

TABLE_135

Let

$$\begin{aligned} F &= x_1^2 + x_1x_2 + 34x_2^2, \Lambda = 5x_1^2 + 5x_1x_2 + 8x_2^2, \\ \Phi &= 4x_1^2 + 3x_1x_2 + 9x_2^2, \Phi^{-1} = 4x_1^2 - 3x_1x_2 + 9x_2^2, \\ \Psi &= 2x_1^2 + x_1x_2 + 17x_2^2, \Psi^{-1} = 2x_1^2 - x_1x_2 + 17x_2^2 \end{aligned}$$

be the reduced forms of binary quadratic forms of discriminant -135.

Consider the following 21 quadratic forms of 4 variables

$$F_2, \Lambda_2, \Phi_2, \Psi_2, F \oplus \Lambda, F \oplus \Phi, F \oplus \Psi, \Lambda \oplus \Phi, \Lambda \oplus \Psi, \Phi \oplus \Psi,$$

where $F_2 := F \oplus F$. We determine the representation numbers by

$$\begin{aligned} r_{F_2}(n) = & \\ & 2/9\sigma(n) - 8/9\sigma\left(\frac{n}{3}\right) + 10/9\sigma\left(\frac{n}{5}\right) + 8/3\sigma\left(\frac{n}{9}\right) - 40/9\sigma\left(\frac{n}{15}\right) - 6\sigma\left(\frac{n}{27}\right) \\ & + 40/3\sigma\left(\frac{n}{45}\right) - 30\sigma\left(\frac{n}{135}\right) + 2/9\tau_{2,15}(n) + 2/9\tau_{2,15}\left(\frac{n}{3}\right) + 2\tau_{2,15}\left(\frac{n}{9}\right) \\ & + 8/9\tau_{2,27}(n) + 40/9\tau_{2,27}\left(\frac{n}{5}\right) + 8/9\tau_{2,135,2}(n) + (16/117\eta + 20/39)\tau_{2,135,3}(n) \\ & + (-16/117\eta + 44/117)\tau_{2,135,4}(n) + (-16/117\eta + 44/117)\tau_{2,135,5}(n) \\ & + (16/117\eta + 20/39)\tau_{2,135,6}(n), \eta \text{ is a root of } x^2 + x - 3. \end{aligned}$$

$$\begin{aligned} r_{\Lambda_2}(n) = & \\ & 2/9\sigma(n) - 8/9\sigma\left(\frac{n}{3}\right) + 10/9\sigma\left(\frac{n}{5}\right) + 8/3\sigma\left(\frac{n}{9}\right) - 40/9\sigma\left(\frac{n}{15}\right) \\ & - 6\sigma\left(\frac{n}{27}\right) + 40/3\sigma\left(\frac{n}{45}\right) - 30\sigma\left(\frac{n}{135}\right) - 2/9\tau_{2,15}(n) - 2/9\tau_{2,15}\left(\frac{n}{3}\right) \\ & - 2\tau_{2,15}\left(\frac{n}{9}\right) + 8/9\tau_{2,27}(n) + 40/9\tau_{2,27}\left(\frac{n}{5}\right) - 8/9\tau_{2,135}\left(\frac{n}{2}\right) \\ & + (-16/117\eta - 20/39)\tau_{2,135,3}(n) + (16/117\eta - 44/117)\tau_{2,135,4}(n) \\ & + (-16/117\eta + 44/117)\tau_{2,135,5}(n) + (16/117 * \eta + 20/39)\tau_{2,135,6}(n) \end{aligned}$$

$$\begin{aligned} r_{\Phi_2}(n) = & \\ & 2/9\sigma(n) - 8/9\sigma\left(\frac{n}{3}\right) + 10/9\sigma\left(\frac{n}{5}\right) + 8/3\sigma\left(\frac{n}{9}\right) - 40/9\sigma\left(\frac{n}{15}\right) \\ & - 6\sigma\left(\frac{n}{27}\right) + 40/3\sigma\left(\frac{n}{45}\right) - 30\sigma\left(\frac{n}{135}\right) + 2/9\tau_{2,15}(n) + 2/9\tau_{2,15}\left(\frac{n}{3}\right) \\ & + 2\tau_{2,15}\left(\frac{n}{9}\right) - 4/9\tau_{2,27}(n) - 20/9\tau_{2,27}\left(\frac{n}{5}\right) - 4/9\tau_{2,135,2}(n) \\ & + (-14/117\eta + 2/39)\tau_{2,135,3}(n) + (14/117\eta + 20/117)\tau_{2,135,4}(n) \\ & + (-22/117\eta + 2/117)\tau_{2,135,5}(n) + (22/117\eta + 8/39)\tau_{2,135,6}(n) \end{aligned}$$

$$r_{\psi_2}(n) =$$

$$\begin{aligned} & 2/9\sigma(n) - 8/9\sigma\left(\frac{n}{3}\right) + 10/9\sigma\left(\frac{n}{5}\right) + 8/3\sigma\left(\frac{n}{9}\right) - 40/9\sigma\left(\frac{n}{15}\right) - 6\sigma\left(\frac{n}{27}\right) \\ & + 40/3\sigma\left(\frac{n}{45}\right) - 30\sigma\left(\frac{n}{135}\right) - 2/9\tau_{2,15}(n) - 2/9\tau_{2,15}\left(\frac{n}{3}\right) - 2\tau_{2,15}\left(\frac{n}{9}\right) \\ & - 4/9\tau_{2,27}(n) - 20/9\tau_{2,27}\left(\frac{n}{5}\right) + 4/9\tau_{2,135,2}(n) + (14/117\eta - 2/39)\tau_{2,135,3}(n) \\ & + (-14/117\eta - 20/117)\tau_{2,135,4}(n) + (-22/117\eta + 2/117)\tau_{2,135,5}(n) \\ & + (22/117\eta + 8/39)\tau_{2,135,6}(n) \end{aligned}$$

$$r_{F\oplus\Lambda}(n) =$$

$$\begin{aligned} & 1/6\sigma(n) + 1/3\sigma\left(\frac{n}{3}\right) - 5/6\sigma\left(\frac{n}{5}\right) + \sigma\left(\frac{n}{9}\right) - 5/3\sigma\left(\frac{n}{15}\right) + 9/2\sigma\left(\frac{n}{27}\right) \\ & - 5\sigma\left(\frac{n}{45}\right) - 45/2\sigma\left(\frac{n}{135}\right) + 17/18\tau_{2,15}(n) - 1/18\tau_{2,15}\left(\frac{n}{3}\right) + 17/2\tau_{2,15}\left(\frac{n}{9}\right) \\ & + 4/9\tau_{2,135,2}(n) + (-4/117\eta + 8/39)\tau_{2,135,3}(n) + (4/117\eta + 28/117)\tau_{2,135,4}(n) \end{aligned}$$

$$r_{F\oplus\Phi}(n) =$$

$$\begin{aligned} & 2/9\sigma(n) - 8/9\sigma\left(\frac{n}{3}\right) + 10/9\sigma\left(\frac{n}{5}\right) + 8/3\sigma\left(\frac{n}{9}\right) - 40/9\sigma\left(\frac{n}{15}\right) \\ & - 6\sigma\left(\frac{n}{27}\right) + 40/3\sigma\left(\frac{n}{45}\right) - 30\sigma\left(\frac{n}{135}\right) + 2/9\tau_{2,15}(n) + 2/9\tau_{2,15}\left(\frac{n}{3}\right) \\ & + 2\tau_{2,15}\left(\frac{n}{9}\right) + 2/9\tau_{2,27}(n) + 10/9\tau_{2,27}\left(\frac{n}{5}\right) + 2/9\tau_{2,135,2}(n) \\ & + (-17/117\eta + 8/39)\tau_{2,135,3}(n) + (17/117\eta + 41/117)\tau_{2,135,4}(n) \\ & + (-1/117\eta + 32/117)\tau_{2,135,5}(n) + (1/117\eta + 11/39)\tau_{2,135,6}(n) \end{aligned}$$

$$r_{F\oplus\Psi}(n) =$$

$$\begin{aligned} & 1/6\sigma(n) + 1/3\sigma\left(\frac{n}{3}\right) - 5/6\sigma\left(\frac{n}{5}\right) + \sigma\left(\frac{n}{9}\right) - 5/3\sigma\left(\frac{n}{15}\right) + 9/2\sigma\left(\frac{n}{27}\right) \\ & - 5\sigma\left(\frac{n}{45}\right) - 45/2\sigma\left(\frac{n}{135}\right) + 5/18\tau_{2,15}(n) + 23/18\tau_{2,15}\left(\frac{n}{3}\right) \\ & + 5/2\tau_{2,15}\left(\frac{n}{9}\right) + 2/3\tau_{2,45}(n) + 2\tau_{2,45}\left(\frac{n}{3}\right) + 4/9\tau_{2,135,2}(n) \\ & + (-1/117\eta + 2/39)\tau_{2,135,3}(n) + (1/117\eta + 7/117)\tau_{2,135,4}(n) + (-1/117\eta \\ & + 2/39)\tau_{2,135,5}(n) + (1/117\eta + 7/117)\tau_{2,135,6}(n) \end{aligned}$$

$$r_{\Lambda\oplus\Phi}(n) =$$

$$\begin{aligned}
& 1/6\sigma(n) + 1/3\sigma\left(\frac{n}{3}\right) - 5/6\sigma\left(\frac{n}{5}\right) + \sigma\left(\frac{n}{9}\right) - 5/3\sigma\left(\frac{n}{15}\right) + 9/2\sigma\left(\frac{n}{27}\right) \\
& - 5\sigma\left(\frac{n}{45}\right) - 45/2\sigma\left(\frac{n}{135}\right) + 5/18\tau_{2,15}(n) + 23/18\tau_{2,15}\left(\frac{n}{3}\right) \\
& + 5/2\tau_{2,15}\left(\frac{n}{9}\right) - 2/3\tau_{2,45}(n) - 2\tau_{2,45}\left(\frac{n}{3}\right) + 4/9\tau_{2,135,2}(n) \\
& + (-1/117\eta + 2/39)\tau_{2,135,3}(n) + (1/117\eta + 7/117)\tau_{2,135,4}(n) \\
& + (1/117\eta - 2/39)\tau_{2,135,5}(n) + (-1/117\eta - 7/117)\tau_{2,135,6}(n)
\end{aligned}$$

$$r_{\Lambda \oplus \Psi}(n) =$$

$$\begin{aligned}
& 2/9\sigma(n) - 8/9\sigma\left(\frac{n}{3}\right) + 10/9\sigma\left(\frac{n}{5}\right) + 8/3\sigma\left(\frac{n}{9}\right) - 40/9\sigma\left(\frac{n}{15}\right) \\
& - 6\sigma\left(\frac{n}{27}\right) + 40/3\sigma\left(\frac{n}{45}\right) - 30\sigma\left(\frac{n}{135}\right) - 2/9\tau_{2,15}(n) - 2/9\tau_{2,15}\left(\frac{n}{3}\right) \\
& - 2\tau_{2,15}\left(\frac{n}{9}\right) + 2/9\tau_{2,27}(n) + 10/9\tau_{2,27}\left(\frac{n}{5}\right) - 2/9\tau_{2,135,2}(n) \\
& + (17/117\eta - 8/39)\tau_{2,135,3}(n) + (-17/117\eta - 41/117)\tau_{2,135,4}(n) \\
& + (-1/117\eta + 32/117)\tau_{2,135,5}(n) + (1/117\eta + 11/39)\tau_{2,135,6}(n)
\end{aligned}$$

$$r_{\Phi \oplus \Psi}(n) =$$

$$\begin{aligned}
& 1/6\sigma(n) + 1/3\sigma\left(\frac{n}{3}\right) - 5/6\sigma\left(\frac{n}{5}\right) + \sigma\left(\frac{n}{9}\right) - 5/3\sigma\left(\frac{n}{15}\right) \\
& + 9/2\sigma\left(\frac{n}{27}\right) - 5\sigma\left(\frac{n}{45}\right) - 45/2\sigma\left(\frac{n}{135}\right) - 7/18\tau_{2,15}(n) \\
& - 25/18\tau_{2,15}\left(\frac{n}{3}\right) - 7/2\tau_{2,15}\left(\frac{n}{9}\right) + 4/9\tau_{2,135,2}(n) + (2/117\eta \\
& - 4/39)\tau_{2,135,3}(n) + (-2/117\eta - 14/117)\tau_{2,135,4}(n)
\end{aligned}$$

Now, consider the following 21 quadratic forms of 8 variables

$$F_2 := F \oplus F \oplus F \oplus F,$$

$$\begin{aligned}
& F_4, \Lambda_4, \Phi_4, \Psi_4, F_3 \oplus \Lambda, F_3 \oplus \Phi, F_3 \oplus \Psi, \\
& \Lambda_3 \oplus F, \Lambda_3 \oplus \Phi, \Lambda_3 \oplus \Psi, \Phi_3 \oplus F, \Phi_3 \oplus \Psi, \\
& \Psi_3 \oplus F, \Psi_3 \oplus \Lambda, \Psi_3 \oplus \Phi, F_2 \oplus \Lambda_2, F_2 \oplus \Phi_2 \\
& F_2 \oplus \Psi_2, \Lambda_2 \oplus \Phi_2, \Lambda_2 \oplus \Psi_2, \Phi_2 \oplus \Psi_2.
\end{aligned}$$

We determine the representation numbers by

$$r_{F_4}(n) =$$

$$\begin{aligned}
& 240/21060\sigma_3(n) + 480/5265\sigma_3\left(\frac{n}{3}\right) + 1200/4212\sigma_3\left(\frac{n}{5}\right) \\
& + 480/585\sigma_3\left(\frac{n}{9}\right) + 2400/1053\sigma_3\left(\frac{n}{15}\right) + 2160\sigma_3\left(\frac{n}{27}\right) \\
& + 2400/117\sigma_3\left(\frac{n}{45}\right) + 10800/52\sigma_3\left(\frac{n}{135}\right) \\
& - 64/1053\tau_{4,5}(n) - 4000/7371\tau_{4,5}\left(\frac{n}{3}\right) - 4000/819\tau_{4,5}\left(\frac{n}{9}\right) \\
& - 576/13\tau_{4,5}\left(\frac{n}{27}\right) + 64/135\tau_{4,9}(n) + 64/15\tau_{4,9}\left(\frac{n}{3}\right) \\
& + 320/27\tau_{4,9}\left(\frac{n}{5}\right) + 320/3\tau_{4,9}\left(\frac{n}{15}\right) + 4/9\tau_{4,15,1}(n) \\
& + 4\tau_{4,15,1}\left(\frac{n}{3}\right) + 36\tau_{4,15,1}\left(\frac{n}{9}\right) + 8/9\tau_{4,15,2}(n) \\
& + 248/21\tau_{4,15,2}\left(\frac{n}{3}\right) + 72\tau_{4,15,1}\left(\frac{n}{9}\right) + 64/135\tau_{4,27,1}(n) \\
& + 320/27\tau_{4,27,1}\left(\frac{n}{5}\right) + (56/459\omega + 8/17)\tau_{4,27,3}(n) + (1400/459\omega \\
& + 200/17)\tau_{4,27,3}\left(\frac{n}{5}\right) + (-56/459\omega + 8/17)\tau_{4,27,4}(n) \\
& + (-1400/459\omega + 200/17)\tau_{4,27,4}\left(\frac{n}{5}\right) + 32/27\tau_{4,45,3}(n) \\
& + 32/3\tau_{4,45,3}\left(\frac{n}{3}\right) + 128/405\tau_{4,45,4}(n) + 128/45\tau_{4,45,4}\left(\frac{n}{3}\right) \\
& + 112/459\tau_{4,135,2}(n) + 1/2282985(-35848\alpha^2 \\
& + 106184\alpha + 1067376)\tau_{4,135,5}(n) + (1/2282985(35848\alpha
\end{aligned}$$

$$\begin{aligned}
& + 70336)\beta + 1/2282985(35848\alpha^2 - 35848\alpha \\
& + 242872))\tau_{4,135,6}(n) + (1/2282985(-35848\alpha - 70336)\beta \\
& + 1/2282985(-70336\alpha + 313208))\tau_{4,135,7}(n) \\
& (1/50733(-104\alpha^2 - 4536\alpha + 23184))\tau_{4,135,8}(n) \\
& + (1/50733(104\alpha - 4640)\beta + 1/50733(104\alpha^2 - 104\alpha \\
& + 20792))\tau_{4,135,9}(n) + (1/50733(-104\alpha + 4640)\beta \\
& + 1/50733(4640\alpha + 16152))\tau_{4,135,10}(n) + (-1928/35883\gamma^2 \\
& + 6200/35883\gamma + 10064/11961)\tau_{4,135,11}(n) \\
& + ((1928/35883\gamma - 3440/35883)\delta + 1928/35883\gamma^2 \\
& - 9640/35883\gamma + 14768/35883)\tau_{4,135,12}(n) + ((-1928/35883\gamma \\
& + 3440/35883)\delta + 3440/35883\gamma - 2432/35883)\tau_{4,135,13}(n), \\
& \omega \text{ is a root of } x^2 - 18, \alpha, \beta \text{ are roots of } x^3 - x^2 - 23x + 6, \\
& \delta, \gamma \text{ are roots of } x^3 + 5x^2 - 8x - 30.
\end{aligned}$$

$$r_{\Lambda_4}(n) =$$

$$\begin{aligned}
& 240/21060\sigma_3(n) + 480/5265\sigma_3\left(\frac{n}{3}\right) + 1200/4212\sigma_3\left(\frac{n}{5}\right) \\
& + 480/585\sigma_3\left(\frac{n}{9}\right) + 2400/1053\sigma_3\left(\frac{n}{15}\right) + 2160/260\sigma_3\left(\frac{n}{27}\right) \\
& + 2400/117\sigma_3\left(\frac{n}{45}\right) + 10800/52\sigma_3\left(\frac{n}{135}\right) \\
& - 64/1053\tau_{4,5}(n) - 4000/7371\tau_{4,5}\left(\frac{n}{3}\right) - 4000/819\tau_{4,5}\left(\frac{n}{9}\right) \\
& - 576/13\tau_{4,5}\left(\frac{n}{27}\right) + 64/135\tau_{4,9}(n) + 64/15\tau_{4,9}\left(\frac{n}{3}\right) \\
& + 320/27\tau_{4,9}\left(\frac{n}{5}\right) + 320/3\tau_{4,9}\left(\frac{n}{15}\right) - 4/9\tau_{4,15,1}(n) \\
& - 4\tau_{4,15,1}\left(\frac{n}{3}\right) - 36\tau_{4,15,1}\left(\frac{n}{9}\right) + 8/9\tau_{4,15,2}(n) \\
& + 248/21\tau_{4,15,2}\left(\frac{n}{3}\right) + 72\tau_{4,15,1}\left(\frac{n}{9}\right) + 64/135\tau_{4,27,1}(n) \\
& + 320/27\tau_{4,27,1}\left(\frac{n}{5}\right) + (56/459\omega + 8/17)\tau_{4,27,3}(n) \\
& + (1400/459\omega + 200/17)\tau_{4,27,3}\left(\frac{n}{5}\right) + (-56/459\omega \\
& + 8/17)\tau_{4,27,4}(n) + (-1400/459\omega + 200/17)\tau_{4,27,4}\left(\frac{n}{5}\right) \\
& - 32/27\tau_{4,45,3}(n) - 32/3\tau_{4,45,3}\left(\frac{n}{3}\right) + 128/405\tau_{4,45,4}(n) \\
& + 128/45\tau_{4,45,4}\left(\frac{n}{3}\right) + \tau_{4,135,1}(n) + 112/459\tau_{4,135,2}(n) \\
& + 1/2282985(-35848\alpha^2 + 106184\alpha + 1067376)\tau_{4,135,5}(n) \\
& + (1/2282985(35848\alpha + 70336)\beta + 1/2282985(35848\alpha^2 \\
& - 35848\alpha + 242872))\tau_{4,135,6}(n) + (1/2282985(-35848\alpha \\
& - 70336)\beta + 1/2282985(-70336\alpha + 313208))\tau_{4,135,7}(n) \\
& (1/50733(104\alpha^2 + 4536\alpha - 23184))\tau_{4,135,8}(n) \\
& + (1/50733(-104\alpha + 4640)\beta + 1/50733(-104\alpha^2 \\
& + 104\alpha - 20792))\tau_{4,135,9}(n) + (1/50733(104\alpha - 4640)\beta \\
& + 1/50733(-4640\alpha - 16152))\tau_{4,135,10}(n) \\
& + (1928/35883\gamma^2 - 6200/35883\gamma - 10064/11961)\tau_{4,135,11}(n) \\
& ((-1928/35883\gamma + 3440/35883)\delta - 1928/35883\gamma^2 \\
& + 9640/35883\gamma - 14768/35883)\tau_{4,135,12}(n) + ((1928/35883\gamma \\
& - 3440/35883)\delta - 3440/35883\gamma + 2432/35883)\tau_{4,135,13}(n)
\end{aligned}$$

$$r_{\Phi_4}(n) =$$

$$\begin{aligned}
& 240/21060\sigma_3(n) + 480/5265\sigma_3\left(\frac{n}{3}\right) + 1200/4212\sigma_3\left(\frac{n}{5}\right) \\
& + 480/585\sigma_3\left(\frac{n}{9}\right) + 2400/1053\sigma_3\left(\frac{n}{15}\right) + 2160/260\sigma_3\left(\frac{n}{27}\right) \\
& + 2400/117\sigma_3\left(\frac{n}{45}\right) + 10800/52\sigma_3\left(\frac{n}{135}\right) - 64/1053\tau_{4,5}(n) \\
& - 4000/7371\tau_{4,5}\left(\frac{n}{3}\right) - 4000/819\tau_{4,5}\left(\frac{n}{9}\right) - 576/13\tau_{4,5}\left(\frac{n}{27}\right) \\
& - 32/135\tau_{4,9}(n) - 32/15\tau_{4,9}\left(\frac{n}{3}\right) - 160/27\tau_{4,9}\left(\frac{n}{5}\right) \\
& - 160/3\tau_{4,9}\left(\frac{n}{15}\right) - 4/3\tau_{4,15,1}\left(\frac{n}{3}\right) + 8/7\tau_{4,15,2}\left(\frac{n}{3}\right) \\
& + (28/459\omega + 4/17)\tau_{4,27,3}(n) + (700/459\omega + 100/17)\tau_{4,27,3}\left(\frac{n}{5}\right) \\
& + (-28/459\omega + 4/17)\tau_{4,27,4}(n) + (-700/459\omega \\
& + 100/17)\tau_{4,27,4}\left(\frac{n}{5}\right) - 4/21\tau_{4,45,2}(n) - 12/7\tau_{4,45,2}\left(\frac{n}{3}\right) \\
& + 64/189\tau_{4,45,3}(n) + 64/21\tau_{4,45,3}\left(\frac{n}{3}\right) + 56/405\tau_{4,45,4}(n) \\
& + 56/45\tau_{4,45,4}\left(\frac{n}{3}\right) - 80/459\tau_{4,135,2}(n) + (1/456597(7708\alpha^2
\end{aligned}$$

$$\begin{aligned}
& + 280\alpha - 143400))\tau_{4,135,5}(n) + (1/456597(-7708\alpha + 7988)\beta \\
& + 1/456597(-7708\alpha^2 + 7708\alpha + 33884))\tau_{4,135,6}(n) \\
& + (1/456597(7708\alpha - 7988)\beta + 1/456597(-7988\alpha \\
& + 41872))\tau_{4,135,7}(n) + (1/152199(1828\alpha^2 - 6416\alpha \\
& - 18984))\tau_{4,135,8}(n) + (1/152199(-1828\alpha - 4588)\beta \\
& + 1/152199(-1828\alpha^2 + 1828\alpha + 23060))\tau_{4,135,9}(n) \\
& + (1/152199(1828\alpha + 4588)\beta + 1/152199(4588\alpha \\
& + 18472))\tau_{4,135,10}(n) + (-4/35883\gamma^2 + 1204/35883\gamma \\
& - 1832/11961)\tau_{4,135,11}(n) + ((4/35883\gamma + 1184/35883)\delta \\
& + 4/35883\gamma^2 - 20/35883\gamma - 5528/35883)\tau_{4,135,12}(n) \\
& + ((-4/35883\gamma - 1184/35883)\delta - 1184/35883\gamma \\
& + 392/35883)\tau_{4,135,13}(n)
\end{aligned}$$

$$r_{\Psi_4}(n) =$$

$$\begin{aligned}
& 240/21060\sigma_3(n) + 480/5265\sigma_3\left(\frac{n}{3}\right) + 1200/4212\sigma_3\left(\frac{n}{5}\right) \\
& + 480/585\sigma_3\left(\frac{n}{9}\right) + 2400/1053\sigma_3\left(\frac{n}{15}\right) + 2160/260\sigma_3\left(\frac{n}{27}\right) \\
& + 2400/117\sigma_3\left(\frac{n}{45}\right) + 10800/52\sigma_3\left(\frac{n}{135}\right) - 64/1053\tau_{4,5}(n) \\
& - 4000/7371\tau_{4,5}\left(\frac{n}{3}\right) - 4000/819\tau_{4,5}\left(\frac{n}{9}\right) - 576/13\tau_{4,5}\left(\frac{n}{27}\right) \\
& - 32/135\tau_{4,9}(n) - 32/15\tau_{4,9}\left(\frac{n}{3}\right) - 160/27\tau_{4,9}\left(\frac{n}{5}\right) \\
& - 160/3\tau_{4,9}\left(\frac{n}{15}\right) + 4/3\tau_{4,15,1}\left(\frac{n}{3}\right) + 8/7\tau_{4,15,2}\left(\frac{n}{3}\right) \\
& + (28/459\omega + 4/17)\tau_{4,27,3}(n) + (700/459\omega \\
& + 100/17)\tau_{4,27,3}\left(\frac{n}{5}\right) + (-28/459\omega + 4/17)\tau_{4,27,4}(n) \\
& + (-700/459\omega + 100/17)\tau_{4,27,4}\left(\frac{n}{5}\right) + 4/21\tau_{4,45,2}(n) \\
& + 12/7\tau_{4,45,2}\left(\frac{n}{3}\right) - 64/189\tau_{4,45,3}(n) - 64/21\tau_{4,45,3}\left(\frac{n}{3}\right) \\
& + 56/405\tau_{4,45,4}(n) + 56/45\tau_{4,45,4}\left(\frac{n}{3}\right) - 80/459\tau_{4,135,2}(n)
\end{aligned}$$

$$\begin{aligned}
& + (1/456597(7708\alpha^2 + 280\alpha - 143400))\tau_{4,135,5}(n) \\
& + (1/456597(-7708\alpha + 7988)\beta + 1/456597(-7708\alpha^2 \\
& + 7708\alpha + 33884))\tau_{4,135,6}(n) + (1/456597(7708\alpha \\
& - 7988)\beta + 1/456597(-7988\alpha + 41872))\tau_{4,135,7}(n) \\
& + (1/152199(-1828\alpha^2 + 6416\alpha + 18984))\tau_{4,135,8}(n) \\
& + (1/152199(1828\alpha + 4588)\beta + 1/152199(1828\alpha^2 \\
& - 1828\alpha - 23060))\tau_{4,135,9}(n) + (1/152199(-1828\alpha \\
& - 4588)\beta + 1/152199(-4588\alpha - 18472))\tau_{4,135,10}(n) \\
& + (4/35883\gamma^2 - 1204/35883\gamma + 1832/11961)\tau_{4,135,11}(n) \\
& + ((-4/35883\gamma - 1184/35883)\delta - 4/35883\gamma^2 \\
& + 20/35883\gamma + 5528/35883)\tau_{4,135,12}(n) + ((4/35883\gamma \\
& + 1184/35883)\delta + 1184/35883\gamma - 392/35883)\tau_{4,135,13}(n)
\end{aligned}$$

$$r_{F_3 \oplus \Lambda_1}(n) =$$

$$\begin{aligned}
& 240/15552\sigma_3(n) - 1200/7776\sigma_3\left(\frac{n}{3}\right) \\
& -6000/15552\sigma_3\left(\frac{n}{5}\right) + 1200/864\sigma_3\left(\frac{n}{9}\right) \\
& +30\,000/7776\sigma_3\left(\frac{n}{15}\right) - 720/64\sigma_3\left(\frac{n}{27}\right) \\
& -30000/864\sigma_3\left(\frac{n}{45}\right) + 18\,000/64\sigma_3\left(\frac{n}{135}\right) - 8/567\tau_{4,5}(n) \\
& -164/567\tau_{4,5}\left(\frac{n}{3}\right) - 164/63\tau_{4,5}\left(\frac{n}{9}\right) - 72/7\tau_{4,5}\left(\frac{n}{27}\right) \\
& +32/405\tau_{4,9}(n) + 88/45\tau_{4,9}\left(\frac{n}{3}\right) + 440/81\tau_{4,9}\left(\frac{n}{5}\right) \\
& +160/9\tau_{4,9}\left(\frac{n}{15}\right) + 19/36\tau_{4,15,1}(n) - 17/12\tau_{4,15,1}\left(\frac{n}{3}\right) \\
& +171/4\tau_{4,15,1}\left(\frac{n}{9}\right) + 2/7\tau_{4,15,2}(n) + 62/21\tau_{4,15,2}\left(\frac{n}{3}\right) \\
& +162/7\tau_{4,15,2}\left(\frac{n}{9}\right) + 4/15\tau_{4,27,1}(n) + 20/3\tau_{4,27,1}\left(\frac{n}{5}\right) \\
& +68/405\tau_{4,27,2}(n) - 340/81\tau_{4,27,2}\left(\frac{n}{5}\right) + (106/1377\omega \\
& +112/459)\tau_{4,27,3}(n) + (2650/1377\omega + 2800/459)\tau_{4,27,3}\left(\frac{n}{5}\right) \\
& +(-106/1377\omega + 112/459)\tau_{4,27,4}(n) + (-2650/1377\omega \\
& +2000/459)\tau_{4,27,4}\left(\frac{n}{5}\right) + 4/9\tau_{4,45,3}(n) + 4\tau_{4,45,3}\left(\frac{n}{3}\right) \\
& +8/81\tau_{4,45,4}(n) + 8/9\tau_{4,45,4}\left(\frac{n}{3}\right) + 16/135\tau_{4,45,5}(n) \\
& -16/15\tau_{4,45,5}\left(\frac{n}{3}\right) 116/459\tau_{4,135,2}(n) + (1/2282985(-41252\alpha^2
\end{aligned}$$

$$\begin{aligned}
& +86716\alpha + 978144))\tau_{4,135,5}(n) + (1/2282985(41252\alpha \\
& +45464)\beta + 1/2282985(41252\alpha^2 - 41252\alpha \\
& +29348))\tau_{4,135,6}(n) + (1/2282985(-41252\alpha - 45464)\beta \\
& +1/2282985(-45464\alpha + 74812))\tau_{4,135,7}(n) \\
& +(1/760995(-15844\alpha^2 - 44088\alpha + 596128))\tau_{4,135,8}(n) \\
& +(1/760995(15844\alpha - 59932)\beta + 1/760995(15844\alpha^2 \\
& -15844\alpha + 231716))\tau_{4,135,9}(n) + (1/760995(-15844\alpha \\
& +59932)\beta + 1/760995(59932\alpha + 171784))\tau_{4,135,10}(n) \\
& +(-1262/35883\gamma^2 + 2426/35883\gamma + 3436/3987)\tau_{4,135,11}(n) \\
& +((1262/35883\gamma - 3884/35883)\delta + 1262/35883\gamma^2 \\
& -6310/35883\gamma + 20828/35883)\tau_{4,135,12}(n) + ((-1262/35883\gamma \\
& +3884/35883)\delta + 3884/35883\gamma + 1408/35883)\tau_{4,135,13}(n)
\end{aligned}$$

$$r_{F_3 \oplus \Phi_1}(n) =$$

$$\begin{aligned}
& 240/21060\sigma_3(n) + 480/5265\sigma_3\left(\frac{n}{3}\right) + 1200/4212\sigma_3\left(\frac{n}{5}\right) \\
& + 480/585\sigma_3\left(\frac{n}{9}\right) + 2400/1053\sigma_3\left(\frac{n}{15}\right) + 2160/260\sigma_3\left(\frac{n}{27}\right) \\
& + 2400/117\sigma_3\left(\frac{n}{45}\right) + 10800/52\sigma_3\left(\frac{n}{135}\right) - 64/1053\tau_{4,5}(n) \\
& - 4000/7371\tau_{4,5}\left(\frac{n}{3}\right) - 4000/819\tau_{4,5}\left(\frac{n}{9}\right) - 576/13\tau_{4,5}\left(\frac{n}{27}\right) \\
& + 8/27\tau_{4,9}(n) + 8/3\tau_{4,9}\left(\frac{n}{3}\right) + 200/27\tau_{4,9}\left(\frac{n}{5}\right) + 200/3\tau_{4,9}\left(\frac{n}{15}\right) \\
& + 1/2\tau_{4,15,1}(n) - 5/3\tau_{4,15,1}\left(\frac{n}{3}\right) + 81/2\tau_{4,15,1}\left(\frac{n}{9}\right) + 1/2\tau_{4,15,2}(n) \\
& + 29/7\tau_{4,15,2}\left(\frac{n}{3}\right) + 81/2\tau_{4,15,1}\left(\frac{n}{9}\right) + 16/45\tau_{4,27,1}(n) \\
& + 80/9\tau_{4,27,1}\left(\frac{n}{5}\right) + (2/27\omega + 1/3)\tau_{4,27,3}(n) + (50/27\omega \\
& + 25/3)\tau_{4,27,3}\left(\frac{n}{5}\right) + (-2/27\omega + 1/3)\tau_{4,27,4}(n) + (-50/27\omega \\
& + 25/3)\tau_{4,27,4}\left(\frac{n}{5}\right) + 5/18\tau_{4,45,1}(n) - 5/2\tau_{4,45,1}\left(\frac{n}{3}\right) + 1/42\tau_{4,45,2}(n) \\
& + 3/14\tau_{4,45,2}\left(\frac{n}{3}\right) + 104/189\tau_{4,45,3}(n) + 104/21\tau_{4,45,3}\left(\frac{n}{3}\right) \\
& + 13/81\tau_{4,45,4}(n) + 13/9\tau_{4,45,4}\left(\frac{n}{3}\right) + 1/9\tau_{4,45,5}(n) - \tau_{4,45,5}\left(\frac{n}{3}\right) \\
& + 8/27\tau_{4,135,2}(n) + (1/2282985(-30316\alpha^2 + 73853)\alpha \\
& + 828012)\tau_{4,135,5}(n) + (1/2282985(30316\alpha + 43537)\beta
\end{aligned}$$

$$\begin{aligned}
& + 1/2282985(30316\alpha^2 - 30316\alpha + 130744))\tau_{4,135,6}(n) \\
& + (1/2282985(-30316\alpha - 43537)\beta + 1/2282985(-43537\alpha \\
& + 174281))\tau_{4,135,7}(n) + (1/50733(-500\alpha^2 - 2295\alpha \\
& + 23004))\tau_{4,135,8}(n) + (1/50733(500\alpha - 2795)\beta \\
& + 1/50733(500\alpha^2 - 500\alpha + 11504))\tau_{4,135,9}(n) \\
& + (1/50733(-500\alpha + 2795)\beta + (1/50733(2795\alpha \\
& + 8709))\tau_{4,135,10}(n) + (-665/35883\gamma^2 + 815/35883\gamma \\
& + 6416/11961)\tau_{4,135,11}(n) + ((665/35883\gamma - 2510/35883)\delta \\
& + 665/35883\gamma^2 - 3325/35883\gamma + 13928/35883)\tau_{4,135,12}(n) \\
& + ((-665/35883\gamma + 2510/35883)\delta + 2510/35883\gamma \\
& + 1378/35883)\tau_{4,135,13}(n)
\end{aligned}$$

$$r_{F_3 \oplus \Psi_1}(n) =$$

$$\begin{aligned}
& 240/15552\sigma_3(n) - 1200/7776\sigma_3\left(\frac{n}{3}\right) - 6000/15552\sigma_3\left(\frac{n}{5}\right) \\
& + 1200/864\sigma_3\left(\frac{n}{9}\right) + 30000/7776\sigma_3\left(\frac{n}{15}\right) - 720/64\sigma_3\left(\frac{n}{27}\right) \\
& - 30000/864\sigma_3\left(\frac{n}{45}\right) + 18000/64\sigma_3\left(\frac{n}{135}\right) + 64/567\tau_{4,5}(n) \\
& + 340/567\tau_{4,5}\left(\frac{n}{3}\right) + 340/63\tau_{4,5}\left(\frac{n}{9}\right) + 576/7\tau_{4,5}\left(\frac{n}{27}\right) \\
& + 32/405\tau_{4,9}(n) + 32/9\tau_{4,9}\left(\frac{n}{3}\right) + 800/81\tau_{4,9}\left(\frac{n}{5}\right) \\
& + 160/9\tau_{4,9}\left(\frac{n}{15}\right) + 7/36\tau_{4,15,1}(n) + 7/12\tau_{4,15,1}\left(\frac{n}{3}\right) \\
& + 63/4\tau_{4,15,1}\left(\frac{n}{9}\right) + 17/63\tau_{4,15,2}(n) + 100/21\tau_{4,15,2}\left(\frac{n}{3}\right) \\
& + 153/7\tau_{4,15,2}\left(\frac{n}{9}\right) + 32/135\tau_{4,27,1}(n) + 160/27\tau_{4,27,1}\left(\frac{n}{5}\right) \\
& - 16/405\tau_{4,27,2}(n) + 80/81\tau_{4,27,2}\left(\frac{n}{5}\right) + (124/1377\omega \\
& + 208/459)\tau_{4,27,3}(n) + (3100/1377\omega \\
& + 5200/459)\tau_{4,27,3}\left(\frac{n}{5}\right) + (-124/1377\omega \\
& + 208/459)\tau_{4,27,4}(n) + (-3100/1377\omega \\
& + 5200/459)\tau_{4,27,4}\left(\frac{n}{5}\right) - 1/9\tau_{4,45,1}(n) + \tau_{4,45,1}\left(\frac{n}{3}\right) \\
& + 13/63\tau_{4,45,2}(n) + 13/7\tau_{4,45,2}\left(\frac{n}{3}\right) \\
& + 4/7\tau_{4,45,3}(n) + 36/7\tau_{4,45,3}\left(\frac{n}{3}\right) + 94/405\tau_{4,45,4}(n) \\
& + 94/45\tau_{4,45,4}\left(\frac{n}{3}\right) - 2/135\tau_{4,45,5}(n) + 2/15\tau_{4,45,5}\left(\frac{n}{3}\right) \\
& + 128/459\tau_{4,135,2}(n) + (1/2282985(-30674\alpha^2
\end{aligned}$$

$$\begin{aligned}
& +93217\alpha + 1086468))\tau_{4,135,5}(n) + (1/2282985(30674\alpha \\
& +62543)\beta + 1/2282985(30674\alpha^2 - 30674\alpha \\
& +380966))\tau_{4,135,6}(n) + (1/2282985(-30674\alpha - 62543)\beta \\
& +1/2282985(-62543\alpha + 443509))\tau_{4,135,7}(n) \\
& +(1/760995(-1322\alpha^2 - 49999\alpha + 338644))\tau_{4,135,8}(n) \\
& +(1/760995(1322\alpha - 51321)\beta + 1/760995(1322\alpha^2 \\
& -1322\alpha + 308238))\tau_{4,135,9}(n) + (1/760995(-1322\alpha \\
& +51321)\beta + 1/253665(17107\alpha + 85639))\tau_{4,135,10}(n) \\
& +(-518/35883\gamma^2 + 1754/35883\gamma + 1840/3987)\tau_{4,135,11}(n) \\
& +((518/35883\gamma - 836/35883)\delta + 518/35883\gamma^2 \\
& -2590/35883\gamma + 12416/35883)\tau_{4,135,12}(n) \\
& +((-518/35883\gamma + 836/35883)\delta + 836/35883\gamma \\
& +8236/35883)\tau_{4,135,13}(n)
\end{aligned}$$

$$r_{\Lambda_3 \oplus F_1}(n) =$$

$$\begin{aligned}
& 240/15552\sigma_3(n) - 1200/7776\sigma_3\left(\frac{n}{3}\right) - 6000/15552\sigma_3\left(\frac{n}{5}\right) \\
& + 1200/864\sigma_3\left(\frac{n}{9}\right) + 30000/7776\sigma_3\left(\frac{n}{15}\right) - 720/64\sigma_3\left(\frac{n}{27}\right) \\
& - 30000/864\sigma_3\left(\frac{n}{45}\right) + 18000/64\sigma_3\left(\frac{n}{135}\right) + 8/567\tau_{4,5}(n) \\
& + 164/567\tau_{4,5}\left(\frac{n}{3}\right) + 164/63\tau_{4,5}\left(\frac{n}{9}\right) + 72/7\tau_{4,5}\left(\frac{n}{27}\right) \\
& - 88/405\tau_{4,9}(n) - 32/45\tau_{4,9}\left(\frac{n}{3}\right) - 160/81\tau_{4,9}\left(\frac{n}{5}\right) \\
& - 440/9\tau_{4,9}\left(\frac{n}{15}\right) + 19/36\tau_{4,15,1}(n) - 17/12\tau_{4,15,1}\left(\frac{n}{3}\right) \\
& + 171/4\tau_{4,15,1}\left(\frac{n}{9}\right) - 2/7\tau_{4,15,2}(n) - 62/21\tau_{4,15,2}\left(\frac{n}{3}\right) \\
& - 162/7\tau_{4,15,2}\left(\frac{n}{9}\right) - 4/15\tau_{4,27,1}(n) - 20/3\tau_{4,27,1}\left(\frac{n}{5}\right) \\
& + 68/405\tau_{4,27,2}(n) - 340/81\tau_{4,27,2}\left(\frac{n}{5}\right) + (-106/1377\omega \\
& - 112/459)\tau_{4,27,3}(n) + (-2650/1377\omega \\
& - 2800/459)\tau_{4,27,3}\left(\frac{n}{5}\right) + (106/1377\omega - 112/459)\tau_{4,27,4}(n) \\
& + (2650/1377\omega - 2800/459)\tau_{4,27,4}\left(\frac{n}{5}\right) + 4/9\tau_{4,45,3}(n) \\
& + 4\tau_{4,45,3}\left(\frac{n}{3}\right) - 8/81\tau_{4,45,4}(n) - 8/9\tau_{4,45,4}\left(\frac{n}{3}\right) \\
& + 16/135\tau_{4,45,5}(n) - 16/15\tau_{4,45,5}\left(\frac{n}{3}\right) - 116/459\tau_{4,135,2}(n)
\end{aligned}$$

$$\begin{aligned}
& + (1/2282985(41252\alpha^2 - 86716\alpha - 978144))\tau_{4,135,5}(n) \\
& + (1/2282985(-41252\alpha - 45464)\beta + 1/2282985(-41252\alpha^2 \\
& + 41252\alpha - 29348))\tau_{4,135,6}(n) + (1/2282985(41252\alpha \\
& + 45464)\beta + 1/2282985(45464\alpha - 74812))\tau_{4,135,7}(n) \\
& + (1/760995(-15844\alpha^2 - 44088\alpha + 596128))\tau_{4,135,8}(n) \\
& + (1/760995(15844\alpha - 59932)\beta + 1/760995(15844\alpha^2 \\
& - 15844\alpha + 231716))\tau_{4,135,9}(n) + (1/760995(-15844\alpha \\
& + 59932)\beta + 1/760995(59932\alpha + 171784))\tau_{4,135,10}(n) \\
& + (-1262/35883\gamma^2 + 2426/35883\gamma + 3436/3987)\tau_{4,135,11}(n) \\
& ((1262/35883\gamma - 3884/35883)\delta + 1262/35883\gamma^2 \\
& - 6310/35883\gamma + 20828/35883)\tau_{4,135,12}(n) + ((-1262/35883\gamma \\
& + 3884/35883)\delta + 3884/35883\gamma + 1408/35883)\tau_{4,135,13}(n)
\end{aligned}$$

$$r_{\Lambda_3 \oplus \Phi_1}(n) =$$

$$\begin{aligned}
& 240/15552\sigma_3(n) - 1200/7776\sigma_3\left(\frac{n}{3}\right) \\
& -6000/15552\sigma_3\left(\frac{n}{5}\right) + 1200/864\sigma_3\left(\frac{n}{9}\right) \\
& +30000/7776\sigma_3\left(\frac{n}{15}\right) - 720/64\sigma_3\left(\frac{n}{27}\right) \\
& -30000/864\sigma_3\left(\frac{n}{45}\right) + 18000/64\sigma_3\left(\frac{n}{135}\right) \\
& -64/567\tau_{4,5}(n) - 340/567\tau_{4,5}\left(\frac{n}{3}\right) - 340/63\tau_{4,5}\left(\frac{n}{9}\right) \\
& -576/7\tau_{4,5}\left(\frac{n}{27}\right) - 32/81\tau_{4,9}(n) - 32/45\tau_{4,9}\left(\frac{n}{3}\right) \\
& -160/81\tau_{4,9}\left(\frac{n}{5}\right) - 800/9\tau_{4,9}\left(\frac{n}{15}\right) + 7/36\tau_{4,15,1}(n) \\
& +7/12\tau_{4,15,1}\left(\frac{n}{3}\right) + 63/4\tau_{4,15,1}\left(\frac{n}{9}\right) - 17/63\tau_{4,15,2}(n) \\
& -100/21\tau_{4,15,2}\left(\frac{n}{3}\right) - 153/7\tau_{4,15,2}\left(\frac{n}{9}\right) - 32/135\tau_{4,27,1}(n) \\
& -160/27\tau_{4,27,1}\left(\frac{n}{5}\right) - 16/405\tau_{4,27,2}(n) + 80/81\tau_{4,27,2}\left(\frac{n}{5}\right) \\
& +(-124/1377\omega - 208/459)\tau_{4,27,3}(n) + (-3100/1377\omega \\
& -5200/459)\tau_{4,27,3}\left(\frac{n}{5}\right) + (124/1377\omega - 208/459)\tau_{4,27,4}(n) \\
& +(3100/1377\omega - 5200/459)\tau_{4,27,4}\left(\frac{n}{5}\right) \\
& +1/9\tau_{4,45,1}(n) - \tau_{4,45,1}\left(\frac{n}{3}\right) + 13/63\tau_{4,45,2}(n) + 13/7\tau_{4,45,2}\left(\frac{n}{3}\right) \\
& +4/7\tau_{4,45,3}(n) + 36/7\tau_{4,45,3}\left(\frac{n}{3}\right) - 94/405\tau_{4,45,4}(n) \\
& -94/45\tau_{4,45,4}\left(\frac{n}{3}\right) - 2/135\tau_{4,45,5}(n) + 2/15\tau_{4,45,5}\left(\frac{n}{3}\right) \\
& -128/459\tau_{4,135,2}(n) + (1/2282985(30674\alpha^2 - 93217\alpha
\end{aligned}$$

$$\begin{aligned}
& -1086468))\tau_{4,135,5}(n) + (1/2282985(-30674\alpha - 62543)\beta \\
& + 1/2282985(-30674\alpha^2 + 30674\alpha - 380966))\tau_{4,135,6}(n) \\
& + (1/2282985(30674\alpha + 62543)\beta + 1/2282985(62543\alpha \\
& - 443509))\tau_{4,135,7}(n) + (1/760995(-1322\alpha^2 - 49999\alpha \\
& + 338644))\tau_{4,135,8}(n) + (1/760995(1322\alpha - 51321)\beta \\
& + 1/760995(1322\alpha^2 - 1322\alpha + 308238))\tau_{4,135,9}(n) \\
& + (1/760995(-1322\alpha + 51321)\beta + 1/253665(17107\alpha \\
& + 85639))\tau_{4,135,10}(n) + (-518/35883\gamma^2 + 1754/35883\gamma \\
& + 1840/3987)\tau_{4,135,11}(n) + ((518/35883\gamma - 836/35883)\delta \\
& + 518/35883\gamma^2 - 2590/35883\gamma + 12416/35883)\tau_{4,135,12}(n) \\
& + ((-518/35883\gamma + 836/35883)\delta + 836/35883\gamma \\
& + 8236/35883)\tau_{4,135,13}(n)
\end{aligned}$$

$$r_{\Lambda_3 \oplus \Psi_1}(n) =$$

$$\begin{aligned}
& 240/21060\sigma_3(n) + 480/5265\sigma_3\left(\frac{n}{3}\right) + 1200/4212\sigma_3\left(\frac{n}{5}\right) \\
& + 480/585\sigma_3\left(\frac{n}{9}\right) + 2400/1053\sigma_3\left(\frac{n}{15}\right) \\
& + 2160/260\sigma_3\left(\frac{n}{27}\right) + 2400/117\sigma_3\left(\frac{n}{45}\right) \\
& + 10800/52\sigma_3\left(\frac{n}{135}\right) - 64/1053\tau_{4,5}(n) - 4000/7371\tau_{4,5}\left(\frac{n}{3}\right) \\
& - 4000/819\tau_{4,5}\left(\frac{n}{9}\right) - 576/13\tau_{4,5}\left(\frac{n}{27}\right) + 8/27\tau_{4,9}(n) \\
& + 8/3\tau_{4,9}\left(\frac{n}{3}\right) + 200/27\tau_{4,9}\left(\frac{n}{5}\right) + 200/3\tau_{4,9}\left(\frac{n}{15}\right) \\
& - 1/2\tau_{4,15,1}(n) + 5/3\tau_{4,15,1}\left(\frac{n}{3}\right) - 81/2\tau_{4,15,1}\left(\frac{n}{9}\right) \\
& + 1/2\tau_{4,15,2}(n) + 29/7\tau_{4,15,2}\left(\frac{n}{3}\right) + 81/2\tau_{4,15,1}\left(\frac{n}{9}\right) \\
& + 16/45\tau_{4,27,1}(n) + 80/9\tau_{4,27,1}\left(\frac{n}{5}\right) + (2/27\omega + 1/3)\tau_{4,27,3}(n) \\
& + (50/27\omega + 25/3)\tau_{4,27,3}\left(\frac{n}{5}\right) + (-2/27\omega + 1/3)\tau_{4,27,4}(n) \\
& + (-50/27\omega + 25/3)\tau_{4,27,4}\left(\frac{n}{5}\right) + 5/18\tau_{4,45,1}(n) - 5/2\tau_{4,45,1}\left(\frac{n}{3}\right) \\
& - 1/42\tau_{4,45,2}(n) - 3/14\tau_{4,45,2}\left(\frac{n}{3}\right) - 104/189\tau_{4,45,3}(n) \\
& - 104/21\tau_{4,45,3}\left(\frac{n}{3}\right) + 13/81\tau_{4,45,4}(n) + 13/9\tau_{4,45,4}\left(\frac{n}{3}\right) \\
& - 1/9\tau_{4,45,5}(n) + \tau_{4,45,5}\left(\frac{n}{3}\right) + 8/27\tau_{4,135,2}(n) \\
& - 13928/35883\tau_{4,135,12}(n) + ((665/35883\gamma - 2510/35883)\delta
\end{aligned}$$

$$\begin{aligned}
& + (1/2282985(-30316\alpha^2 + 73853\alpha + 828012))\tau_{4,135,5}(n) \\
& + (1/2282985(30316\alpha + 43537)\beta + 1/2282985(30316\alpha^2 \\
& - 30316\alpha + 130744))\tau_{4,135,6}(n) + (1/2282985(-30316\alpha \\
& - 43537)\beta + 1/2282985(-43537\alpha + 174281))\tau_{4,135,7}(n) \\
& + (1/50733(500\alpha^2 + 2295\alpha - 23004))\tau_{4,135,8}(n) \\
& + (1/50733(-500\alpha + 2795)\beta + 1/50733(-500\alpha^2 + 500\alpha \\
& - 11504))\tau_{4,135,9}(n) + (1/50733(500\alpha - 2795)\beta \\
& + 1/50733(-2795\alpha - 8709))\tau_{4,135,10}(n) + (665/35883\gamma^2 \\
& - 815/35883\gamma - 6416/11961)\tau_{4,135,11}(n) + ((-665/35883\gamma \\
& + 2510/35883)\delta - 665/35883\gamma^2 + 3325/35883\gamma \\
& - 2510/35883\gamma - 1378/35883)\tau_{4,135,13}
\end{aligned}$$

$$r_{\Phi_3 \oplus F_1}(n) =$$

$$\begin{aligned}
& 240/21060\sigma_3(n) + 480/5265\sigma_3\left(\frac{n}{3}\right) + 1200/4212\sigma_3\left(\frac{n}{5}\right) \\
& + 480/585\sigma_3\left(\frac{n}{9}\right) + 2400/1053\sigma_3\left(\frac{n}{15}\right) \\
& + 2160/260\sigma_3\left(\frac{n}{27}\right) + 2400/117\sigma_3\left(\frac{n}{45}\right) \\
& + 10800/52\sigma_3\left(\frac{n}{135}\right) + 488/7371\tau_{4,5}(n) + 2552/7371\tau_{4,5}\left(\frac{n}{3}\right) \\
& + 2552/819\tau_{4,5}\left(\frac{n}{9}\right) + 4392/91\tau_{4,5}\left(\frac{n}{27}\right) - 8/135\tau_{4,9}(n) \\
& - 8/15\tau_{4,9}\left(\frac{n}{3}\right) - 40/27\tau_{4,9}\left(\frac{n}{5}\right) - 40/3\tau_{4,9}\left(\frac{n}{15}\right) + 1/18\tau_{4,15,1}(n) \\
& - 1/3\tau_{4,15,1}\left(\frac{n}{3}\right) + 9/2\tau_{4,15,1}\left(\frac{n}{9}\right) + 19/26\tau_{4,15,2}(n) \\
& - 1/21\tau_{4,15,2}\left(\frac{n}{3}\right) + 171/14\tau_{4,15,1}\left(\frac{n}{9}\right) + 16/135\tau_{4,27,1}(n) \\
& + 80/27\tau_{4,27,1}\left(\frac{n}{5}\right) + (20/459\omega + 11/51)\tau_{4,27,3}(n) + (500/459\omega \\
& + 275/51)\tau_{4,27,3}\left(\frac{n}{5}\right) + (-20/459\omega + 11/51)\tau_{4,27,4}(n) \\
& + (-500/459\omega + 275/51)\tau_{4,27,4}\left(\frac{n}{5}\right) + 1/18\tau_{4,45,1}(n) \\
& - 1/2\tau_{4,45,1}\left(\frac{n}{3}\right) + 3/14\tau_{4,45,2}(n) + 27/14\tau_{4,45,2}\left(\frac{n}{3}\right) \\
& - 16/189\tau_{4,45,3}(n) - 16/21\tau_{4,45,3}\left(\frac{n}{3}\right) + 29/405\tau_{4,45,4}(n) \\
& + 29/45\tau_{4,45,4}\left(\frac{n}{3}\right) + 1/9\tau_{4,45,5}(n) - \tau_{4,45,5}\left(\frac{n}{3}\right) + 40/459\tau_{4,135,2}(n)
\end{aligned}$$

$$\begin{aligned}
& + (1/2282985(15278\alpha^2 + 14921\alpha - 193596))\tau_{4,135,5}(n) \\
& + (1/2282985(-15278\alpha + 30199))\beta + 1/2282985(-15278\alpha^2 \\
& + 15278\alpha + 157798))\tau_{4,135,6}(n) + (1/2282985(15278\alpha \\
& - 30199))\beta + 1/2282985(-30199\alpha + 187997))\tau_{4,135,7}(n) \\
& + (1/50733(-566\alpha^2 + 897\alpha + 11700))\tau_{4,135,8}(n) \\
& + (1/50733(566\alpha + 331))\beta + 1/50733(566\alpha^2 - 566\alpha \\
& - 1318))\tau_{4,135,9}(n) + (1/50733(-566\alpha - 331))\beta \\
& + 1/50733(-331\alpha - 987))\tau_{4,135,10}(n) + (301/35883\gamma^2 \\
& - 2887/35883\gamma + 2300/11961)\tau_{4,135,11}(n) + ((-301/35883\gamma \\
& - 1382/35883)\delta - 301/35883\gamma^2 + 1505/35883\gamma \\
& + 9308/35883)\tau_{4,135,12}(n) + (301/35883\gamma + 1382/35883)\delta \\
& + 1382/35883\gamma + 2398/35883)\tau_{4,135,13}
\end{aligned}$$

$$r_{\Phi_3 \oplus \Lambda_1}(n) =$$

$$\begin{aligned}
& 240/15552\sigma_3(n) - 1200/7776\sigma_3\left(\frac{n}{3}\right) - 6000/15552\sigma_3\left(\frac{n}{5}\right) \\
& + 1200/864\sigma_3\left(\frac{n}{9}\right) + 30000/7776\sigma_3\left(\frac{n}{15}\right) \\
& - 720/64\sigma_3\left(\frac{n}{27}\right) - 30000/864\sigma_3\left(\frac{n}{45}\right) \\
& + 18000/64\sigma_3\left(\frac{n}{135}\right) - 8/567\tau_{4,5}(n) - 164/567\tau_{4,5}\left(\frac{n}{3}\right) \\
& - 164/63\tau_{4,5}\left(\frac{n}{9}\right) - 72/7\tau_{4,5}\left(\frac{n}{27}\right) - 112/405\tau_{4,9}(n) \\
& + 16/45\tau_{4,9}\left(\frac{n}{3}\right) + 80/81\tau_{4,9}\left(\frac{n}{5}\right) - 560/9\tau_{4,9}\left(\frac{n}{15}\right) \\
& + 1/12\tau_{4,15,1}(n) - 1/12\tau_{4,15,1}\left(\frac{n}{3}\right) + 27/4\tau_{4,15,1}\left(\frac{n}{9}\right) \\
& - 1/21\tau_{4,15,2}(n) - 64/21\tau_{4,15,2}\left(\frac{n}{3}\right) - 27/7\tau_{4,15,2}\left(\frac{n}{9}\right) \\
& - 16/135\tau_{4,27,1}(n) - 80/27\tau_{4,27,1}\left(\frac{n}{5}\right) - 16/405\tau_{4,27,2}(n) \\
& + 80/81\tau_{4,27,2}\left(\frac{n}{5}\right) + (-62/1377\omega - 104/459)\tau_{4,27,3}(n) \\
& + (-1550/1377\omega - 2600/459)\tau_{4,27,3}\left(\frac{n}{5}\right) + (162/1377\omega \\
& - 104/459)\tau_{4,27,4}(n) + (1550/1377\omega - 2600/459)\tau_{4,27,4}\left(\frac{n}{5}\right) \\
& + 1/9\tau_{4,45,1}(n) - \tau_{4,45,1}\left(\frac{n}{3}\right) + 8/63\tau_{4,45,2}(n) + 8/7\tau_{4,45,2}\left(\frac{n}{3}\right) \\
& + 20/63\tau_{4,45,3}(n) + 20/7\tau_{4,45,3}\left(\frac{n}{3}\right) - 32/405\tau_{4,45,4}(n) \\
& - 32/45\tau_{4,45,4}\left(\frac{n}{3}\right) - 2/135\tau_{4,45,5}(n) + 2/15\tau_{4,45,5}\left(\frac{n}{3}\right) \\
& - 64/459\tau_{4,135,2}(n) + (1/2282985(15712\alpha^2 - 37841\alpha
\end{aligned}$$

$$\begin{aligned}
& -340644))\tau_{4,135,5}(n) + (1/2282985(-15712\alpha - 22129)\beta \\
& + 1/2282985(-15712\alpha^2 + 15712\alpha + 20732))\tau_{4,135,6}(n) \\
& + (1/2282985(15712\alpha + 22129)\beta + 1/2282985(22129\alpha \\
& - 1397))\tau_{4,135,7}(n) + (1/760995(2668\alpha^2 - 27739\alpha \\
& + 99604))\tau_{4,135,8}(n) + (1/760995(-2668\alpha - 25071)\beta \\
& + 1/760995(-2668\alpha^2 + 2668\alpha + 160968))\tau_{4,135,9}(n) \\
& + (1/760995(2668\alpha + 25071)\beta + 1/253665(8357\alpha \\
& + 45299))\tau_{4,135,10}(n) + (178/35883\gamma^2 - 418/35883\gamma \\
& + 4/3987)\tau_{4,135,11}(n) + (((-178/35883\gamma + 472/35883)\delta \\
& - 178/35883\gamma^2 + 890/35883\gamma + 1460/35883)\tau_{4,135,12}(n) \\
& + ((178/35883\gamma - 472/35883)\delta - 472/35883\gamma \\
& + 3820/35883)\tau_{4,135,13}
\end{aligned}$$

$$r_{\Phi_3 \oplus \Psi_1}(n) =$$

$$\begin{aligned}
& 240/15552\sigma_3(n) - 1200/7776\sigma_3\left(\frac{n}{3}\right) - 6000/15552\sigma_3\left(\frac{n}{5}\right) \\
& + 1200/864\sigma_3\left(\frac{n}{9}\right) + 30\,000/7776\sigma_3\left(\frac{n}{15}\right) \\
& - 720/64\sigma_3\left(\frac{n}{27}\right) - 30000/864\sigma_3\left(\frac{n}{45}\right) \\
& + 18\,000/64\sigma_3\left(\frac{n}{135}\right) + 4/81\tau_{4,5}(n) + 88/567\tau_{4,5}\left(\frac{n}{3}\right) \\
& + 88/63\tau_{4,5}\left(\frac{n}{9}\right) + 36\tau_{4,5}\left(\frac{n}{27}\right) + 32/405\tau_{4,9}(n) \\
& + 16/45\tau_{4,9}\left(\frac{n}{3}\right) + 80/81\tau_{4,9}\left(\frac{n}{5}\right) + 160/9\tau_{4,9}\left(\frac{n}{15}\right) \\
& - 1/36\tau_{4,15,1}(n) - 3/4\tau_{4,15,1}\left(\frac{n}{3}\right) - 9/4\tau_{4,15,1}\left(\frac{n}{9}\right) \\
& - 10/21\tau_{4,15,2}\left(\frac{n}{3}\right) + 8/405\tau_{4,27,2}(n) - 40/81\tau_{4,27,2}\left(\frac{n}{5}\right) \\
& + (-38/1377\omega - 44/459)\tau_{4,27,3}(n) + (-950/1377\omega \\
& - 1100/459)\tau_{4,27,3}\left(\frac{n}{5}\right) + (38/1377\omega - 44/459)\tau_{4,27,4}(n) \\
& + (950/1377\omega - 1100/459)\tau_{4,27,4}\left(\frac{n}{5}\right) - 1/7\tau_{4,45,2}(n) \\
& - 9/7\tau_{4,45,2}\left(\frac{n}{3}\right) + 16/63\tau_{4,45,3}(n) + 16/7\tau_{4,45,3}\left(\frac{n}{3}\right) \\
& - 14/405\tau_{4,45,4}(n) - 14/45\tau_{4,45,4}\left(\frac{n}{3}\right) + 4/135\tau_{4,45,5}(n) \\
& - 4/15\tau_{4,45,5}\left(\frac{n}{3}\right) + 20/459\tau_{4,135,2}(n) + (1/456597(-1510\alpha^2
\end{aligned}$$

$$\begin{aligned}
& -4864\alpha + 36552))\tau_{4,135,5}(n) + (1/456597(1510\alpha - 6374)\beta \\
& + 1/456597(1510\alpha^2 - 1510\alpha + 1822))\tau_{4,135,6}(n) \\
& + (1/456597(-1510\alpha + 6374)\beta + 1/456597(6374\alpha \\
& - 4552))\tau_{4,135,7}(n) + (1/760995(8086\alpha^2 - 38158\alpha \\
& - 81392))\tau_{4,135,8}(n) + (1/760995(-8086\alpha - 30072)\beta \\
& + 1/760995(-8086\alpha^2 + 8086\alpha + 104586))\tau_{4,135,9}(n) \\
& + (1/760995(8086\alpha + 30072)\beta + 1/253665(10024\alpha \\
& + 24838))\tau_{4,135,10}(n) + (334/35883\gamma^2 + 470/35883\gamma \\
& - 988/3987)\tau_{4,135,11}(n) + ((-334/35883\gamma + 2140/35883)\delta \\
& - 334/35883\gamma^2 + 1670/35883\gamma - 6220/35883)\tau_{4,135,12}(n) \\
& + (334/35883\gamma - 2140/35883)\delta - 2140/35883\gamma \\
& + 4480/35883)\tau_{4,135,13}
\end{aligned}$$

$$r_{\Psi_3 \oplus F_1}(n) =$$

$$\begin{aligned}
& 240/15552\sigma_3(n) - 1200/7776\sigma_3\left(\frac{n}{3}\right) - 6000/15552\sigma_3\left(\frac{n}{5}\right) \\
& + 1200/864\sigma_3\left(\frac{n}{9}\right) + 30000/7776\sigma_3\left(\frac{n}{15}\right) \\
& - 720/64\sigma_3\left(\frac{n}{27}\right) - 30000/864\sigma_3\left(\frac{n}{45}\right) \\
& + 18000/64\sigma_3\left(\frac{n}{135}\right) + 8/567\tau_{4,5}(n) + 164/567\tau_{4,5}\left(\frac{n}{3}\right) \\
& + 164/63\tau_{4,5}\left(\frac{n}{9}\right) + 72/7\tau_{4,5}\left(\frac{n}{27}\right) - 16/405\tau_{4,9}(n) \\
& + 112/45\tau_{4,9}\left(\frac{n}{3}\right) + 560/81\tau_{4,9}\left(\frac{n}{5}\right) - 80/9\tau_{4,9}\left(\frac{n}{15}\right) \\
& + 1/12\tau_{4,15,1}(n) - 1/12\tau_{4,15,1}\left(\frac{n}{3}\right) + 27/4\tau_{4,15,1}\left(\frac{n}{9}\right) \\
& + 1/21\tau_{4,15,2}(n) + 64/21\tau_{4,15,2}\left(\frac{n}{3}\right) + 27/7\tau_{4,15,2}\left(\frac{n}{9}\right) \\
& + 16/135\tau_{4,27,1}(n) + 80/27\tau_{4,27,1}\left(\frac{n}{5}\right) - 16/405\tau_{4,27,2}(n) \\
& + 80/81\tau_{4,27,2}\left(\frac{n}{5}\right) + (62/1377\omega + 104/459)\tau_{4,27,3}(n) \\
& + (1550/1377\omega + 2600/459)\tau_{4,27,3}\left(\frac{n}{5}\right) + (-62/1377\omega \\
& + 104/459)\tau_{4,27,4}(n) + (-1550/1377\omega + 2600/459)\tau_{4,27,4}\left(\frac{n}{5}\right) \\
& - 1/9\tau_{4,45,1}(n) + \tau_{4,45,1}\left(\frac{n}{3}\right) + 8/63\tau_{4,45,2}(n) + 8/7\tau_{4,45,2}\left(\frac{n}{3}\right) \\
& + 20/63\tau_{4,45,3}(n) + 20/7\tau_{4,45,3}\left(\frac{n}{3}\right) + 32/405\tau_{4,45,4}(n) \\
& + 32/45\tau_{4,45,4}\left(\frac{n}{3}\right) - 2/135\tau_{4,45,5}(n) + 2/15\tau_{4,45,5}\left(\frac{n}{3}\right) \\
& + 64/459\tau_{4,135,2}(n) + (1/2282985(-15712\alpha^2 + 37841\alpha
\end{aligned}$$

$$\begin{aligned}
& +340644))\tau_{4,135,5}(n) + (1/2282985(15712\alpha + 22129)\beta \\
& +1/2282985(15712\alpha^2 - 15712\alpha - 20732))\tau_{4,135,6}(n) \\
& + (1/2282985(-15712\alpha - 22129)\beta + 1/2282985(-22129\alpha \\
& +1397))\tau_{4,135,7}(n) + (1/760995(2668\alpha^2 - 27739\alpha \\
& +99604))\tau_{4,135,8}(n) + (1/760995(-2668\alpha - 25071)\beta \\
& +1/760995(-2668\alpha^2 + 2668\alpha + 160968))\tau_{4,135,9}(n) \\
& + (1/760995(2668\alpha + 25071)\beta + 1/253665(8357\alpha \\
& +45299))\tau_{4,135,10}(n) + (178/35883\gamma^2 - 418/35883\gamma \\
& +4/3987)\tau_{4,135,11}(n) + ((-178/35883\gamma + 472/35883)\delta \\
& -178/35883\gamma^2 + 890/35883\gamma + 1460/35883)\tau_{4,135,12}(n) \\
& + ((178/35883\gamma - 472/35883)\delta - 472/35883\gamma \\
& +3820/35883)\tau_{4,135,13}
\end{aligned}$$

$$r_{\Psi_3 \oplus \Lambda_1}(n) =$$

$$\begin{aligned}
& 240/21060\sigma_3(n) + 480/5265\sigma_3\left(\frac{n}{3}\right) + 1200/4212\sigma_3\left(\frac{n}{5}\right) \\
& + 480/585\sigma_3\left(\frac{n}{9}\right) + 2400/1053\sigma_3\left(\frac{n}{15}\right) \\
& + 2160/260\sigma_3\left(\frac{n}{27}\right) + 2400/117\sigma_3\left(\frac{n}{45}\right) \\
& + 10800/52\sigma_3\left(\frac{n}{135}\right) + 488/7371\tau_{4,5}(n) \\
& + 2552/7371\tau_{4,5}\left(\frac{n}{3}\right) + 2552/819\tau_{4,5}\left(\frac{n}{9}\right) \\
& + 4392/91\tau_{4,5}\left(\frac{n}{27}\right) - 8/135\tau_{4,9}(n) - 8/15\tau_{4,9}\left(\frac{n}{3}\right) \\
& - 40/27\tau_{4,9}\left(\frac{n}{5}\right) - 40/3\tau_{4,9}\left(\frac{n}{15}\right) - 1/18\tau_{4,15,1}(n) \\
& + 1/3\tau_{4,15,1}\left(\frac{n}{3}\right) - 9/2\tau_{4,15,1}\left(\frac{n}{9}\right) \\
& + 19/26\tau_{4,15,2}(n) - 1/21\tau_{4,15,2}\left(\frac{n}{3}\right) + 171/14\tau_{4,15,1}\left(\frac{n}{9}\right) \\
& + 16/135\tau_{4,27,1}(n) + 80/27\tau_{4,27,1}\left(\frac{n}{5}\right) + (20/459\omega \\
& + 11/51)\tau_{4,27,3}(n) + (500/459\omega + 275/51)\tau_{4,27,3}\left(\frac{n}{5}\right) \\
& + (-20/459\omega + 11/51)\tau_{4,27,4}(n) + (-500/459\omega \\
& + 275/51)\tau_{4,27,4}\left(\frac{n}{5}\right) + 1/18\tau_{4,45,1}(n) - 1/2\tau_{4,45,1}\left(\frac{n}{3}\right) \\
& - 3/14\tau_{4,45,2}(n) - 27/14\tau_{4,45,2}\left(\frac{n}{3}\right) + 16/189\tau_{4,45,3}(n) \\
& + 16/21\tau_{4,45,3}\left(\frac{n}{3}\right) + 29/405\tau_{4,45,4}(n) + 29/45\tau_{4,45,4}\left(\frac{n}{3}\right) \\
& - 1/9\tau_{4,45,5}(n) + \tau_{4,45,5}\left(\frac{n}{3}\right) + 40/459\tau_{4,135,2}(n)
\end{aligned}$$

$$\begin{aligned}
& +(1/2282985(15278\alpha^2 + 14921\alpha - 193596))\tau_{4,135,5}(n) \\
& +(1/2282985(-15278\alpha + 30199)\beta + 1/2282985(-15278\alpha^2 \\
& + 15278\alpha + 157798))\tau_{4,135,6}(n) + (1/2282985(15278\alpha \\
& - 30199)\beta + 1/2282985(-30199\alpha + 187997))\tau_{4,135,7}(n) \\
& +(1/50733(566\alpha^2 - 897\alpha - 11700))\tau_{4,135,8}(n) \\
& +(1/50733(-566\alpha - 331)\beta + 1/50733(-566\alpha^2 + 566\alpha \\
& + 1318))\tau_{4,135,9}(n) + (1/50733(566\alpha + 331)\beta \\
& + 1/50733(331\alpha + 987))\tau_{4,135,10}(n) + (-301/35883\gamma^2 \\
& + 2887/35883\gamma - 2300/11961)\tau_{4,135,11}(n) + ((301/35883\gamma \\
& + 1382/35883)\delta + 301/35883\gamma^2 - 1505/35883\gamma \\
& - 9308/35883)\tau_{4,135,12}(n) + ((-301/35883\gamma \\
& - 1382/35883)\delta - 1382/35883\gamma - 2398/35883)\tau_{4,135,13}
\end{aligned}$$

$$r_{\Psi_3 \oplus \Phi_1}(n) =$$

$$\begin{aligned}
& 240/15552\sigma_3(n) - 1200/7776\sigma_3\left(\frac{n}{3}\right) - 6000/15552\sigma_3\left(\frac{n}{5}\right) \\
& + 1200/864\sigma_3\left(\frac{n}{9}\right) + 30000/7776\sigma_3\left(\frac{n}{15}\right) \\
& - 720/64\sigma_3\left(\frac{n}{27}\right) - 30000/864\sigma_3\left(\frac{n}{45}\right) \\
& + 18000/64\sigma_3\left(\frac{n}{135}\right) - 4/81\tau_{4,5}(n) - 88/567\tau_{4,5}\left(\frac{n}{3}\right) \\
& - 88/63\tau_{4,5}\left(\frac{n}{9}\right) - 36\tau_{4,5}\left(\frac{n}{27}\right) - 16/405\tau_{4,9}(n) \\
& - 32/45\tau_{4,9}\left(\frac{n}{3}\right) - 160/81\tau_{4,9}\left(\frac{n}{5}\right) - 80/9\tau_{4,9}\left(\frac{n}{15}\right) \\
& - 1/36\tau_{4,15,1}(n) - 3/4\tau_{4,15,1}\left(\frac{n}{3}\right) - 9/4\tau_{4,15,1}\left(\frac{n}{9}\right) \\
& + 10/21\tau_{4,15,2}\left(\frac{n}{3}\right) + 8/405\tau_{4,27,2}(n) - 40/81\tau_{4,27,2}\left(\frac{n}{5}\right) \\
& + (38/1377\omega + 44/459)\tau_{4,27,3}(n) + (950/1377\omega \\
& + 1100/459)\tau_{4,27,3}\left(\frac{n}{5}\right) + (-38/1377\omega + 44/459)\tau_{4,27,4}(n) \\
& + (-950/1377\omega + 1100/459)\tau_{4,27,4}\left(\frac{n}{5}\right) - 1/7\tau_{4,45,2}(n) \\
& - 9/7\tau_{4,45,2}\left(\frac{n}{3}\right) + 16/63\tau_{4,45,3}(n) + 16/7\tau_{4,45,3}\left(\frac{n}{3}\right) \\
& + 14/405\tau_{4,45,4}(n) + 14/45\tau_{4,45,4}\left(\frac{n}{3}\right) + 4/135\tau_{4,45,5}(n) \\
& - 4/15\tau_{4,45,5}\left(\frac{n}{3}\right) - 20/459\tau_{4,135,2}(n) + (1/456597(1510\alpha^2
\end{aligned}$$

$$\begin{aligned}
& +4864\alpha - 36552))\tau_{4,135,5}(n) + (1/456597(-1510\alpha + 6374)\beta \\
& +1/456597(-1510\alpha^2 + 1510\alpha - 1822))\tau_{4,135,6}(n) \\
& +(1/456597(1510\alpha - 6374)\beta + 1/456597(-6374\alpha \\
& +4552))\tau_{4,135,7}(n) + (1/760995(8086\alpha^2 - 38158\alpha \\
& -81392))\tau_{4,135,8}(n) + (1/760995(-8086\alpha - 30072)\beta \\
& +1/760995(-8086\alpha^2 + 8086\alpha + 104586))\tau_{4,135,9}(n) \\
& +(1/760995(8086\alpha + 30072)\beta + 1/253665(10024\alpha \\
& +24838))\tau_{4,135,10}(n) + (334/35883\gamma^2 + 470/35883\gamma \\
& -988/3987)\tau_{4,135,11}(n) + ((-334/35883\gamma + 2140/35883)\delta \\
& -334/35883\gamma^2 + 1670/35883\gamma - 6220/35883)\tau_{4,135,12}(n) \\
& +(334/35883\gamma - 2140/35883)\delta - 2140/35883\gamma \\
& +4480/35883)\tau_{4,135,13}
\end{aligned}$$

$$r_{F_2 \oplus \Lambda_2}(n) =$$

$$\begin{aligned}
& 240/21060\sigma_3(n) + 480/5265\sigma_3\left(\frac{n}{3}\right) + 1200/4212\sigma_3\left(\frac{n}{5}\right) \\
& + 480/585\sigma_3\left(\frac{n}{9}\right) + 2400/1053\sigma_3\left(\frac{n}{15}\right) \\
& + 2160/260\sigma_3\left(\frac{n}{27}\right) + 2400/117\sigma_3\left(\frac{n}{45}\right) \\
& + 10800/52\sigma_3\left(\frac{n}{135}\right) + 440/2457\tau_{4,5}(n) - 1888/2457\tau_{4,5}\left(\frac{n}{3}\right) \\
& - 1888/273\tau_{4,5}\left(\frac{n}{9}\right) + 11880/91\tau_{4,5}\left(\frac{n}{27}\right) + 32/135\tau_{4,9}(n) \\
& + 32/15\tau_{4,9}\left(\frac{n}{3}\right) + 160/27\tau_{4,9}\left(\frac{n}{5}\right) + 160/3\tau_{4,9}\left(\frac{n}{15}\right) \\
& + 44/63\tau_{4,15,2}(n) + 24/7\tau_{4,15,2}\left(\frac{n}{3}\right) + 396/7\tau_{4,15,1}\left(\frac{n}{9}\right) \\
& + 16/45\tau_{4,27,1}(n) + 80/9\tau_{4,27,1}\left(\frac{n}{5}\right) + (92/1377\omega \\
& + 128/459)\tau_{4,27,3}(n) + (2300/1377\omega + 3200/459)\tau_{4,27,3}\left(\frac{n}{5}\right) \\
& + (-92/1377\omega + 128/459)\tau_{4,27,4}(n) + (-2300/1377\omega \\
& + 3200/459)\tau_{4,27,4}\left(\frac{n}{5}\right) + 4/9\tau_{4,45,1}(n) - 4\tau_{4,45,1}\left(\frac{n}{3}\right) \\
& + 8/135\tau_{4,45,4}(n) + 8/15\tau_{4,45,4}\left(\frac{n}{3}\right) + 32/51\tau_{4,135,2}(n) \\
& + (1/760995(-5272\alpha^2 - 3304\alpha + 294144))\tau_{4,135,5}(n) \\
& + (1/760995(5272\alpha - 8576)\beta + 1/760995(5272\alpha^2 \\
& - 5272\alpha + 172888))\tau_{4,135,6}(n) + (1/760995(-5272\alpha \\
& + 8576)\beta + 1/760995(8576\alpha + 164312))\tau_{4,135,7}(n)
\end{aligned}$$

$$r_{F_2 \oplus \Phi_2}(n) =$$

$$\begin{aligned}
& 240/21060\sigma_3(n) + 480/5265\sigma_3\left(\frac{n}{3}\right) + 1200/4212\sigma_3\left(\frac{n}{5}\right) \\
& + 480/585\sigma_3\left(\frac{n}{9}\right) + 2400/1053\sigma_3\left(\frac{n}{15}\right) \\
& + 2160/260\sigma_3\left(\frac{n}{27}\right) + 2400/117\sigma_3\left(\frac{n}{45}\right) \\
& + 10800/52\sigma_3\left(\frac{n}{135}\right) + 176/7371\tau_{4,5}(n) \\
& + 368/7371\tau_{4,5}\left(\frac{n}{3}\right) + 368/819\tau_{4,5}\left(\frac{n}{9}\right) + 1584/91\tau_{4,5}\left(\frac{n}{27}\right) \\
& + 16/135\tau_{4,9}(n) + 16/15\tau_{4,9}\left(\frac{n}{3}\right) + 80/27\tau_{4,9}\left(\frac{n}{5}\right) \\
& + 80/3\tau_{4,9}\left(\frac{n}{15}\right) + 8/27\tau_{4,15,1}(n) - 20/9\tau_{4,15,1}\left(\frac{n}{3}\right) \\
& + 24\tau_{4,15,1}\left(\frac{n}{9}\right) + 2/7\tau_{4,15,2}(n) + 4/7\tau_{4,15,2}\left(\frac{n}{3}\right) \\
& + 162/7\tau_{4,15,1}\left(\frac{n}{9}\right) + 32/135\tau_{4,27,1}(n) + 160/27\tau_{4,27,1}\left(\frac{n}{5}\right) \\
& + (22/459\omega + 38/153)\tau_{4,27,3}(n) + (550/459\omega \\
& + 950/153)\tau_{4,27,3}\left(\frac{n}{5}\right) + (-22/459\omega + 38/153)\tau_{4,27,4}(n) \\
& + (-550/459\omega + 950/153)\tau_{4,27,4}\left(\frac{n}{5}\right) + 2/9\tau_{4,45,1}(n) \\
& - 2\tau_{4,45,1}\left(\frac{n}{3}\right) + 8/63\tau_{4,45,2}(n) + 8/7\tau_{4,45,2}\left(\frac{n}{3}\right) \\
& + 32/189\tau_{4,45,3}(n) + 32/21\tau_{4,45,3}\left(\frac{n}{3}\right) + 32/405\tau_{4,45,4}(n) \\
& + 32/45\tau_{4,45,4}\left(\frac{n}{3}\right) + 4/27\tau_{4,45,5}(n) - 4/3\tau_{4,45,5}\left(\frac{n}{3}\right) \\
& + 112/459\tau_{4,135,2}(n) + (1/2282985(-2074\alpha^2 + 34712\alpha
\end{aligned}$$

$$\begin{aligned}
& +235128))\tau_{4,135,5}(n) + (1/2282985(2074\alpha + 32638)\beta \\
& +1/2282985(2074\alpha^2 - 2074\alpha + 187426))\tau_{4,135,6}(n) \\
& + (1/2282985(-2074\alpha - 32638)\beta + 1/2282985(-32638\alpha \\
& +220064))\tau_{4,135,7}(n) + (1/456597(-3646\alpha^2 - 6100\alpha \\
& +138072))\tau_{4,135,8}(n) + (1/456597(3646\alpha - 9746)\beta \\
& +1/456597(3646\alpha^2 - 3646\alpha + 54214))\tau_{4,135,9}(n) \\
& + (1/456597(-3646\alpha + 9746)\beta + 1/456597(9746\alpha \\
& +44468))\tau_{4,135,10}(n) + (26/11961\gamma^2 - 82/1329\gamma \\
& +1276/3987)\tau_{4,135,11}(n) + ((-26/11961\gamma - 608/11961)\delta \\
& -26/11961\gamma^2 + 130/11961\gamma + 4036/11961)\tau_{4,135,12}(n) \\
& + ((26/11961\gamma + 608/11961)\delta + 608/11961\gamma \\
& +332/3987)\tau_{4,135,13}
\end{aligned}$$

$$r_{F_2 \oplus \Psi_2}(n) =$$

$$\begin{aligned}
& 240/21060\sigma_3(n) + 480/5265\sigma_3\left(\frac{n}{3}\right) + 1200/4212\sigma_3\left(\frac{n}{5}\right) \\
& + 480/585\sigma_3\left(\frac{n}{9}\right) + 2400/1053\sigma_3\left(\frac{n}{15}\right) + 2160/260\sigma_3\left(\frac{n}{27}\right) \\
& + 2400/117\sigma_3\left(\frac{n}{45}\right) + 10800/52\sigma_3\left(\frac{n}{135}\right) \\
& + 16/117\tau_{4,5}(n) + 1000/819\tau_{4,5}\left(\frac{n}{3}\right) + 1000/91\tau_{4,5}\left(\frac{n}{9}\right) \\
& + 1296/13\tau_{4,5}\left(\frac{n}{27}\right) + 16/135\tau_{4,9}(n) + 16/15\tau_{4,9}\left(\frac{n}{3}\right) \\
& + 80/27\tau_{4,9}\left(\frac{n}{5}\right) + 80/3\tau_{4,9}\left(\frac{n}{15}\right) + 4/27\tau_{4,15,1}(n) \\
& + 8/9\tau_{4,15,1}\left(\frac{n}{3}\right) + 12\tau_{4,15,1}\left(\frac{n}{9}\right) + 8/27\tau_{4,15,2}(n) \\
& + 272/63\tau_{4,15,2}\left(\frac{n}{3}\right) + 24\tau_{4,15,1}\left(\frac{n}{9}\right) + 64/405\tau_{4,27,1}(n) \\
& + 320/81\tau_{4,27,1}\left(\frac{n}{5}\right) + (8/153\omega + 146/459)\tau_{4,27,3}(n) \\
& + (200/153\omega + 3650/459)\tau_{4,27,3}\left(\frac{n}{5}\right) + (-8/153\omega \\
& + 146/459)\tau_{4,27,4}(n) + (-200/153\omega + 3650/459)\tau_{4,27,4}\left(\frac{n}{5}\right) \\
& + 4/21\tau_{4,45,2}(n) + 12/7\tau_{4,45,2}\left(\frac{n}{3}\right) \\
& + 104/189\tau_{4,45,3}(n) + 104/21\tau_{4,45,3}\left(\frac{n}{3}\right) + 8/45\tau_{4,45,4}(n) \\
& + 8/5\tau_{4,45,4}\left(\frac{n}{3}\right) + 16/153\tau_{4,135,2}(n) + (1/760995(1938\alpha^2 \\
& + 3296\alpha + 59984))\tau_{4,135,5}(n) + (1/760995(-1938\alpha + 5234)\beta \\
& + 1/760995(-1938\alpha^2 + 1938\alpha + 104558))\tau_{4,135,6}(n) \\
& + (1/760995(1938\alpha - 5234)\beta + 1/760995(-5234\alpha \\
& + 109792))\tau_{4,135,7}(n) + (1/456597(2086\alpha^2 - 16844\alpha \\
& + 74400))\tau_{4,135,8}(n) + (1/456597(-2086\alpha - 14758)\beta \\
& + 1/456597(-2086\alpha^2 + 2086\alpha + 122378))\tau_{4,135,9}(n) \\
& + (1/456597(2086\alpha + 14758)\beta + 1/456597(14758\alpha \\
& + 107620))\tau_{4,135,10}(n) + (-446/35883\gamma^2 + 1346/35883\gamma \\
& + 3056/11961)\tau_{4,135,11}(n) + ((446/35883\gamma - 884/35883)\delta \\
& + 446/35883\gamma^2 - 2230/35883\gamma + 5600/35883)\tau_{4,135,12}(n) \\
& + ((-446/35883\gamma + 884/35883)\delta + 884/35883\gamma \\
& + 1180/35883)\tau_{4,135,13}
\end{aligned}$$

$$r_{\Lambda_2 \oplus \Phi_2}(n) =$$

$$\begin{aligned}
& 240/21060\sigma_3(n) + 480/5265\sigma_3\left(\frac{n}{3}\right) + 1200/4212\sigma_3\left(\frac{n}{5}\right) \\
& + 480/585\sigma_3\left(\frac{n}{9}\right) + 2400/1053\sigma_3\left(\frac{n}{15}\right) + 2160/260\sigma_3\left(\frac{n}{27}\right) \\
& + 2400/117\sigma_3\left(\frac{n}{45}\right) + 10800/52\sigma_3\left(\frac{n}{135}\right) \\
& + 16/117\tau_{4,5}(n) + 1000/819\tau_{4,5}\left(\frac{n}{3}\right) + 1000/91\tau_{4,5}\left(\frac{n}{9}\right) \\
& + 1296/13\tau_{4,5}\left(\frac{n}{27}\right) + 16/135\tau_{4,9}(n) + 16/15\tau_{4,9}\left(\frac{n}{3}\right) \\
& + 80/27\tau_{4,9}\left(\frac{n}{5}\right) + 80/3\tau_{4,9}\left(\frac{n}{15}\right) - 4/27\tau_{4,15,1}(n) \\
& - 8/9\tau_{4,15,1}\left(\frac{n}{3}\right) - 12\tau_{4,15,1}\left(\frac{n}{9}\right) + 8/27\tau_{4,15,2}(n) \\
& + 272/63\tau_{4,15,2}\left(\frac{n}{3}\right) + 24\tau_{4,15,1}\left(\frac{n}{9}\right) + 64/405\tau_{4,27,1}(n) \\
& + 320/81\tau_{4,27,1}\left(\frac{n}{5}\right) + (8/153\omega + 146/459)\tau_{4,27,3}(n) \\
& + (200/153\omega + 3650/459)\tau_{4,27,3}\left(\frac{n}{5}\right) \\
& + (-8/153\omega + 146/459)\tau_{4,27,4}(n) + (-200/153\omega \\
& + 3650/459)\tau_{4,27,4}\left(\frac{n}{5}\right) - 4/21\tau_{4,45,2}(n) - 12/7\tau_{4,45,2}\left(\frac{n}{3}\right) \\
& - 104/189\tau_{4,45,3}(n) - 104/21\tau_{4,45,3}\left(\frac{n}{3}\right) + 8/45\tau_{4,45,4}(n) \\
& + 8/5\tau_{4,45,4}\left(\frac{n}{3}\right) + 16/153\tau_{4,135,2}(n) + ((446/35883\gamma \\
& - 884/35883)\delta - 884/35883\gamma - 1180/35883)\tau_{4,135,13} \\
& + (1/760995(1938\alpha^2 + 3296\alpha + 59984))\tau_{4,135,5}(n) \\
& + (1/760995(-1938\alpha + 5234)\beta + 1/760995(-1938\alpha^2 \\
& + 1938\alpha + 104558))\tau_{4,135,6}(n) + (1/760995(1938\alpha \\
& - 5234)\beta + 1/760995(-5234\alpha + 109792))\tau_{4,135,7}(n) \\
& + (1/456597(-2086\alpha^2 + 16844\alpha + 74400))\tau_{4,135,8}(n) \\
& + (1/456597(2086\alpha + 14758)\beta + 1/456597(2086\alpha^2 \\
& - 2086\alpha + 122378))\tau_{4,135,9}(n) + (1/456597(-2086\alpha \\
& - 14758)\beta + 1/456597(-14758\alpha - 107620))\tau_{4,135,10}(n) \\
& + (446/35883\gamma^2 - 1346/35883\gamma - 3056/11961)\tau_{4,135,11}(n) \\
& + ((-446/35883\gamma + 884/35883)\delta - 446/35883\gamma^2 \\
& + 2230/35883\gamma - 5600/35883)\tau_{4,135,12}(n)
\end{aligned}$$

$$r_{\Lambda_2 \oplus \Psi_2}(n) =$$

$$\begin{aligned}
& 240/21060\sigma_3(n) + 480/5265\sigma_3\left(\frac{n}{3}\right) + 1200/4212\sigma_3\left(\frac{n}{5}\right) \\
& + 480/585\sigma_3\left(\frac{n}{9}\right) + 2400/1053\sigma_3\left(\frac{n}{15}\right) + 2160/260\sigma_3\left(\frac{n}{27}\right) \\
& + 2400/117\sigma_3\left(\frac{n}{45}\right) + 10800/52\sigma_3\left(\frac{n}{135}\right) \\
& + 16/117\tau_{4,5}(n) + 1000/819\tau_{4,5}\left(\frac{n}{3}\right) + 1000/91\tau_{4,5}\left(\frac{n}{9}\right) \\
& + 1296/13\tau_{4,5}\left(\frac{n}{27}\right) + 16/135\tau_{4,9}(n) + 16/15\tau_{4,9}\left(\frac{n}{3}\right) \\
& + 80/27\tau_{4,9}\left(\frac{n}{5}\right) + 80/3\tau_{4,9}\left(\frac{n}{15}\right) + 4/27\tau_{4,15,1}(n) \\
& + 8/9\tau_{4,15,1}\left(\frac{n}{3}\right) + 12\tau_{4,15,1}\left(\frac{n}{9}\right) + 8/27\tau_{4,15,2}(n) \\
& + 272/63\tau_{4,15,2}\left(\frac{n}{3}\right) + 24\tau_{4,15,1}\left(\frac{n}{9}\right) + 64/405\tau_{4,27,1}(n) \\
& + 320/81\tau_{4,27,1}\left(\frac{n}{5}\right) + (8/153\omega + 146/459)\tau_{4,27,3}(n) \\
& + (200/153\omega + 3650/459)\tau_{4,27,3}\left(\frac{n}{5}\right) + (-8/153\omega \\
& + 146/459)\tau_{4,27,4}(n) + (-200/153\omega + 3650/459)\tau_{4,27,4}\left(\frac{n}{5}\right) \\
& + 3650/459\tau_{4,27,4}\left(\frac{n}{5}\right) + 4/21\tau_{4,45,2}(n) + 12/7\tau_{4,45,2}\left(\frac{n}{3}\right) \\
& + 104/189\tau_{4,45,3}(n) + 104/21\tau_{4,45,3}\left(\frac{n}{3}\right) + 8/45\tau_{4,45,4}(n) \\
& + 8/5\tau_{4,45,4}\left(\frac{n}{3}\right) + 16/153\tau_{4,135,2}(n) + (1/760995)(1938\alpha^2
\end{aligned}$$

$$\begin{aligned}
& +3296\alpha + 59984))\tau_{4,135,5}(n) + (1/760995(-1938\alpha \\
& +5234)\beta + 1/760995(-1938\alpha^2 + 1938\alpha \\
& +104558))\tau_{4,135,6}(n) + (1/760995(1938\alpha - 5234)\beta \\
& +1/760995(-5234\alpha + 109792))\tau_{4,135,7}(n) + \\
& (1/456597(2086\alpha^2 - 16844\alpha - 74400))\tau_{4,135,8}(n) \\
& +(1/456597(-2086\alpha - 14758)\beta + 1/456597(-2086\alpha^2 \\
& +2086\alpha - 122378))\tau_{4,135,9}(n) + (1/456597(2086\alpha \\
& +14758)\beta - 1/456597(14758\alpha + 107620))\tau_{4,135,10}(n) \\
& +(-446/35883\gamma^2 + 1346/35883\gamma \\
& +3056/11961)\tau_{4,135,11}(n) + ((446/35883\gamma \\
& -884/35883)\delta + 446/35883\gamma^2 - 2230/35883\gamma \\
& +5600/35883)\tau_{4,135,12}(n) + ((-446/35883\gamma \\
& +884/35883)\delta + 884/35883\gamma + 1180/35883)\tau_{4,135,13}
\end{aligned}$$

$$r_{\Phi_2 \oplus \Psi_2}(n) =$$

$$\begin{aligned}
& 240/21060\sigma_3(n) + 480/5265\sigma_3\left(\frac{n}{3}\right) + 1200/4212\sigma_3\left(\frac{n}{5}\right) \\
& +480/585\sigma_3\left(\frac{n}{9}\right) + 2400/1053\sigma_3\left(\frac{n}{15}\right) + 2160/260\sigma_3\left(\frac{n}{27}\right) \\
& +2400/117\sigma_3\left(\frac{n}{45}\right) + 10800/52\sigma_3\left(\frac{n}{135}\right) \\
& -56/351\tau_{4,5}(n) - 2096/2457\tau_{4,5}\left(\frac{n}{3}\right) - 2096/273\tau_{4,5}\left(\frac{n}{9}\right) \\
& -1512/13\tau_{4,5}\left(\frac{n}{27}\right) - 16/135\tau_{4,9}(n) - 16/15\tau_{4,9}\left(\frac{n}{3}\right) \\
& -80/27\tau_{4,9}\left(\frac{n}{5}\right) - 80/3\tau_{4,9}\left(\frac{n}{15}\right) + 16/21\tau_{4,15,2}\left(\frac{n}{3}\right) \\
& +(4/81\omega + 4/27)\tau_{4,27,3}(n) + (100/81\omega + 100/27)\tau_{4,27,3}\left(\frac{n}{5}\right) \\
& +(-4/81\omega + 4/27)\tau_{4,27,4}(n) + (-100/81\omega \\
& +100/27)\tau_{4,27,4}\left(\frac{n}{5}\right) + 16/135\tau_{4,45,4}(n) + 16/15\tau_{4,45,4}\left(\frac{n}{3}\right) \\
& +(1/152199(1228\alpha^2 + 2104\alpha - 27456))\tau_{4,135,5}(n) \\
& +(1/152199(-1228\alpha + 3332)\beta + 1/152199(-1228\alpha^2 \\
& +1228\alpha + 788))\tau_{4,135,6}(n) + (1/152199(1228\alpha - 3332)\beta \\
& +1/152199(-3332\alpha + 4120))\tau_{4,135,7}(n)
\end{aligned}$$