

HALL CURRENT EFFECTS ON UNSTEADY MHD FLUID FLOW WITH RADIATIVE HEATFLUX AND HEAT SOURCE OVER A POROUS MEDIUM

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ABSTRACT

In this paper the study of unsteady hydromagnetic free flow of viscoelastic fluid (Walter's B) past an infinite vertical plate through porous medium was conducted. The temperature is assumed to be oscillating with time, also the effects of hall-current is taken in to account. The solution of velocity, temperature and concentration profiles have been obtained. The effects of various parameters on temperature, concentration primary and secondary velocity profiles were presented graphically.

1. INTRODUCTION

The effects of hall current in hydromagnetic fluid have attracted the attention of large number of scholars, particularly in the field of fluid dynamics. This may be as a result of wider range of its applications in this era of modern science, technology and vast industrialisation. The flow of an electrically conducting fluid has important applications in many branches of engineering science such as magneto hydrodynamics (MHD) generators, plasma studies, nuclear reactors, geothermal energy extraction, electromagnetic propulsion and the boundary layer control in the field of aerodynamics. Magneto hydrodynamics is currently undergoing a period of great enlargement and differentiation of subject matter.[1].

In addition to this, the relevance of fluid flow in the health sector can never be overemphasised. As we all know, the cardiovascular system is sensitive to changes in the environment, and flow characteristics of blood are modified to satisfy changing demands of the organisms. In addition to transporting of oxygen, metabolites, and other dissolved substances to and from the tissues, blood flow alters heat transfer within the body. [2].

Hydro magnetic flow and heat transfer between two horizontal, the lower plate being a stretching sheet had been studied by [3]. [4] made similarity analysis in magneto hydrodynamics, Hall effects on forced convective heat and mass transfer of non Newtonian power law fluids past semi infinite vertical flat plate. Heat and mass transfer in elastico-viscous fluid past an impulsively started infinite vertical plate with hall effects was examined by [5]. [6] investigated the effects of chemical reaction and radiation absorption on an unsteady MHD convective heat and mass transfer flow past semi infinite vertical moving plate embedded in a porous medium. [7] analysed the radiation and mass transfer effects on MHD free convection flow past an exponentially accelerated vertical plate with variable temperature.[8], described the effects of heat transfer to MHD oscillatory flow in a channel filled with porous medium. [9] investigated the thermal radiation and buoyancy effects on MHD free convective heat generating flow over an accelerating permeable surface with temperature dependent viscosity. Heat and mass transfer effects on unsteady MHD free convection flow near moving vertical plate in porous medium had been studied by [10]. The demonstration of effects of unsteady two dimensional hydro magnetic flow and heat transfer of a fluid [2]. This shows that an external magnetic field has the same effects on the flow as viscoelasticity. [7] analysed the radiation and mass transfer effects on MHD free convectional flow past an exponentially accelerated vertical plate with variable temperature. Kumar and Chand [11] illustrated the effects of slip condition and hall current on unsteady MHD flow of a visco-elastic fluid past an infinite porous vertical plate through porous medium. [11] further illustrated hall effects on heat and mass transfer in the flow of oscillating viscoelastic fluid through porous medium with slip condition.

This paper aimed to extend the study of heat and mass transfer for an electrically conducting incompressible fluid past a continuously moving plate to include hall current, radiative flux, heat source, mass flux and viscoelasticity through a porous medium in the presence of magnetic field.

2. PROBLEM FORMULATION

We consider the unsteady flow of a viscous incompressible and electrically conducting viscoelastic fluid with oscillating temperature. The flow occurs over an infinite vertical porous plate. The x^* axis is assumed to be oriented vertically upward along the plate and y^* axis taken normal to the plane of the plate. It is assumed that the plate is electrically none conducting and a uniform magnetic field of strength B_0 is applied normal to the plate. The induced magnetic field is assumed to be negligible so that $\vec{B}(0, B_0, 0)$. The plate is subjected to a constant suction velocity V_0 .

Since the plate is infinite in extend all physical quantities are functions of y^* and t^* only. Thus the governing equations of the flow under the usual Boussinesq approximation are:

The Momentum Equations:

$$\begin{aligned} \frac{\partial u'}{\partial t} - v_0 \frac{\partial u'}{\partial y'} &= v \frac{\partial^2 u'}{\partial y'^2} - v \frac{u'}{K_l} - g\beta(T' - T'_\infty) + \\ &g\beta^*(C' - C'_\infty) - \frac{\sigma\beta_0^2}{\rho(1+m^2)}(u' + mw') - K_0 \left\{ \frac{\partial^3 u'}{\partial t \partial y'^2} - v_0 \frac{\partial^3 u'}{\partial y'^3} \right\} \end{aligned} \quad (1)$$

$$\begin{aligned} \frac{\partial w'}{\partial t} - v_0 \frac{\partial w'}{\partial y'} &= v \frac{\partial^2 w'}{\partial y'^2} - v \frac{w'}{K_l} + \frac{\sigma\beta_0^2}{\rho(1+m^2)}(mu' - w') \\ &- K_0 \left\{ \frac{\partial^3 w'}{\partial t \partial y'^2} - v_0 \frac{\partial^3 w'}{\partial y'^3} \right\} \end{aligned} \quad (2)$$

Energy Equation:

$$\begin{aligned} \frac{\partial T'}{\partial t} - v_0 \frac{\partial T'}{\partial y'} &= \frac{k}{\rho c p} \frac{\partial^2 T'}{\partial y'^2} - \frac{Q_0}{\rho c p} (T' - T'_\infty) - \frac{1}{\rho c p} \frac{\partial q_r}{\partial y} \\ &\sigma^* k^* \end{aligned} \quad (3)$$

Concentration Equation:

$$\frac{\partial C'}{\partial t} - v_0 \frac{\partial C'}{\partial y'} = D \frac{\partial^2 C'}{\partial y'^2} - K_c (C' - C'_\infty) \quad (4)$$

The initial boundary conditions are :

$$\begin{aligned} u' = 0, \quad w' = 0, \quad \theta = 1 + \varepsilon e^{\Omega t}, \quad C' = 1 + \varepsilon e^{\Omega t}, \quad y=0 \\ u' \rightarrow 0, \quad w' \rightarrow 0, \quad T' \rightarrow T'_\infty, \quad C' \rightarrow C'_\infty, \quad y \rightarrow \infty \end{aligned} \quad (5)$$

Introducing the following non dimensional quantities

$$y = \frac{v_0 y'}{v}, \quad u' = U u_0, \quad u'_1 = \frac{U}{e} u_1, \quad w'_0 = U w_0, \quad w'_1 = \frac{U}{e} w_1,$$

$$\begin{aligned}
81 \quad y &= \frac{v_0 y'}{v} \quad \theta = \frac{T' - T'_\infty}{T'_\omega - T'_\infty} \quad C = \frac{C' - C'_\infty}{C'_\omega - C'_\infty} \quad M = \frac{\sigma \beta_0^2 v}{\rho v_0^2} \\
82 \quad Gr &= \frac{v \beta (T'_\omega - T'_\infty)}{U v_0^2} \quad S = \frac{v^2 Q_0}{v_0^2} \quad Gc = \frac{v \beta^* v (C'_\omega - C'_\infty)}{U v_0^2} \quad Km = \frac{K_0 v_0^2}{v^2} \quad Sc = \frac{v}{D} \\
83 \quad Pr &= \frac{\mu C_p}{k} \quad K = \frac{K_I v}{v_0^2}
\end{aligned} \tag{6}$$

84 Where, β volumetric coefficient of the thermal expansion, v the kinematic viscosity, ρ is density,
85 μ the coefficient of viscosity, β^* is volumetric coefficient of expansion with concentration,
86 u_0 the velocity of the plate, y the coordinate axis normal to the plate, g acceleration due to
87 gravity, q_r the radiation heat flux in the y direction, C_p is specific heat at constant pressure, C' is
88 specific concentration in the fluid, C'_∞ the concentration in the fluid far away from the plate,
89 C'_ω the concentration on the plate, D the mass diffusion coefficient, t' is time, T the temperature
90 of the fluid near the plate, T_ω the temperature of the plate, T_∞ the temperature of the fluid far
91 away from the plate. M the Hartmann number, Km viscoelastic parameter, Ω , is the frequency of
92 oscillation.

94 Let the assumed solutions be,

$$\begin{aligned}
95 \quad u' &= u'_0 + \varepsilon u'_1 e^{i\Omega t} \\
96 \quad \theta' &= \theta'_0 + \varepsilon \theta'_1 e^{i\Omega t}
\end{aligned} \tag{7}$$

$$\begin{aligned}
97 \quad w' &= w'_0 + \varepsilon w'_1 e^{i\Omega t} \\
98 \quad C' &= C'_0 + \varepsilon C'_1 e^{i\Omega t}
\end{aligned}$$

100 Substituting equation (7) in (1) above
Equations (1) to (4) using (6) and (7) reduced to

$$101 \quad \frac{Km \partial^3 u_0}{\partial y^3} + \frac{\partial^2 u_0}{\partial y^2} + \frac{\partial u_0}{\partial y} - L u_0 - J w_0 + Gr \theta_0 + Gc C_0 = 0 \tag{8}$$

$$102 \quad Km \frac{d^3 w_0}{dy^3} + \frac{d^2 w_0}{dy^2} + \frac{dw_0}{dy} + L w_0 - J u_0 = 0 \tag{9}$$

$$103 \quad \Rightarrow Km \frac{d^3 u_1}{dy^3} + (1 - i\Omega Km) \frac{d^2 u_1}{dy^2} + \frac{du_1}{dy} - L n u_1 + J w_1 = -Gr \theta'_1 - Gc C'_1 \tag{10}$$

$$104 \quad \Rightarrow Km \frac{d^3 w_1}{dy^3} + (1 - i\Omega Km) \frac{d^2 w_1}{dy^2} + \frac{dw_1}{dy} - (Ln) w_1 + J u_1 = 0 \tag{11}$$

$$105 \quad Z \frac{\partial^2 \theta'_0}{\partial y^2} + Pr \frac{\partial \theta'_0}{\partial y} - S \theta'_0 = 0 \tag{12}$$

$$106 \quad Z \frac{\partial^2 \theta'_1}{\partial y^2} + Pr \frac{\partial \theta'_1}{\partial y} - q \theta'_1 = 0 \tag{13}$$

$$\frac{1}{SC} \frac{\partial^2 C'_0}{\partial y^2} + \frac{\partial C'_0}{\partial y} - KC'_0 = 0 \quad (14)$$

$$\frac{\partial^2 C'_1}{\partial y^2} + SC \frac{\partial C'_1}{\partial y} - SC(K + i\Omega)C'_1 = 0 \quad (15)$$

The corresponding boundary conditions are:

$$\begin{aligned} U'_0 = U'_1 = 0, \quad w'_0 = w'_1 = 0, \quad \theta'_0 = \theta'_1 = 1, \quad C'_0 = C'_1 = 0 \quad \text{at } y = 0 \\ U'_1 \rightarrow 0, \quad w'_0 \rightarrow 0, \quad U'_0 \rightarrow 0, \quad w'_1 \rightarrow 0, \quad \theta'_0 \rightarrow 0, \quad \theta'_1 \rightarrow 0, \\ C'_0 \rightarrow 0, \quad C'_1 \rightarrow 0 \quad \text{as } y \rightarrow \infty \end{aligned} \quad (16)$$

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3.METHOD OF SOLUTION

116 Introducing $F = (u_0 + iw_0)$, and $i = \sqrt{-1}$ also $H = (u_1 + iw_1)$ equation (8) to (11)
117 transformed to

$$Km \frac{d^3 F}{dy^3} + \frac{d^2 F}{dy^2} + \frac{dF}{dy} - FL - FJ = -Gr\theta_0 - GcC_0 \quad (17)$$

$$Km \frac{d^3 H}{dy^3} + E \frac{d^2 H}{dy^2} + \frac{dH}{dy} - NnH = -Gr\theta_1 - GcC_1 \quad (18)$$

120 Equations (17) and (18) are third order differential equations due to the presence of viscoelasticity.

121 Therefore, F_0 and H_0 are expanded using [12] in terms of Km

$$F_0 = F_{00} + KmF_{01} \quad \text{and} \quad H_1 = H_{11} + KmH_{12}$$

123

Zeroth-order

$$F_{00}'''' + F_{01}'' + F_{01}' - NF_{01} = 0 \quad (19)$$

$$F_{01}'' + F_{01}' - NF_{01} = -F_{00}''' \quad (20)$$

$$H_{11}'''' + EH_{12}'' + H_{11}' - NnH_{12} = 0 \quad (21)$$

$$EH_{12}'' + H_{12}' - NnH_{12} = -H_{11}''' \quad (22)$$

128 The corresponding boundary condition transformed to

$$Y = 0 : F_{00} = F_{01} = 0$$

129

$$Y \rightarrow \infty : F_{00} = F_{01} \rightarrow 0$$

130

Similarly,

$$Y = 0 : H_{00} = H_{01} = 0$$

131

$$Y \rightarrow \infty : H_{00} = H_{01} \rightarrow 0 \quad (23)$$

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Solving equations (19) to (22) under the boundary condition (23) to obtained

133 The primary velocity is given by,

$$\begin{aligned}
 U = & (A_{10} + KmA_{15})e^{-n_5y} - (A_{11} + KmA_{16})e^{-n_5y} - \\
 & (A_{12} + KmA_{17})e^{-n_3y} + KmA_{14}e^{-n_6y} + \\
 & \varepsilon e^{\Omega t} \left[(B_2 + KmB_7)e^{-f_1y} - (B_3 + KmB_8)e^{-n_2y} \right. \\
 & \left. - (B_4 + KmB_9)e^{-n_4y} + KmB_6e^{-f_2y} \right]
 \end{aligned}$$

The secondary velocity is also given by

$$\begin{aligned}
 W = & (A_{11} + KmA_{16})e^{-n_1y} + (A_{12} + KmA_{17})e^{-n_3y} - \\
 & KmA_{14}e^{-n_6y} - (A_{10} + KmA_{15})e^{-n_5y} + \\
 & i \varepsilon e^{\Omega t} \left[(B_3 + KmB_8)e^{-n_2y} + (B_4 + KmB_9)e^{-n_4y} \right. \\
 & \left. - KmB_6e^{-f_2y} - (B_2 + KmB_7)e^{-f_1y} \right]
 \end{aligned}$$

The temperature expression
 $\theta'(y) = e^{-n_1y} + \varepsilon e^{(i\pi t - n_2y)}$

The concentration expression
 $\therefore C'(y) = e^{-n_3y} + \varepsilon e^{(i\pi t - n_4y)}$

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4. RESULTS AND DISCUSSION

In order to illustrate the influence of various parameters which include Grashof number Gr, mass Grashof number Gc, magnetic number M, chemical reaction parameter R, Schmits number Sc, Prandtl number Pr, radiation parameter R, heat source s, hall current parameter m and viscoelastic parameter Km on velocity, temperature and concentration profiles. Computations were carried out using Gr=2, Gc=2, Pr=0.71, Sc=0.6, R=0.4, M=5, s=0.2, m=0.2 Km=0.05 various values based on physical quantities are computed and presented in figure.1-16.

5. CONCLUSIONS

The hall current effects on unsteady MHD fluid flow with radiative heat flux and heat source over a porous medium is investigated by transforming the governing partial differential equations into ordinary differential equations which are then solved using perturbation techniques. The result of the flow variables indicates that the fluid temperature is reduced by increasing Prandtl number (Pr) and radiation parameter (R). Concentration is reduced with

160

161 increase in Schmidt number (Sc) and chemical reaction parameter (K). The primary velocity
 162 decrease with increasing prandtl (Pr), radiation parameter and hall-current while the opposite
 163 trend is observed in secondary velocity. The primary velocity increases with increase in
 164 mass Grashoof number (Gr) and thermal Grashoof number (Gc) also the reverse is the
 165 case in secondary velocity. The primary velocity decreases with increase in M , s and Sc

166 Temperature profiles

167 Figure.1 illustrates the effects of Prandtl number on temperature field. It is noticed that as the
 168 Prandtl number increases the temperature decreases. The effects of Radiation parameter
 169 have been illustrated in figure.2 Increase in radiation parameter R is accompanied with
 increase in temperature profile.

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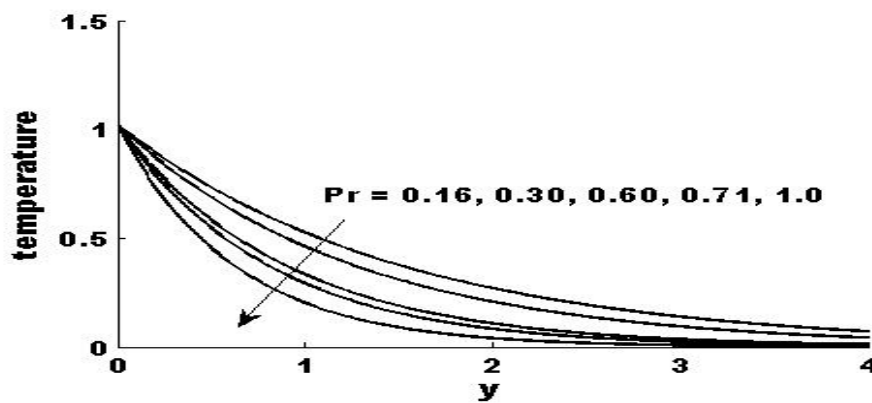
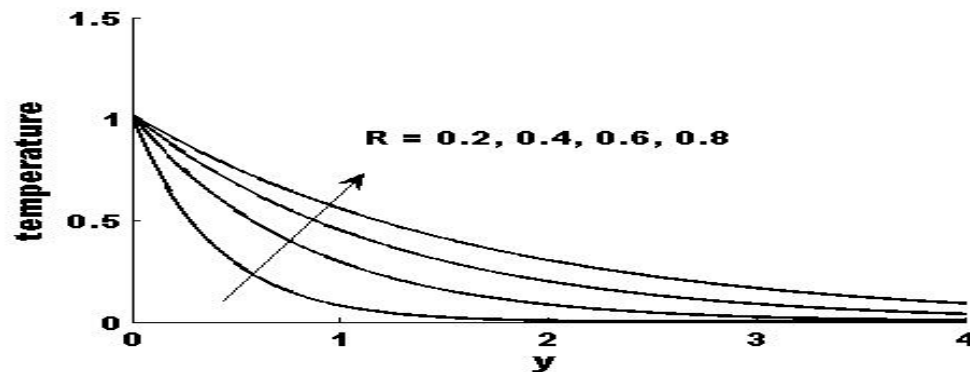


Figure: 1. Effects of Pr on temperature profile

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174 Figure: 2. Effects of R radiation parameter on temperature

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176 Concentration profiles

177 Figure 3 Shows the effects of Schmidt number on concentration profile with fixed values of
 178 $t=0.2$, $K=0.22$. The graph shows that, increase in the values of Schmidt number Sc results
 in decrease in the concentration profile. Figure 4 demonstrates the effects of chemical

179

180 reaction parameter K , on concentration field. It also exhibits the same behaviour as Schmidt
 181 number.
 182

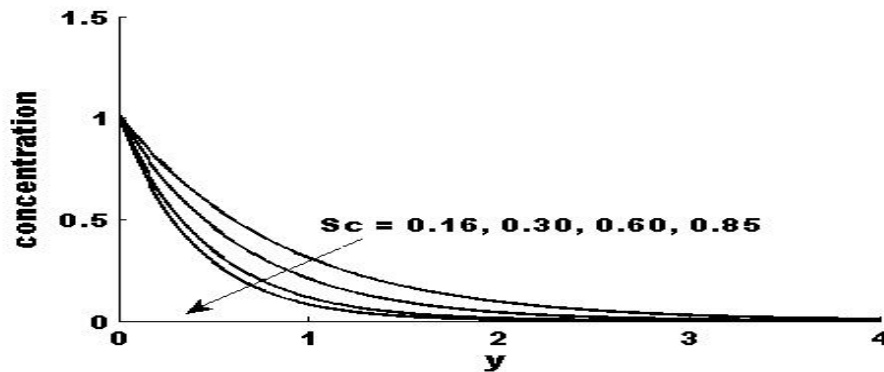


Figure 3.Effects of Sc on concentration profile.

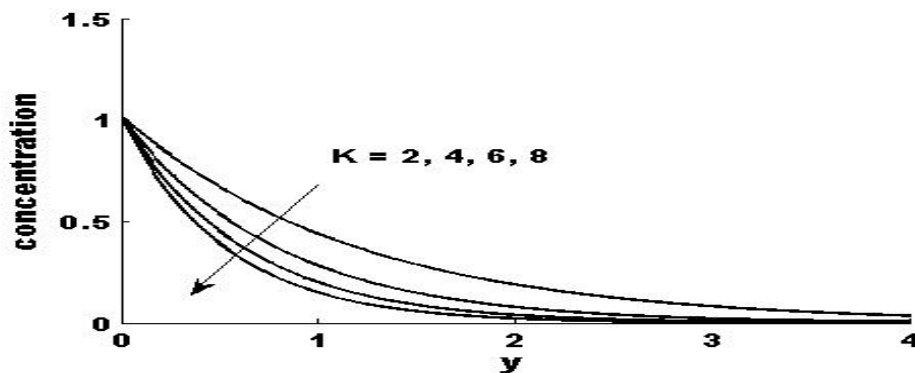


Figure 4 Effects of K on concentration profile.

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187 Velocity profiles

188 Figure 5 and figure 6, demonstrate the effects of primary velocity on Gr and Gc , while figure
 189 7 and figure 8 show the effects of secondary velocity on Gr and Gc , in the former cases the
 190 velocity decrease with increase in the values of both Gr and Gc . In the later cases the
 velocity is directly proportional to the values of both Gr and Gc .

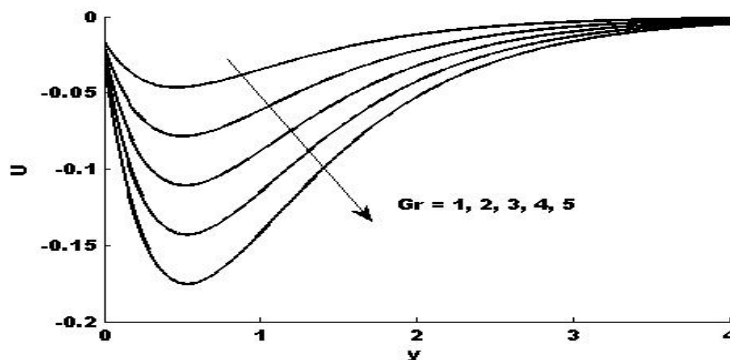


Figure: 5. Effects of Thermal Grashof number Gr on Primary velocity.

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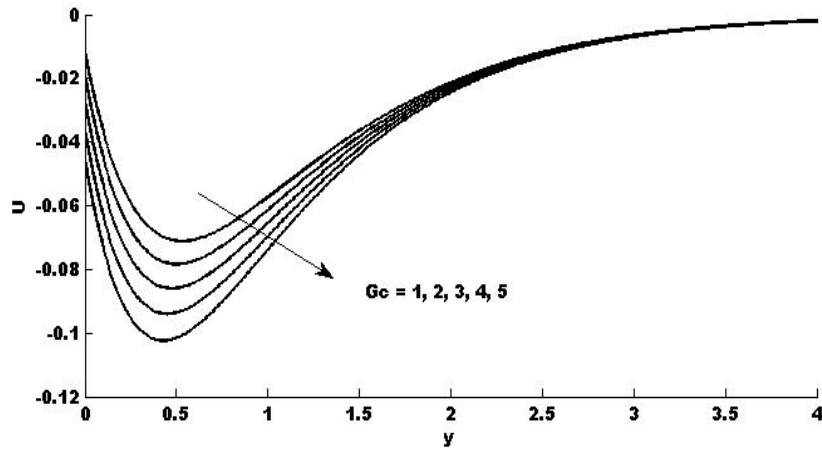


Figure6. Effects of Mass Grashof number G_c on Primary velocity

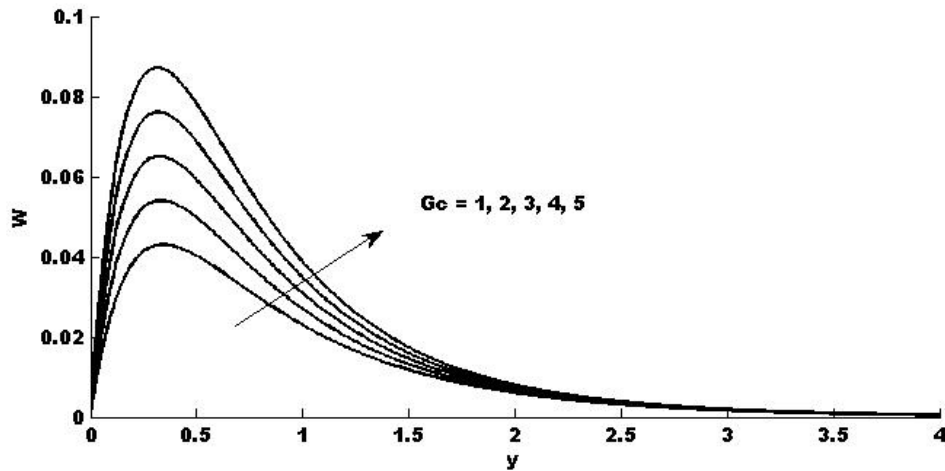


Figure:7.Effects of Thermal Grashof number Gr on secondary velocity

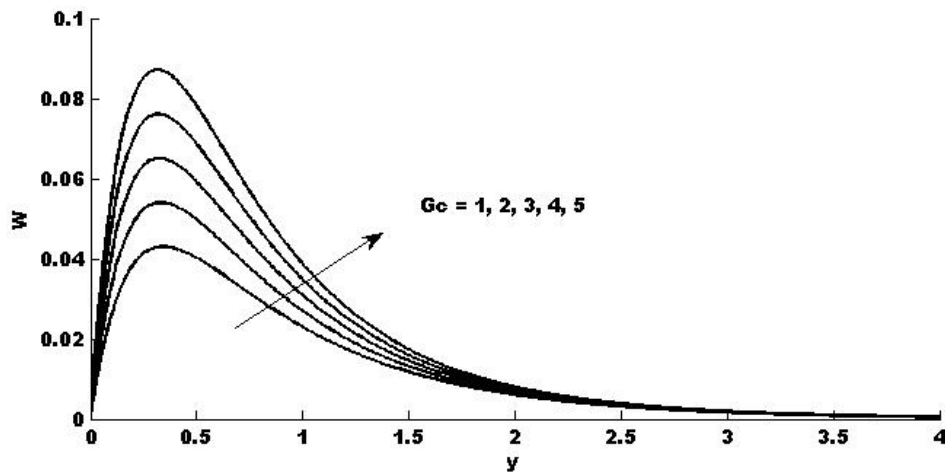


Figure: 8 Effects of G_c on secondary velocity.

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201 Figure 9 and 10 describe the effects of Hartmann number M , on primary and secondary
 202 velocity respectively. Increase in the value of M increases the primary velocity profile.
 203 Likewise decrease in M results increase in profile of W . Figure 11 and 12 illustrate the
 204 effects of heat source (s) on both primary and secondary velocity. Increase in heat source
 205 increases the primary velocity, while decrease in s accompanied with increases secondary
 206 velocity. Figure 13 and 14 demonstrates the effects of K_m on both primary and secondary
 207 velocities. K_m exhibit the same behaviour in both primary and secondary velocity as Gr and
 208 G_c . Increase in the values of K_m accompanies with decrease in primary velocity and
 209 increase secondary velocity. Figure 15 and 16 illustrate the effects of hall current on both
 210 primary and secondary velocity. Increase in hall current parameter results to decrease in
 primary velocity and vice versa in secondary velocity as it is expected.

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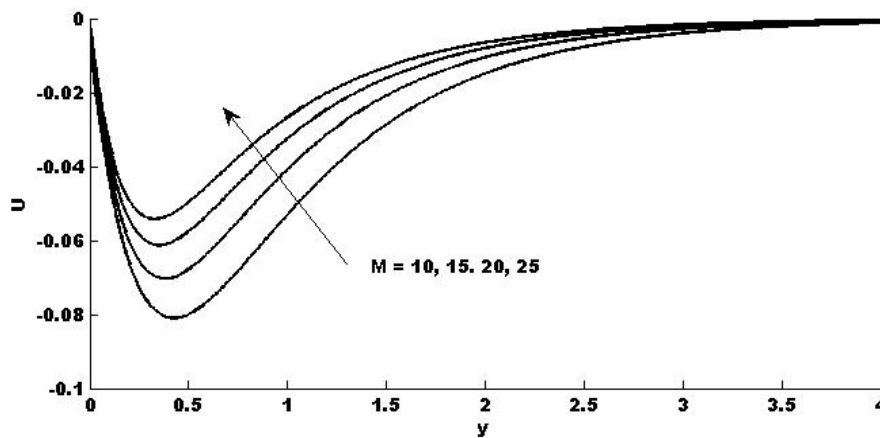


Figure: 9. Effects of Magnetic field M on primary velocity.

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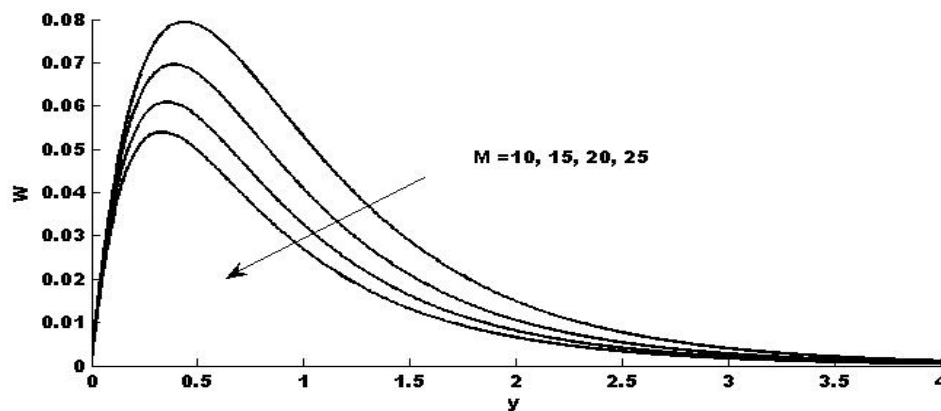


Figure: 10. Effects of Magnetic field M on secondary velocity.

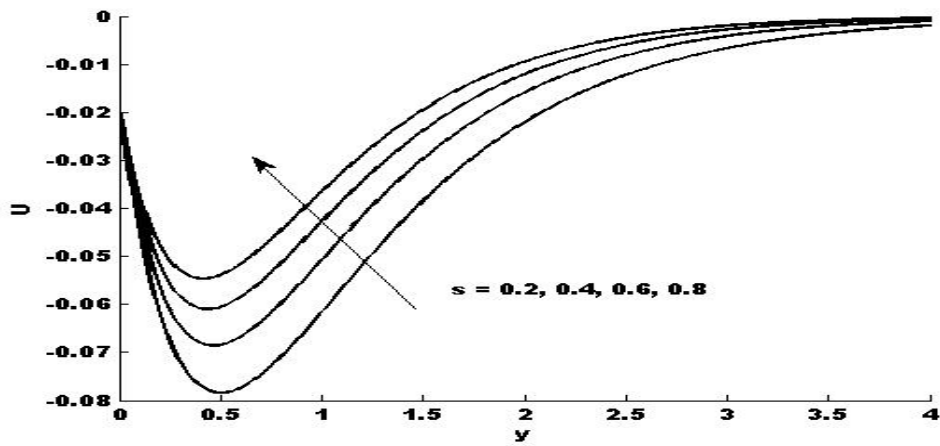


Figure: 11.Effects of heat source s on primary velocity.

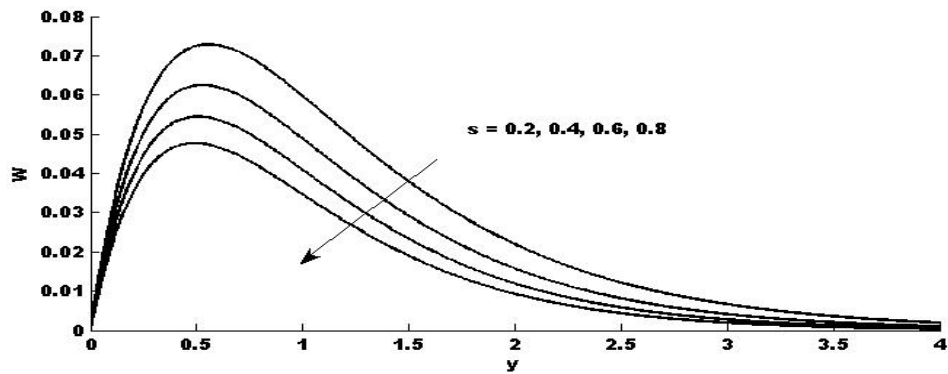


Figure: 12. Effects of heat source s on secondary velocity.

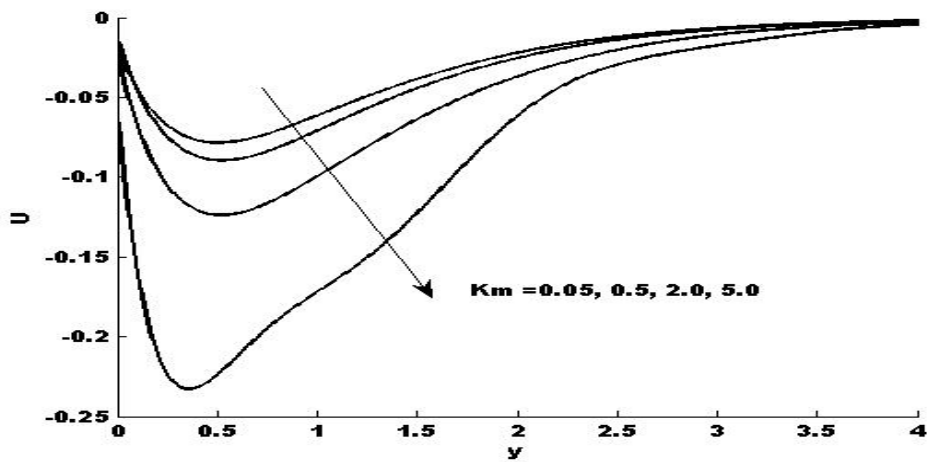


Figure 13. Effects of K_m on primary velocity.

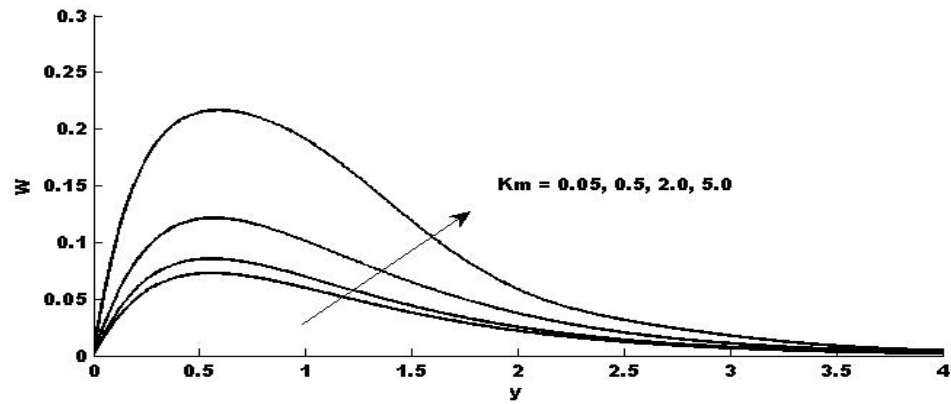


Figure: 14, Effects of Km on secondary velocity.

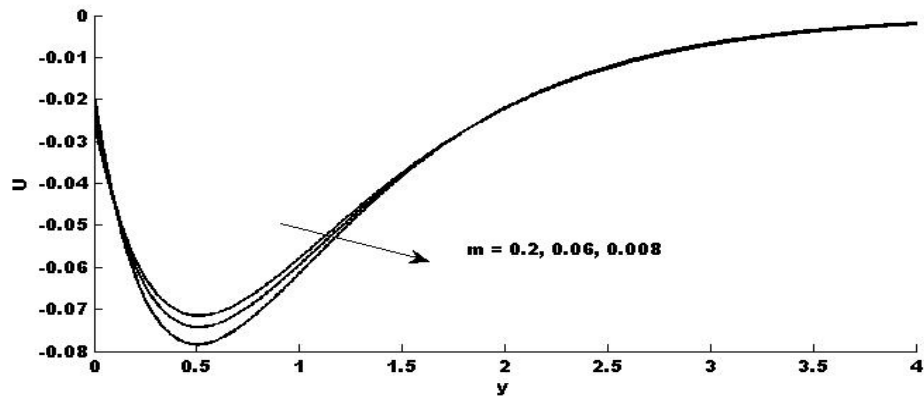


Figure: 15. Effects of hall current m on primary velocity

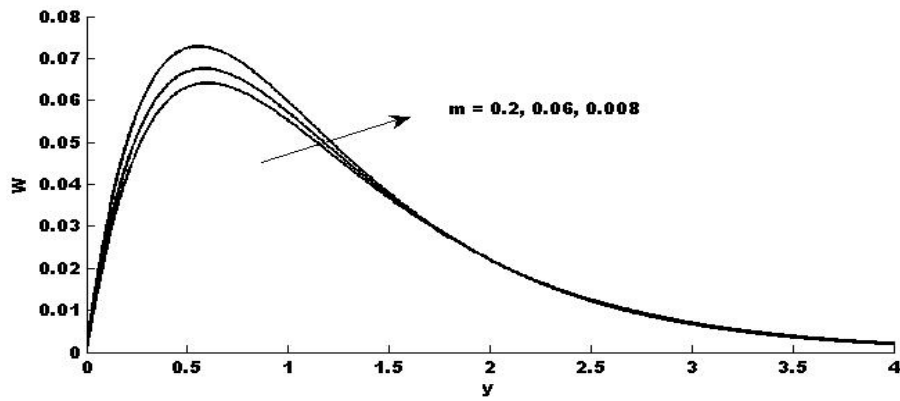


Figure: 16. Effects of hall current (m) on secondary velocity.

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APPENDIX

$$L = \left(\frac{1}{K_s} + \frac{M}{(1+m^2)} \right) \quad J = \frac{Mm}{(1+m^2)} \quad Z = \left(\frac{3+4R}{3Pr} \right) \quad n_1 = \frac{-\frac{Pr}{Z} \pm \sqrt{\left(\frac{Pr}{Z}\right)^2 + 4\frac{S}{Z}}}{2}$$

$$n_3 = \frac{-SC \pm \sqrt{(SC)^2 + 4SCK}}{2} \quad n_5 = \frac{-1 \pm \sqrt{1+4N}}{2} \quad A_{11} = \frac{-Gr}{(n_1^2 - n_1 - N)} \therefore A_{12} = \frac{-Gc}{(n_3^2 - n_3 - N)}$$

$$283 \quad A_{10} = A_{11} + A_{12} \quad \therefore A_{15} = \frac{n_5^3 A_{10}}{(n_5^2 - n_5 - N)} \quad \therefore A_{17} = \frac{-n_5^3 A_{12}}{(n_5^2 - n_5 - N)}, \quad A_{14} = -A_{15} + A_{16} + A_{17}$$

$$284 \quad n_2 = \frac{-\frac{1}{2} \pm \sqrt{\left(\frac{1}{2}\right)^2 + \frac{4q}{Z}}}{2}, \quad q = (K + i\pi) \quad n_4 = \frac{-SC \pm \sqrt{(SC)^2 + 4SCq}}{2} \quad f_1 = \frac{-1 \pm \sqrt{1 + 4ENn}}{2E}$$

$$285 \quad B_3 = -\frac{Gr}{En_2^2 - n - Nn} \quad B_4 = -\frac{Gc}{En_4^2 - n_4 - Nn} \quad B_2 = B_3 + B_4 \quad f_2 = \frac{-1 \pm \sqrt{1 + 4ENn}}{2E}$$

$$286 \quad B_7 = \frac{f_1^3 B_2}{Ef_1^2 - f_1 - Nn} \quad B_8 = \frac{-n_2^3 B_3}{En_2^2 - n - Nn}, \quad B_9 = \frac{-n_4^3 B_4}{En_4^2 - n_4 - Nn}, \quad B_6 = -B_7 + B_8 + B_9$$

$$287 \quad Nn = Ln + J \quad Ln = L + i\Omega \quad N = (L + J) \quad n_5 = n_6$$