

Original Research Article

On the application of the multistage laplace Adomian decomposition method to the Chaotic Financial system

Abstract

In this paper, a new reliable algorithm, multistage laplace Adomian decomposition method (MLADM) based on standard laplace – Adomian method, is presented to solve a time-fractional financial model for both chaotic and non- chaotic. The new algorithm is just a simple modification of laplace- Adomian method (LAM). This method is considered as an algorithm in a sequence of small intervals for obtaining accurate approximate solutions. The study depicts that the LADM provides reliable results for $t \ll 1$. Numerical comparisons between the MLADM and the classical Runge- Kutta fourth order method (RK4) in the case of integer-order derivatives solutions indicates that the MLADM gives better output with high accuracy and is a promising technique for nonlinear systems of integer and fractional order.

Keywords: Laplace adomian decomposition method; Runge- Kutta fourth order method; laplace- Adomian method; Riemann Liouville integral; Chaos theory.

1. Introduction

For the past four decades, a phenomenon called chaos has been investigated by many researchers including mathematicians, engineers and others [7, 8]. The first classical chaotic system was discovered in 1963 by Lorenz when he studied atmospheric convection [17]. In 1984, Chau introduced the first chaotic circuit which connects the chaos theory and the nonlinear circuit theory. In 1999, a dual system of Lorenz system was identified by Chen and Ueta through a technique called chaotification [5]. In 2002, a new chaotic system was identified and named as Lü system [18]. Lü *et al.* put together all the three systems described above into one chaotic called unified chaotic system [18]. Chaos theory deals with behaviour of dynamical systems has insightful effects on the numerical solutions based on its initial conditions [2, 25]. This is sometimes sensitive to time step sizes with regard to its numerical solutions. It has been identified that the majority real life phenomena and some scientific problem could be modeled by chaotic systems of ordinary differential equations (ODEs). For instance, Chaos theory has been employed in many fields of endeavour such as

33 population dynamics, mathematics, biology, computer science, economics engineering,
34 finance, meteorology and economics [11, 12]. Interestingly, many chaotic systems cannot be
35 solved analytically via their complexities. It is, therefore, prudent to explore the power of
36 numerical methods to obtain approximation solutions of such problems. Consider a financial
37 system [6, 26]. The system comprise three state variables as

$$\begin{aligned} \frac{dx}{dt} &= z + (y - d)x \\ \frac{dy}{dt} &= 1 - ey - x^2 \\ \frac{dz}{dt} &= -x - fz \end{aligned} \quad (1.1)$$

39 where x, y and z are the state variables d, e and f are positive real numbers. Here d
40 denotes the amount of saving, e represents the cost per investment, and the commercial
41 markets elasticity demand is denoted by f , $i = 1, 2, 3$, are parameters dealing with the order of
42 the fractional time derivative in the Caputo perspective. Clearly, the classical integer-order
43 financial can be seen as exceptional situation from fractional α -order system by putting
44 $\alpha_1 = \alpha_2 = \alpha_3 = 1$ where the chaotic occurs when $d = 3, e = 0.1$ and $f = 1$. We obtain the
45 following system of fractional time derivatives equation;

$$\begin{aligned} D_*^{\alpha_1} x(t) &= z + (y - d)x \\ D_*^{\alpha_2} y(t) &= 1 - ey - x^2 \\ D_*^{\alpha_3} z(t) &= -x - fz \end{aligned} \quad (1.2)$$

47 Li and Chen, [19] examined the dynamics of Rössler equation in fractional-order
48 generalizations. They observed in their studies that chaotic behaviour exists as low as 2.4 in
49 Rössler equation. Moaddy et al., [20] studied fractional order dynamics behaviour Rössler
50 chaotic using the nonstandard finite difference scheme. The study shows that smallest value
51 that chaotic occurs is 2.1

52 In recent times, Roslan et al., [24] explored Euler's method to obtain solution to chaotic
53 system. Even though the results obtained led to butterfly- shape but cannot provide good
54 accuracy. One of the common and widely used numerical methods is RK4 for simulating the

55 solution of chaotic system [19, 23, 27] and has been mostly used as comparison method
 56 [1,4,9,10]. There are other powerful methods of solving chaotic systems such as Laplace
 57 Adomian decomposition method (LADM) [13, 14]. This method has been applied in solving
 58 different kinds of differential equations. Unfortunately, it has some setbacks including most
 59 semi- analytic scheme. By using the LADM, we obtain a series solution which is, essentially
 60 a truncated series solution. This obtained solution does not reveal the real characteristics of
 61 the given problem but provides a good approximation to the exact solution in a small area. By
 62 the fact that the LADM leads to small convergence area, a multi-staging technique referred to
 63 as the Multistage Laplace Adomian Decomposition Method (MLADM) is proposed.
 64 In this paper we intend to apply the LADM and MLADM to solve financial system [6, 26],
 65 using the concept of fractional calculus for both chaotic and non-chaotic scenarios and
 66 compare to fourth order Runge-Kutta method.

67

68 2. Basic Definitions and Notations

69 This section deals with a number of basic definitions and notations of fractional calculus that
 70 will help in the subsequent sections [12].

71 **Definition 1.** A function $f(x)$ having the positive values of x is recognized in the space
 72 D_α ($\alpha \in \mathbb{R}$) if it is expressed in the form $f(x) = x^q f_1(x)$ and for some $q > \alpha$, where $f_1(x)$ is
 73 continuous in $[0, \infty)$, and it is known to be in the space D_α^n if $f^{(n)} \in D_n, n \in \mathbb{N}$.

74 **Definition 2.** The Riemann Liouville integral operator of a given order $\alpha > 0$ with $e \geq 0$ is
 75 expressed as

$$\begin{aligned} (J_\alpha^\alpha p)(x) &= \frac{1}{\Gamma(\alpha)} \int_e^x (x-t)^{\alpha-1} f(t) dt, \quad x > \alpha, \\ (J_e^0 f)(x) &= f(x), \end{aligned} \quad (2.1)$$

77 For the properties of the operator see [22], we need only the following. For
 78 $p \in B_n, \alpha > 0, \beta > 0, c \in \mathbb{R}$ and $\gamma > -1$, one gets

$$\begin{aligned} (J_e^\alpha J_e^\beta f)(x) &= (J_e^\beta J_e^\alpha f)(x) = (J_e^{\alpha+\beta} f)(x), \\ J_e^\alpha x^\gamma &= \frac{x^{\gamma+\alpha}}{\Gamma(e)} \beta_{(x-e)/x}(\alpha, \gamma+1), \end{aligned} \quad (2.2)$$

where $\beta_\tau(\alpha, \gamma+1)$ characterizes the incomplete beta function stated as

$$\begin{aligned} D_\tau(\alpha, \gamma+1) &= \int_0^\tau t^{\alpha_1} (1-t)^\gamma dt, \\ J_e^\alpha f^{cx} &= f^{cx}(x-e)^\alpha \sum_{k=0}^{\infty} \frac{[c(c-e)]^k}{\Gamma(\alpha+k+1)}. \end{aligned} \quad (2.3)$$

The Riemann Liouville derivative has some challenges when applied to real life situations with fractional differential equations. Thus, at this point we explore a modified version of fractional differential operator D_e^α which has been used in Caputo work on the theory of viscoelasticity.

Definition 3. The Caputo fractional derivative of $p(x)$ of order $\alpha > 0$ with $a \geq 0$ is expressed as

$$(D_e^\alpha f)(x) = (J_e^{m-\alpha} f^{(m)})(x) = \frac{1}{\Gamma(m-\alpha)} \int_e^x \frac{f^{(m)}(t)}{(x-t)^{(\alpha+1-m)}} dt, \quad (2.4)$$

for $m-1 < \alpha \leq m$, $m \in N$, $x \geq e$, $f(x) \in D_{-1}^m$. Many researchers employed the Caputo's fractional order derivatives for $m-1 < \alpha \leq m$, $f(x) \in D_\alpha^m$, and $\alpha \geq -1$, one obtains

$$(J_e^\alpha D_e^\alpha f)(x) = J_e^{(m)} D_e^{(m)} p(x) = f(x) - \sum_{k=0}^{m-1} f^{(k)}(e) \frac{(x-e)^k}{k!}, \quad (2.5)$$

3. Laplace Adomian Decomposition Method (LADM)

In this section, we present a Laplace Adomian decomposition method for obtaining a solution of differential equations expressed operator form:

$$L_t u + S(u) + M(n) = f \quad (3.1)$$

with initial condition

$$u(x, 0) = g(x) \quad (3.2)$$

where L_t denotes a first-order differential operator, P represents a linear operator, M denotes a non-linear operator and f provides the source term. We initially explore and apply Laplace transform to both faces of equation (3.1) and then substitute the initial condition (3.2).

$$L[L_t u] + L[P(u)] + L[M(u)] = L[f] \quad (3.3)$$

$$sL[u] - g(x) = L[f] - L[P(u)] - L[M(u)]$$

$$L[u] = \frac{g(x)}{s} + \frac{L[f]}{s} - \frac{L[P(u)]}{s} - \frac{L[M(u)]}{s} \quad (3.4)$$

The infinite series in terms of LADM solution $u(x, t)$ is defined as

$$u(x, t) = \sum_{m=0}^{\infty} u_m \quad (3.5)$$

The Adomian polynomials technique is used to write the non-linear term which is expressed as [3]:

$$M(u) = \sum_{m=0}^{\infty} A_m \quad (3.6)$$

$$A_m = \frac{1}{m!} \left[\frac{d^m}{d\lambda^m} [M \sum_{i=0}^{\infty} \lambda^i u_i] \right] \quad (3.7)$$

111

By substituting (3.5) and (3.6) into (3.3) leads to:

$$L[\sum_{m=0}^{\infty} u_m] = \frac{g(x)}{s} + \frac{L[f]}{s} - \frac{L[P(u)]}{s} - \frac{L[\sum_{m=0}^{\infty} A_m]}{s} \quad (3.8)$$

Following, (3.8), the following recursive formula can be stated:

$$L[u_0] = \frac{g(x)}{s} + \frac{L[f]}{s}$$

$$(3.9)$$

$$L[u_{n+1}] = -\frac{L[P(u)]}{s} - \frac{L[A_n]}{s} \quad (3.10)$$

By carefully taking the inverse Laplace transform to both ends of (3.9) and (3.10) we get

$u_m (m \geq 0)$ subsequently substituted into (3.5).

3.2 Multistage Laplace Adomian Decomposition Method (MLADM)

121 A robust and efficient method is required to provide approximate solutions to system of
122 differential equations for large t ($t \geq 0$) both linear and non-linear, multi- staging the
123 solution approach is one of the methods.

124 The interval with which the differential equation (1) solution is to be determined is made to
125 be $[0, T]$. The solution $[0, T]$ is further divided in M subintervals ($m = 1, 2, \dots, M$) given
126 equal step size $h = T/M$ having the interval at end points $t_m = mh$.

127 The LADM scheme is applied initially to determine the approximate solutions x, y and z of
128 (1), over the interval $[0, t_1]$ by employing the initial condition $x(0), y(0)$ and $z(0)$ in that
129 order. In order to determine the approximate solution of (1) with respect to the next interval
130 $[t_1, t_2]$, the $x(t_1), y(t_1)$ and $z(t_1)$ are considered as the new initial condition. In general terms
131 the iterative scheme is continued for any m with the right endpoints $x(t_{n-1}), y(t_{n-1})$ and $z(t_{n-1})$
132 at the previous interval being considered as the new initial condition for the interval $[t_{n-1}, t_n]$.

133 4. Application

134 In this section, we explore and employ the Laplace Adomian decomposition method to
135 financial system (1). The basic procedure of Laplace Adomian decomposition method is
136 employed to the fractional financial model and is expressed as

$$137 \quad \begin{aligned} \mathcal{L}[D^{\alpha_1} x] &= \mathcal{L}[z] + \mathcal{L}[xy] - d\mathcal{L}[x] \\ \mathcal{L}[D^{\alpha_2} y] &= \mathcal{L}[1] - e\mathcal{L}[y] - \mathcal{L}[x^2] \end{aligned} \quad (4.1)$$

$$\mathcal{L}[D^{\alpha_3} z] = -\mathcal{L}[x] - f\mathcal{L}[z]$$

$$s^{\alpha_1} \mathcal{L}[x] - s^{\alpha_1-1} x(0) = \mathcal{L}[z] + \mathcal{L}[xy] - d\mathcal{L}[x]$$

$$s^{\alpha_2} \mathcal{L}[y] - s^{\alpha_2-1} y(0) = \mathcal{L}[1] - e\mathcal{L}[y] - \mathcal{L}[x^2]$$

$$s^{\alpha_3} \mathcal{L}[z] - s^{\alpha_3-1} z(0) = -\mathcal{L}[x] - f\mathcal{L}[z]$$

$$\mathcal{L}[x] = \frac{x(0)}{s} + \frac{1}{s^{\alpha_1}} \mathcal{L}[z] + \frac{1}{s^{\alpha_1}} \mathcal{L}[xy] - \frac{d}{s^{\alpha_1}} \mathcal{L}[x]$$

$$138 \quad \mathcal{L}[y] = \frac{y(0)}{s} + \frac{1}{s^{\alpha_2}} \mathcal{L}[1] - \frac{e}{s^{\alpha_2}} \mathcal{L}[y] - \frac{1}{s^{\alpha_2}} \mathcal{L}[x^2] \quad (4.2)$$

$$\mathcal{L}[z] = \frac{z(0)}{s} - \frac{1}{s^{\alpha_3}} \mathcal{L}[x] - \frac{f}{s^{\alpha_3}} \mathcal{L}[z]$$

139 The solution of the financial system takes the form

$$140 \quad x(t) = \sum_n^\infty X_n \quad y(t) = \sum_n^\infty Y_n \quad z(t) = \sum_n^\infty Z_n$$

141 The non-linear Adomian polynomials are expressed as:

$$142 \quad xy = \sum_n^\infty A_n \quad x^2 = \sum_n^\infty B_n$$

143 Then (4.2) can be written as a recursive formula in parameterized form as:

$$\mathcal{L} \left[\sum \lambda^n X_n \right] = \frac{x(0)}{s} + \frac{\lambda}{s^{\alpha_1}} \mathcal{L} \left[\sum \lambda^n Z_n \right] + \frac{\lambda}{s^{\alpha_1}} \mathcal{L} \left[\sum \lambda^n A_n \right] - \frac{d\lambda}{s^{\alpha_1}} \mathcal{L} \left[\sum \lambda^n X_n \right]$$

$$\mathcal{L} \left[\sum \lambda^n Y_n \right] = \frac{y(0)}{s} + \frac{\lambda}{s^{\alpha_2}} \mathcal{L}[1] - \frac{e\lambda}{s^{\alpha_2}} \mathcal{L} \left[\sum \lambda^n Y_n \right] - \frac{\lambda}{s^{\alpha_2}} \mathcal{L} \left[\sum \lambda^n B_n \right]$$

$$144 \quad \mathcal{L}[\sum \lambda^n Z_n] = \frac{z(0)}{s} - \frac{\lambda}{s^{\alpha_3}} \mathcal{L}[\sum \lambda^n X_n] - \frac{f\lambda}{s^{\alpha_3}} \mathcal{L}[\sum \lambda^n Z_n] \quad (4.3)$$

145 Comparing equal powers of λ in equation (4.3), we have:

$$146 \quad \mathcal{L}[X_0] = \frac{x(0)}{s} \quad \mathcal{L}[Y_0] = \frac{y(0)}{s} + \frac{1}{s^{\alpha_2+1}} \quad \mathcal{L}[Z_0] = \frac{z(0)}{s}$$

$$147 \quad X_0 = \mathcal{L}^{-1} \left[\frac{x(0)}{s} \right] \quad Y_0 = \mathcal{L}^{-1} \left[\frac{y(0)}{s} + \frac{1}{s^{\alpha_2+1}} \right] \quad Z_0 = \mathcal{L}^{-1} \left[\frac{z(0)}{s} \right] \quad (4.4)$$

148

$$\mathcal{L}[X_{n+1}] = \frac{1}{s^{\alpha_1}} \mathcal{L}[Z_n] + \frac{1}{s^{\alpha_1}} \mathcal{L}[A_n] - \frac{d}{s^{\alpha_1}} \mathcal{L}[X_n]$$

$$\mathcal{L}[Y_{n+1}] = -\frac{e}{s^{\alpha_2}} \mathcal{L}[Y_n] - \frac{1}{s^{\alpha_2}} \mathcal{L}[B_n]$$

$$\mathcal{L}[Z_{n+1}] = -\frac{1}{s^{\alpha_3}} \mathcal{L}[X_n] - \frac{f}{s^{\alpha_3}} \mathcal{L}[Z_n]$$

149

$$150 \quad X_{n+1} = \mathcal{L}^{-1} \left[\frac{1}{s^{\alpha_1}} \mathcal{L}[Z_n] + \frac{1}{s^{\alpha_1}} \mathcal{L}[A_n] - \frac{d}{s^{\alpha_1}} \mathcal{L}[X_n] \right] \quad (4.5)$$

$$Y_{n+1} = \mathcal{L}^{-1} \left[-\frac{e}{s^{\alpha_2}} \mathcal{L}[Y_n] - \frac{1}{s^{\alpha_2}} \mathcal{L}[B_n] \right]$$

$$Z_{n+1} = \mathcal{L}^{-1} \left[-\frac{1}{s^{\alpha_3}} \mathcal{L}[X_n] - \frac{f}{s^{\alpha_3}} \mathcal{L}[Z_n] \right]$$

151 The system is solved with the initial condition $x(0) = 2$, $y(0) = 3$ and $z(0) = 2$. For $d =$
 152 3 , $e = 0.1$ and $f = 1$ we have a chaotic system and $d = 2$, $e = -0.1$ and $f = 1.6$ correspond
 153 to a non-chaotic system. The recursive relations (4.3) and (4.5) are evaluated with the aid of
 154 Mathematica 10.0 version to obtain the solution up to a 10 terms approximation for the time
 155 range $[0, 300]$ with a time step size 0.01. MLADM is implemented by dividing the solution
 156 interval $[0, 300]$

157 5. Results and discussions

158 The chaotic case of the classical finance system was obtained by applying the LADM and
 159 MLADM which is then implemented in Mathematica 10.0 version, the 21 terms approximate
 160 LADM solution was arrived at. The first few terms of the recursive relation (4.5) are obtained
 161 as

$$162 X_0 = 2$$

$$Y_0 = \frac{3 + t^{\alpha_2}}{\Gamma(1 + \alpha_2)}$$

$$Z_0 = 2$$

$$X_1 = 2t^{\alpha_1} \left(\frac{1}{\Gamma(1 + \alpha_1)} + \frac{t^{\alpha_2}}{\Gamma(1 + \alpha_1 + \alpha_2)} \right)$$

$$Y_1 = t^{\alpha_2} \left(-\frac{4.3}{\Gamma(1 + \alpha_2)} - \frac{0.1 t^{\alpha_2}}{\Gamma(1 + 2\alpha_2)} \right)$$

$$Z_1 = \frac{-4t^{\alpha_3}}{\Gamma(1 + \alpha_3)}$$

$$X_2 = t^{\alpha_1} \left[\frac{2 t^{\alpha_1 + \alpha_2} \Gamma(1 + \alpha_1 + \alpha_2)}{\Gamma(1 + \alpha_1) \Gamma(1 + \alpha_2) \Gamma(1 + 2\alpha_1 + \alpha_2)} - \frac{0.2 t^{2\alpha_2}}{\Gamma(1 + \alpha_1 + 2\alpha_2)} \right. \\ \left. + t^{\alpha_2} \left(\frac{-8.6 + \frac{2 t^{\alpha_1 + \alpha_2} \Gamma(1 + \alpha_1 + 2\alpha_2)}{\Gamma(1 + \alpha_2) \Gamma(1 + 2\alpha_1 + 2\alpha_2)}}{\Gamma(1 + \alpha_1 + \alpha_2)} \right) - \frac{4t^{\alpha_3}}{\Gamma(1 + \alpha_1 + \alpha_3)} \right]$$

$$Y_2 = t^{\alpha_2} \left[-\frac{8t^{\alpha_1}}{\Gamma(1 + \alpha_1 + \alpha_2)} + t^{\alpha_2} \left(\frac{0.43}{\Gamma(1 + 2\alpha_2)} - \frac{8t^{\alpha_1}}{\Gamma(1 + \alpha_1 + 2\alpha_2)} + \frac{0.01 t^{\alpha_2}}{\Gamma(1 + 3\alpha_2)} \right) \right]$$

$$Y_2 = t^{\alpha_2} \left[-\frac{8t^{\alpha_1}}{\Gamma(1 + \alpha_1 + \alpha_2)} + t^{\alpha_2} \left(\frac{0.43}{\Gamma(1 + 2\alpha_2)} - \frac{8t^{\alpha_1}}{\Gamma(1 + \alpha_1 + 2\alpha_2)} + \frac{0.01 t^{\alpha_2}}{\Gamma(1 + 3\alpha_2)} \right) \right]$$

$$Z_2 = 2t^{\alpha_3} \left[-\frac{t^{\alpha_1}}{\Gamma(1 + \alpha_1 + \alpha_3)} - \frac{t^{\alpha_1 + \alpha_2}}{\Gamma(1 + \alpha_1 + \alpha_2 + \alpha_3)} + \frac{2t^{\alpha_3}}{\Gamma(1 + 2\alpha_3)} \right]$$

163 into 3000 subintervals ($n = 1, 2, \dots, 3000$) of equal step size given by $h = 0.1$

164

165 For the chaotic case $d = 3$, $e = 0.1$ and $f = 1$, we have the series solution as:

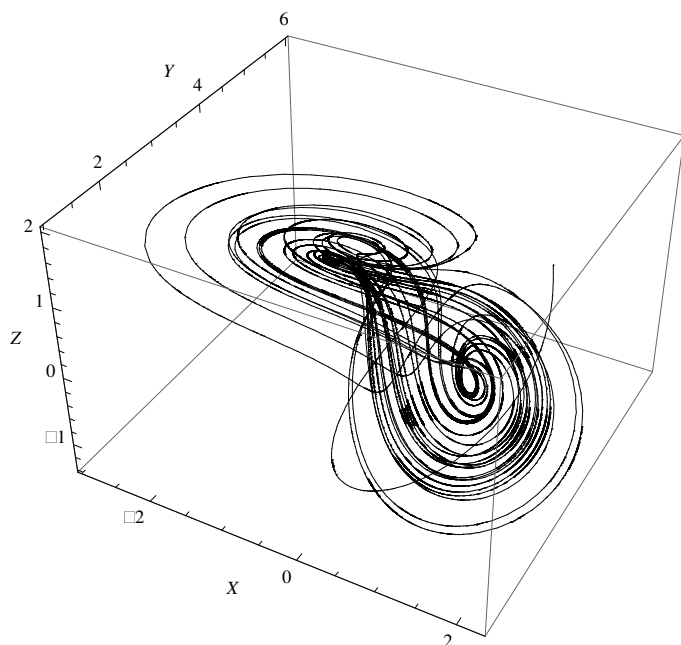
$$\begin{aligned} x = & 2 + 2t - 5.3t^2 - 4.42333t^3 + 5.74392t^4 + 13.3092t^5 - 4.82205t^6 - 28.4528t^7 \\ & - 5.57176t^8 + 52.3603t^9 + 38.865t^{10} + 96.7929t^{11} + 4.95108t^{12} \\ & - 11.4009t^{13} - 1.1977t^{14} + 0.175363t^{15} + 0.019144t^{16} \\ & - 0.000347514t^{17} - 0.0000423286t^{18} - 1.74675e - 10t^{19} + 5.38229e \\ & - 10t^{20} \end{aligned}$$

$$\begin{aligned} y = & 3 - 3.3t - 3.835t^2 + 5.86117t^3 + 9.5768t^4 - 6.866t^5 - 20.4022t^6 + 1.34447t^7 \\ & + 40.6071t^8 + 18.4081t^9 - 68.6147t^{10} + 14.9803t^{11} + 44.9862t^{12} \\ & + 4.17385t^{13} - 1.75135t^{14} - 0.188676t^{15} + 0.0104597t^{16} \\ & + 0.00115873t^{17} - 3.16501e - 6t^{18} - 5.80154e - 7t^{19} \end{aligned}$$

$$\begin{aligned} z = & 2 - 4t + t^2 + 1.43333t^3 + 0.7475t^4 - 1.29828t^5 - 2.00182t^6 + 0.974838t^7 \\ & + 3.43474t^8 + 0.237447t^9 - 5.25977t^{10} - 0.483038t^{11} + 2.04308t^{12} \\ & + 0.258554t^{13} - 0.0481953t^{14} - 0.00576637t^{15} + 0.000138421t^{16} \\ & + 0.0000174021t^{17} + 2.7936e - 10t^{18} - 5.66557e - 10t^{19} \end{aligned}$$

166 The effectiveness and accuracy of the LADM and MLADM is studied by comparing their
167 solutions to the RK4 solution for the parameter where the system is chaotic with the initial
168 conditions $x(0) = 2$, $y(0) = 3$ and $z(0) = 2$. The RK4 with time steps $\Delta t = 0.01$ with the
169 number of significant digits set to 16 is used. Table 1 presents the absolute errors between the

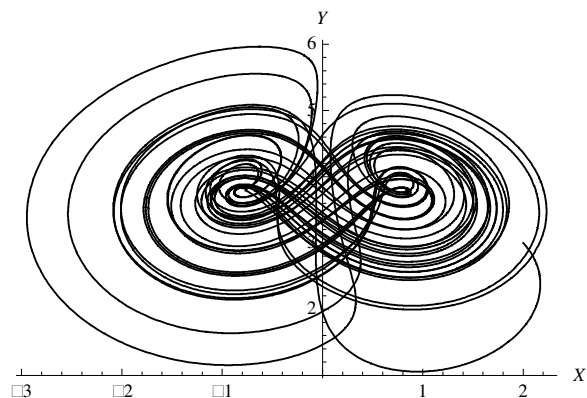
170 21-term LADM solutions and the 21-term MLADM solutions for $d=3, e=0.1, f=1$ and
171 the RK4 solutions.



172

173 **Figure 1: X-Y-Z Phase portrait using 10-term MLADM on $\Delta t = 0.01$ for $d = 3$,**
174 **$e = 0.1$ and $f = 1$.**

175



176

177 **Figure 2: X-Y Phase portrait using 10-term MLADM on $\Delta t = 0.01$ for $d = 3, e = 0.1$**
178 **and $f = 1, \alpha = 1$.**

179

180

181

182

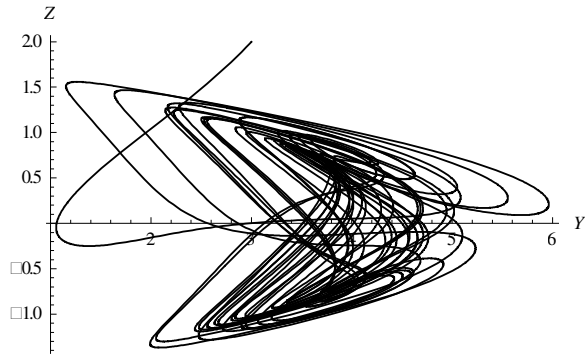


Figure 3: Y-Z Phase portrait using 10-term MLADM on $\Delta t = 0.01$ for $d = 3$, $e = 0.1$ and $f = 1$, $\alpha = 1$.

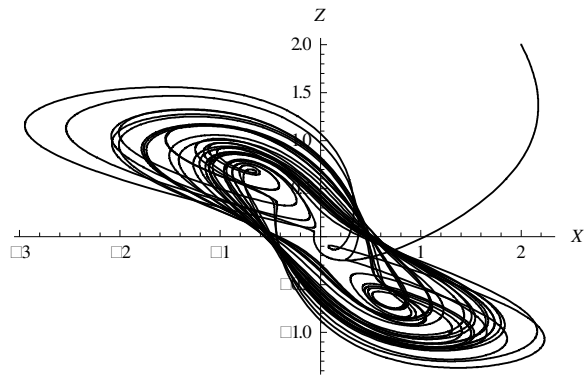


Figure 4: X-Z Phase portrait using 10-term MLADM on $\Delta t = 0.01$ for $d = 3$, $e = 0.1$ and $f = 1$, $\alpha = 1$.

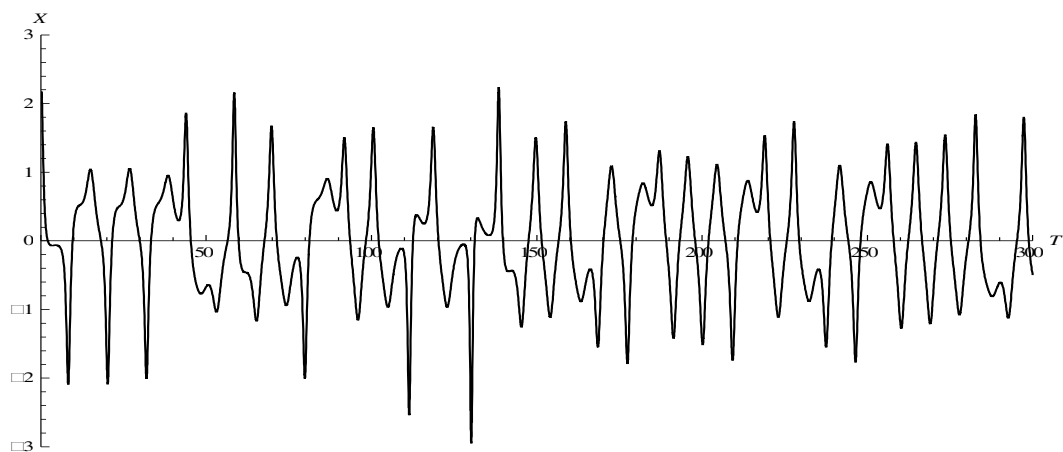


Figure 5: $X(t)$ time series using 10-term MLADM for $d = 3$, $e = 0.1$ and $f = 1$.

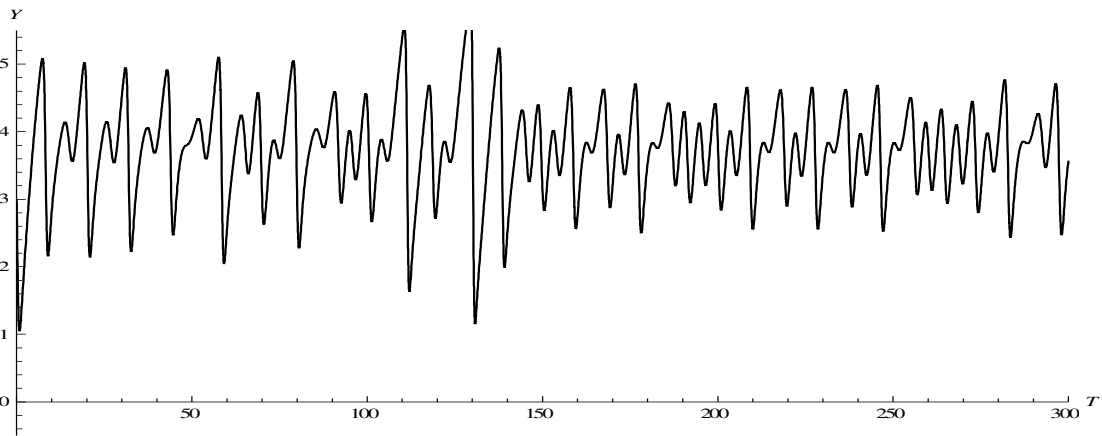


Figure 6: $Y(t)$ time series using 10-term MLADM for $d = 3$, $e = 0.1$ and $f = 1$.

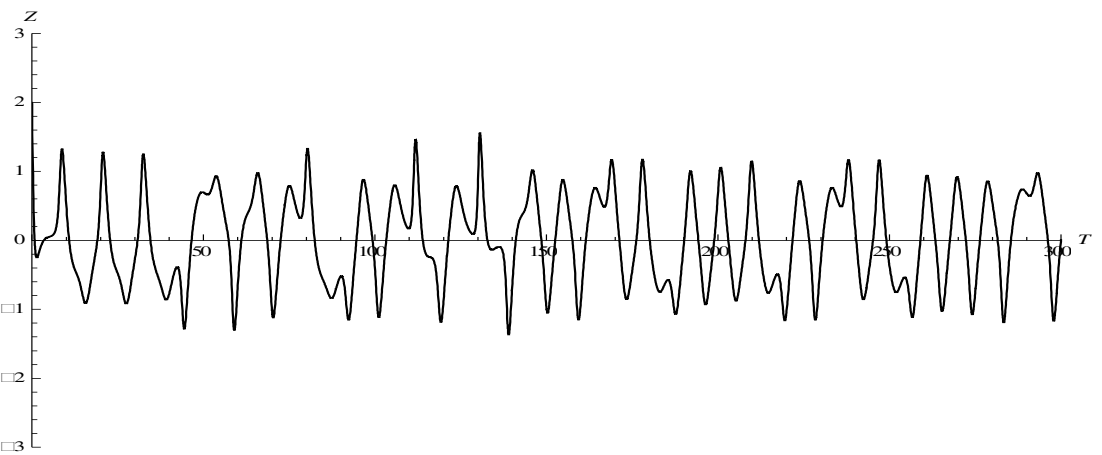


Figure 7: $Z(t)$ time series using 10-term MLADM for $d = 3$, $e = 0.1$ and $f = 1$.

Table 1: Absolute differences between 10-term LADM and 10-term MLADM with RK4 solutions ($\Delta t = 0.01$) for $d = 3$, $e = 0.1$ and $f = 1$.

t	$ \text{LADM}_{0.01} - \text{RK4}_{0.01} $			$ \text{MLADM}_{0.01} - \text{RK4}_{0.01} $		
	Δx	Δy	Δz	Δx	Δy	Δz
0.15	4.623E-10	2.812E-11	3.594E-09	4.623E-10	2.812E-11	3.594E-09
4.43	8.647E-05	1.801E-05	6.452E-04	1.008E-09	1.297E-09	1.512E-09
22.04	1.066E+00	1.166E+00	4.594E+00	4.494E-09	8.654E-09	7.228E-10
46.71	5.340E+02	2.479E+03	5.249E+03	4.254E-10	7.486E-09	1.570E-09
71.13	1.972E+05	2.159E+06	3.763E+06	5.482E-09	2.697E-08	9.049E-09
95.52	3.741E+07	6.570E+08	1.016E+09	2.785E-10	2.058E-08	6.010E-11
120.04	3.169E+09	7.231E+10	1.046E+11	4.052E-10	1.922E-08	1.051E-10
156.33	9.462E+10	2.484E+12	3.468E+12	3.465E-09	1.601E-08	4.111E-09
185.51	2.147E+12	6.196E+13	8.447E+13	2.261E-09	1.567E-08	4.535E-09

212.12	9.249E+12	2.767E+14	3.738E+14	1.928E-09	1.130E-08	2.424E-09
278.23	4.314E+13	1.333E+15	1.787E+15	6.767E-10	1.211E-08	2.196E-10

202

203 In Table 1, we can see that the LADM only provides valid result for $t \ll 1$. The MLADM
 204 solutions on the time step $\Delta t = 0.01$ for the chaotic case matches with the RK4 solutions on
 205 the time step $\Delta t = 0.01$ to at least 4 decimal places. Therefore, for the classical finance
 206 chaotic system we notice that the MLADM solutions matches with the RK4 solutions to a
 207 significant degree. The $z + (y - d)x$, $1 - ey - x^2$ and $-x - fz$ phase portraits for the non-
 208 chaotic case is determined employing the 21-term MLADM solutions are depicted in Fig. 1
 209 to Fig. 4. In figure 5, 6 and 7 we show the time series plots of the model equations
 210 respectively. These graphs depict oscillatory variations in the time interval of $0 \leq t \leq 30$
 211 seconds using MLADM when $d = 3$, $e = 0.1$ and $f = 1$.

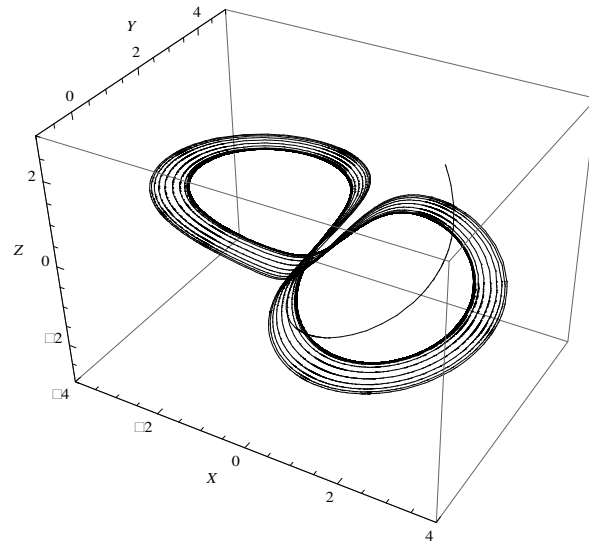
212 For the non-chaotic case $d = 2$, $e = -0.1$ and $f = 1.6$, we have the series solution
 213 as:

$$\begin{aligned} x = & 2 + 4t - 3.3t^2 - 9.40333t^3 - 8.87358t^4 + 14.5722t^5 + 37.4136t^6 + 13.0765t^7 \\ & - 70.9391t^8 - 119.882t^9 + 17.3112t^{10} - 158.999t^{11} - 89.8305t^{12} \\ & - 4.81831t^{13} + 2.84324t^{14} + 0.383951t^{15} + 0.00429468t^{16} \\ & - 0.0012362t^{17} - 0.0000420489t^{18} + 1.70486e - 8t^{19} + 5.38229e \\ & - 10t^{20} \end{aligned}$$

$$\begin{aligned} y = & 3 - 2.7t - 8.135t^2 - 1.2045t^3 + 15.9732t^4 + 20.2857t^5 - 7.88893t^6 \\ & - 59.1442t^7 - 53.5294t^8 + 68.4484t^9 + 210.243t^{10} + 154.278t^{11} \\ & - 25.6593t^{12} - 22.3443t^{13} - 2.38427t^{14} + 0.145652t^{15} + 0.0302603t^{16} \\ & + 0.00928876t^{17} - 0.0000171922t^{18} - 5.80154e - 7t^{19} \end{aligned}$$

$$\begin{aligned} z = & 2 - 5.2t + 2.16t^2 - 0.052t^3 + 2.37163t^4 + 1.01579t^5 - 2.69958t^6 - 4.72776t^7 \\ & - 0.689006t^8 + 8.00462t^9 + 10.7075t^{10} + 10.8071t^{11} + 0.721589t^{12} \\ & - 0.652907t^{13} - 0.0998574t^{14} - 0.000655983t^{15} + 0.00047164t^{16} \\ & + 0.0000171717t^{17} - 1.58464e - 8t^{18} - 5.66557e - 10t^{19} \end{aligned}$$

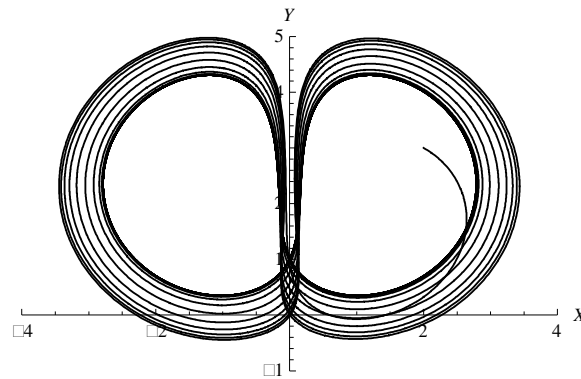
214



215

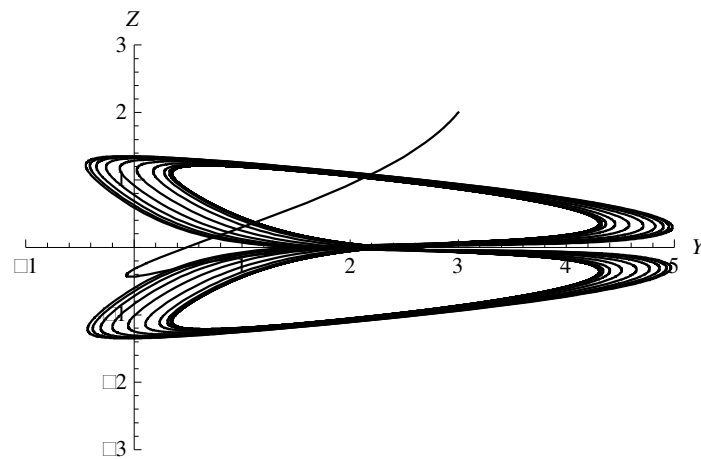
216 **Figure 8: X-Y-Z Phase portrait using 10-term MLADM on $\Delta t = 0.01$ for $d = 2$, $e = -0.1$**
 217 **and $f = 1.6$, $\alpha = 1$.**

218



219

220 **Figure 9: X-Y Phase portrait using 10-term MLADM on $\Delta t = 0.01$ for $d = 2$, $e = -0.1$**
 221 **and $f = 1.6$, $\alpha = 1$.**



222

Figure 10: Y-Z Phase portrait using 10-term MLADM on $\Delta t = 0.01$ for $d = 2$, $e = -0.1$ and $f = 1.6$, $\alpha = 1$.

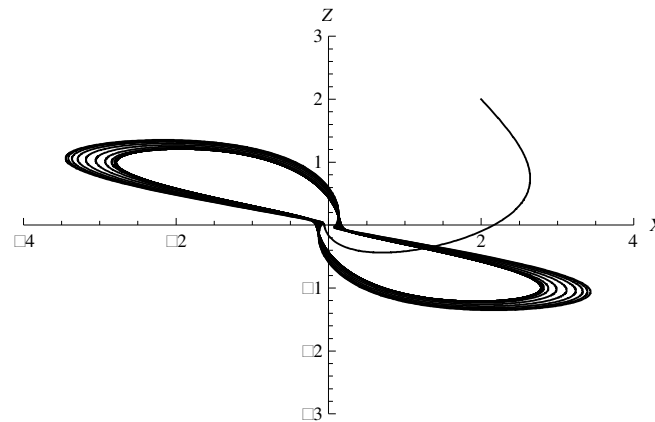


Figure 11: X-Z Phase portrait using 10-term MLADM on $\Delta t = 0.01$ for $d = 2$, $e = -0.1$ and $f = 1.6$, $\alpha = 1$.

Table 2: Absolute differences between 10-term LADM and 10-term MLADM with RK4 solutions ($\Delta t = 0.01$) for $d = 2$, $e = -0.1$ and $f = 1.6$

t	$ \text{LADM}_{0.01} - \text{RK4}_{0.01} $			$ \text{MLADM}_{0.01} - \text{RK4}_{0.01} $		
	Δx	Δy	Δz	Δx	Δy	Δz
0.15	2.407E-10	4.552E-09	6.828E-10	2.407E-10	4.552E-09	6.828E-10
4.43	4.003E-04	1.049E-03	8.516E-05	4.064E-10	3.177E-10	4.378E-10
22.04	1.067E+01	2.777E+00	5.612E-01	7.022E-11	5.363E-10	3.718E-10
46.71	5.788E+03	6.718E+03	4.383E+02	2.435E-10	4.909E-11	4.626E-10
71.13	2.885E+06	4.328E+06	1.786E+05	2.417E-11	1.541E-11	1.839E-11
95.52	6.804E+08	1.110E+09	2.574E+07	1.224E-09	1.553E-09	9.840E-10
120.04	6.613E+10	1.117E+11	1.381E+09	4.385E-10	1.947E-11	1.583E-10
156.33	2.136E+12	3.668E+12	2.377E+10	8.462E-10	7.770E-10	6.206E-10
185.51	5.118E+13	8.884E+13	2.212E+11	3.171E-11	7.643E-11	3.571E-11
212.12	2.252E+14	3.924E+14	3.903E+11	6.414E-12	9.170E-12	3.612E-11
278.23	1.071E+15	1.872E+15	6.907E+11	1.762E-06	8.250E-06	1.779E-08

In Table 2, we can also notice that the LADM only provides valid result for $t \ll 1$. The MLADM solutions for the non-chaotic case is in agreement with the RK4 solutions to at least 4 decimal places. The $z + (y - d)x$, $1 - ey - x^2$ and $-x - fz$ depict the phase portraits for the

non-chaotic classical finance system obtained employing the 21-term MLADM solutions and are shown in Fig. 8 to Fig. 11.

6. Conclusion

In this work, a new numerical technique to deal with time- fractional classical financial model is proposed. The method is a only modification of the standard laplace-Adomian method. Comparisons were made among the LADM, MLADM and the fourth-order Runge-Kutta (RK4) method. Following the numerical results obtained in the case of chaotic and non-chaotic the MLADM and RK4 were consistent. However, in the case of chaotic solution the MLADM performed relatively better than that of RK4.

References

- [1] Alomari, A.K., M.S.M. Noorani and R. Nazar, 2009. Adaptation of homotopy analysis method for the numeric-analytic solution of Chen system, *Communications in Nonlinear Science and Numerical Simulation*, 14: 2336-2346.
- [2] Asad Freihat and Shaher Momani. 2012. Adaptation of differential transform method for the numeric-analytic solution of fractional-order Rossler chaotic and hyperchaotic systems, 1934219
- [3] Adomian G. Solving Frontier Problems in Physics.1994. The Decomposition Method. Boston: Kluwer Academic Publishers
- [4] Al-Sawalha, M.M. and M.S.M. Noorani, 2009. A numeric-analytic method for approximating the chaotic Chen system, *Chaos, Solitons and Fractals*, 42: 1784-1791.
- [5] Chen, G.R., and T. Ueta, 1999. Yet another chaotic attractor, *Int. J. Bifur. Chaos*, vol.9, pp.1465-66
- [6] Chen Wei-Chin. 2008. Nonlinear dynamics and chaos in a fractional-order financial system, *Chaos, Solutions & Fractals*, 36, 1305-1314.
- [7] Elwakil A. S. and Kennedy M. P .2001. Construction of classes of circuit independent chaotic oscillators using passive-only nonlinear devices, *IEEE Trans. Circuits Syst. I*, vol. 48, no. 3, pp. 289-307.
- [8] Elwakil A. S., Ozoguz S., and Kennedy M. P.2002. Creation of a complex butterfly attractor using a novel Lorenz-type system,” *IEEE Trans. Circuits Syst. I*, vol. 49, no. 4, pp. 527-530.
- [9] Goh, S.M., M.S.M. Noorani and I. Hashim. 2009. A new application of variational iteration method for the chaotic Rössler system, *Chaos, Solitons and Fractals*, 42: 1604-1610

- [10] Hashim, I. and M.S.H. Chowdhury.2009.Application of multistage homotopy-perturbation method for the solutions of the Chen system, *Nonlinear Analysis: Real World Applications* 10: 381-391.
- [11] Kyrtsov, C. and Vorlow, C. 2005. Complex dynamics in macroeconomics: A novel approach. *New Trends in Macroeconomics*; 223-238.
- [12] Kyrtsov C. and Labys W.2006. Evidence for chaotic dependence between US inflation and commodity prices. *Journal of Macroeconomics* 28: 256–266.
- [13] S.A. Khuri. 2001. A Laplace decomposition algorithm applied to class of nonlinear differential equations. *J Math. Appl*; 4: 141 - 155.
- [14] S.A. Khuri. 2004. A new approach to Bratu’s problem. *Appl. Math. Comput*; 147: 131 - 136.
- [15] Lu, J., T. Zhou, G. Chen and S. Zhang, 2002. Local bifurcations of the Chen system, *International Journal of Bifurcation and Chaos*, 12(10): 2257-2270.
- [16] Lü, J.H., and G.R. Chen. 2002. A new chaotic attractor coined, *Int. J. Bifur. Chaos*, vol.12, pp.659-661.
- [17] Lorenz, E.N.1963. Deterministic nonperiodic flow, *J. Atmos. Sci.*, vol.20, pp.130-141.
- [18] Lü, J.H., G.R.Chen. 2002. Bridge the gap between the Lorenz system and the Chen system, *Int. J. Bifur. Chaos*, vol.12, pp.2917-2926.
- [19] C. P. Li and G. Chen. 2004. Chaos and hyperchaos in the fractional-order Rössler equations,” *Physica A*, vol. 341, no. 1–4, pp. 55–61.
- [20] K. Moaddy, S. Momani, and I. Hashim, 2001. Non-standard finite difference schemes for solving fractional–order Rössler chaotic and hyperchaotic system, ,” *Computers and Mathematics with Applications*, vol. 61, no. 4, pp. 1209–1216.
- [21] R. L. Magin.2006. *Fractional Calculus in Bioengineering*, Begell House, Redding, Mass, USA.
- [22] Podlubny.1999. *Fractional Differential Equations*, Academic Press, New York, NY, USA.
- [23] Park, J.H. 2006. Chaos synchronization of nonlinear. Bloch equations, *Chaos, Solitons and Fractals*, 27: 357-361.
- [24] Roslan, U.A.M., Z. Salleh and A. Kılıçman. 2010. Solving Zhou’s chaotic system using Euler’s method, *Thai Journal of Mathematics*, 8(2): 299-309
- [25] Saheed O. Ajibola, Olusola.T Kolebaje, Samuel O. Sedara .2013. On the application of the multistage laplace Adomian decomposition method to the chaotic Chen system. *International Journal of Applied Mathematical Research*, 2(1) 116-124

- 329 [26] Zhen Wang, Xia Huang & Guodong Shi. 2011. Analysis of nonlinear dynamics and
330 chaos in a fractional order financial system with time delay, Computers & Mathematics with
331 Applications, 62, 1531-1539.
- 332 [27] Yassen, M.T. 2003. Chaos control of Chen chaotic dynamical system, Chaos, Solitons
333 and Fractals, 15: 271-283
334