \frac{p_{\Omega_{j,t}}}{p_\Omega} dt \right) \right) \quad (31)$$

But

$$p_{\Omega_{j,t}} = p_\Omega e^{((\alpha_\Omega - \frac{1}{2}\sigma_\Omega^2)t - \sigma z_{\Omega_t})} \quad (32)$$

and

$$\begin{aligned} E(p_{\Omega_{j,t}} | \vartheta_{\Omega_{j,t-1}}) &= E(p_\Omega e^{((\alpha_\Omega - \frac{1}{2}\sigma_\Omega^2)t - \sigma z_{\Omega_t})} | \vartheta_{\Omega_{j,t-1}}) \\ &= p_\Omega e^{(\alpha_\Omega - \frac{1}{2}\sigma_\Omega^2)t} E(e^{\sigma z_{\Omega_t}} | \vartheta_{\Omega_{j,t-1}}) \\ &= p_\Omega e^{(\alpha_\Omega - \frac{1}{2}\sigma_\Omega^2)t} E(e^{\sigma(z_{\Omega_t} - z_{\Omega_{t-1}} + z_{\Omega_{t-1}})} | \vartheta_{\Omega_{j,t-1}}) \\ &= p_\Omega e^{(\alpha_\Omega - \frac{1}{2}\sigma_\Omega^2)t} e^{\sigma z_{\Omega_{t-1}}} E(e^{\sigma(z_{\Omega_t} - z_{\Omega_{t-1}})} | \vartheta_{\Omega_{j,t-1}}) \\ &= p_\Omega e^{(\alpha_\Omega - \frac{1}{2}\sigma_\Omega^2)t + \sigma z_{\Omega_{t-1}}} \mathbb{E}(e^{\sigma(z_{\Omega_t} - z_{\Omega_{t-1}})}) \end{aligned} \quad (33)$$

where $\mathbb{E}$ is "the mathematical expectation of" and $\vartheta_{\Omega_{j,t-1}}$ denote history of information available at j^{th} year, t^{th} month (i.e. sub-sigma algebra) with respect to the investment.

But

$$\mathbb{E}(e^{\sigma(z_{\Omega_t} - z_{\Omega_{t-1}})}) = \frac{1}{\varrho \sqrt{2\Pi}} \int_{-\infty}^{\infty} e^{\sigma(z_{\Omega_t} - z_{\Omega_{t-1}})} e^{-\frac{((z_{\Omega_t} - z_{\Omega_{t-1}}) - \varsigma)^2}{2\varrho^2}} d(z_{\Omega_t} - z_{\Omega_{t-1}}) \quad (35)$$

Let

$$\Lambda = \frac{((z_{\Omega_t} - z_{\Omega_{t-1}}) - \varsigma)}{\varrho} \quad (36)$$

so that

$$d(z_{\Omega_t} - z_{\Omega_{t-1}}) = \varrho d\Lambda \quad (37)$$

Substituting equations (36),(37) in equation (35), we obtain

$$\begin{aligned} \mathbb{E}(e^{\sigma_{\Omega}(z_{\Omega_t} - z_{\Omega_{t-1}})}) &= \frac{1}{\varrho\sqrt{2\Pi}} \int_{-\infty}^{\infty} e^{\sigma_{\Omega}(\varrho\Lambda + \varsigma)} e^{-\frac{\Lambda^2}{2}} \varrho d\Lambda \\ &= \frac{1}{\sqrt{2\Pi}} \int_{-\infty}^{\infty} e^{\sigma_{\Omega}\varrho\Lambda + \sigma_{\Omega}\varsigma} e^{-\frac{\Lambda^2}{2}} d\Lambda \\ &= \frac{e^{\sigma_{\Omega}\varsigma}}{\sqrt{2\Pi}} \int_{-\infty}^{\infty} e^{\sigma_{\Omega}\varrho\Lambda - \frac{\Lambda^2}{2}} d\Lambda \\ &= \frac{e^{\sigma_{\Omega}\varsigma}}{\sqrt{2\Pi}} \int_{-\infty}^{\infty} e^{-(\frac{\Lambda^2}{2} - \sigma_{\Omega}\varrho\Lambda)} d\Lambda \end{aligned} \quad (38)$$

but

$$\sigma_{\Omega}\varrho\Lambda - \frac{\Lambda^2}{2} = \frac{-(\Lambda - \sigma_{\Omega}\varrho)^2}{2} + \frac{\sigma_{\Omega}^2\varrho^2}{2} \quad (39)$$

and substituting equation (39) in equation (38) resulted in

$$\begin{aligned} \mathbb{E}(e^{\sigma_{\Omega}(z_{\Omega_t} - z_{\Omega_{t-1}})}) &= \frac{e^{\sigma_{\Omega}\varsigma}}{\sqrt{2\Pi}} \int_{-\infty}^{\infty} e^{\frac{-(\Lambda - \sigma_{\Omega}\varrho)^2}{2} + \frac{\sigma_{\Omega}^2\varrho^2}{2}} d\Lambda \\ &= \frac{e^{\sigma_{\Omega}\varsigma + \frac{\sigma_{\Omega}^2\varrho^2}{2}}}{\sqrt{2\Pi}} \int_{-\infty}^{\infty} e^{\frac{-(\Lambda - \sigma_{\Omega}\varrho)^2}{2}} d\Lambda \end{aligned} \quad (40)$$

Let

$$\tilde{\varrho} = \Lambda - \sigma_{\Omega}\varrho \quad (41)$$

so that

$$d\tilde{\varrho} = d\Lambda \quad (42)$$

Substituting equations (41) and (42) in equation (40) resulted in

$$\mathbb{E}(e^{\sigma_{\Omega}(z_{\Omega_t} - z_{\Omega_{t-1}})}) = \frac{e^{\sigma_{\Omega}\varsigma + \frac{\sigma_{\Omega}^2\varrho^2}{2}}}{\sqrt{2\Pi}} \int_{-\infty}^{\infty} e^{\frac{\tilde{\varrho}^2}{2}} d\tilde{\varrho} \quad (43)$$

$$= e^{\sigma_{\Omega}\varsigma + \frac{\sigma_{\Omega}^2\varrho^2}{2}} \sim N(0, 1) \quad (44)$$

$$= e^{\frac{\sigma_{\Omega}^2}{2}t} \quad (45)$$

Therefore, from equation (34), we have

$$E(p_{\Omega_{j,t}}|\vartheta_{\Omega_{j,t}}) = p_{\Omega}e^{\alpha_{\Omega}t} \quad (46)$$

Substituting equation (35) in equation (31), we have

$$\frac{F_{\Omega}(p)}{p_{\Omega}} = \left(\frac{1}{(1+r_{\Omega})^T} \sum_{j=1}^T \left(\int_{t=0}^{\infty} e^{-r_{\Omega}t} \left[\beta_{\Omega_1} \frac{u_{\Omega}(f_{\Omega_{j,t}})}{p_{\Omega}e^{\alpha_{\Omega}t}} - u'_{\Omega}(f_{\Omega_{j,t}}) \right] \frac{p_{\Omega}e^{\alpha_{\Omega}t}}{p_{\Omega}} dt \right) \right) \quad (47)$$

$$\frac{F_{\Omega}(p)}{p_{\Omega}} = \left(\frac{1}{(1+r_{\Omega})^T} \sum_{j=1}^T \left(\int_{t=0}^{\infty} \left[\beta_{\Omega_1} \frac{u_{\Omega}(f_{\Omega_{j,t}})}{p_{\Omega}e^{(\alpha_{\Omega}+r_{\Omega})t}} - \frac{u'_{\Omega}(f_{\Omega_{j,t}})}{e^{(r_{\Omega}-\alpha_{\Omega})t}} \right] dt \right) \right) \quad (48)$$

As each $p_{\Omega} \rightarrow \infty$ the ratio in equation (37) converges to:

$$\frac{F_{\Omega}(p)}{p_{\Omega}} = \left(\frac{1}{(1+r_{\Omega})^T} \sum_{j=1}^T \left(\int_{t=0}^{\infty} \left[- \frac{u'_{\Omega}(f_{\Omega_{j,t}})}{e^{(r_{\Omega}-\alpha_{\Omega})t}} \right] dt \right) \right) \quad (49)$$

$$\begin{aligned} &= \left(\frac{1}{(1+r_{\Omega})^T} \sum_{j=1}^T \left(\left(\frac{1}{r_{\Omega}-\alpha_{\Omega}} \right) \left[- \frac{u'_{\Omega}(f_{\Omega_{j,t}})}{e^{(r_{\Omega}-\alpha_{\Omega})t}} \right]_{t=0}^{t=\infty} \right) \right) \\ &= \left(\frac{1}{(1+r_{\Omega})^T} \sum_{j=1}^T \left(\frac{1}{r_{\Omega}-\alpha_{\Omega}} \lim_{p_{\Omega} \rightarrow \infty} u'_{\Omega}(f_{\Omega_{j,t}}) \right) \right) \end{aligned} \quad (50)$$

which with the condition $F^{\Omega}(0) < 0$ ensures there exist a $p_{\Omega}^* \in (0, \infty)$ such that $F_{\Omega}(p_{\Omega}^*) > 0$. Hence, the set $\mathfrak{P}^{**} = \{p_{\Omega}^* : p_{\Omega}^* \in (0, \infty) \text{ for } \Omega = 1, 2, \dots, N\}$ is unique. i.e. the expected threshold revenue for each $p_{\Omega}^* \in \mathfrak{P}^{**}$ is unique for the corresponding Ω - irreversible investment.

2.5 Equipment Value

We consider a brand new equipment with life-span J. Let:

- W_j denotes the Equipment value at the j^{th} year, $j \geq 1$ and W_0 equipment value at the date of manufacture.

We have $W_0 > W_j$, $j = 1, 2, \dots, T$.

- e denote the depreciation rate of the equipment

- $D_{j,n}$ denote the n th component of the equipment that is faulty in the j^{th} year
- $E_{j,n}$ the unit cost and $\Theta_{j,n}$ the number of component(s) replaced at the j th year.

The cost for a faulty component, $D_{j,n}$ at the j^{th} year is

$$= E_{j,n} \Theta_{j,n} \quad n = 1, 2, \dots, z < \infty$$

The total maintenance cost at the j^{th} year is

$$\mathbb{B} = \begin{cases} \sum_{n=1}^z [E_{j,n} \Theta_{j,n}] + \Gamma_j, & \text{when there are service charges, } \Gamma_j, \text{ for the } j\text{th year.} \\ \Gamma_j, & \text{when there is no component replaced in the } j\text{th year} \end{cases}$$

Let Φ_j denote the percentage ratio of maintenance cost to worth of equipment in the j th year, .i.e

$$\Phi_j = \frac{\mathbb{B}}{W_{j-1}} \times 100 \quad (51)$$

Then,

$$\tilde{\Phi}_T = \max(\{\Phi_j : 1 \leq j \leq T\}) \quad (52)$$

The value of the equipment at the end of the first year, i.e $j = 1$ is

$$W_1 = W_0 - eW_0 = W_0(1 - e) \quad (53)$$

. The value of the equipment at the end of the second year, i.e. $j = 2$ is

$$W_2 = W_1 - eW_1 = W_1(1 - e) = W_0(1 - e)^2 + \tilde{\Phi}_j W_1 \quad (54)$$

For $j = 3$, we have

$$W_3 = W_2 - eW_2 = W_2(1 - e) = W_0(1 - e)^3 + 2\tilde{\Phi}_j W_{j-2} \quad (55)$$

In general, the value of the equipment at the j th year is

$$W_j = W_0(1 - e)^j + (j - 1)\tilde{\Phi}_j W_{j-1} \quad (56)$$

3 Reserve Equation

Let $h_{\varphi_{j,t}}$ denote the amount held by the investor at month $t \geq 0$ of the j^{th} year ($j \in [0, T]$) for the N different riskless accounts. Let $h_{\Omega_{j,t}}$ denote the amount held by the investor at any time $t \geq 0$ in the N different riskless accounts. Let h_{Ω_0} denote the amount in the Investment Reserve (IR) with instantaneous interest rate of r_{Ω_f} % per annum at the beginning of month (i.e. $t = 0$) with respect to Ω -riskless investment. Let $q_{\Omega_{\bar{n},t}}$ denote the transaction charge with respect to Ω -riskless investment for the t^{th} month. Then, the transaction charges for the t^{th} month is $\sum_{\bar{n}=1}^z q_{\Omega_{\bar{n},t}}$. Let $S_{\Omega} = (s_{\Omega_{j,t}})$ where $s_{\Omega_{j,t}} = \sum_{\bar{n}=1}^z q_{\Omega_{\bar{n},t}}$. Let ℓ_{Ω} be withdrawal made from the reserve for time $t > 0$. Thus,

$$h_{\Omega_{j,t}} = h_0(t-1) + x_{\Omega} h_{\Omega_0}(t-1) + v_{\Omega_{j,t}} - \ell_{\Omega_{j,t}} - s_{\Omega_{j,t}} - \frac{\mathbb{B}_{\Omega}}{12} \quad (57)$$

where $x_{\Omega} = \frac{r_{\Omega_f}}{12}$. For $t = 1, 2, \dots, \hat{m}, \dots$, we have

$$h_{\Omega_{T,t}} = h_{\Omega_0}(1+x_{\Omega})^T + \sum_{g=0}^{T-1} v_{\Omega_{T-g}}(1+x_{\Omega})^g - \ell_{\Omega} \sum_{g=0}^{T-1} (1+x_{\Omega})^g - s_{\Omega} \sum_{g=0}^{T-1} (1+x_{\Omega})^g - \frac{\mathbb{B}_{\Omega}}{12} \quad (58)$$

provided; $\ell_{\Omega_{j,t}} = \ell_{\Omega_{k,w}}$ and $s_{\Omega_{j,t}} = s_{\Omega_{k,w}}$ for $j \neq k, t \neq w$

$$h_{\Omega_{T,t}} = h_0(1+x_{\Omega})^T + \sum_{g=0}^{T-1} v_{\Omega_{T-g}}(1+x_{\Omega})^g - (\ell_{\Omega} + s_{\Omega}) \frac{(1+x_{\Omega})^T - 1}{x_{\Omega}} - \frac{\mathbb{B}_{\Omega}}{12} \quad (59)$$

such that

$$\sum_{\Omega=1}^N h_{\Omega_{T,t}} = \sum_{\Omega=1}^N h_0(1+x_{\Omega})^T + \sum_{\Omega=1}^N \left(\sum_{g=0}^{T-1} v_{\Omega_{T-g}}(1+x_{\Omega})^g - (\ell_{\Omega} + s_{\Omega}) \frac{(1+x_{\Omega})^T - 1}{x_{\Omega}} - \frac{\mathbb{B}_{\Omega}}{12} \right) \quad (60)$$

Let

$$\Delta(T) = \sum_{\Omega=1}^N h_{\Omega_{T,t}} \quad (61)$$

The cash in the IR (investor's account) at T -years of investment is $\Delta(T)$

Theorem 3.1

If $\Delta(T) > 0$, then the portfolio investment is worthwhile, in money provided T is within the period for the loan repayment and other conditions remain constant.

Proof

Let

$$\mathfrak{H}_j^{h_{\Omega_j,t}} := \Upsilon \left(h_{\Omega_j,t}(w), w \in [0, t] \right) \text{ for each } j = 1, 2, \dots, T < \infty$$

denote the history of the reserve process within the expiration of the loan repayment, $T < \infty$

Assume

$$h_{\Omega_j,t} \geq 0 \text{ for each } \Omega$$

is adapted process with respect to the filtered space $(\mathfrak{F}, \chi(\mathfrak{F}), F(\chi))$.

$F(\chi)$ is the filtration of the process. Then, by equations (40) and (41),

$$\Delta(T) > 0$$

Hence, the portfolio is worthwhile, in money provided T is within the period for the loan repayment and other conditions remain constant

4 The Nigerian Maritime Industry Problem

In this section, we consider an investor that intends to venture into maritime business. After a thorough market survey, he realizes that a twelve year old single hull product tanker with a capacity of 7500mt dwt (metric tonne deadweight) cost \$ 4.00M, company registration and other expenses is #20.11 million (\$0.1105 million). Also, the investor discovered that a ten year old single hull crude tanker with capacity of 15,000mt dwt cost \$5.5 M and other expenses cost \$1.3 M all expenditure in US dollar.

The banks (bank A, bank B) he approached for loans of \$4.30 M and \$5.50 M

allow six years and ten years periods for repayment at instantaneous riskless interest rate of 10 % per annum and 12 % per annum respectively. He bided for two six years contract for time charter of current revenue of \$ 140,000.00 and \$280,000.00 for transporting product and crude oil respectively. Both tankers have twenty-five years life-span from their manufacturing date.

The banks he contacted for loans allow four months moratorium on the loan of \$4.30M and six months moratorium on the loan of \$5.50 M.

The investor has the following decisions to take:

1. Should he consider the investments?
2. If yes, when is it appropriate if the contracts are awarded for six years?

The Bank COT are as follows:

For bank A, we have;

- #1 000.00 on request for withdrawal booklet.
- 10% VAT on issuance of withdrawal booklet.
- #1 100.00 for monthly statement of Account.
- 1% of every withdrawal as withdrawal fees.
- Management fees is 0.25% of the loan
- Facility fees is 0.25% of the loan
- Processing fees is 0.75% of the loan

and

For bank B, we have;

- #1 000.00 on request for withdrawal booklet.
- 8% VAT on issuance of withdrawal booklet.
- #1 000.00 for monthly statement of Account.

- 1% of every withdrawal as withdrawal fees.
- Management fees is 0.25% of the loan
- Facility fees is 0.25% of the loan
- Processing fees is 0.75% of the loan

4.1 Problem Statement

We desire to determine for each irreversible investment considered, the monthly:

- (i) Revenue
- (ii) Expenses
- (iii) Reserve
- (iv) Investment value
- (v) Threshold revenue
- (vi) The penalty for default in repayment if any and
- (vii) Yearly depreciation value of the equipment purchased

4.2 Problem Formulation

We define the decision variable as follows:

For $\Omega = 1, 2$,

Let:

- $p_{\Omega_{j,t}} :=$ investment revenue at month i , $t > 0$ for j th year
- $k_{\Omega_{j,t}} :=$ Investment expenditure at month, t , $t > 0$ j th year
- $p_{\Omega_{j,t}}^* :=$ threshold revenue j th year and t month
- $a_{\Omega_{j,t}} :=$ investment loan repayment at month t , $t > 0$ for j th year

- $\epsilon(a_{\Omega_{j,t}}) :=$ loan default investment payment at month t , $t > 0$ for j th year

The Portfolio optimization problems to be solved are:

$$\begin{aligned} \max \left(\sum_{\Omega=1}^2 v_{\Omega_{j,t}} \right) = \sum_{\Omega=1}^2 \left(e^{-r_{\Omega}} [(1 - \eta_{\Omega})(p_{\Omega_{j,t}} - m_{\Omega_{j,t}}) + \right. \\ \left. \eta_{\Omega}(1 - (p_{\Omega_{j,t}} - m_{\Omega_{j,t}})^2)] \right) \end{aligned} \quad (62)$$

subject to:

$$\begin{aligned} p_{\Omega_{j,t}} &= p_{\Omega_0} e^{(\alpha - 0.5\sigma^2)\Delta + \sigma N(0,1)} > p_{\Omega_{j,t}}^* > 0; \\ p_{1_0} &= \#49.14 \text{ million } (270,000.00); \quad \text{for } \alpha_1 = 0; \quad \sigma_1 = 10\% ; \quad \eta_1 = 20\%; \\ m_{\Omega_{j,t}} &= k_{\Omega_{j,t}} + a_{\Omega_{j,t}} + \epsilon(a_{\Omega_{j,t}}) \\ p_{2_0} &= \#49.14 \text{ million } (370,000.00); \quad \text{for } \alpha_2 = 0; \quad \sigma_2 = 10\% ; \quad \eta_2 = 20\%; \\ m_{\Omega_{j,t}} &= k_{\Omega_{j,t}} + a_{\Omega_{j,t}} + \epsilon(a_{\Omega_{j,t}}); \quad p_{j,t} > p^* > m_{j,t}; \\ \{\mathbf{I}_{\Omega} : \mathbf{I}_{\Omega=1} &= \#728.00 \text{ million } (4.20\text{million}), \quad \mathbf{I}_{\Omega=2} = \#928.00 \text{ million } (5.20\text{million})\} \\ \text{for } r_1 &= 12\%; \quad a_{\Omega=1_{j,t}} = \frac{\mathbf{I}_{\Omega=1}(1+\frac{r_1}{12})}{d}; \quad d_1 = 60 - 4 \text{ and} \\ r_2 &= 10\%; \quad a_{\Omega=2_{j,t}} = \frac{\mathbf{I}_{\Omega=2}(1+\frac{r_2}{12})}{d}; \quad d_2 = 60 - 4 \end{aligned}$$

$$\begin{aligned} \max \left(\sum_{\Omega=1}^N h_{\Omega_{T,t}} \right) = \sum_{\Omega=1}^N h_0(1 + x_{\Omega})^T + \sum_{\Omega=1}^N \left(\sum_{g=0}^{T-1} v_{\Omega_{T-g}}(1 + x_{\Omega})^g - \right. \\ \left. (\ell_{\Omega} + s_{\Omega}) \frac{(1 + x_{\Omega})^T - 1}{x_{\Omega}} + \frac{\mathbb{B}_{\Omega}}{12} \right) \end{aligned} \quad (63)$$

subject to:

$$\begin{aligned} h_0 &= \mathbf{I} - \text{equipment value at purchase}; \quad x_{\Omega} = \frac{r_{\Omega_f}}{12}; \quad r_{\Omega=10_f} \%; \\ v_{\Omega_{i-g}} &> 0; \quad \ell_{\Omega=1,2} = \{\#1.82\text{million}, \#2.82\text{million}\}; \\ s_{\Omega_{j,t}} &= \sum_{n=1}^{\nu} m_{\Omega_{n,t}}. \end{aligned}$$

Yearly equipment value after purchase:

$$\max(W_j) = W_0(1 - \frac{1}{j})^j + (j - 1)\tilde{\Phi}_j W_{j-1} \quad (64)$$

subject to:

- $J = 25$ years from build.
- $W_0 \equiv 1$ (Value at manufactured date)
- $\tilde{\Phi}$ is replacement per cent per year
- $j \geq 1$

We used Microsoft-Excel package to solve the optimization problems 62-64:

5 Analysis of Result

The investment is worthwhile based on the loans, the interest rates, exchange rate of about #156.00 to a dollar, calculated threshold revenue of #41.20 for product tanker and #42.49 for crude oil tanker, predicted monthly revenue subject to volatility of 10% for product and crude oil tanker respectively (figures 1,2 in the appendix) , the Bank COT, and 20% level of risk aversion on monthly generated revenue. However, the liquidity of his proposition depends on the status of his investment reserve, provided all other conditions mention earlier are satisfied. Expected volume of the reserve is displace on figure 5 and figure 6 in the appendix.

Furthermore, having serviced the loan obtained, the equipment purchased for product tanker would have spent 47.96% of its life-span but when maintained periodically, it would have spent 28.46%. That of crude oil tanker would have spent 50.04% of its life span but with periodic maintenance, it would have spent 19.39%. This is illustrated by figures 3,4 in the Appendix Thus, there is the need to use the tankers for few years with the requisite mandatory special surveys and drydock respectively (i.e regular inspections) subject to :

- The Classification society and flag administration requirements.
- Comparative cost of total overhauling to cost of purchasing a new one.
- Comparative benefit for selling it off as a scrap.

- Cost of maintenance and docking.

before selling off the tankers as scrap.

The investor benefits from maintaining the capital equipments periodically (figures 3,4 in the appendix) and repaying the loan as at when due but not after the due date. From figures 6,8 in the appendix, the expected cashflow for the portfolio is strictly positive though the product tanker proposition is more solvent to crude oil proposition.

The investment proposition is affected by exchange **rate**. For the Maritime investment **problem in Nigeria**, the proposition is worthwhile for exchange rate $\in (N150, N175]$ to a US dollar for **the** product oil tanker proposition and worthwhile for exchange rate $\in (N150, N180)$ for **the** crude oil tanker proposition. At **an** exchange rate of N175 to a dollar, the product oil tanker proposition is not worthwhile since the threshold revenue do not exist. (Figure 9)

Therefore, the investments decision can be made since the following guaranteed investments profit:

- **Fluctuated sets of revenues.**
- **Sets of; loans, interest rates and moratorium periods**
- **Value of the capital equipments at investment decision and the value at a future date for the duration for servicing the loans.**
- **Benefits and costs for maintaining the capital equipments.**
- **Liquidity of the investments propositions.**
- **Effect of exchange rate on the investments proposition.**
- **Set of vectors of expenditures.**

Generally, the results from this study are applicable to irreversible investment in incomplete markets. For example, in the aviation industry where the decision to invest either in a cargo or passenger plane is irreversible. This is also applicable to cement industry in which the decision to invest in the cement milling plant is irreversible. In poultry farming, the decision to invest incubator for hatching is irreversible since the incubator can only be used for hatching birds. In any of these areas, the investment analysis for a worthwhile venture is focus on the; expected incomes, expected expenses, present and future values of the capital equipment, required loans, interests and mode of servicing the loans, interests within the agreed periods, moratorium periods and finally investment liquidity. There are also many other areas where these can be applied.

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Appendix

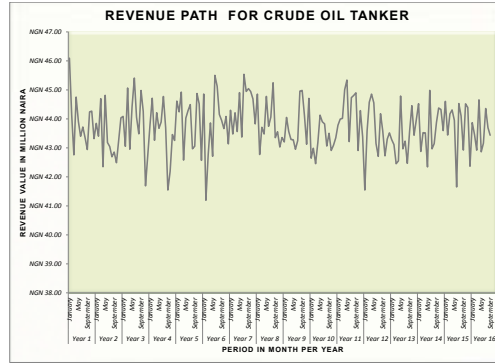


Figure 1: $Volatility = 0.10$, $current\ investment\ value = N21.84M$

Figure 1 display expected revenue for crude oil proposition with expected initial revenue of N43.83 million at exchange rate of N156 to a dollar.

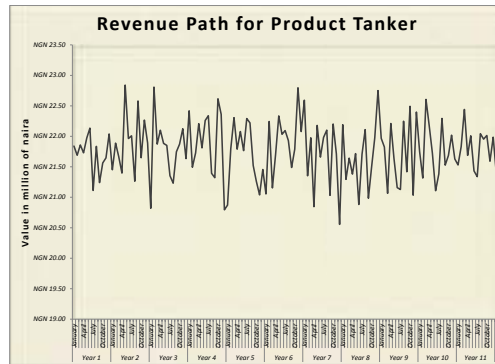


Figure 2: $Volatility = 0.10$, $current\ investment\ value = N21.84M$

Figure 2 display expected revenue for product oil proposition with expected initial revenue of N21.84 million at exchange rate of N156 to a dollar.

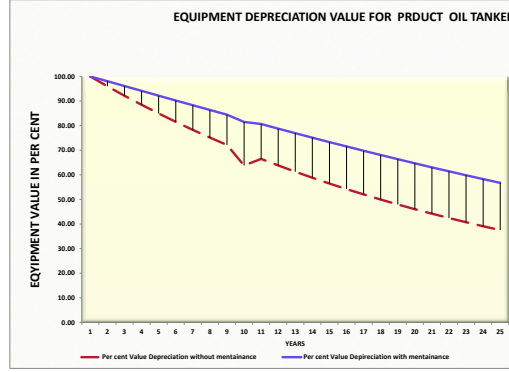


Figure 3: $Depreciation\ rate = 0.04, Maintenance = 0.35$

Figures 3 and 4 display the equipments (product tanker and crude tanker) depreciation rates of 0.4 each and maintenance rates of 0.35, 0.01 respectively. The gaps between the curves respectively are the expected benefits from maintaining the equipments.

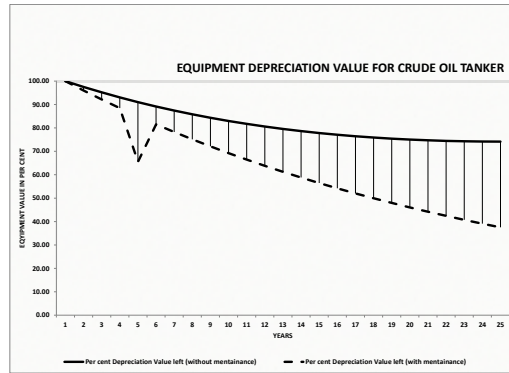


Figure 4: $Depreciation\ rate = 0.04, Maintenance = 0.01$

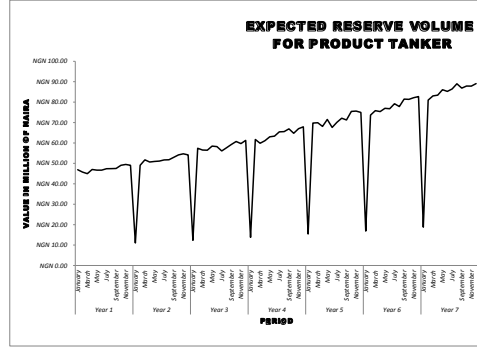


Figure 5: *Moratorium* = 4months, Bank interest rate = 0.055, Risk avoidance = 0.20, Reserve rate = 0.08 at N156 per \$1

Figures 5,6 display the graph of expected reserve as a function of loan moratorium, loan interest rate, level of risk avoidance, reserve rate, exchange rate of N156 to a dollar respectively.

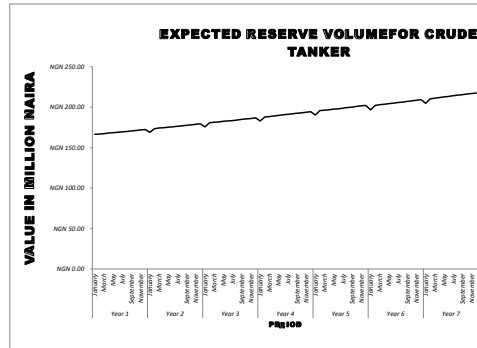


Figure 6: *Moratorium* = 6months, Bank interest rate = 0.11, Risk avoidance = 0.18, Reserve rate = 0.04 at N156 per \$1

The figures 7,8 show the cashflow of the investment propositions.

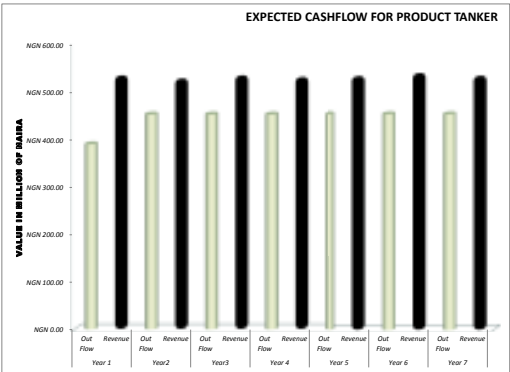


Figure 7: EXPECTED CASHFLOW FOR PRODUCT TANKER

The investment propositions are solvent and worthwhile.

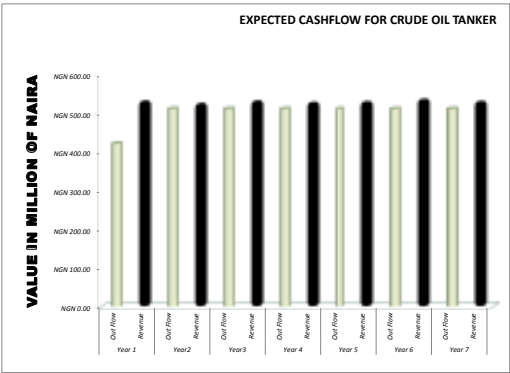


Figure 8: EXPECTED CASHFLOW FOR CRUDE OIL TANKER

Exchange Rate	Investment Reserve		Calculated Threshold Revenue:	
	Tanker	Tanker	Product Oil Tanker	Crude Oil Tanker
N156 / \$1	N91.31 M	N217.72 M	N41.20 M	N42.49 M
N160 / \$1	N91.52 M	N223.25 M	N45.00 M	N46.50 M
N165 / \$1	N95.33 M	N230.08 M	47	{N47.5 M, N50.1 M}
N170 / \$1	N99.95 M	N237.29 M	{N47 M, N48.5 M}	{N47.8 M, N50.5 M}
N1750 / \$1	N98.94 M	N243.97 M	{N48.05 M, N48.5 M}	N51.00 M
N180 / \$1	N100.84 M	N250.87 M	Nil	N50 M - N50.5

Effect of Exchange Rate on the Investment Proposition

Figure 9:

From figure 9, as the exchange rate of naira to a dollar increases, the expected reserve and threshold revenue rises for both product oil and crude oil proposition but between exchange rate of N170 to N175 to a dollar the expected reserve for product oil proposition drops. This is due to expected high threshold revenue. Although, the expected investment reserve for product oil proposition is at the peak compare to exchange rate $\in (155, 181)$ to a dollar, the investment is not worthwhile since there the expected threshold revenue do not exist at exchange rate of N180 to a dollar.