

Original Research Article

A new approach to H_∞ filtering for switched T-S fuzzy systems with interval time varying delays and parameter uncertainties

Abstract

This paper deals with the problem of delay-dependent robust H_∞ filter for switched T-S fuzzy systems with interval time-varying delay and parameter uncertainties. By choosing a new Lyapunov function and using the convexity property of matrix inequalities, some delay-dependent stability conditions are obtained for the asymptotical stability of the filtering-error system. Finally, tree examples are provided to demonstrate the effectiveness and the superiority of the proposed design methods.

Keywords: T-S fuzzy systems; switched systems; interval time-delay systems; H_∞ filter; linear matrix inequality (LMI).

1. Introduction:

H_∞ filter has came to play an important role in fuzzy model during the past years, especially; in the control and filtering problems [1], [2] and [3]. H_∞ filtering of time-delay T-S fuzzy systems based on piecewise Lyapunov-Krasovskii functional was investigated in [4]. The problem of delay-dependent robust H_∞ filter for T-S fuzzy time-delay systems with exponential stability was studied in [5]. The delay-dependent robust H_∞ filter for a class of Takagi-Sugeno (T-S) fuzzy descriptor time-delay systems with parameter uncertainties has been discussed in [6].

Switched systems arise in many practical models in manufacturing, communication networks, automotive engine control, chemical processes etc [7-9]. The idea of controller switching is introduced in order to overcome the shortcomings of the single controller and improve system performance [10–12]. The stability analysis of switched time delay systems has attracted lot of attention. In [13], the stabilization via switching is investigated by using a multiple Lyapunov function. In [14], exponential stability and stabilization are designed in terms of the solution of Riccati- type equations. In [15-16], the exponential stability of switched systems with mixed time delay is studied. In [17], the exponential stability and guaranteed cost of switched linear systems with mixed varying delays is studied. For T-S fuzzy switched systems with time delays, a few results are encountered in the literature. The delay dependent exponential stability is studied for systems with norm bounded uncertainties in [18] and in [19] the stability

robustness of T-S fuzzy systems with two additive time varying delays is investigated by using the free weighting approach. In addition, the state estimation problem has been investigated for switched systems with different performance indexes including H^∞ filter design [20, 21], where in [20] the linear matrix inequality (LMI) based existence condition of such a filter is derived by the introduction of slack variables to eliminate the cross coupling of system matrices and Lyapunov matrices among different subsystems. An H^∞ filtering design is developed by using switched Lyapunov functional approach for discrete time switched systems with time delay in [21]. Switched linear filters are designed to ensure that the filtering error systems under switching signal with average dwell time are exponentially stable with a prescribed H^∞ noise attenuation level, which is studied in [22]. But at present, the problem of delay-dependent robust H^∞ filter for T-S fuzzy time delay systems with exponential stability has rarely been reported and remains an open research domain.

For uncertain switched T-S fuzzy time delay systems, this paper discusses the design methods of delay dependent robust H^∞ filter. The parameter uncertainties are assumed to be time varying but norm-bounded. The designed filter is obtained by constructing a new Lyapunov krasovskii functional and guarantees the asymptotic stability of the error system, some improved LMI-based robust stability criteria are obtained without ignoring any useful terms in the derivative of the Lyapunov functional, the obtained criterion is less conservative than the existing ones. Finally, three numerical examples are given to show the effectiveness and the superiority of the proposed design methods.

2. Preliminaries and Problem Formulation:

Consider a class of switched systems with T-S fuzzy subsystems as follows

R^i : If $z_1(t)$ is W_1^i , $z_2(t)$ is $W_2^i, \dots, z_s(t)$ is W_s^i

Then

$$\begin{cases} \dot{x}(t) = (A_{i\sigma} + \Delta A_{i\sigma}(t))x(t) + (A_{di\sigma} + \Delta A_{di\sigma}(t))x(t - \tau(t)) + (B_{wi\sigma} + \Delta B_{wi\sigma}(t))\omega(t) \\ y(t) = (C_{i\sigma} + \Delta C_{i\sigma}(t))x(t) + (C_{di\sigma} + \Delta C_{di\sigma}(t))x(t - \tau(t)) + (D_{wi\sigma} + \Delta D_{wi\sigma}(t))\omega(t) \\ z(t) = (L_{i\sigma} + \Delta L_{i\sigma}(t))x(t) + (L_{di\sigma} + \Delta L_{di\sigma}(t))x(t - \tau(t)) + (G_{wi\sigma} + \Delta G_{wi\sigma}(t))\omega(t) \\ x(t) = \phi(t) \quad t \in [-h_b, 0] \quad i = 1, \dots, r \end{cases} \quad (2.1)$$

Where $x(t) \in \mathbb{R}^n$ is the state vector; $\omega(t) \in \mathbb{R}^p$ is the disturbance input that belongs to $L_2 [0, \infty)$; $y(t) \in \mathbb{R}^m$ is the measurements vector; $z(t) \in \mathbb{R}^q$ is the controlled output, $z_1(t), z_2(t), \dots, z_s(t)$ are the premise variables and $W_l^i, l=1, \dots, s$ are the fuzzy sets, $\sigma(t) := j \in \{1, \dots, N\}$ is the switching sequence and r is the number of IF-THEN rules. $\phi \in ([-h_b, 0], \mathbb{R}^n)$ is the initial

63 function, with norm $\|\phi\| = \sup_{\theta \in [-h_b, 0]} \|\phi(\theta)\|$. $\tau(t)$ is a bounded time varying delay
 64 satisfying:

$$65 \quad 0 \leq h_a \leq \tau(t) \leq h_b \quad (2.2)$$

66 The time varying delay $\tau(t)$ is assumed to be differentiable $\dot{\tau}(t) = d$ with

$$67 \quad d_1 \leq d \leq d_2 \quad (2.3)$$

68 $[A_{i\sigma}, A_{di\sigma}, B_{\omega i\sigma}, C_{i\sigma}, C_{di\sigma}, D_{\omega i\sigma}, L_{i\sigma}, L_{di\sigma}, G_{\omega i\sigma}, \Delta A_{i\sigma}(t), \Delta A_{di\sigma}(t), \Delta B_{\omega i\sigma}(t), \Delta C_{i\sigma}(t), \Delta C_{di\sigma}(t), \Delta D_{\omega i\sigma}(t),$
 69 $\Delta L_{i\sigma}(t), \Delta L_{di\sigma}(t), \Delta G_{\omega i\sigma}(t)] \in \{[A_{ij}, A_{dij}, B_{\omega ij}, C_{ij}, C_{dij}, D_{\omega ij}, L_{ij}, L_{dij}, G_{\omega ij}, \Delta A_{ij}(t), \Delta A_{dij}(t), \Delta B_{\omega ij}(t),$
 70 $\Delta C_{ij}(t), \Delta C_{dij}(t), \Delta D_{\omega ij}(t), \Delta L_{ij}(t), \Delta L_{dij}(t), \Delta G_{\omega ij}(t)], j=1 \dots N\}$, $A_{ij}, A_{dij}, B_{\omega ij}, C_{ij}, C_{dij}, D_{\omega ij}, L_{ij},$
 71 $L_{dij}, G_{\omega ij}$ are constant matrices, $\Delta A_{ij}(t), \Delta A_{dij}(t), \Delta B_{\omega ij}(t), \Delta C_{ij}(t), \Delta C_{dij}(t), \Delta D_{\omega ij}(t), \Delta L_{ij}(t),$
 72 $\Delta L_{dij}(t), \Delta G_{\omega ij}(t)$ are unknown perturbation matrices. The matrix $A_{i\sigma}$ switches between
 73 matrices $A_{i1}, A_{i2}, \dots, A_{iN}$, the matrix $A_{di\sigma}$ switches between matrices $A_{di1}, A_{di2}, \dots, A_{diN}$, the
 74 matrix $B_{\omega i\sigma}$ switches between matrices $B_{\omega i1}, B_{\omega i2}, \dots, B_{\omega iN}$, the matrix $C_{i\sigma}$ switches between
 75 matrices $C_{i1}, C_{i2}, \dots, C_{iN}$, the matrix $C_{di\sigma}$ switches between matrices $C_{di1}, C_{di2}, \dots, C_{diN}$, the
 76 matrix $D_{\omega i\sigma}$ switches between matrices $D_{\omega i1}, D_{\omega i2}, \dots, D_{\omega iN}$, the matrix $L_{i\sigma}$ switches between
 77 matrices $L_{i1}, L_{i2}, \dots, L_{iN}$, the matrix $L_{di\sigma}$ switches between matrices $L_{di1}, L_{di2}, \dots, L_{diN}$ and the
 78 matrix $G_{\omega i\sigma}$ switches between matrices $G_{\omega i1}, G_{\omega i2}, \dots, G_{\omega iN}$. The perturbation matrices $\Delta A_{i\sigma}(t),$
 79 $\Delta A_{di\sigma}(t), \Delta B_{\omega i\sigma}(t), \Delta C_{i\sigma}(t), \Delta C_{di\sigma}(t), \Delta D_{\omega i\sigma}(t), \Delta L_{i\sigma}(t), \Delta L_{di\sigma}(t)$ and $\Delta G_{\omega i\sigma}(t)$ switches
 80 between matrices $\Delta A_{i1}(t), \Delta A_{i2}(t), \dots, \Delta A_{iN}(t), \Delta A_{di1}(t), \Delta A_{di2}(t), \dots, \Delta A_{diN}(t), \Delta B_{\omega i1}(t),$
 81 $\Delta B_{\omega i2}(t), \dots, \Delta B_{\omega iN}(t), \Delta C_{i1}(t), \Delta C_{i2}(t), \dots, \Delta C_{iN}(t), \Delta C_{di1}(t), \Delta C_{di2}(t), \dots, \Delta C_{diN}(t), \Delta D_{\omega i1}(t),$
 $\Delta D_{\omega i2}(t), \dots, \Delta D_{\omega iN}(t), \Delta L_{i1}(t), \Delta L_{i2}(t), \dots, \Delta L_{iN}(t), \Delta L_{di1}(t), \Delta L_{di2}(t), \dots, \Delta L_{diN}(t)$ and
 $\Delta G_{\omega i1}(t), \Delta G_{\omega i2}(t), \dots, \Delta G_{\omega iN}(t)$ respectively.

82 Hence the system (2.1) can be represented as follows:

83 Plant rule i for individual subsystem j :

84 R^i : If $z_1(t)$ is W_1^i , $z_2(t)$ is $W_2^i, \dots, z_s(t)$ is W_s^i

85 Then

$$86 \quad \begin{cases} \dot{x}(t) = (A_{ij} + \Delta A_{ij}(t))x(t) + (A_{dij} + \Delta A_{dij}(t))x(t - \tau(t)) + (B_{\omega ij} + \Delta B_{\omega ij}(t))\omega(t) \\ y(t) = (C_{ij} + \Delta C_{ij}(t))x(t) + (C_{dij} + \Delta C_{dij}(t))x(t - \tau(t)) + (D_{\omega ij} + \Delta D_{\omega ij}(t))\omega(t) \\ z(t) = (L_{ij} + \Delta L_{ij}(t))x(t) + (L_{dij} + \Delta L_{dij}(t))x(t - \tau(t)) + (G_{\omega ij} + \Delta G_{\omega ij}(t))\omega(t) \\ x(t) = \phi(t) \quad t \in [-h_b, 0] \quad i = 1, \dots, r \end{cases} \quad (2.4)$$

87 **Assumption 2.1.** The perturbation matrices are assumed to be norm bounded and given by:

$$88 \quad [\Delta A_{ij}(t) \quad \Delta A_{dij}(t) \quad \Delta B_{\omega ij}(t)] = D_{1ij} F_{ij}(t) [E_{10ij} \quad E_{11ij} \quad E_{12ij}]$$

$$89 \quad [\Delta C_{ij}(t) \quad \Delta C_{dij}(t) \quad \Delta D_{\omega ij}(t)] = D_{2ij} F_{ij}(t) [E_{20ij} \quad E_{21ij} \quad E_{22ij}] \quad (2.5)$$

$$[\Delta L_{ij}(t) \quad \Delta L_{dij}(t) \quad \Delta G_{\omega ij}(t)] = D_{3ij} F_{ij}(t) [E_{30ij} \quad E_{31ij} \quad E_{32ij}]$$

Where $E_{10ij}, E_{11ij}, E_{12ij}, E_{20ij}, E_{21ij}, E_{22ij}, E_{30ij}, E_{31ij}, E_{32ij}, D_{1ij}, D_{2ij}$ and D_{3ij} are known constant matrices with appropriate dimensions, $F_{ij}(t)$ are unknown time-varying matrices with Lebesgue measurable elements bounded by:

$$F_{ij}^T(t) F_{ij}(t) \leq I \quad (2.6)$$

By using the common used center-average defuzzifier, product interference and singleton fuzzifier, the global dynamics of T-S fuzzy system (2.4) can be inferred as

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^r \mu_i(z(t)) [(A_{ij} + \Delta A_{ij}(t)) x(t) + (A_{dij} + \Delta A_{dij}(t)) x(t - \tau(t)) + (B_{\omega ij} + \Delta B_{\omega ij}(t)) \omega(t)] \\ y(t) = \sum_{i=1}^r \mu_i(z(t)) [(C_{ij} + \Delta C_{ij}(t)) x(t) + (C_{dij} + \Delta C_{dij}(t)) x(t - \tau(t)) + (D_{\omega ij} + \Delta D_{\omega ij}(t)) \omega(t)] \\ z(t) = \sum_{i=1}^r \mu_i(z(t)) [(L_{ij} + \Delta L_{ij}(t)) x(t) + (L_{dij} + \Delta L_{dij}(t)) x(t - \tau(t)) + (G_{\omega ij} + \Delta G_{\omega ij}(t)) \omega(t)] \\ x(t) = \phi(t) \quad t \in [-h_b, 0] \end{cases} \quad (2.7)$$

Where $\mu_i(z(t)) = w_i(z(t)) / \sum_{i=1}^r w_i(z(t))$, $w_i(z(t)) = \prod_{j=1}^s W_j^i(z_j(t))$

$W_j^i(z_j(t))$ being the grade of the membership function of $z_j(t)$ in W_j^i , some basic properties of

$\mu_i(z(t))$ are: $\mu_i(z(t)) \geq 0$, $\sum_{i=1}^r \mu_i(z(t)) = 1$

In this paper, we consider the following fuzzy filter:

R^i : If $z_1(t)$ is W_1^i , $z_2(t)$ is $W_2^i, \dots, z_s(t)$ is W_s^i

$$\text{Then } \begin{cases} \hat{\dot{x}}(t) = A_{fij} \hat{x}(t) + B_{fij} y(t) \\ \hat{z}(t) = C_{fij} \hat{x}(t) + D_{fij} y(t) \\ \hat{x}(0) = 0 \end{cases} \quad (i=1, 2, \dots, r), \quad (2.8)$$

Where $\hat{x}(t) \in R^n$ is the filter state, $\hat{z}(t) \in R^q$ is the estimation of $z(t)$ in fuzzy system (2.1), $A_{fij}, B_{fij}, C_{fij}$ and D_{fij} are the filter matrices with appropriate dimensions, which are to be designed. The final fuzzy filter of fuzzy system (2.1) is thus inferred as follows

$$\begin{cases} \hat{\dot{x}}(t) = \sum_{i=1}^r \mu_i(z(t)) [A_{fij} \hat{x}(t) + B_{fij} y(t)] \\ \hat{z}(t) = \sum_{i=1}^r \mu_i(z(t)) [C_{fij} \hat{x}(t) + D_{fij} y(t)] \\ \hat{x}(0) = 0 \end{cases} \quad (2.9)$$

Augmenting system (2.1) to include switched T-S fuzzy filter (2.9), we can obtain the filtering error system

$$\begin{cases} \tilde{\dot{x}}(t) = (\tilde{A}_j + \Delta \tilde{A}_j(t)) \tilde{x}(t) + (\tilde{A}_{dj} + \Delta \tilde{A}_{dj}(t)) \tilde{x}(t - \tau(t)) + (\tilde{B}_{\omega j} + \Delta \tilde{B}_{\omega j}(t)) \omega(t) \\ e(t) = (\tilde{C}_j + \Delta \tilde{C}_j(t)) \tilde{x}(t) + (\tilde{C}_{dj} + \Delta \tilde{C}_{dj}(t)) \tilde{x}(t - \tau(t)) + (\tilde{D}_{\omega j} + \Delta \tilde{D}_{\omega j}(t)) \omega(t) \\ \tilde{x}(t) = [\Phi^T(t) \quad 0]^T, \quad \forall t \in [-h_b, 0], \end{cases} \quad (2.10)$$

111

112 Where $\tilde{x}(t) := \text{col} \{x(t) \quad \hat{x}(t)\}$, $e(t) := z(t) - \hat{z}(t)$

$$\begin{aligned}
 113 \quad \tilde{A}_j &= \sum_{i=1}^r \mu_i(z(t)) \sum_{k=1}^r \mu_k(z(t)) \begin{bmatrix} A_{kj} & 0 \\ B_{fij} C_{kj} & A_{fij} \end{bmatrix}; \\
 114 \quad \tilde{A}_{dj} &= \sum_{i=1}^r \mu_i(z(t)) \sum_{k=1}^r \mu_k(z(t)) \begin{bmatrix} A_{dkj} & 0 \\ B_{fij} C_{dkj} & 0 \end{bmatrix}; \\
 115 \quad \tilde{B}_{\omega j} &= \sum_{i=1}^r \mu_i(z(t)) \sum_{k=1}^r \mu_k(z(t)) \begin{bmatrix} B_{\omega kj} & 0 \\ B_{fij} D_{\omega kj} & 0 \end{bmatrix}; \\
 116 \quad \tilde{C}_j &= \sum_{i=1}^r \mu_i(z(t)) \sum_{k=1}^r \mu_k(z(t)) [L_{ij} - D_{fkj} C_{ij} - C_{fkj}]; \\
 117 \quad \tilde{C}_{dj} &= \sum_{i=1}^r \mu_i(z(t)) \sum_{k=1}^r \mu_k(z(t)) [L_{dij} - D_{fkj} C_{dij}]; \\
 118 \quad \tilde{D}_{\omega j} &= \sum_{i=1}^r \mu_i(z(t)) \sum_{k=1}^r \mu_k(z(t)) [G_{\omega ij} - D_{fkj} D_{\omega ij}];
 \end{aligned} \tag{2.11}$$

$$\begin{aligned}
 119 \quad \text{And } \Delta \tilde{A}_j(t) &= \sum_{i=1}^r \mu_i(z(t)) \sum_{k=1}^r \mu_k(z(t)) \begin{bmatrix} \Delta A_{kj}(t) & 0 \\ B_{fij} \Delta C_{kj}(t) & 0 \end{bmatrix}; \\
 120 \quad \Delta \tilde{A}_{dj}(t) &= \sum_{i=1}^r \mu_i(z(t)) \sum_{k=1}^r \mu_k(z(t)) \begin{bmatrix} \Delta A_{dkj}(t) & 0 \\ B_{fij} \Delta C_{dkj}(t) & 0 \end{bmatrix}; \\
 121 \quad \Delta \tilde{B}_{\omega j}(t) &= \sum_{i=1}^r \mu_i(z(t)) \sum_{k=1}^r \mu_k(z(t)) \begin{bmatrix} \Delta B_{\omega kj}(t) & 0 \\ B_{fij} \Delta D_{\omega kj}(t) & 0 \end{bmatrix}; \\
 122 \quad \Delta \tilde{C}_j(t) &= \sum_{i=1}^r \mu_i(z(t)) \sum_{k=1}^r \mu_k(z(t)) [\Delta L_{ij}(t) - D_{fkj} \Delta C_{ij}(t) - \Delta C_{fkj}(t)]; \\
 123 \quad \Delta \tilde{C}_{dj}(t) &= \sum_{i=1}^r \mu_i(z(t)) \sum_{k=1}^r \mu_k(z(t)) [\Delta L_{dij}(t) - D_{fkj} \Delta C_{dij}(t)]; \\
 124 \quad \Delta \tilde{D}_{\omega j}(t) &= \sum_{i=1}^r \mu_i(z(t)) \sum_{k=1}^r \mu_k(z(t)) [\Delta G_{\omega ij}(t) - D_{fkj} \Delta D_{\omega ij}(t)];
 \end{aligned} \tag{2.12}$$

125 The robust H^∞ filtering problem to be addressed in this paper is formulated as follows:
 126 Given the switched T-S fuzzy time-delay system (2.7) and a prescribed level of noise
 127 attenuation $\gamma > 0$, determine a filter in the form of (2.9) such that the following requirements are
 128 satisfied:

- 129 (a) The system (2.10) with $\omega(t) = 0$ is asymptotically stable,
 130 (b) The H^∞ performance $\|e\|_2 < \gamma \|\omega(t)\|_2$ is guaranteed for all nonzero $\omega(t) \in L_2[0, \infty]$
 131 under the condition $\tilde{x}(t) = 0$, for all $t \in [-h_b, 0]$. If this is the case, we say that the fuzzy H^∞
 132 filter design problem is solvable.

133 To present our objective more precisely, the following lemma are introduced.

134 **Lemma 2.2 [23]:** Let $Q = Q^T$, Ω , E , and $F(t)$ satisfying $F^T(t)F(t) \leq I$ are appropriately
 135 dimensioned matrices, the following inequality:

$$136 \quad Q + \Omega F(t)E + E^T F^T(t) \Omega^T < 0$$

137 Is true, if and only if the following inequality holds for any scalar $\varepsilon > 0$

$$138 \quad Q + \varepsilon^{-1} \Omega \Omega^T + \varepsilon E^T E < 0$$

139 **Lemma 2.3 [24]:** For any two matrices X and Y , we have

$$X^T Y + Y^T X \leq \varepsilon X^T X + \varepsilon^{-1} Y^T Y$$

140 Where $X \in \mathbb{R}^{m \times n}$, $Y \in \mathbb{R}^{m \times n}$, and ε is any positive constant.

141

142 3. Fuzzy H_∞ performance analysis:

143 In this section, we will present an efficient LMI- based method for the design of an H_∞
144 fuzzy filter and we propose sufficient stability criterion for the **filter** error system (2.10).

145 **Remark 3.1.** The nominal system of (2.10), i.e., $\Delta A_{ij}(t) = \Delta A_{dij}(t) = \Delta B_{\omega ij}(t) = \Delta C_{ij}(t) =$
146 $\Delta C_{dij}(t) = \Delta D_{\omega ij}(t) = \Delta L_{ij}(t) = \Delta L_{dij}(t) = \Delta G_{\omega ij}(t) = 0$ is reduced to the system as follows

$$147 \begin{cases} \tilde{x}(t) = \tilde{A}_j \tilde{x}(t) + \tilde{A}_{dj} \tilde{x}(t - \tau(t)) + \tilde{B}_{\omega j} w(t) \\ e(t) = \tilde{C}_j \tilde{x}(t) + \tilde{C}_{dj} \tilde{x}(t - \tau(t)) + \tilde{D}_{\omega j} w(t) \\ \tilde{x}(t) = [\Phi^T(t) \ 0]^T, \forall t \in [-h_b, 0], \end{cases} \quad (3.1)$$

148 Where $\tilde{A}_j, \tilde{A}_{dj}, \tilde{B}_{\omega j}, \tilde{C}_j, \tilde{C}_{dj}$ and $\tilde{D}_{\omega j}$ are defined in (2.11). In this case, we have the following
149 theorem.

150 **Theorem 3.2 :** Given scalars $0 \leq h_a \leq h_b, d_1 \leq d_2$ and $\gamma > 0$, the H_∞ filter error system (3.1),
151 for all differentiable delay $\tau(t) \in [h_a, h_b]$, is asymptotically stable and has a prescribed H_∞
152 performance level γ if there exist real symmetric matrices $R_0 > 0, R_\lambda > 0, Q_0 > 0, Q_\lambda >$
153 $0, P = \begin{bmatrix} P_1 & P_2 \\ * & P_3 \end{bmatrix} > 0, R_d \geq 0, P_d \geq 0$, and real matrices $\chi_{pq}, (p = 1, 2; q = 1, 2, \dots, 6)$ with
154 appropriate dimensions such that the following LMIs (3.2) are feasible where $d = d_1, d_2$

$$155 \Sigma_{pik}^j = \begin{bmatrix} \Pi_{ik}^j + [-I_p^T Q_\lambda I_p + \lambda_h \chi_p I_p + \lambda_h I_p^T \chi_p^T] & h_a Y_{1j}^T Q_0 & \delta_h Y_{1j}^T Q_\lambda & \lambda_h \chi_p & Y_{3j}^T \\ * & -Q_0 & 0 & 0 & 0 \\ * & * & -Q_\lambda & 0 & 0 \\ * & * & * & -Q_\lambda & 0 \\ * & * & * & * & -I \end{bmatrix} < 0 \quad (3.2)$$

156

157 Where

$$\Pi_{ik}^j = \begin{bmatrix} \varphi_{11j} & \varphi_{12j} & Q_0 & 0 & \varphi_{15j} & \varphi_{16j} \\ * & \varphi_{22j} & 0 & 0 & \varphi_{25j} & \varphi_{26j} \\ * & * & R_d - R_0 + R_\lambda - Q_0 - Q_\lambda & 0 & Q_\lambda & 0 \\ * & * & * & -R_\lambda - Q_\lambda - P_d & Q_\lambda & 0 \\ * & * & * & * & -(1-d)(R_d - P_d) - 2Q_\lambda & 0 \\ * & * & * & * & * & -\gamma^2 I \end{bmatrix}$$

$$158 \varphi_{11j} = P_1 A_{kj} + A_{kj}^T P_1 + P_2 B_{fij} C_{kj} + C_{kj}^T B_{fij}^T P_2^T + R_0 - Q_0$$

$$\begin{aligned} 159 \quad \varphi_{12j} &= P_2 A_{fij} + A_{kj}^T P_2^T + C_{kj}^T B_{fij}^T P_3; & \varphi_{22j} &= P_3 A_{fij} + A_{fij}^T P_3 \\ 160 \quad \varphi_{15j} &= P_1 A_{dkj} + P_2 B_{fij} C_{dkj}; & \varphi_{25j} &= P_2^T A_{dkj} + P_3 B_{fij} C_{dkj} \end{aligned} \quad (3.3)$$

$$161 \quad \varphi_{16j} = P_1 B_{\omega ij} + P_2 B_{fij} D_{\omega kj}; \quad \varphi_{26j} = P_2^T B_{\omega kj} + P_3 B_{fij} D_{\omega kj}$$

$$162 \quad I_1 = [0 \ 0 \ 0 \ -I \ I \ 0]; \ I_2 = [0 \ 0 \ I \ 0 \ -I \ 0];$$

$$163 \quad \chi_p := \{\chi_{p1} \ \chi_{p2} \ \chi_{p3} \ \chi_{p4} \ \chi_{p5} \ \chi_{p6}\}, p=1,2$$

164

$$165 \quad Y_{1j} := [A_{kj} \ 0 \ 0 \ 0 \ A_{dkj} \ B_{\omega kj}] \quad (3.4)$$

$$Y_{3j} := [L_{kj} - D_{fij} C_{kj} \ -C_{fij} \ 0 \ 0 \ L_{dkj} - D_{fij} C_{dkj} \ G_{\omega kj} - D_{fij} D_{\omega kj}]$$

166 **Proof:** We introduce the following **lyapunov krasovskii** functional:

$$167 \quad V(t, \tilde{x}_t) = V_p(t, \tilde{x}_t) + V_h(t, x_t) \quad (3.5)$$

168 Where $\tilde{x}_t$ denotes the function $\tilde{x}(s)$ defined on $[t-h_b, t]$,

$$169 \quad V_p(t, \tilde{x}_t) = \tilde{x}^T(t) P \tilde{x}(t) \quad (3.6)$$

$$170 \quad V_h(t, x_t) = \int_{t-h_a}^t x^T(s) R_0 x(s) ds + \int_{t-h_b}^{t-h_a} x^T(s) R_\lambda x(s) ds + \int_{t-\tau(t)}^{t-h_a} x^T(s) R_d x(s) ds + \quad (3.7)$$

$$171 \quad \int_{t-h_b}^{t-\tau(t)} x^T(s) P_d x(s) ds + h_a \int_{-h_a}^0 \int_{t+\theta}^t \dot{x}^T(s) Q_0 \dot{x}(s) ds d\theta + \lambda_h \int_{-h_b}^{-h_a} \int_{t+\theta}^t \dot{x}^T(s) Q_\lambda \dot{x}(s) ds d\theta$$

172 With $R_\lambda > 0$, $Q_\lambda > 0$, $R_0 > 0$, $R_d \geq 0$, $P_d \geq 0$, $Q_0 > 0$, $P = \begin{bmatrix} P_1 & P_2 \\ * & P_3 \end{bmatrix} > 0$ being real symmetric

173 matrices with appropriate dimensions and $\lambda_h = h_b - h_a$.

174 Introduce the following vector:

$$175 \quad \xi := \text{col}\{x(t) \ \hat{x}(t) \ x(t-h_a) \ x(t-h_b) \ x(t-\tau(t)) \ \omega(t)\} \quad (3.8)$$

176 Then, rewrite error system (2.10) as

$$177 \quad \begin{cases} \tilde{\dot{x}}(t) = \sum_{i=1}^r \mu_i(z(t)) \sum_{k=1}^r \mu_k(z(t)) \begin{bmatrix} Y_{1j} \\ Y_{2j} \end{bmatrix} Y \\ e(t) = \sum_{i=1}^r \mu_i(z(t)) \sum_{k=1}^r \mu_k(z(t)) Y_{3j} Y \\ \tilde{x}(t) = [\Phi^T(t) \ 0] \end{cases} \quad (3.9)$$

178 Where Y_{1j} are defined in (3.4) and $Y_{2j} := [B_{fij} C_{kj} \ A_{fij} \ 0 \ 0 \ B_{fij} C_{dkj} \ B_{fij} D_{\omega kj}]$.

179 Using the expressions in (3.5) and taking the time-derivative of $V(t, \tilde{x}_t)$ along the solution of
180 (3.1) we obtain

$$181 \quad \dot{V}_p(t, \tilde{x}_t) = 2 \sum_{i=1}^r \mu_i(z(t)) \sum_{k=1}^r \mu_k(z(t)) \tilde{x}^T(t) P \begin{bmatrix} Y_{1j} \\ Y_{2j} \end{bmatrix} Y \quad (3.10)$$

$$\begin{aligned} \dot{V}_h(t, x_t) &= x^T(t) R_0 x(t) - x^T(t-h_a) R_0 x(t-h_a) + x^T(t-h_a) R_\lambda x(t-h_a) \\ 182 \quad &- x^T(t-h_b) R_\lambda x(t-h_b) + x^T(t-h_a) R_d x(t-h_a) - (1-d) x^T(t-\tau(t)) R_d x(t-\tau(t)) \end{aligned}$$

$$\begin{aligned}
 & + (1-d)x^T(t - \tau(t)) P_d x(t - \tau(t)) - x^T(t - h_b)P_d x(t - h_b) + h_a^2 \dot{x}^T(t) Q_0 \dot{x}(t) - \\
 & h_a \int_{t-h_a}^t \dot{x}^T(s) Q_0 \dot{x}(s) ds + \lambda_h^2 \dot{x}^T(t) Q_\lambda \dot{x}(t) - \lambda_h \int_{t-h_b}^{t-h_a} \dot{x}^T(s) Q_\lambda \dot{x}(s) ds
 \end{aligned} \tag{3.11}$$

Since $\tau(t) \in [h_a, h_b]$, we define $\rho(t) = (h_b - \tau(t))/\lambda_h$, $\lambda_h = h_b - h_a$, and we use lemma 2.3 to yield the following inequalities:

$$-h_a \int_{t-h_a}^t \dot{x}^T(s) Q_0 \dot{x}(s) ds \leq \begin{bmatrix} x(t) \\ x(t - h_a) \end{bmatrix}^T \begin{bmatrix} -Q_0 & Q_0 \\ * & -Q_0 \end{bmatrix} \begin{bmatrix} x(t) \\ x(t - h_a) \end{bmatrix} \tag{3.12}$$

$$\begin{aligned}
 & -\lambda_h \int_{t-h_b}^{t-h_a} \dot{x}^T(s) Q_\lambda \dot{x}(s) ds = -\lambda_h \int_{t-h_b}^{t-\tau(t)} \dot{x}^T(s) Q_\lambda \dot{x}(s) ds - \lambda_h \int_{t-\tau(t)}^{t-h_a} \dot{x}^T(s) Q_\lambda \dot{x}(s) ds \\
 & = - (h_b - \tau(t)) \int_{t-h_b}^{t-\tau(t)} \dot{x}^T(s) Q_\lambda \dot{x}(s) ds - (\tau(t) - h_a) \int_{t-\tau(t)}^{t-h_a} \dot{x}^T(s) Q_\lambda \dot{x}(s) ds \\
 & - (1 - \rho(t)) \lambda_h \int_{t-h_b}^{t-\tau(t)} \dot{x}^T(s) Q_\lambda \dot{x}(s) ds - \rho(t) \lambda_h \int_{t-\tau(t)}^{t-h_a} \dot{x}^T(s) Q_\lambda \dot{x}(s) ds \\
 & \leq \begin{bmatrix} x(t - h_a) \\ x(t - h_b) \\ x(t - \tau(t)) \end{bmatrix}^T \begin{bmatrix} -Q_\lambda & 0 & Q_\lambda \\ * & -Q_\lambda & Q_\lambda \\ * & * & -2Q_\lambda \end{bmatrix} \begin{bmatrix} x(t - h_a) \\ x(t - h_b) \\ x(t - \tau(t)) \end{bmatrix} \\
 & - (1 - \rho(t)) \lambda_h \int_{t-h_b}^{t-\tau(t)} \dot{x}^T(s) Q_\lambda \dot{x}(s) ds - \rho(t) \lambda_h \int_{t-\tau(t)}^{t-h_a} \dot{x}^T(s) Q_\lambda \dot{x}(s) ds
 \end{aligned} \tag{3.13}$$

In addition, by the Leibniz- Newton formula, we obtain the following equations for any real matrices χ_{pq} , $p = 1, 2$; $q = 1, 2, \dots, 6$ with appropriate dimensions:

$$\begin{aligned}
 0 & = 2\lambda_h (1 - \rho(t)) \xi^T \chi_1 \left[x(t - \tau(t)) - x(t - h_b) - \int_{t-h_b}^{t-\tau(t)} \dot{x}(s) ds \right], \\
 0 & = 2\lambda_h \rho(t) \xi^T \chi_2 \left[x(t - h_a) - x(t - \tau(t)) - \int_{t-\tau(t)}^{t-h_a} \dot{x}(s) ds \right],
 \end{aligned} \tag{3.14}$$

$$\chi_p := \text{col}\{\chi_{p1} \quad \chi_{p2} \quad \chi_{p3} \quad \chi_{p4} \quad \chi_{p5} \quad \chi_{p6}\},$$

Adding the right hand side of (3.14) and (3.13), and combining with (3.10)-(3.12) yield the following inequality:

$$\begin{aligned}
 \dot{V}(t, x_t) - \gamma^2 \omega^T(t) \omega(t) & \leq \sum_{i=1}^r \mu_i(z(t)) \sum_{k=1}^r \mu_k(z(t)) \{ \xi^T \Pi_{dj} \xi \\
 & - \lambda_h (1 - \rho(t)) \int_{t-h_b}^{t-\tau(t)} (\xi^T \chi_1 + \dot{x}^T(t) Q_\lambda) Q_\lambda^{-1} (\chi_1^T \xi + Q_\lambda \dot{x}(s)) ds - \\
 & \lambda_h \rho(t) \int_{t-\tau(t)}^{t-h_a} (\xi^T \chi_2 + \dot{x}^T(t) Q_\lambda) Q_\lambda^{-1} (\chi_2^T \xi + \\
 & Q_\lambda \dot{x}(s)) ds \}
 \end{aligned} \tag{3.15}$$

Where

$$\begin{aligned}
 \Pi_{dj} & = (1 - \rho(t)) \Pi_{1ik}^j + \rho(t) \Pi_{2ik}^j \\
 \Pi_{pik}^j & := \Pi_{ik}^j + [-I_p^T Q_\lambda I_p + \lambda_h \chi_p + \delta_h I_p^T \chi_p^T] + h_a^2 \gamma_{1j}^T Q_0 \gamma_{1j} + \delta_h^2 \gamma_{1j}^T Q_\lambda \gamma_{1j} + \lambda_h^2 \chi_p^T Q_\lambda^{-1} \chi_p
 \end{aligned} \tag{3.16}$$

With χ_p , ($p = 1, 2$), Π_{ik}^j , I_1 and I_2 are defined in (3.3).

Notice that, since $Q_\lambda > 0$, $\rho(t) \in [0, 1]$, (3.15) implies the following:

$$\dot{V}(t, x_t) - \gamma^2 \omega^T(t) \omega(t) \leq \sum_{i=1}^r \mu_i(z(t)) \sum_{k=1}^r \mu_k(z(t)) \xi^T \Pi_{dj} \xi \quad (3.17)$$

Due to $\rho(t) \in [0, 1]$, $\Pi_{dj} < 0$ if $\Pi_{pik}^j < 0$, $p=1, 2$. According to **schur's complement**, $\Pi_{pik}^j < 0$ is equivalent to the following LMIs:

$$\Psi_{pik}^j = \begin{bmatrix} \Pi_{ik}^j + [-I_p^T Q_\lambda I_p + \lambda_h \chi_p I_p + \lambda_h I_p^T \chi_p^T] & h_a Y_{1j}^T Q_0 & \lambda_h Y_{1j}^T Q_\lambda & \delta_h \chi_p \\ * & -Q_0 & 0 & 0 \\ * & * & -Q_\lambda & 0 \\ * & * & * & -Q_\lambda \end{bmatrix} < 0, p=1, 2 \quad (3.18)$$

And $\Psi_{pik}^j < 0$ leads for $d = d_1$ or d_2 , to the following:

$$\Psi_{pik}^{jl} = \Psi_{pik}^j|_{d=d_l} < 0; l=1, 2 \quad (3.19)$$

Notice that

$$\Psi_{pik}^j = \frac{d_2 - d}{d_2 - d_1} \Psi_{pik}^{j1} + \frac{d - d_1}{d_2 - d_1} \Psi_{pik}^{j2} \quad (3.20)$$

Therefore, the **two LMIs (3.19)** imply (3.18), and Ψ_{pik}^j are **thus convex in** $d \in [d_1, d_2]$, for $p=1, 2$.

Then if the two LMIs (3.18) are feasible, then $\Pi_{dj} < 0$. It follows from (3.17) that

$$\dot{V}(t, x_t) - \gamma^2 \omega^T(t) \omega(t) < 0$$

From the above analysis, it can be easily seen that the stability **asymptotic** of the error system can be obtained for $\omega(t) = 0$, that is

$$\dot{V}(t, x_t) < 0$$

Next, assuming that $\tilde{x}(t) = 0$, for all $t \in [-h_p, 0]$, we prove that the H_∞ performance $\|e\|_2 < \gamma \|\omega(t)\|_2$ is also guaranteed for all nonzero $\omega(t) \in L_2[0, \infty)$ and prescribed performance level $\gamma > 0$.

Since $\sum_{i=1}^r \mu_i(z(t)) = 1$, we can write $e^T(t)e(t) = \sum_{i=1}^r \mu_i(z(t)) \sum_{k=1}^r \mu_k(z(t)) \xi^T Y_{3j}^T Y_{3j} \xi$, one rewrites (3.17) in the following:

$$\dot{V}(t, x_t) \leq \sum_{i=1}^r \mu_i(z(t)) \sum_{k=1}^r \mu_k(z(t)) \{ \xi^T \hat{\Pi}_{dj} \xi - e^T(t)e(t) + \gamma^2 \omega^T(t) \omega(t) \}, \quad (3.21)$$

Where

$$\begin{aligned} \hat{\Pi}_{dj} &= (1 - \rho(t)) \hat{\Pi}_{1ik}^j + \rho(t) \hat{\Pi}_{2ik}^j \\ \hat{\Pi}_{pik}^j &:= \Pi_{ik}^j + [-I_p^T Q_\lambda I_p + \lambda_h \chi_p + \lambda_h I_p^T \chi_p^T] + h_a^2 Y_{1j}^T Q_0 Y_{1j} + \lambda_h^2 Y_{1j}^T Q_\lambda Y_{1j} + \lambda_h^2 \chi_p^T Q_\lambda^{-1} \chi_p + Y_{3j}^T Y_{3j} \end{aligned} \quad (3.22)$$

If the condition (3.2) holds, then by using the schur complement we obtain $\hat{\Pi}_{pik}^j < 0$ for $p=1, 2$ and consequently $\hat{\Pi}_{dj} < 0$ because of $\rho(t) \in [0, 1]$.

230 In the same way as above, we can show that $\Psi_{pik}^j < 0, (p = 1, 2)$ are also convex in

231 $d \in [d_1, d_2]$. Thus, we have

$$232 \quad \dot{V}(t, x_t) \leq -e^T(t)e(t) + \gamma^2 \omega^T(t)\omega(t) \quad (3.23)$$

233 Integrating both sides of (3.23) from 0 to ∞ on t , and considering the zero initial condition, we

234 obtain

$$235 \quad \int_0^\infty e^T(t)e(t) < \gamma^2 \int_0^\infty \omega^T(t)\omega(t) \quad (3.24)$$

236 That is $\|e\|_2 < \gamma \|\omega\|_2$,

$$237 \quad \tilde{\Sigma}_i^j = \Pi_{ik}^j - I_p^T Q_\lambda I_p + \lambda_h \chi_p I_p + \lambda_h I_p^T \chi_p^T$$

$$\hat{\Sigma}_i^j = \begin{bmatrix} h_a \gamma_{1j}^T Q_0 & \lambda_h \gamma_{1j}^T Q_\lambda & \lambda_h \chi_p & \gamma_{3j}^T \end{bmatrix}$$

$$\tilde{Q} = \text{diag} \{Q_0, Q_\lambda, Q_\lambda, I\}$$

$$\theta_i^j = \tilde{\Sigma}_i^j + \hat{\Sigma}_i^j \tilde{Q}^{-1} \hat{\Sigma}_i^{jT}$$

238 And let

$$S_j = \{(x^T, y^T, z^T, u^T, v^T, q^T)^T / (x^T, y^T, z^T, u^T, v^T, q^T)^T \theta_i^j (x^T, y^T, z^T, u^T, v^T, q^T)^T < 0\}$$

$$239 \quad \text{Then } \cup_{j=1}^N S_j = \mathbb{R}^{5n+p} / \{0\}$$

240 Construct the following sets:

$$241 \quad \bar{S}_1 = S_1, \bar{S}_2 = S_2 - \bar{S}_1, \dots,$$

$$242 \quad \bar{S}_i = S_i - \cup_{j=1}^{i-1} \bar{S}_j, \dots, \bar{S}_N = S_N - \cup_{j=1}^{N-1} \bar{S}_j$$

$$243 \quad \text{We get } \cup_{j=1}^{N-1} \bar{S}_j = \mathbb{R}^{5n+p} / \{0\}, \bar{S}_i \cap \bar{S}_j = \emptyset \quad i \neq j \quad (3.25)$$

244 Now we construct the switching rule SR as $\sigma = j$

245 When $\xi = (x^T(t), \hat{x}^T(t), x^T(t - h_a), x^T(t - h_b), x^T(t - \tau), \omega^T(t))^T \in \bar{S}_j, j \in \{1, 2, \dots, N\}$. So,

246 when $\xi \in \bar{S}_j$, the j^{th} subsystem is activated.

247 This completes the proof. ■

248 In the following part, we present a new stability result for the filter error system (2.10) with

249 uncertainties satisfying (2.5).

250 **Corollary 3.2 :** Given a scalars $0 \leq h_a \leq h_b, d_1 \leq d_2$ and $\gamma > 0$, the filter error system (2.10)

251 is **robustly stable**, for all differentiable delay $\tau(t) \in [h_a, h_b]$ and has a prescribed H_∞

252 performance level γ if there exist real symmetric matrices $R_0 > 0, R_\lambda > 0, Q_0 > 0, Q_\lambda >$

$$253 \quad 0, P = \begin{bmatrix} P_1 & P_2 \\ * & P_3 \end{bmatrix} > 0, R_d \geq P_d \geq 0, \text{ real matrices } X_{pq}, (p = 1, 2; q = 1, 2, \dots, 6) \text{ with}$$

254 appropriate dimensions, and positive scalars $\varepsilon_1, \varepsilon_2, \varepsilon_3$ such that the following LMIs are feasible

255 where $d = d_1, d_2$

$$\Sigma_{pik}^{jc} = \begin{bmatrix} \Sigma_{pik}^j & H_1 & \varepsilon_1 U_1 & H_2 & \varepsilon_2 U_2 & H_3 & \varepsilon_3 U_3 \\ * & -\varepsilon_1 I & 0 & 0 & 0 & 0 & 0 \\ * & * & -\varepsilon_1 I & 0 & 0 & 0 & 0 \\ * & * & * & -\varepsilon_2 I & 0 & 0 & 0 \\ * & * & * & * & -\varepsilon_2 I & 0 & 0 \\ * & * & * & * & * & -\varepsilon_3 I & 0 \\ * & * & * & * & * & * & -\varepsilon_3 I \end{bmatrix} < 0 \quad (3.26)$$

Where Σ_{pik}^j are define in (3.2), (3.3), (3.4) and

$$\begin{aligned} H_1 &= [D_{1ij}^T P_1 \quad D_{1ij}^T P_2^T \quad 0 \quad 0 \quad 0 \quad 0 \quad h_a D_{1ij}^T Q_0 \quad \delta_h D_{1ij}^T Q_\lambda \quad 0 \quad 0]^T \\ H_2 &= [D_{2ij}^T B_{fij}^T P_2^T \quad D_{2ij}^T B_{fij}^T P_3 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad -D_{2ij}^T D_{fij}^T]^T \\ H_3 &= [0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad D_{3ij}^T]^T \\ U_i &= [E_{k0ij} \quad 0 \quad 0 \quad 0 \quad E_{k1ij} \quad E_{k2ij} \quad 0 \quad 0 \quad 0 \quad 0]^T, \end{aligned} \quad (3.27)$$

Proof: Replacing $A_{kj}, A_{dkj}, B_{\omega kj}, C_{kj}, C_{dkj}, D_{\omega kj}, L_{kj}, L_{dkj}$ and $G_{\omega kj}$ by $A_{kj} + \Delta A_{kj}(t), A_{dkj} + \Delta A_{dkj}(t), B_{\omega kj} + \Delta B_{\omega kj}(t), C_{kj} + \Delta C_{kj}(t), C_{dkj} + \Delta C_{dkj}(t), D_{\omega kj} + \Delta D_{\omega kj}(t), L_{kj} + \Delta L_{kj}(t), L_{dkj} + \Delta L_{dkj}(t)$ and $G_{\omega kj} + \Delta G_{\omega kj}(t)$ in (3.2) respectively, using lemma 2.2 and lemma 2.3 and by using the schur complement we obtain the LMI defined in (3.26). This completes the proof. ■

4. Fuzzy H_∞ filter design:

In the sequel, the following theorem provides sufficient conditions for the existence of a filter in the form of (2.9) assuring an H_∞ performance for system (3.1) and shows how to determine the filter parameter matrices.

Theorem 4.1 : Given scalars $0 \leq h_a \leq h_b, d_1 \leq d_2$ and $\gamma > 0$, the H_∞ filter error design problem is solvable, for all differentiable delay $\tau(t) \in [h_a, h_b]$, if there exist matrices $P_1 > 0, U > 0, R_0 > 0, R_\lambda > 0, Q_0 > 0, Q_\lambda > 0, P_\lambda \geq 0, R_\lambda \geq 0$, and real matrices $N_{1ij}, N_{2ij}, N_{3ij}, N_{4ij}$ ($i = 1, \dots, r; j = 1, \dots, N$), $\hat{\chi}_p^i = \text{col}\{\chi_{p1}^i \quad \chi_{p2}^i \quad \chi_{p3}^i \quad \chi_{p4}^i \quad \chi_{p5}^i \quad \chi_{p6}^i\}$, ($p = 1, 2; i = 1, \dots, r$) with appropriate dimensions such that the following LMIs are feasible where $d = d_1, d_2$

$$\text{i. } U - P_1 < 0 \quad (4.1)$$

$$\text{ii. } \Pi_p^j(m, n) =$$

$$\begin{bmatrix} \hat{\Pi}_{mn}^j + [-I_p^T Q_\lambda I_p + \lambda_h \hat{\chi}_p^m I_p + \lambda_h I_p^T (\hat{\chi}_p^m)^T] & h_a (\gamma_{1j}^m)^T Q_0 & \lambda_h (\gamma_{1j}^m)^T Q_\lambda & \lambda_h \hat{\chi}_p^m & (\gamma_{3j}^m)^T \\ * & -Q_0 & 0 & 0 & 0 \\ * & * & -Q_\lambda & 0 & 0 \\ * & * & * & -Q_\lambda & 0 \\ * & * & * & * & -I \end{bmatrix} < 0$$

279 (4.2)

280 Where

$$\hat{\Pi}_{mn}^j = \begin{bmatrix} \hat{\varphi}_{11j} & \hat{\varphi}_{12j} & Q_0 & 0 & \hat{\varphi}_{15j} & \hat{\varphi}_{16j} \\ * & \hat{\varphi}_{22j} & 0 & 0 & \hat{\varphi}_{25j} & \hat{\varphi}_{26j} \\ * & * & R_d - R_0 + R_\lambda - Q_0 - Q_\lambda & 0 & Q_\lambda & 0 \\ * & * & * & -R_\lambda - Q_\lambda - P_d & Q_\lambda & 0 \\ * & * & * & * & -(1-d)(R_d - P_d) - 2Q_\lambda & 0 \\ * & * & * & * & * & -\gamma^2 I \end{bmatrix}$$

$$\begin{aligned} 281 \quad \hat{\varphi}_{11j} &= P_1 A_{mj} + A_{mj}^T P_1 + N_{2nj} C_{mj} + C_{mj}^T N_{2nj}^T + R_0 - Q_0 \\ 282 \quad \hat{\varphi}_{12j} &= N_{1nj} + A_{mj}^T U + C_{mj}^T N_{2nj}^T; \quad \hat{\varphi}_{22j} = N_{1nj} + N_{1nj}^T \\ 283 \quad \hat{\varphi}_{15j} &= P_1 A_{dm} + N_{2nj} C_{dmj}; \quad \hat{\varphi}_{25j} = U A_{dmj} + N_{2nj} C_{mj} \end{aligned} \quad (4.3)$$

$$\begin{aligned} 284 \quad \hat{\varphi}_{16j} &= P_1 B_{wmj} + N_{2nj} D_{wmj}; \quad \hat{\varphi}_{26j} = U B_{wmj} + N_{2nj} D_{wmj} \\ Y_{1j}^m &:= [A_{mj} \quad 0 \quad 0 \quad 0 \quad A_{\tau mj} \quad B_{wmj}] \\ Y_{3j}^m &:= [L_{mj} - N_{4nj} C_{mj} \quad -N_{3nj} \quad 0 \quad 0 \quad L_{dmj} - N_{4nj} C_{dmj} \quad G_{wmj} - N_{4nj} D_{wmj}] \end{aligned}$$

285 Moreover, the filter matrices can be constructed by

$$286 \quad A_{fij} = N_{1ij} U^{-1}, B_{fij} = N_{2ij}, C_{fij} = N_{3ij} U^{-1}, D_{fij} = N_{4ij} \quad (i=1,2,\dots,r; j=1,2,\dots,N) \quad (4.4)$$

287 **Proof:** Due to $U > 0$, there exist a nonsingular real $n \times n$ matrix P_2 and a real $n \times n$ matrix $P_3 > 0$
288 such that $U = P_2 P_3^{-1} P_2^T$. Defining

$$289 \quad J := \text{diag} \{I, P_2^{-T} P_3, I, I, I, I, I, I, I\} \quad (4.5)$$

290 Left and right multiply $\Pi_p^j(i, k)$, $p = 1, 2$ defined in (4.2) by J^T and J , respectively, and take

$$291 \quad \chi_{p2} := P_3 P_2^{-1} \hat{\chi}_{p2}, \text{ and}$$

$$\begin{aligned} \bar{A}_{ij} &= P_2^{-1} N_{1ij} U^{-1} P_2, \quad \bar{B}_{ij} = P_2^{-1} N_{2ij}, \\ \bar{C}_{ij} &= N_{3ij} U^{-1} P_2, \quad \bar{D}_{ij} = N_{4ij}, \end{aligned} \quad (4.6)$$

293 By replacing $(A_{fij}, B_{fij}, C_{fij}, D_{fij})$ in Σ_{pik}^j , $p=1, 2$ defined in (3.2) with $(\bar{A}_{ij}, \bar{B}_{ij}, \bar{C}_{ij}, \bar{D}_{ij})$, one

$$294 \quad \text{yields } \Sigma_{pik}^j = J^T \Pi_p^j(i, k) J, \quad p=1, 2 \quad (4.7)$$

295 So $\Pi_p^j(i, k) < 0$ if $\Sigma_{pik}^j < 0$. On the other hand, it is clear from (4.1) that

$$296 \quad P_1 - U = P_1 - P_2 P_3^{-1} P_2^T > 0, \text{ applying schur complement yields } \begin{bmatrix} P_1 & P_2 \\ * & P_3 \end{bmatrix} > 0. \text{ Therefore, we can}$$

297 conclude from theorem 3.2 that the filter, that is,

$$\begin{cases} \dot{\bar{x}}(t) = \sum_{i=1}^r \mu_i(z(t)) (\bar{A}_{ij} \bar{x}(t) + \bar{B}_{ij} y(t)) \\ \bar{z}(t) = \sum_{i=1}^r \mu_i(z(t)) (\bar{C}_{ij} \bar{x}(t) + \bar{D}_{ij} y(t)) \end{cases} \quad (4.8)$$

299 With $(\bar{A}_{ij}, \bar{B}_{ij}, \bar{C}_{ij}, \bar{D}_{ij})$ are defined in (4.6), guarantees that the filtering error system (3.1) to be
 300 asymptotically stable with a prescribed H_∞ noise attenuation level γ .

301 Next, let $\hat{x}(t) = P_2 \bar{x}(t)$ then from (4.8) we obtain the filter (2.9) with the matrices given by
 302 (4.4)

303 Let $\tilde{\Sigma}_i^j = \hat{\Pi}_{ik}^j - I_p^T Q_\lambda I_p + \lambda_h \hat{\chi}_p^i I_p + \lambda_h I_p^T \hat{\chi}_p^i$
 $\hat{\Sigma}_i^j = [h_a Y_{1ij}^T Q_0 \quad \lambda_h Y_{1ij}^T Q_\lambda \quad \lambda_h \hat{\chi}_p \quad Y_{3ij}^T]$

$$\tilde{Q} = \text{diag} \{Q_0, Q_\lambda, Q_\lambda, I\}$$

$$\theta_{ik}^j = \tilde{\Sigma}_{ik}^j + \hat{\Sigma}_{ik}^j \tilde{Q}^{-1} \hat{\Sigma}_{ik}^{jT}$$

304 And let

$$S_j = \{(x^T, y^T, z^T, u^T, v^T, q^T)^T / (x^T, y^T, z^T, u^T, v^T, q^T) \theta_{ik}^j (x^T, y^T, z^T, u^T, v^T, q^T)^T < 0\}$$

305 Then $\cup_{j=1}^N S_j = \mathbb{R}^{5n+p} / \{0\}$

306 Then we construct the set $\bar{S}_j, j=1, \dots, N$, and the switching rule in the same way as in the
 307 previous section.

308 this completes the proof. ■

309 Similar to corollary 3.2, the following results is then obtained.

310 **Corollary 4.2:** Given scalars $0 \leq h_a \leq h_b, d_1 \leq d_2$ and $\gamma > 0$, the H_∞ filter error design
 311 problem, for all differentiable delay $\tau(t) \in [h_a, h_b]$, is **solvable** if there exist matrices $P_1 > 0$,

312 $U > 0, R_0 > 0, R_\lambda > 0, Q_0 > 0, Q_\lambda > 0, P_d \geq 0, R_d \geq 0$, and real matrices

313 $N_{1ij}, N_{2ij}, N_{3ij}, N_{4ij} (i = 1, \dots, r; j = 1, \dots, N), \hat{\chi}_p^i =$

314 $\text{col}\{\chi_{p1}^i \quad \chi_{p2}^i \quad \chi_{p3}^i \quad \chi_{p4}^i \quad \chi_{p5}^i \quad \chi_{p6}^i\}, (p = 1, 2; i = 1, 2, \dots, r)$ with appropriate dimensions,

315 and positive scalars $\varepsilon_1, \varepsilon_2, \varepsilon_3$ such that the following LMIs are feasible where $d = d_1, d_2$

316 i. $U - P_1 < 0$ (4.9)

317 ii. $\Pi_{pik}^{jc} = \begin{bmatrix} \Pi_p^j(i, k) & H_1 & \varepsilon_1 U_1 & H_2 & \varepsilon_2 U_2 & H_3 & \varepsilon_3 U_3 \\ * & -\varepsilon_1 I & 0 & 0 & 0 & 0 & 0 \\ * & * & -\varepsilon_1 I & 0 & 0 & 0 & 0 \\ * & * & * & -\varepsilon_2 I & 0 & 0 & 0 \\ * & * & * & * & -\varepsilon_2 I & 0 & 0 \\ * & * & * & * & * & -\varepsilon_3 I & 0 \\ * & * & * & * & * & * & -\varepsilon_3 I \end{bmatrix} < 0$ (4.10)

318 Where $\Pi_p^j(i, k), H_i$ and U_i are defined in (4.2) and (3.29) respectively.

319 5. Examples:

320 In this section, **tree** examples are given to show the effectiveness of the proposed method and to
 321 compare with the literature results.

322 **Example 5.1 ([25], [26]):** Consider the following time-delay fuzzy system

323 Rule 1: If $z_1(t)$ is W_1^i ,

324 Then

$$325 \begin{cases} \dot{x}(t) = A_{11}x(t) + A_{d11}x(t - \tau(t)) + B_{w11}w(t) \\ y(t) = C_{11}x(t) + C_{d11}x(t - \tau(t)) + D_{w11}w(t) \\ z(t) = L_{11}x(t) + L_{d11}x(t - \tau(t)) + G_{w11}w(t) \\ x(t) = \phi(t) \quad t \in [-h_b, 0] \end{cases}$$

326 And

327 Rule 2: If $z_1(t)$ is W_1^i ,

328 Then

$$329 \begin{cases} \dot{x}(t) = A_{21}x(t) + A_{d21}x(t - \tau(t)) + B_{w21}w(t) \\ y(t) = C_{21}x(t) + C_{d21}x(t - \tau(t)) + D_{w21}w(t) \\ z(t) = L_{21}x(t) + L_{d21}x(t - \tau(t)) + G_{w21}w(t) \\ x(t) = \phi(t) \quad t \in [-h_b, 0] \end{cases}$$

330 With $A_{11} = \begin{bmatrix} -2.1 & 0.1 \\ 1 & -2 \end{bmatrix}$; $A_{21} = \begin{bmatrix} -1.9 & 0 \\ -0.2 & -1.1 \end{bmatrix}$; $A_{d11} = \begin{bmatrix} -1.1 & 0.1 \\ -0.8 & -0.9 \end{bmatrix}$;

331 $A_{d21} = \begin{bmatrix} -0.9 & 0 \\ -1.1 & -1.2 \end{bmatrix}$; $B_{w11} = \begin{bmatrix} 1 \\ -0.2 \end{bmatrix}$; $B_{w21} = \begin{bmatrix} 0.3 \\ 0.1 \end{bmatrix}$; $C_{11} = [1 \quad 0]$; $C_{21} = [0.5 \quad -0.6]$;

332 $C_{d11} = [-0.8 \quad 0.6]$; $C_{d21} = [-0.2 \quad 1]$; $D_{w11} = 0.3$; $D_{w21} = -0.6$; $L_{11} = [1 \quad -0.5]$;

333 $L_{21} = [-0.2 \quad 0.3]$; $L_{d11} = [0.1 \quad 0]$; $L_{d21} = [0 \quad 0.2]$; $G_{w11} = 0$; $G_{w21} = 0$;

334 Our purpose is to determine the minimum H_∞ prescribed attenuation level γ for which the
 335 system is asymptotically stable. Table 1 lists the results from various values of h_b when $h_a =$
 336 0.3 . Table 2 lists the results from various values of h_b when $h_a = 0.2$. It is clear that the results
 337 obtained from our method are better than those obtained from existing methods [25], [26].

338 **Table1:** Minimum values of H_∞ performance γ for given delay h_b with $h_a = 0.3$

	$h_b=1$	$h_b=0.8$	$h_b=0.6$	$h_b=0.5$
γ [25]	0.37	0.35	0.34	0.34
γ [26]	0.3481	0.3405	0.3322	0.3264
γ (theorem 4.1)	0.2116	0.2007	0.1853	0.1750

339

340

341

Table2: Minimum values of H_∞ performance γ for given delay h_b with $h_a = 0.2$

	$h_b=1$	$h_b=0.8$	$h_b=0.6$	$h_b=0.5$
γ [26]	0.3307	0.3244	0.3158	0.3148
γ (theorem 4.1)	0.2124	0.2014	0.1857	0.1751

For $h_a = 0.3, h_b = 1$, the filter parameters as given as follows:

$$A_{f11} = \begin{bmatrix} -7.9117 & -3.7469 \\ 5.5546 & 0.9479 \end{bmatrix}; A_{f21} = \begin{bmatrix} -2.2407 & -0.3109 \\ 0.0384 & -1.9337 \end{bmatrix}; B_{f11} = \begin{bmatrix} -0.4551 \\ 0.3802 \end{bmatrix};$$

$$B_{f21} = \begin{bmatrix} -0.1321 \\ 0.2398 \end{bmatrix}; C_{f11} = [-7.7611 \quad -2.3596]; C_{f21} = [0.0481 \quad -1.9574];$$

Example 5.2 [22]: In this example two cases are presented to demonstrate the effectiveness of the proposed approach presented in this paper.

Case 1: consider a time varying delay switched nominal system (3.1), where $h_a = 0.2$,

$d_1 = 0, d_2 = 0.4, \gamma=2$ with the following two subsystems

Subsystem 1 is described by

$$A_{11} = \begin{bmatrix} -2 & 0 \\ 0 & -0.9 \end{bmatrix}; A_{d11} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}; B_{w11} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}; C_{11} = [1 \quad 0]; C_{d11} = [0 \quad 1];$$

$$D_{w11} = 1; L_{11} = [1 \quad 2]; L_{d11} = [1 \quad 0]; G_{w11} = 0.1;$$

And subsystem 2 is described by

$$A_{12} = \begin{bmatrix} -2 & 0 \\ 0 & -0.7 \end{bmatrix}; A_{d12} = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix}; B_{w12} = \begin{bmatrix} -0.5 \\ 2 \end{bmatrix}; C_{12} = [0 \quad 1]; C_{d12} = [1 \quad 2];$$

$$D_{w12} = 1; L_{12} = [2 \quad 1]; L_{d12} = [0 \quad 1]; G_{w12} = 0.2;$$

Applying the results of [22], the maximum time delay h_b is found to be $h_b = 0.6$. Applying theorem 4.1 of this paper we obtain $h_b = 0.9030$. It is easy to verify that the results in this paper are less conservative than the ones obtained in [22].

And the filter parameters as follows:

$$A_{f11} = 10^3 \times \begin{bmatrix} 0.5513 & -7.4723 \\ 0.2242 & -3.0191 \end{bmatrix}; B_{f11} = \begin{bmatrix} -4.2492 \\ -1.1140 \end{bmatrix}$$

$$C_{f11} = 10^3 \times [0.1523 \quad -2.0473]; D_{f11} = -0.2272$$

$$A_{f12} = 10^4 \times \begin{bmatrix} 0.0948 & -1.2768 \\ 0.0408 & -0.5489 \end{bmatrix}; B_{f12} = \begin{bmatrix} -3.3350 \\ -0.9112 \end{bmatrix}$$

$$C_{f12} = 10^3 \times [0.2306 \quad -3.0986]; D_{f12} = 0.6106$$

358 **Case 2:** consider a time varying delay switched nominal system (3.1), where $h_a = 0.3$, $d_1 =$
 359 0 , $d_2 = 0$, $\gamma=8$. with the following two subsystems
 360 Subsystem 1 is described by

$$A_{11} = \begin{bmatrix} -0.2 & 0 \\ 0 & -0.8 \end{bmatrix}; A_{d11} = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix}; B_{w11} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}; C_{11} = [1 \quad 0]; C_{d11} = [0 \quad 1];$$

$$D_{w11} = 1; L_{11} = [1 \quad 0]; L_{d11} = [1 \quad 0]; G_{w11} = 0.1;$$

361 And subsystem 2 is described by

$$A_{12} = \begin{bmatrix} -0.1 & 0 \\ 0 & -1 \end{bmatrix}; A_{d12} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}; B_{w12} = \begin{bmatrix} -0.5 \\ 1 \end{bmatrix}; C_{12} = [0 \quad 1]; C_{d12} = [1 \quad 0];$$

$$D_{w12} = 1; L_{12} = [1 \quad 1]; L_{d12} = [0 \quad 1]; G_{w12} = 0.1;$$

362 Computing the upper bound h_b of time delay, we obtain by applying our results and those of
 363 [22], $h_b = 1.4270$ and $h_b = 1$ respectively. It is easy to verify that the results in this paper are
 364 less conservative than the ones obtained in [22]. The filter parameters as follows:

$$A_{f11} = \begin{bmatrix} 0 - 3.2600 & 1.5006 \\ -1.2327 & 0.0318 \end{bmatrix}; B_{f11} = \begin{bmatrix} -0.2092 \\ 1.1343 \end{bmatrix}$$

$$C_{f11} = [-1.8983 \quad 0.4018]; D_{f11} = 0.0290$$

$$A_{f12} = \begin{bmatrix} -1.0478 & 0.8190 \\ 2.3136 & -3.5177 \end{bmatrix}; B_{f12} = \begin{bmatrix} 1.0905 \\ -0.8014 \end{bmatrix}$$

$$C_{f12} = [-0.3187 \quad -0.5311]; D_{f12} = -0.6148$$

365 **Example 5.3:** In this section, we provide a simulation example to illustrate the H^∞ filter design
 366 approach developed in the previous section. The uncertain switched time-delay T-S fuzzy
 367 system considered in this example is described with two rules:

368 For the subsystem 1

369 Plant Rule 1: If $z_1(t)$ is W_1

$$\begin{cases} \dot{x}(t) = (A_{11} + \Delta A_{11}(t))x(t) + (A_{d11} + \Delta A_{d11}(t))x(t - \tau(t)) + (B_{w11} + \Delta B_{w11}(t))\omega(t) \\ y(t) = (C_{11} + \Delta C_{11}(t))x(t) + (C_{d11} + \Delta C_{d11}(t))x(t - \tau(t)) + (D_{w11} + \Delta D_{w11}(t))\omega(t) \\ z(t) = (L_{11} + \Delta L_{11}(t))x(t) + (L_{d11} + \Delta L_{d11}(t))x(t - \tau(t)) + (G_{w11} + \Delta G_{w11}(t))\omega(t) \\ x(t) = \phi(t) \quad t \in [-h_b, 0] \quad i = 1, \dots, r \end{cases}$$

370 And

371 Plant Rule 2: If $z_2(t)$ is W_2

$$\begin{cases} \dot{x}(t) = (A_{21} + \Delta A_{21}(t))x(t) + (A_{d21} + \Delta A_{d21}(t))x(t - \tau(t)) + (B_{w21} + \Delta B_{w21}(t))\omega(t) \\ y(t) = (C_{21} + \Delta C_{21}(t))x(t) + (C_{d21} + \Delta C_{d21}(t))x(t - \tau(t)) + (D_{w21} + \Delta D_{w21}(t))\omega(t) \\ z(t) = (L_{21} + \Delta L_{21}(t))x(t) + (L_{d21} + \Delta L_{d21}(t))x(t - \tau(t)) + (G_{w21} + \Delta G_{w21}(t))\omega(t) \\ x(t) = \phi(t) \quad t \in [-h_b, 0] \quad i = 1, \dots, r \end{cases}$$

372 For the subsystem 2

373 Plant Rule 1: If $z_1(t)$ is W_1

$$\begin{cases} \dot{x}(t) = (A_{12} + \Delta A_{12}(t))x(t) + (A_{d12} + \Delta A_{d12}(t))x(t - \tau(t)) + (B_{w12} + \Delta B_{w12}(t))\omega(t) \\ y(t) = (C_{12} + \Delta C_{12}(t))x(t) + (C_{d12} + \Delta C_{d12}(t))x(t - \tau(t)) + (D_{w12} + \Delta D_{w12}(t))\omega(t) \\ z(t) = (L_{12} + \Delta L_{12}(t))x(t) + (L_{d12} + \Delta L_{d12}(t))x(t - \tau(t)) + (G_{w12} + \Delta G_{w12}(t))\omega(t) \\ x(t) = \phi(t) \quad t \in [-h_b, 0] \quad i = 1, \dots, r \end{cases}$$

374 And

375 Plant Rule 2: If $z_2(t)$ is W_2

$$\begin{cases} \dot{x}(t) = (A_{22} + \Delta A_{22}(t))x(t) + (A_{d22} + \Delta A_{d22}(t))x(t - \tau(t)) + (B_{w22} + \Delta B_{w22}(t))\omega(t) \\ y(t) = (C_{22} + \Delta C_{22}(t))x(t) + (C_{d22} + \Delta C_{d22}(t))x(t - \tau(t)) + (D_{w22} + \Delta D_{w22}(t))\omega(t) \\ z(t) = (L_{22} + \Delta L_{22}(t))x(t) + (L_{d22} + \Delta L_{d22}(t))x(t - \tau(t)) + (G_{w22} + \Delta G_{w22}(t))\omega(t) \\ x(t) = \phi(t) \quad t \in [-h_b, 0] \quad i = 1, \dots, r \end{cases}$$

376 Where $A_{11} = \begin{bmatrix} 17 & -1 \\ 0 & -9 \end{bmatrix}$; $A_{d11} = \begin{bmatrix} -10 & 0 \\ -0.15 & -1 \end{bmatrix}$; $B_{w11} = \begin{bmatrix} 0 \\ -2 \end{bmatrix}$; $C_{11} = \begin{bmatrix} -1 & 0 \end{bmatrix}$;

377 $C_{d11} = \begin{bmatrix} 0 & 1 \end{bmatrix}$; $D_{w11} = 0.3$; $L_{11} = \begin{bmatrix} -0.2 & 0.3 \end{bmatrix}$; $L_{d11} = \begin{bmatrix} 1 & 0 \end{bmatrix}$; $G_{w11} = -0.5$;

378 $A_{21} = \begin{bmatrix} -19 & 0 \\ 0 & -9 \end{bmatrix}$; $A_{d21} = \begin{bmatrix} 1 & 0 \\ 0.05 & -1 \end{bmatrix}$; $B_{w21} = \begin{bmatrix} 0 \\ -2 \end{bmatrix}$; $C_{21} = \begin{bmatrix} -0.21 & 0 \end{bmatrix}$;

$C_{d21} = \begin{bmatrix} 0 & 1 \end{bmatrix}$; $D_{w21} = 0.3$; $L_{21} = \begin{bmatrix} -0.2 & 0.3 \end{bmatrix}$; $L_{d21} = \begin{bmatrix} 0 & 1 \end{bmatrix}$; $G_{w21} = -0.02$;

379 $A_{12} = \begin{bmatrix} -12 & 0 \\ -0.5 & -9 \end{bmatrix}$; $A_{d12} = \begin{bmatrix} -10 & 0 \\ -1 & -1 \end{bmatrix}$; $B_{w12} = \begin{bmatrix} -0.5 \\ -2 \end{bmatrix}$; $C_{12} = \begin{bmatrix} 0 & 1 \end{bmatrix}$;

$C_{d12} = \begin{bmatrix} 1 & 2 \end{bmatrix}$; $D_{w12} = 0.3$; $L_{12} = \begin{bmatrix} -0.2 & 0.3 \end{bmatrix}$; $L_{d12} = \begin{bmatrix} 1 & 0 \end{bmatrix}$; $G_{w12} = -0.15$;

380 $A_{22} = \begin{bmatrix} -12 & 0 \\ 0 & -9 \end{bmatrix}$; $A_{d22} = \begin{bmatrix} -0.0021 & 0 \\ -1 & -1 \end{bmatrix}$; $B_{w22} = \begin{bmatrix} -0.5 \\ -0.2 \end{bmatrix}$; $C_{22} = \begin{bmatrix} 0 & 1 \end{bmatrix}$;

$C_{d22} = \begin{bmatrix} 1 & 2 \end{bmatrix}$; $D_{w22} = -0.6$; $L_{22} = \begin{bmatrix} -0.2 & 0.3 \end{bmatrix}$; $L_{d22} = \begin{bmatrix} 0 & -1 \end{bmatrix}$; $G_{w22} = 0.6$;

381 The membership functions of μ_1 and μ_2 are, respectively, described by

$$382 \quad \mu_1(z_1(t)) = \begin{cases} -1 & \text{for } z_1 < -1 \\ \frac{1}{2} + \frac{1}{2}z_1 & \text{for } |z_1| \leq 1 \\ 1 & \text{for } z_1 > 1 \end{cases} \quad \text{And} \quad \mu_2(z_1(t)) = \begin{cases} 0 & \text{for } z_1 < -1 \\ \frac{1}{2} + \frac{1}{2}z_1 & \text{for } |z_1| \leq 1 \\ 1 & \text{for } z_1 > 1 \end{cases}$$

383 The parameter uncertainties $\Delta A_{ij}(t), \Delta A_{\tau ij}(t), \Delta B_{wij}(t), \Delta C_{ij}(t), \Delta C_{dij}(t), \Delta D_{wij}(t), \Delta L_{ij}(t),$
 384 $\Delta L_{dij}(t)$ and $\Delta G_{wij}(t)$, $i=1,2, j=1,2$ are assumed to satisfy (2.5) and (2.6) with

$$D_{111} = \begin{bmatrix} 0 \\ -0.5 \end{bmatrix}; E_{1011} = [0 \quad 0.3]; E_{1111} = [0.2 \quad 0]; E_{1211} = 0.1; D_{211} = 0.8;$$

$$E_{2011} = [0 \quad 0.3]; E_{2111} = [0.2 \quad 0]; E_{2211} = 0.1; D_{311} = 0.002;$$

$$E_{3011} = [0.1 \quad 0]; E_{3111} = [0.1 \quad 0]; E_{3211} = 0.1;$$

$$D_{121} = \begin{bmatrix} 0 \\ 0.3 \end{bmatrix}; E_{1021} = [0.5 \quad 0]; E_{1121} = [0 \quad -0.2]; E_{1221} = 0.2; D_{221} = 0.6;$$

$$E_{2021} = [0.5 \quad 0]; E_{2121} = [0 \quad -0.2]; E_{2221} = 0.2; D_{321} = 0.001; E_{3021} = [0.2 \quad 0];$$

$$E_{3121} = [0.2 \quad 0]; E_{3221} = -0.5;$$

$$D_{112} = \begin{bmatrix} 0 \\ -0.5 \end{bmatrix}; E_{1012} = [0 \quad 0.3]; E_{1112} = [0.2 \quad 0]; E_{1212} = 0.1; D_{212} = 0.8;$$

$$E_{2012} = [0 \quad 0.3]; E_{2112} = [0.2 \quad 0]; E_{2212} = 0.1; D_{312} = 0.004; E_{3012} = [0.1 \quad 0];$$

$$E_{3112} = [0.1 \quad 0]; E_{3212} = -0.2; D_{122} = \begin{bmatrix} 0 \\ 0.3 \end{bmatrix}; E_{1022} = [0.5 \quad 0]; E_{1122} = [0 \quad -0.2];$$

$$E_{1222} = 0.2; D_{222} = 0.6; E_{2122} = [0.5 \quad 0]; E_{2222} = [0 \quad -0.2]; E_{2222} = 0.2;$$

$$D_{322} = 0.002; E_{3022} = [0.2 \quad 0]; E_{3122} = [0.2 \quad 0]; E_{3222} = -0.2;$$

385 Given $h_a = 0, d_1 = 0, d_2 = 0.2$ and the upper bound $h_b = 4.2$

386 The purpose of this example is to design a stable switched fuzzy filter such that the filtering
 387 error system is stable and satisfies a prescribed H^∞ performance level by using corollary 4.2. In
 388 this example, the H^∞ performance level is specified to be 0.98 : For this purpose, we resort to
 389 the Matlab LMI Control Toolbox to solve the LMIs in (4.16), we can obtain the filter
 390 parameters as follows:

$$\begin{aligned} A_{f11} &= \begin{bmatrix} -217.0968 & -124.6463 \\ -14.7098 & -125.1392 \end{bmatrix}; & B_{f11} &= \begin{bmatrix} 0.0147 \\ 0.0052 \end{bmatrix} \\ C_{f11} &= [-3.7125 \quad 5.7636]; & D_{f11} &= -0.0025 \end{aligned}$$

$$\begin{aligned} A_{f21} &= \begin{bmatrix} -283.1316 & -67.5379 \\ 1.6472 & -135.6281 \end{bmatrix}; & B_{f21} &= \begin{bmatrix} 0.0038 \\ 0.0086 \end{bmatrix} \\ C_{f21} &= [1.0019 \quad -10.2763]; & D_{f21} &= 0.0140 \end{aligned}$$

$$\begin{aligned} A_{f12} &= \begin{bmatrix} -13.8393 & 58.5614 \\ -0.7136 & -136.5757 \end{bmatrix}; & B_{f12} &= \begin{bmatrix} 0.041 \\ -0.0073 \end{bmatrix} \\ C_{f12} &= [7.6017 \quad -25.5315]; & D_{f12} &= -4.5541 \times 10^{-5} \end{aligned}$$

$$\begin{aligned} A_{f22} &= \begin{bmatrix} -153.6803 & -20.050 \\ -4.1752 & -154.7375 \end{bmatrix}; & B_{f22} &= \begin{bmatrix} -0.0102 \\ -0.0178 \end{bmatrix} \\ C_{f22} &= [3.5613 \quad -24.1776]; & D_{f22} &= -0.0392 \end{aligned}$$

391

392 **6. Conclusion:**

393 In the paper, we have studied the problem of H_∞ filter for switched T-S fuzzy system with
 394 interval time-varying delay is investigated. A new method has been proposed to solve H_∞ filter
 395 design problem by employing convexity of the matrix function. Attention has been focused on
 396 the design of an H_∞ filter so that the filter error system is asymptotically stable and a given H_∞
 397 disturbance attenuation level is guaranteed. The obtained results are less conservative than the
 398 existing ones. Numerical examples show the effectiveness and the less conservativeness of the
 399 proposed method.

400

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