

Original Research Article

Maize Price Stabilization in Ghana: An Application of Continuous-Time Delay Differential Model with Buffer Stock

Abstract

The paper is intended to use mathematical models (which mimic model developed by Soltes et al. [16]) for controlling prices of maize by employing a nonlinear continuous time delay differential equation derived from linear demand and nonlinear supply functions of price with delay parameters reflecting the realities prevailing at the local market. These models would be formulated from parameters estimated from real economic data of maize price demand and production in the Ashanti Region, of Ghana through the use of regression. The data is obtained from the Ministry of Food and Agriculture, Statistical Directorate Kumasi-Ghana, from 1994 to 2013.

The results of the study dispute assertion by Mackey[18], who based his argument on the fact commodity price is dependent on planting time, storage time, relaxation time and total production time. If all these individual time parameters are combined as one for supply delay and make storage time, the delay for the buffer, then price oscillations would be drastically reduced as shown in this paper. The study also improved on the work done by Athanasiou et al. [9],[16] and others, whose models are discrete time dependent which is a limit case of the delay buffer stock model used in this study. The efficiency of buffer system is proven to be dependent on delay variation suitably enough to be used by buffer stock operator.

It is noted that, farther the buffer stock delay and supply delay are reduced, more stable the price becomes and the effects of buffer stock are felt more by stakeholders. The results of the analysis provide an average stable price of maize as GH¢ 30.49 compared to the actual average price of GH¢30.27. The equilibrium price in turn provide the average equilibrium metric tons of 2931.6 and 8217.6, for demand and supply respectively. The excess supply is kept in stock and when needed it is released in next market period. The standard deviation also is reduced to 0.1602 compared to 29.48 in the table 2, and the same applies to the variance.

However, before the application of buffer stock scheme, price oscillated between two price points and could not converge. This affirms the fact buffer stock acts as a reserve against short-term shortages and dampens down excessive fluctuations.

We draw inferences from this study that researchers should rather use continuous time nonlinear delay models as they reflect the realities in most real-life economic problems.

Keywords: buffer stock, price stabilization, delay-differential equation

1 Introduction

The prices of agricultural products such as maize, wheat and the likes tend to fluctuate more than prices of manufactured products and services. This is largely due to the instability in the supply of agricultural products coupled with the fact that demand and supply are pricing inelastic [1]. It is believed that in agricultural industry, Price volatility and scarcity is a great threat in the distribution system of seasonal staple foods including maize produced by farmers. There is a lot of disparities in term of supply during the harvest and planting seasons which in effect could create inconveniences to stakeholders such as producers, consumers, and government [2]. Thus supplies of staple food in agro-industry are extremely different between the harvest season and the planting season, and condition triggers price volatility and shortage of food which in turn leads to food security problems such as scarcity and price hikes for poor households. In almost all business environments, extensive studies have been done on demand because demand uncertainty is inherent. However, supply uncertainty has received less research attention and yet supply has a significant impact on achieving price stability. It is therefore critical for any profit oriented businesses to move the right quantity of commodities to the right location at the right time in order to turn potential consumer demand into revenue [3].

In agricultural economics, detecting the source of price fluctuations is one of the most important subjects. In the quest to study the reason for the recurring cycles in the production and prices of commodities on regular bases in the thirties, gave birth to the cobweb model[4], which describes price fluctuations in a single market for a commodity that takes one unit of time to produce.

In Ghana like the other developing countries, staple food such as maize normally accounts for quite significant share of poor households' budgets. The poor have limited possibilities to insure against adverse price shocks. Price spikes are very problematic for this class of people who are not self-sufficient, and can often jeopardize their capacity to feed themselves. An implement of food price stabilization policies are the only important way by which governments in these countries can express their concern to the problem [5].

1.1 Price Stabilization Systems

Many policy interventions have been used by stakeholders to stabilize price, improve quality of maize and increase maize production in Ghana for both local consumption and export to the world market. Price stability depends upon matching supply with demand because both supply and demand uncertainties can present a major obstacle to achieving this goal.

One way to smooth out the fluctuations in prices is to operate price support schemes through the use of buffer stocks. These schemes seek to stabilize the market price of agricultural products by buying up supplies of the product when harvests are plentiful and selling stocks of the product onto the market when supplies are low [1]. Thus the basic function of a buffer stocks policy is to store a certain amount of commodity in boom periods, when the price is low, and to release a certain amount of the stored commodity in bust periods, when the price is high.

1.1.1 Buffer Stock System

The empirical results from Myers et al. [6] also paint a relatively favourable picture of the buffer stock scheme. In absence of the scheme, demand shocks are the dominant source of variability, but stockholding did blunt the effects of these shocks. This requires a reliable supply process that usually consists of many intermediate steps, starting from producing the commodities at the supplier to shelving the commodities in the retailer's store.

There is scientific raging debate regarding the effects of storage, considered by most as a stabilizer on commodity prices and storage is placed at the core of the explanation of intriguing characteristics of price dynamics. Recently, it is hypothesized that endogenous chaotic behavior of markets is the cause of commodity price fluctuations. A nonlinear cobweb (that shows chaotic price series) model with storage, risk averse agents and adaptive expectations was developed, and like the theory of competitive storage, the nonlinear cobweb model found to reduce price variability [7].

Sutopo [8] observed in previous researches that, buffer stock models have been developed separately based on nonlinear optimization and econometrics methods. While optimizations methods had been used to determine the level of availability with schemes consisting of time and quantity of buffer stock, the econometrics methods on the other hand had been used to determine the equilibrium price using the selling-price and the amount of buffer stock. Therefore they developed a buffer stock model that integrates both methods using decision variables which consist of quantity, time and price to achieve stability.

Athanasioa et al [9] presented a nonlinear cobweb model with supply increasing according to certain a piecewise linear supply function. Athanasioa et al [9,10] also proposed naive expectation to test effect of governmental interventions through the provision of reserve from buffer stocks operation to weakening commodity price fluctuations. Their model proved that if the storage capacity for a particular commodity is sufficiently large then there exists a simple stabilization policy, such that the equilibrium price is a global attractor for the corresponding closed-loop system. Athanasioa et al[9] in addition, showed that if the government approximates the equilibrium supply with the average supply, stabilization is guaranteed.

Anokye and Oduro [11] developed linear cobweb model which studied the phenomenon of commodity price fluctuations with the assumptions that fresh tomatoes have no equal substitutes and also no exogenous shocks needed to generate price fluctuations. The model showed unstable price oscillations around the equilibrium point. However, after incorporating buffer stock scheme into the model, price stability was achieved in the short run while in the long run buffer system suffered instability until the average supply was reviewed.

Edwards and Hallwood [12] developed linear econometric models (linear supply and demand) to determine the amount of commodities that Government must store, so that the amount of stocks to be released to in order stabilize prices could also be determined. The decision variables of these models are the parameter values of functions and their performance criteria are evaluated based on the level of government expenses.

Price dynamics of a rice market was examined by Brennan [13] using dynamic programming techniques that parameterized the case of Bangladesh that is characterized with high price elasticity (due to income effects) and high storage and interest costs. In this model various storage invention (both public and private) were explored, and the results show that both interventions have positive impact in ensuring food security and fair prices for the poor.

Nguyen [14] proposed a simple rule for the buffer stock authorities to stabilize both price and earnings in all circumstances, except when market is instable. The model dampens price fluctuations in all periods and not only in the periods when a freely fluctuating price would fall but also outside the chosen limits.

Jha and Srinivasan [15] evaluate the impact of food buffer stocks in stabilizing prices of food commodities when private external trade is liberalized with multi-market equilibrium framework adopted in the model as endogenous factor. The results from the analysis indicated that under liberalized trade, buffer stock scheme is ineffective for stabilizing domestic prices. It is also found that variables such as levies and/or subsidies on trade are cost effective. It is therefore prudent to use trade restrictions when buffer scheme is implemented to ensure price stabilization.

This assertion is buttressed by Supoto et al [2] who developed a buffer stock model under free-trade that parameterized trade restrictions, tariffs and an indirect market intervention in accordance with a warehouse receipt and collateral management system.

The results from this single objective mixed integer nonlinear programming model with multi-criteria decision variables showed that indirect market intervention is more efficient than its counterpart, the direct market intervention. Soltes et al [16] on the other hand, used inventory (buffer stock) model in which price tends to equilibrium point not only monotonically but also oscillates around equilibrium point, and they attributed the cause of the oscillations to the order of the differential equations.

1.1.2 Time Varying Effect System

The effect of time varying velocity on output responses to policies for reducing and/or stopping inflation was explored through the use of dynamic general equilibrium model in which time-varying component (effect) was introduced as endogenous parameter for analyzing optimal speed of disinflation. The solution of this nonlinear model revealed that output losses would be much larger when disinflation boom disappears. It was also found from the analysis that gradual disinflation of low inflation is undesirable given its overall impact on the economy [17].

Mackey [18] developed a price adjustment model with state dependent production and storage delays for the study of price dynamics in a single commodity market. Conditions for stable equilibrium price were derived in terms of a variety of economic parameters and was found that, when stability of the equilibrium price is lost, Hopf bifurcation occurs, giving rise to oscillatory commodity price with a period between two and four times the equilibrium production-storage delay.

Liz and Rost [19] studied the price dynamics in a commodity market governed by a balance between demand and supply with delay, by employing a delay differential model. They also did a thorough study of the discrete time case and then used the results obtained to get new sufficient conditions for the global convergence of the solution to the positive equilibrium in the continuous time case. When the delay is large and the price system is unstable, they provide bounds to limit amplitude of the oscillations that are quite sharp. Matsumoto [20] used nonlinear delay differential and found out that time delay effects has strong stabilizing effect in minimizing cyclic oscillations.

Ruediger et al [21] accessed weather, time-varying business volatility affect the price setting of firms and thus the transmission of monetary policy into the real economy. Data analysis were performed to evaluate the impact of idiosyncratic volatility on the price setting behavior of firms and then used a calibrated business cycle model to measure the effects of time-varying volatility on the transmission of monetary policy to output. The results in twofold suggest that, sharp business volatility increases the probability of a price change, tripling of volatility caused the average quarterly price change to increase from 31.6% to 32.3%. Secondary, the increase in volatility caused monetary policy output to decline from 0.347% to 0.341%.

Anokye and Oduro [22] also assessed the effects of delay parameter τ on price oscillations and it is observed that oscillations (price fluctuations) are suppressed for $\tau \leq 0.5$, using the nonlinear delay equation. This is an indication that price fluctuations are reduced, if and only if, factors affected by time lag, such as the time necessary for increasing supply and installing buffer stock scheme are improved.

All the models reviewed so far (under buffer stock systems) have failed to consider effects of time that elapsed before supply change in response to market dynamics. They also fail to consider the effects of this time (delay) on buffer stock operation in stabilizing prices of food commodities. Mackey [18] on the other hand used delay parameter and has proven that, storage delay can rather be a destabilizing factor for price.

However, this paper is intended to use mathematical models for controlling prices of maize at the market and yet require that the structure and parameters of the model reflect the realities prevailing at the local market so as to prove it that the time varying parameter (delay) is price stabilizer. We therefore introduce time varying parameter model by employing a nonlinear continuous time delay differential equation derived from linear demand function of

price and nonlinear supply function of price with delay. This model would mimic the model used by Soltes [16] to prove of the positive impact of the time varying effects on price because according to Papachristodoulou [23], analysis of nonlinear delay differential equations is difficult and so researchers are constrained to the investigation of the properties of their nonlinear undelayed versions, as was done in Slotes work.

The model would also have its parameters estimated from real economic data of maize price demand and production in the Ashanti Region, of Ghana through the use of regression (the same data are used in Anokye and Oduro [22]; Anokye et al. [24]). The model would then be integrated with buffer stock model that has supply delay parameter.

It is believed that by the introduction of time varying parameter in the models, stakeholders will appreciate its dynamics on prices and be encouraged to adopt this model to devise mechanisms to ensure food availability and the stabilization of prices.

1.2 Effects of Nonlinear Delay Differential Equations

In fact, a great deal of recent economic modelling has almost entirely ignored the potential role of delays in generating economic fluctuations and a good mathematical setting in which to consider this gap is provided by delay-differential equations [25,19]. While it is assumed that consumers base their buying decisions on the current market price, for most commodities there is actually a finite time τ that elapses before a change in production occurs. This time lag can be affected by several factors, such as the time necessary for increasing production, buying new machines, hiring workers or building factories; in the case of agricultural commodities, there are also natural constraints that affect the delay [18].

Price dynamics of maize in Ghana was studied based on the price stability conditions of two continuous time delay differential equations using numerical approach. The results from the analysis recommended that researchers use nonlinear models instead of linear models in solving most real-life economic problems to avoid misleading conclusions. The recommendation was deduced from the fact that, the results from the nonlinear delay differential model seemed more realistic than that of the linear differential model [22].

The dynamics of human endocrine level was studied over time using nonlinear time delayed dependent model and it was found that daily variation quantified by time-delayed mutual information system intuitively provided the expected diurnal variation in glucose levels amongst a random population of humans as opposed to the population with no diurnal variation [26].

Also time-dependent stimulation using microfluidics technology has proven valuable in eliciting previously unseen cellular responses in physiological and cellular processes within a framework in an HIV models, thereby potentially allowing researchers to observe new mechanisms in the pathway [27].

2 Method

In order to study price dynamics of maize in Ghana, records of price and production of maize from seventeen major market centres in the Ashanti Region were selected. This secondary data of maize are gathered from the year 1994 to 2013, from the Ministry of Food and Agriculture, Statistical Directorate Kumasi-Ghana as shown in table 1. Nonlinear delay differential equations would be derived from linear demand and nonlinear supply (with delay) functions whose parameters are formulated from the data. This model would then be incorporated with buffer stock scheme to study the dynamics of price stabilization.

Table 1: Price and Production of Maize in Ashanti Region

Prices of Maize (Bag of 100 kg) in GH¢					Production of Maize (Metric Tons)				
Year	1 st Quarter	2 nd Quarter	3 rd Quarter	4 th Quarter	1 st Quarter	2 nd Quarter	3 rd Quarter	4 th Quarter	Total
1994	1.23	1.78	1.49	1.67	10198.13	14830.01	12403.34	13878.19	51309.67
1995	2.69	3.53	1.64	2.22	15947.12	20917.51	9730.17	13171.87	59766.67
1996	2.80	3.41	4.61	4.45	10152.87	12357.34	16710.75	16125.71	55346.67
1997	6.51	8.73	6.74	-	17268.13	23180.01	17886.86	-	58335.00
1998	5.06	5.62	4.43	4.26	19354.45	21514.8	16930.53	16287.52	74087.30
1999	4.64	4.94	5.76	4.60	15016.85	15978.78	18646.42	14875.28	64517.33
2000	9.32	11.05	9.90	10.39	14663.71	17381.97	15579.11	16342.87	63967.66
2001	13.87	18.76	12.98	13.04	13403.83	18127.28	12539.71	12595.85	56666.67

2002	14.83	15.38	10.97	11.36	25347.56	26300.98	18760.61	19417.51	89826.66
2003	14.03	17.06	16.32	14.49	14649.34	17818.76	17039.9	15132	64640.00
2004	17.80	21.67	23.46	23.06	12629.82	15372.77	16646.07	16362.01	61010.67
2005	29.67	45.02	34.25	28.56	11638.64	17660.19	13436.17	11203.67	53938.67
2006	25.91	27.39	20.98	18.87	15226.02	16097.1	12328.32	11090.56	54742.00
2007	25.85	32.34	26.41	26.06	13190.26	16500.28	13474.43	13296.04	56461.01
2008	32.34	56.66	56.72	49.39	10103.38	17700.44	17717.1	15428.41	60949.33
2009	60.79	71.77	55.13	52.21	15780.74	18631.95	14311.43	13552.55	62276.67
2010	53.29	55.51	52.76	46.82	21597.84	22497.63	21385.72	18976.81	84458.00
2011	55.10	75.41	81.34	89.95	10572.89	14470.08	15607.96	17260.74	57911.67
2012	110.69	124.44	90.97	76.00	18169.81	20426.27	14932.32	12475.61	66004.01
2013	77.09	76.24	74.86	81.67	17036.49	16847.17	16542.22	18047.12	68473.00

The table 1 contains the quarterly price and production of maize data at the right and left sections of the table respectively. They are average price and production points across the seventeen (17) market centres. Find beneath details of the data contained in this table.

Table 2: Descriptive Statistics

	N	Range	Mean	Std. Deviation	Skewness	Kurtosis
	Statistic	Statistic	Statistic	Std. Error	Statistic	Std. Error
Price	79	123.21	30.27	3.31709	29.48293	1.160
Production	79	16570.81	16008.72	383.99735	3413.04306	.556
Valid N	79					
(listwise)						

Table 2 shows the statistic values of the data in table 1 which includes range, mean, skewness over 1 (moderate), kurtosis greater than 0 but within the expected value of 3 and their respective standard errors.

2.1 Continuous time non-linear Model

Considering a simple nonlinear delay differential equation (of quadratic form) for the supply function of price as, $S(p(t)) = b + \beta p(t - \tau) - \delta p^2(t - \tau)$ and linear demand function of price as $D(p(t)) = a - \alpha p(t)$, where a, b, α, β and τ are positive constants. The supply of maize is assumed to increase with increase in price until supply exceeds demand where the trend changes. The models also perform on the assumptions that, maize has no equal substitutes and there are no exogenous shocks needed to generate price fluctuations. The market price is determined by only the available supply in a single market and the rate of change of the price is proportional to the difference between supply and demand function. [28,18,11].

We therefore have;

$$p'(t) = [(a - b) - \alpha p(t) - \beta p(t - \tau) + \delta p^2(t - \tau)] \text{ where, } \tau > 0, \text{ on } [0, b], b > 0 \quad (1)$$

This is simple nonlinear delay differential equation which mimics discrete time nonlinear difference equation modelled by [29] and cannot have an exact solution in practice using an analytical method. Therefore, a numerical method is applied using MatLab solver dde23 (code attached at the appendix 1).

This equation with a single delay like all DDEs are mostly solved in a stepwise fashion with a principle called the method of steps: equation (1) would have initial function (also known as history function) as $p(t) = s(t)$ defined over the interval $[-\tau, 0]$ and then its solution is mapped onto solutions of other functions. Thus the solution of this equation is going to be a mapping from functions on the interval $[t-\tau, t]$ into functions on the interval $[t, t+\tau]$,

[$t+\tau$, $t+2\tau$], etc., from time points $t=0, \tau, 2\tau, \dots$

In other words, the solutions of this dynamical system can be considered as sequence of functions $p_0(t)$, $p_1(t)$, $p_2(t), \dots$ defined over contiguous time interval of length τ [30].

2.2 Continuous time non-linear Model with Buffer Stock

We shall study how the dynamics of price adjustment will be affected when buffer stock is incorporated into equation (1). It is assumed that buffer has negative effect on price. Buffer stock is mathematically formulated as given below:

$$p'(t) = [D(p(t)) - S(p(t))] - \int_{\tau}^t [S(p(u)) - D(p(u))] du \quad (2)$$

The second term is the integral of past differences which expresses the accumulated stock (say G) in the buffer. If $G > 0$, then G causes downward adjustment of price because government releases stocks to the market. If $G < 0$ he rather buys from the market to adjust price upward while $G=0$ denotes no interference from the government [9]. Equation (2) is a price adjustment model obtained from continuous time delay integro-differential equation that mimics the integro-differential model used by [16]. This equation can be transformed into differentiation by simplifying it into the form given by:

$$p''(t) = [D'(p(t)) - S'(p(t))] - [S(p(t)) - D(p(t))] \quad (3)$$

The following equation is obtained when the equation (1) fixed into the equation (3);

$$p''(t) = p'(t) - [(b-a) + \alpha p(t) + \beta p(t-\tau) - \delta p^2(t-\tau)] \quad (4)$$

Equation (4) is a second order continuous time nonlinear delay differential equation which also mimics the second order linear differential equation used by [16] which can also be solved numerically using MatLab solver dde23. This equation has to be converted into two first order delay differential equations before one can use MatLab dde23 to solve it (code attached at appendix 2).

The Model Performs with the following Assumptions:

- No equal substitutes for maize, no exogenous shocks to cause price fluctuations and market price is determined by the available supply in the market
- The system relies on starting with a good harvest, indeed, without stocks in the system it is not possible to react to a poor harvest.
- Buffer stocks do not prevent the initial problem from arising.
- A government or agency can establish a target price, and then guarantee to pay farmers and growers this price, whatever output is produced. If the market price rises above this guaranteed price, the market price will prevail. But if the market price falls below the guaranteed price, then the guaranteed price will prevail.
- There are no extra costs of storage which the government or the agency has to incur.
- Government is the only one who operates the inventory and controls the market

3 Results and Discussion

This paper seeks to study whether the delay in supply affects price of maize and this mathematical model used in achieving price stability using real economic data of maize price and production in Ashanti Region, Ghana. Modelling of various functions and their parameter estimates would be done by the use of SPSS and then the numerical solution of this continuous time delay differential equation run using MatLab.

3.1 Preliminary Analysis of Price and Production Data

The data are checked to correct any errors and then SPSS and MatLab used to verify the stationary status for both price and production data sets before formulating the demand and supply functions of price using regression analysis.

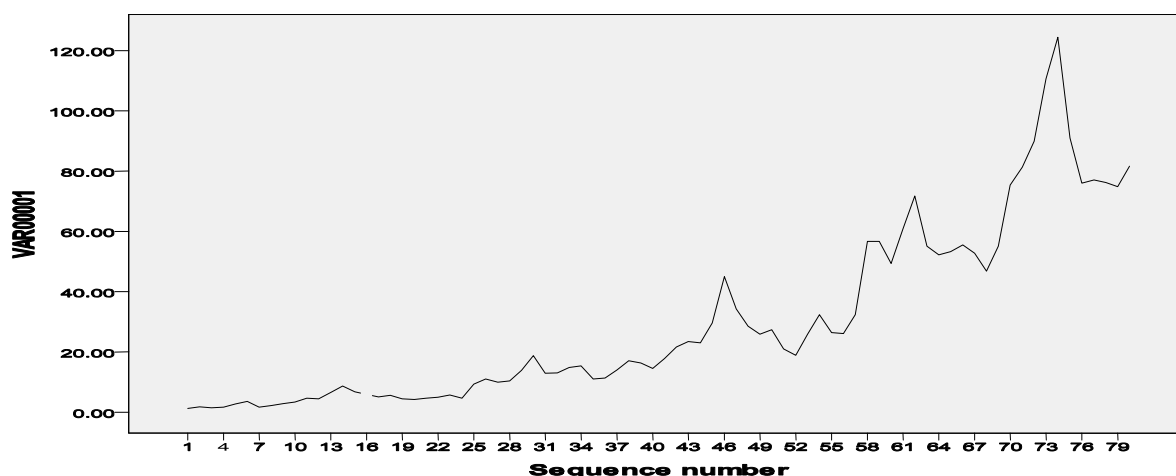


Figure 1: Time Series plot of price

The above figure shows the time series plot (behaviour) of price. It indicates that price data is non-stationary and also exhibits nonlinearity characteristics in the form close to quadratic.

3.2 Parameter Estimates

The following table contains the coefficients of the parameters estimated from the data analyzed. All the parameter values are checked and they are statistically significant.

Table 3: Coefficients of demand and supply functions

Table 1: Coefficients of Demand and Supply Functions						
Model	Unstandardized Coefficients		Standardized Coefficients			
	B	Std. Error	Beta	t	sig.	
1. Price	-96.16	20.26	-0.49	-4.75	0.00	Demand Function
a. Dependent Variable: Production			b. Linear Regression through the Origin			
2. Price	354.28	36.89	1.72	9.60	0.00	Supply Function
Price**2	-2.78	0.45	-1.10	-6.14	0.00	
a. Dependent Variable: Production_Lag			b. Linear Regression through the Origin			

The table 3 above contains the parameter values of demand and supply functions. The regression was carried out through the origin because in both cases the intercepts were not statistically significant. It also contains parameter values of linear demand function of price (1), while (2) is parameter values for quadratic supply function of price.

3.3 Demand Function of Price

The demand function of price from the table 3 is given below with its parameter estimates checked to be statistically significant. This function was obtained from price data of order two (2) differencing and production data of order one (1) differencing.

$$D(p(t)) = -96.16 p(t) \text{ where } a = 0 \quad (5)$$

3.4 Supply Function of Price

295 Similarly supply functions with time delay τ are given below. Equation (6) was obtained with no order of
296 differencing from price or the production data.

$$297 \quad S(p(t)) = 354.28p(t - \tau) - 2.78p^2(t - \tau) \quad \text{where } b = 0 \quad (6)$$

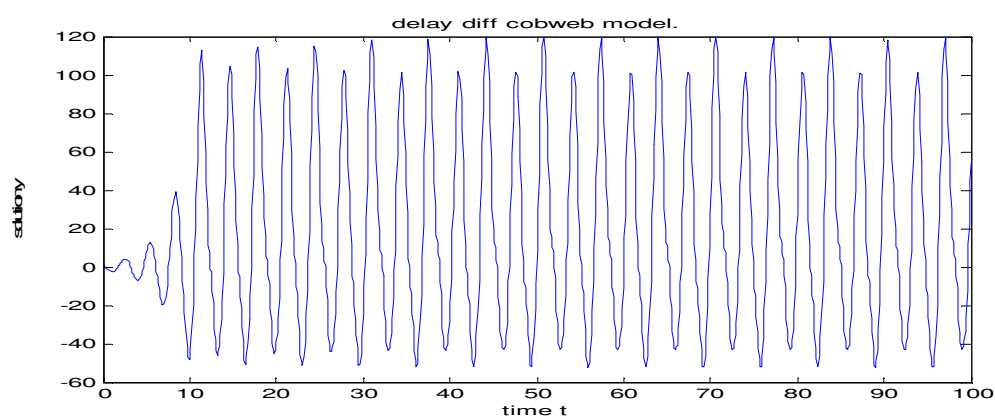
298 The delay τ expresses time that is needed to realise change of supply in dependence on trend of price. Thus current
299 production depends on the past price.

300 **3.4.1 Analysis of Nonlinear Model**

301 From equations (5) and (6), the rate of change of price is given by the following equation, which is in the form of
302 equation (1) where $\tau=1$ and $a=b=0$:

$$303 \quad \frac{1}{96.16} P'(t) = -P(t) - 3.68P(t-1) + 0.03P^2(t-1) \quad (7)$$

304 Based on same assumption for smoothness of the solution using MatLab solver dde23, equation (7) is divided by
305 96.162 and the history function set at $P(t)=1.23$, when $t \leq 0$, and on the interval $[0, 100]$. The solution of equation (7)
306 in numerical form is presented graphically as follows (see fig. 2 below):



307

308 **Fig. 2. Oscillations of price around 2 equilibrium points for $\tau=1$**

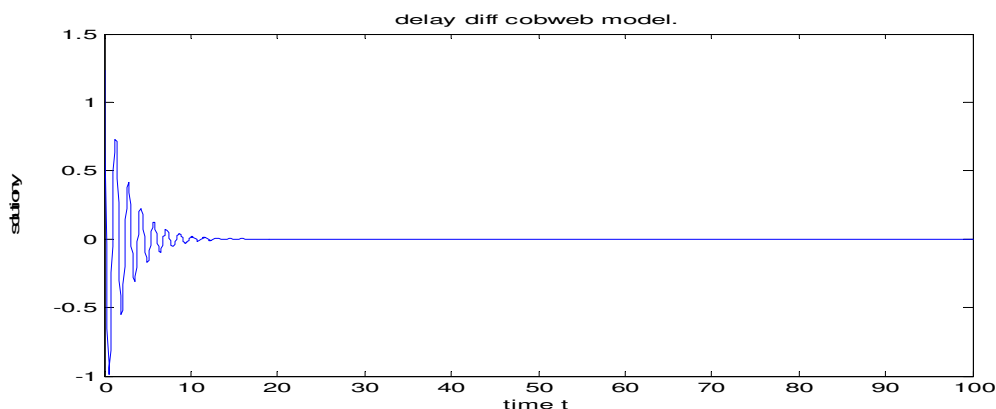
309 It is clear from the Fig. 2, above that the solution of equation (7) oscillates between two (2) equilibrium (price)
310 points and would never converge to either equilibrium price due perhaps to inflation, food insufficiency and/or
311 producers' sensitivity towards price.

312

313 **3.5 Time Varying Effects on Price Stability**

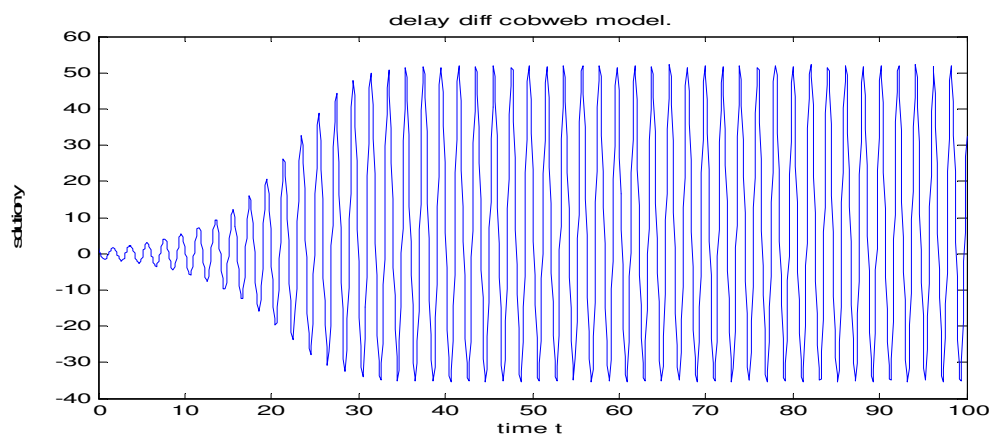
314 The varying effects of delay parameter τ on price oscillations (fluctuations) are discussed. The delay parameter can
315 decrease or increase fluctuations of price (Liz and Rost, 2013). This time varying analysis helps one to have fore
316 knowledge of time needed to respond to supply (market) dynamics and keep the system symmetrical about the
317 equilibrium price.

318 The oscillations with time are suppressed for $\tau \leq 0.5$, as shown in Fig. 3, below using equation (7).



320 **Fig. 3. Oscillation of price equilibrium with delay $\tau \leq 0.5$**

321 Fig. 3. indicates that price would be in stable equilibrium when factors affected by time lag in the system are
322 improved.



324 **Fig. 4. Oscillation of price equilibrium with delay $\tau \leq 0.5$**

325 The price oscillations (fluctuations) start to increase as seen in fig. 4, and with time become asymmetric (just like
326 fig. 2) about the equilibrium for $\tau > 0.5$, using the same equation (7). This indicates that price would never be in
327 stable equilibrium until factors affected by time lag, such as that required for increasing supply, transporting
328 commodities to market centres and increasing of storage capacity are considered and improved [18].

330 3. 6 Dynamics of Price Stabilization with Buffer Stock

331 The buffer stock operator is always committed to price stability in the strict sense of achieving and maintaining a
332 constant price level.

333 It is clear from fig. 3, above that if delay in response to supply dynamics is improved then price stability can be
334 achieved. From equation (4), the buffer stock equation is obtained as follows;

$$335 \quad \frac{1}{96.16} p''(t) = \left[((-p(t) - 3.68p(t - \tau) + 0.03p^2(t - \tau))' \right] - \left[p(t) + 3.68p(t - \tau) - 0.03p^2(t - \tau) \right] \quad (8)$$

337 Where $b=a=0$. If delay is fixed at 0.45 for the inventory (buffer stock) delay and supply delay system (without buffer
338 scheme), the following price oscillation graph (fig. 5, below) is obtained:

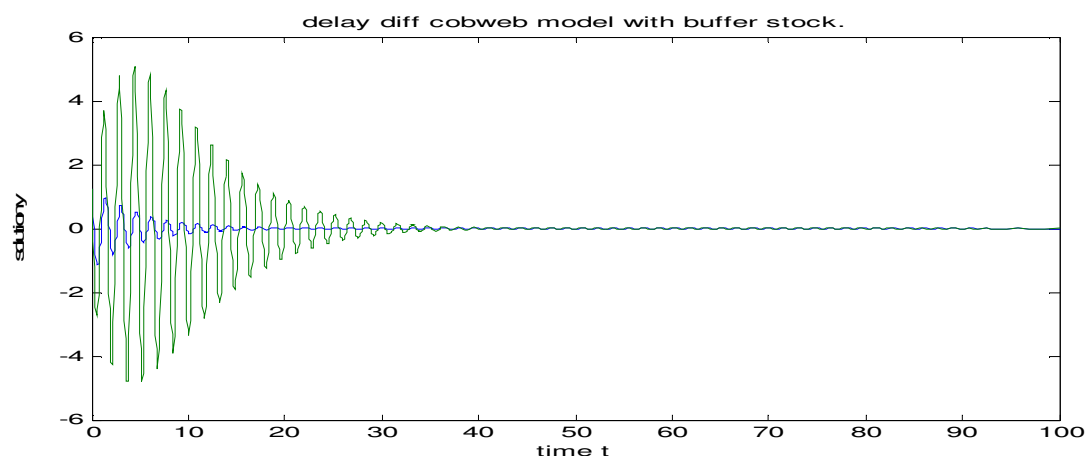


Fig. 5. Oscillations of Price about equilibrium with Buffer Stock, $\tau=0.45$

From fig. 5, above, the buffer stock delay and the supply delay are of different oscillation length which signifies no effects on price from the buffer operation. For the buffer stock to have greater impact, the two systems (buffer stock and supply) should synchronize.

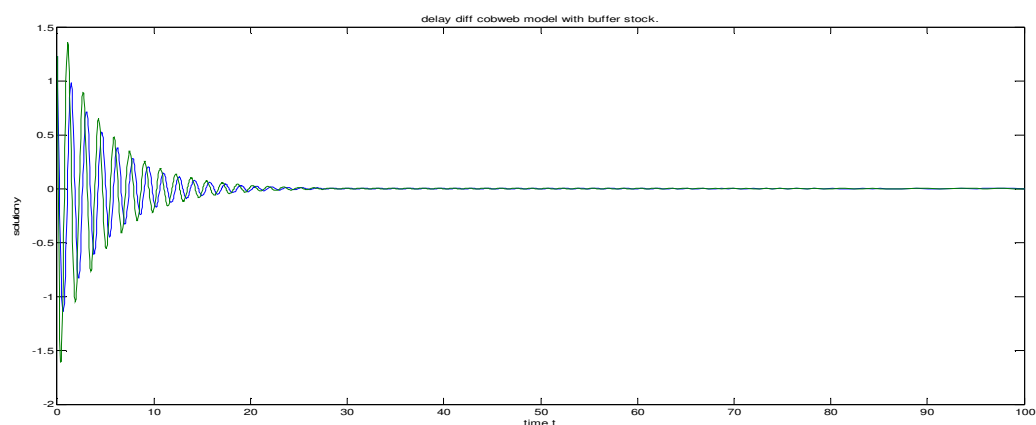


Fig. 6. Oscillations of Price about equilibrium with Buffer Stock, $\tau=0.22$

Now from fig.6, when the delay for the buffer stock is reduced to $\tau = 0.22$, the two systems are synchronized which shows the impact of the buffer stock scheme on the price oscillations of maize. The oscillations as well as amplitude (wave length) are also reduced.

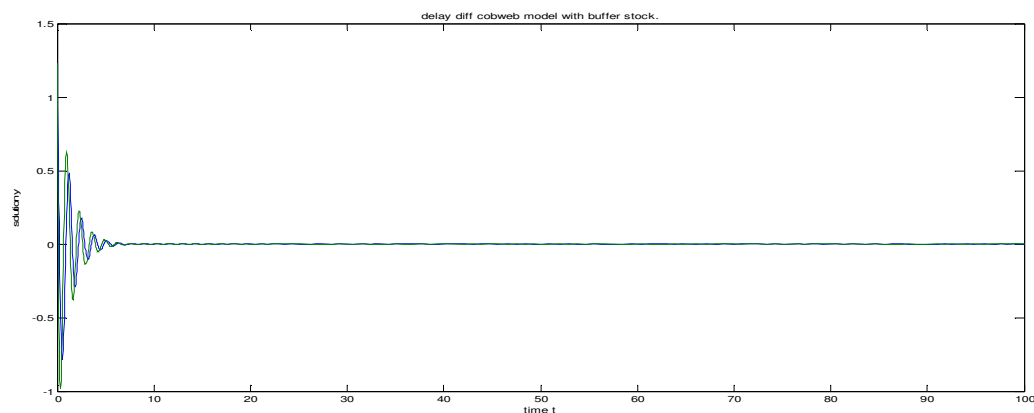


Fig. 7. Oscillations of Price about equilibrium with Buffer Stock Delay, $\tau=0.20$

It is clear from fig. 7, that farther the buffer stock delay ($\tau=0.20$) and supply delay ($\tau=0.35$) are reduced, more stable the price becomes. Now the effects of buffer stock are felt more by stakeholders.

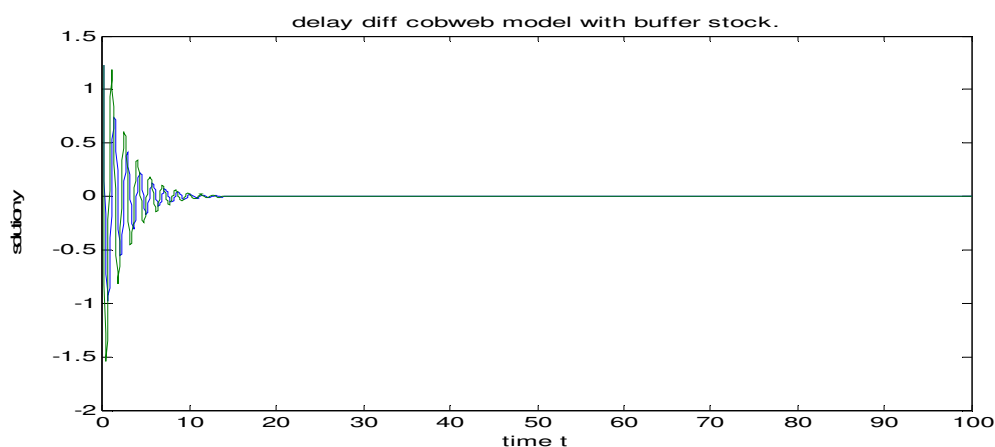


Fig. 8. Oscillations of Price about equilibrium with Buffer Stock Delay, $\tau=0.21$

From fig. 6 and fig. 7, we get obtain best average delay times of $\tau=0.21$ and $\tau=0.40$ for buffer and supply respectively. These delay times gives the fig. 8 which provide an average stable price of maize (from the simulated data in table 3 below) as GH¢ 30.49, that is very close to the mean maize price (from the raw data) of GH¢ 30.27, obtained in table 2.

Thus the mean price value (GH¢ 0.3170) from the table 4 below is multiplied by 96.16 to give us the stable equilibrium price of GH¢ 30.49, which in turns provide the equilibrium average demand and supply respectively as 2931. 6 metric tons and 8217.6 metric tons. The excess supply is kept in stock for next market period. When at another period demand exceeds supply, then appropriate difference is released form buffer to the market in order to keep price in equilibrium.

Table 4: Descriptive Statistics of Maize Price

	N	Range	Minimum	Maximum	Mean	Std.		Variance	Skewness	Kurtosis		
						Std.	Error			Std.	Error	
												Statistic
Price	7	.49	.05	.54	.3170	.06054	.16016	.026	-.398	.794	.336	1.587
Valid N (listwise)	7											

Table 4 shows the statistic values of the simulated data (within time points of stability) when the the buffer stock was run with Matlab dde23 solver. The values include range, mean, skewness less than 1 (moderate), kurtosis greater than 0 but within the expected value of 3 and their respective standard errors. The standard deviation is reduced to 0.1602 compared to 29.48 in the table 2, and the same applies to the variance.

This affirms the fact buffer stock acts as a reserve against short-term shortages and dampens down excessive fluctuations [31], which in turn disputes assertion by Mackey [18], who based his augment on the fact that commodity price is dependent on planting time, storage time, relaxation time and total production time. However, if all these individual time parameters are combined as one for supply delay and make storage time, the delay for the buffer, then price oscillations would be drastically reduced as shown in this paper.

The efficiency of buffer system is dependent on delay variation suitably enough to be used by buffer stock operator. The models used by Athanasiou et al. [9] and others are discrete time dependent which is a limit case of the delay buffer stock model used in this study. Thus their models lack the time varying parameter. This study has also proven Mackey[18] wrong, in that, when storage delay is used as buffer stock delay that is well managed, it could rather be a commodity price stabilizer. This study has also disputed Soltes et al [16]'s assertion and proven the model to be good for price stabilization.

4 Conclusion

The paper is intended to use mathematical models (which mimics model developed by Soltes et al. [16]) for controlling prices of maize by employing a nonlinear continuous time delay differential equation derived from linear demand and nonlinear supply functions of price with delay parameters reflecting the realities prevailing at the local market. These models would be formulated from parameters estimated from real economic data of maize price demand and production in the Ashanti Region, of Ghana through the use of regression. The data is obtained from the Ministry of Food and Agriculture, Statistical Directorate Kumasi-Ghana, from 1994 to 2013.

The results of the study dispute assertion by Mackey [18], who based his augment on the fact commodity price is dependent on planting time, storage time, relaxation time and total production time. If all these individual time parameters are combined as one for supply delay and make storage time, the delay for the buffer, then price oscillations would be drastically reduced as shown in this paper. The study also improved on the work done by Athanasiou et al.[9], [16] and others, whose models are discrete time dependent which is a limit case of the delay buffer stock model used in this study. The efficiency of buffer system is proven to be dependent on delay variation suitably enough to be used by buffer stock operator.

It is noted that, farther the buffer stock delay and supply delay are reduced, more stable the price becomes and the effects of buffer stock are felt more by stakeholders. The results of the analysis provide an average stable price of maize as GH¢ 30.49 compared to the actual average price of GH¢ 30.27. The equilibrium price in turn provide the average equilibrium metric tons of 2931. 6 and 8217.6, for demand and supply respectively. The excess supply is kept in stock and when needed it is released in next market period.The standard deviation also is reduced to 0.1602 compared to 29.48 in the table 2, and the same applies to the variance.

However, before the application of buffer stock scheme, price oscillated between two price points and could not converge. This affirms the fact buffer stock acts as a reserve against short-term shortages and dampens down excessive fluctuations.

Our study findings need to be interpreted in the light of its limitations, some of which could be addressed in further research. Further analysis is required on the impact of buffer stocks on price of the maize when individual storage are allowed to compete with the government in context of the price stabilization. Also foreign substitutes should be considered in the model formulation to assess its impact.

We draw inferences from this study that researchers should rather use continuous time nonlinear delay models as they reflect the realities in most real-life economic problems.

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Appendix 1

The code similar to the one in appendix 2 without the buffer stock

Appendix 2

```
function ddebuffer2
%DDEPRICE Solution for DDE23.
% This code for delay differential equations with buffer stock
% ddeprice are solved on [0, 100] with history y_1(t) = 1.23 and y'_1(0) =
0 ,for t <= 0.
% the buffer used the same history function
% The lags are specified as a vector [a,b] (they are varied), the delay
differential
% equations are coded in the subfunction DDEPRICE, and the history is
% evaluated by the function histprice. Because the history is constant it
% could be supplied as a vector:
% sol = dde23(@ddeprice,[a,b],[1.23;0],[0, 100]);
sol = dde23(@ddebuffer,[0.40,0.21],@histbuffer,[0, 100]);
```

```
512 figure;
513 plot(sol.x,sol.y)
514 title('delay diff cobweb model with buffer stock. ');
515 xlabel('time t');
516 ylabel('solution y');
517 x = linspace(0,20,100);
518 y = deval(sol,x,1);
519 % -----
520
521 function s = histbuffer(t)
522 % Constant history function for ddeprice.
523 s = [1.23,1.23];
524
525 % -----
526 function dydt = ddebuffer(t,y,Z)
527 % Delay Differential equations function for DDEPRICE.
528 ylag1 = Z(:,1);
529 ylag2 = Z(:,2);
530 dydt = [ (-1*y(1)-3.68*ylag1(1)+0.029*ylag1(1)^2);
531          ((-1*y(1)-3.68*ylag1(1)+0.029*ylag1(1)^2)-(1*y(2)+3.68*ylag2(2)-
532          0.029*ylag2(2)^2))];
533
534
535
536
537
```