

A CRITICAL REVIEW OF COMPETITIVE FIRM'S THEORY^{*}

Fernando Antonio Noriega Ureña¹

*Department of Economics
Autonomous Metropolitan University
Campus Azcapotzalco
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The aim of this research is, in the first place, to demonstrate that the economic behavior that neoclassical theory attributes to competitive firms is technically inefficient since it does not correspond to the highest possible internal rate of return, which implies the violation of the first theorem of welfare. We show that with any internal rate of return higher than the one inherent to the maximization of profits, and the same amount of resources determined by current walrasian prices, it is possible to produce more in a more competitive industry, which in turn means higher financing levels for consumers and therefore better situations in the sense of Pareto. It thus implies that neoclassical theory explains the operation of a market economy in which firms operate inefficiently even though they could overcome their own results, that is acting irrationally. The demonstration is carried out through a theorem based on the free entry and exit criterion, fully respecting the initial conditions and hypotheses of neoclassical theory. Secondly, overcoming error in the economic behavior of competitive firms gives rise to the basic results of the theory of nonexistence of the labor market (TNLM), on which the theorem of superiority, a basic element of its construction, is finally proved. For all these effects the mathematics of restricted maximization and some concepts of convex optimization are used.

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¹ Professor of Economic Theory, Member of the Research Group (Área de Investigación), on *International Economics and Integration*, UAM-A. <noriega@correo.azc.uam.mx>

1. INTRODUCTION

The behavior of any competitive firm in neoclassical theory is explained upon three hypotheses: first, that prices are publicly known and available data for everyone in the system, and any firm, being price-taker, can freely buy or sale any desired amount of inputs and outputs; second, that its objective function is to maximize its profits (which are precisely defined as the total revenue minus the total cost), a goal that will pursue at current prices; and third, that it is subject to its technological constraint, which is given by the set of all production feasible plans. The firms will maximize its profits at the point of all its technical possibilities of production in which the difference between the total value of its output and the cost of its inputs is the highest, given the prices.²

The third hypothesis implies that profits are a technical residue that at competitive prices will define its sign depending on the type of returns to scale. If inputs are paid according to their marginal productivities and the set of technical possibilities of production is of increasing returns, profits will be negative, whereas under constant returns those will be zero, but with –and only with– decreasing returns to scale, profits will be strictly positive.³

On this bases, it has become common in theorems of existence of general competitive equilibrium to assume constant returns to scale. This assumption is usually accepted as an implication of the *ad hoc* hypothesis that under free entry and exit there will be no incentives for admission nor expulsion of any productive units. Size of firms and industry is assumed to be optimal under such arbitrary conditions. The general equilibrium analytical framework is thus completed assuming constant returns to scale, and the size and number of production units is then exogenously defined.⁴

In sharp contrast to the above, this research aims to demonstrate that maximizing profits as an objective function is inefficient for both the individual competitive firm as for the industry as an aggregate, in the sense that neoclassical theory itself indicates: with the same amount of resources that firms decide to buy at current prices, it is possible to reach a higher production level, a higher mass of profits, and a larger and more competitive industry size by maximizing instead the profit rate, basic concept of the theory of nonexistence of the labor market (TNLM). The demonstration derives only and exclusively from the initial conditions established by neoclassical theory, and even though it will be later related to the *superiority theorem* of the TNLM, this is not required for being performed.

2. THE OBJECTIVE FUCTION

To explain in the very terms of neoclassical theory why firms seek to maximize their profits mass in a fully decentralized and competitive system ruled by private property,

² Lucas (2003), Estola (2014) and He, Y., X.M. Gong & G. Zhao (2012), are clear and robust references to the state currently held by the criticism of firm theory in the endorsement of mainstream. The reader will realize that the method and results achieved here differ substantially from those indicated in these publications.

³ In Solow (1956), constant returns to scale are a basic hypothesis of growth theory.

⁴ In Arrow (1971: 2-405), we find fully exposed the methodological foundations and axiomatic proofs of existence of a competitive general equilibrium (CGE). Specifically, the Arrow-Debreu type equilibria are currently considered as the methodological basis of any microfounded analysis. Macroeconomic models that propose explanations of the great social pathologies on the basis of economic behavior of individual agents, do commonly refer to CGE as the norm that should guide the orientation of economic policy due to the qualitative properties of it in terms welfare.

given the price vector p , $p \gg 0$, suppose the existence of any firm, being q its production vector –with positive sign in its elements for outputs and negative for inputs– and Q its set of technical possibilities of production, which is assumed to be strictly convex. Under such conditions it will perform the following maximizing calculus:

$$\begin{aligned} & \underset{q}{\text{Max}} \quad p \cdot q \\ & \text{s.t.} \quad q \in Q \end{aligned} \quad (1)$$

In a system consisting of only two goods, and being Π the amount of profits, its figure will be:

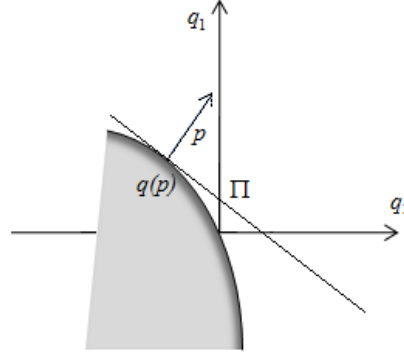


Figure 1
Competitive firm equilibrium

Similarly, the following calculation of a representative consumer, who is a partly owner of the existing firms in the industry, will define the reasons to indicate the managers of any firm to follow (1) as its main criterion of action in the system:

$$\begin{aligned} & \underset{x_i \geq 0}{\text{Max}} \quad u_i(x_i) \\ & \text{s.t.} \\ & \quad p \cdot x_i \leq w_i + \theta_i p \cdot q \end{aligned} \quad (2)$$

On the right side of the budget constraint in (2), we distinguish the revenues from the sales of factors to firms accomplished by the i -th consumer (w_i), added to those from their property rights on firms ($\theta_i p \cdot q$), according to the following rule of partial ownership: $\sum_i \theta_i = 1$. So all the owners, who are also consumers, seek through the maximization of corporate profits, to obtain the highest possible budget to finance their consumption decisions.

3. INEFFICIENCY IN A SIMPLE SCENARIO

Suppose now, for simplicity, an economy in which there is just one non-durable product (q),⁵ and labor as the only production factor (L), i.e. a scenario of two goods for consumers: the product and leisure time (S , $S = \tau - L_s$),⁶ In this expression, τ denotes the

⁵ For every variable, the subscripts "d" and "s" denote demand and supply, respectively.

⁶ A similar case to that shown in Figure 1.

maximum time biologically available to work for every individual –an initial endowment naturally given to everyone–, L_s is the time supplied for labor, and subtraction between the two concepts, i.e. S , refers to leisure time. There is perfect divisibility, complete information and free entry and exit of production units, and every firm is constituted by one or more of them, according to its maximizing behavior.

In adherence to the methodological guidelines of neoclassical theory, assume initially that there exist n firms, $n>0$, all of them price-takers, with production functions of diminishing marginal returns of the form: $q_s = L_d^\alpha$, $\alpha \in (0,1)$

The nominal price of product is equal to one, and the real wage (w) is a positive quantity equal to the marginal product of labor. The economy is at full employment.

Consumers, owners of all firms according to the rule of ownership set out for (2), prepare to compare the results achieved if instead of staying on the plan maximizing the mass of profits, seek an internal rate of return each time higher than that corresponding to that plan (1), using for it the same amount of labor or social effort determined in maximizing profits. The rate of profit or internal rate of return is defined as the mass of profits divided by the total cost of each possible production plan. To make the comparison, consumers consider simultaneously both functions: mass and rate of profit.

The *real mass of profits* Π of any of the n firms, expressed as a function of the labor employed in it (L_d), at the real wage w , is:⁷

$$\Pi(L_d) = L_d^\alpha - wL_d ; \quad \alpha \in (0,1) \quad (3)$$

By (3) it is known that the first and second derivatives of this function are given by:

$$\frac{\partial \Pi(L_d)}{\partial L_d} = \alpha L_d^{\alpha-1} - w \geq 0 \quad , \quad (3')$$

and

$$\frac{\partial^2 \Pi(L_d)}{\partial L_d^2} = -(1-\alpha)\alpha L_d^{\alpha-2} < 0 \quad (3'')$$

It means that (3) has an absolute maximum at the point where the first derivative is zero, which corresponds to the maximum mass of profits.

The rate of profit or internal rate of return of any technologically feasible production plan, denoted by π , is expressed as follows:

$$0 = -wL_d + \frac{L_d^\alpha}{1 + \pi(L_d)} \quad (4)$$

⁷ The properties of the profit function have been sufficiently developed, as highlighted by Varian (1992, pp. 49-58), by Hotelling (1932), Hicks (1946) and Samuelson (1947).

This means that:

$$\pi(L_d) = \frac{1}{wL_d^{(1-\alpha)}} - 1 \quad (5)$$

Note in (3') that the *maximum mass of profits* condition is reached at the point of (3) which verifies that the marginal productivity of labor equals the real wage:

$$\alpha L_d^{-(1-\alpha)} = w \quad (6)$$

Thus, labor demand turns out to be an increasing and negative slope function in w , which means that the higher the real wage, the lower will be the employment level:

$$L_d = (\alpha^{-1} w)^{-(1-\alpha)^{-1}} \quad (7)$$

With these elements, it can already be shown the research problem: the inefficiency of firm's traditional calculation represented in (1).

- From the maximum mass of profits function (3) as the initial situation, consumers will evaluate now the results of each production unit employing in a second scenario only a fraction of the amount of labor initially used. However full employment will remain on the aggregate, due to the entry of enough additional productive units, up to the point of exhaustion of productive resources available at current prices. This means that the number of production units will grow up attracted by the higher profitability, employing, all units together, the same amount of labor than in the initial situation. The evaluation will then consist in comparing levels of internal rate of return (5), and profits mass (3), between the two situations.

- Let λ , $0 < \lambda > 1$, be a pure number that makes it possible to determine the level of employment that every productive unit achieves in the second scenario. Then, if the initial level of employment per productive unit that guarantees the maximum mass of profits is L_d , the employment level in the second situation for each production unit will be λL_d ; magnitude that corresponds to just a fraction of the employment level reached by each at the initial situation. The number of incoming production units that will enable in the new situation to keep the same employment level per unit initially reached, will be λ^{-1} , which means that the economy will be, in the new situation, more competitive than in the previous one, since the number of production units will increase, each of them more profitable and smaller than before, thus preserving full employment in the aggregate.

- The comparison between the mass of profits of the first situation and of the second is given by the following inequality, in whose left member is shown the mass of profits of each productive unit, which are now smaller than in the initial situation, multiplied by the total number of these, and in whose right side is exhibited the situation already shown in (3):

$$\frac{1}{\lambda} [(\lambda L_d)^{\alpha} - (\lambda L_d)w] > L_d^{\alpha} - wL_d; \quad (8)$$

which means that:

$$\left(\frac{1}{\lambda^{(1-\alpha)}} - 1 \right) L_{d1}^{\alpha} > 0 \quad (9)$$

- Note that in both cases the total costs are calculated at prices determined by the profit-maximizing plan, which means adopting the assumption that the new size of the industry would not change prices. This means that any position on the left of the maximum mass of profits for any of the n firms –and therefore also for the representative one– will be of higher internal rate of return, greater mass of profits and a larger number of production units, at current prices that remain unchanged. All this means that maximizing the mass of profits does not imply that a maximum mass is gotten, as usually assumed. There will be countless more efficient situations or scenarios with greater mass of profits and higher internal rate of return derived from the employment of a single and unchanging volume of resources, as shown in the following inequality:

$$\frac{(\lambda L_{d1})^{\alpha}}{w(\lambda L_{d1})} - 1 > \frac{L_{d1}^{\alpha}}{wL_{d1}} - 1 \quad (10)$$

- The following figure displays, for a single firm, the superiority of the internal rate of return (lower graph), in a situation in which each production unit uses only partially the amount of labor used in the initial situation, which is that in which maximizes the mass of profit:

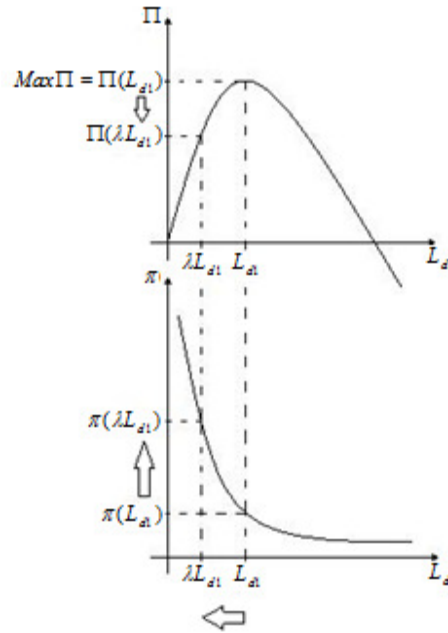


Figure 2
Mass of profits versus internal rate of return

- Since each production unit that uses only a fraction of the labor available at the current real wage will reveal a higher rate of return than that corresponding to the maximum mass of profit, and also a higher average product,⁸ once all labor is employed in productive units of smaller size and higher average productivity, the number of production units in the economy will increase, and thus the competitiveness; the aggregate volume of product and the mass of profits for the economy as a whole will be higher, even though the product and the profits generated by each unit will be lower.

- The increase in the total amount of profits will result from the growth in the volume of output of the whole economy, due to the higher average labor productivity in each production unit, with the use of the same volume of social working effort than in the initial situation. This is shown in the following expression, where the left side of the production function is multiplied by the inverse of the fraction of labor employed by each unit:

$$\frac{1}{\lambda} L_d^\alpha > L_d^\alpha \quad (11)$$

- This demonstrates, in a simple scenario with invariable prices, that the calculation the traditional theory attributed to competitive firms is technically inefficient: with any internal rate of return higher than the initial one and the same social working effort determined by the prices stated by the neoclassical theory, it is possible to produce more, which in turn means higher financing levels for consumers and therefore better situations in the sense of Pareto. It thus appears that the neoclassical theory explains the operation of a market economy in which firms operate inefficiently even though they could overcome their own results, that is acting irrationally, and they do so in a less competitive economy than that achieved under free entry and exit.

However, there remains an important question that must be answered: Does the outcome change if prices are modified for each of the situations compared, since the size of the industry and therefore the conditions for labor payment are different in each case?

To answer it is necessary to remove the condition that prices governing the comparison are those determined in the maximization of profit, and allow price variation for the production units. So, each unit of those using lower amounts of labor and defining their production plans in the frontier points of technical possibilities in which the marginal productivity of labor is higher than in the profit-maximizing plan, will remunerate labor hired with a real wage w^* such that $w^* > w$. Formally, the real wage paid by each production unit or firm employing λL_{d1} of labor, now will be given by:

$$\alpha \lambda^{-(1-\alpha)} L_d^{-(1-\alpha)} = w^* \quad (12)$$

Calculating the total cost of production of all firms in this situation, and consequently redefining the inequality (8), we obtain:

⁸ As shown in the graph following the arrows.

$$\frac{1}{\lambda} [(\lambda L_{d1})^\alpha - (\lambda L_{d1})w^*] > L_{d1}^\alpha - wL_{d1} . \quad (13)$$

And replacing (6) and (12) into (13), it follows that:

$$\frac{1}{\lambda^{(1-\alpha)}} (1-\alpha)L_{d1}^\alpha > (1-\alpha)L_{d1}^\alpha \quad (14)$$

This proves once again, now with differentiated prices, that both the product and the total profits, as well as the number of production units will increase if consumers, as firm's owners, seek higher rates of return than that corresponding to the profit maximization plan, using the same volume of resources than in that plan. However, the rate of return is now equal to that resulting from the maximization of the profits mass, equality that has been reached thanks to the free entry of new production units.⁹ This turns such a plan into an inefficient situation as long as the system allows free entry and exit. The consequence of this demonstration is that consumers committed ultimately with the efficient operation of firms, will not accept the mass function for profits as the objective to pursue on their part.

4. A SCENARIO OF n INPUTS AND ONE PRODUCT

Let it be a competitive economy composed of a large number m of firms, each of which uses non-negative amounts of the n existing productive inputs, $n-1$ of which are produced by the production system itself, and an n -th –labor– assumed homogeneous and perfectly divisible, not produced by firms, exclusively offered by consumers and indispensable into all production processes.

The production functions of all firms are homogeneous of a positive degree greater than one and less than zero, i.e. decreasing returns to scale, ensuring that at competitive prices all reveal positive benefits. Thus, the maximizing behavior of the k -th firm, $k = 1, 2, \dots, m-1, m$, corresponds to the following expression:

$$\text{Max } \Pi_k = f(L_{1k}, L_{2k}, \dots, L_{n-1k}, L_{kn}) - \sum_j w_j L_{jk} \quad (15)$$

The degree of homogeneity of $f(\cdot)$ is given by $\sum_{j=1}^n \alpha_j \in (0,1)$, which is the sum of the elasticities of n inputs, and L_{jk} the amount of the j -th input used by the k -th firm.

Due to Euler's theorem for homogeneous functions, we know that competitive prices for any technique besides the maximizing one will be given by:

⁹ Its expression is: $(1-\alpha)\alpha^{-1}$, as can be seen by dividing each member (14) between its total costs. This is the technical result achieved by A. Pigeon for this analytical scenario and for the one referred to n inputs and one output. The implication of this is that any alternative plan more efficient than the original, being the latter inherent to profits maximizing firms, will reach the same internal rate of return of it but with higher levels of output, profits and factor payments.

$$w_j = \alpha_j \frac{f(L_{1k}, L_{2k}, \dots, L_{n-1k}, L_{nk})}{L_{jk}}, \quad (16)$$

Full payment of the j -th factor will therefore be:

$$w_j L_{jk} = \alpha_j f(L_{1k}, L_{2k}, \dots, L_{n-1k}, L_{nk}) \quad (17)$$

Then substituting (17) into (15) we obtain:

$$\Pi_k = f(L_{1k}, L_{2k}, \dots, L_{n-1k}, L_{nk}) (1 - \sum_j \alpha_j), \quad (18)$$

which means that the profits-product ratio is a constant given by:

$$\frac{\Pi_k}{f(L_{1k}, L_{2k}, \dots, L_{n-1k}, L_{nk})} = (1 - \sum_j \alpha_j) \quad (19)$$

This means that profits are a fixed proportion of the product, whatever the technique used.

Since profit rate has been defined as the share of profits amount over total costs, according to (19) we have that its expression is given by:

$$\frac{\Pi_k}{\sum_j \alpha_j f(L_{1k}, L_{2k}, \dots, L_{n-1k}, L_{nk})} = (1 - \sum_j \alpha_j) \left(\sum_j \alpha_j \right)^{-1} \quad (20)$$

This means that the rate of profit also happens to be a constant, regardless of the technique chosen by the producer, as long as inputs are paid at competitive prices.

Suppose now, on the same grounds of the previous section, that consumers, into their role as owners of firms, decide to compare the results that would be obtained if instead of using the profit-maximizing technique, apply any other in which will be employed only a fraction of inputs than in the aforementioned, being λ , $1 > \lambda > 0$, this fraction. Then, analogous to the expression (15) is:

$$\Pi_{\lambda k} = f(\lambda L_{1k}, \lambda L_{2k}, \dots, \lambda L_{n-1k}, \lambda L_{nk}) - \sum_j w_j \lambda L_{jk}, \quad (21)$$

finally resulting into:

$$\Pi_{\lambda k} = \lambda^{\sum_j \alpha_j} f(L_{1k}, L_{2k}, \dots, L_{n-1k}, L_{nk}) \left(\lambda^{\sum_j \alpha_j} - \lambda \sum_j \alpha_j \right) \quad (22)$$

That is, analogously to (19), the mass of profits as a proportion of the product is given by the following expression:

$$\frac{\Pi_{\lambda k}}{\lambda^{\sum_j \alpha_j} f(L_{1k}, L_{2k}, \dots, L_{n-1k}, L_{nk})} = \left(1 - \lambda^{1 - \sum_j \alpha_j} \sum_j \alpha_j \right) \quad (23)$$

Meanwhile, the rate of profit is given by:

$$\frac{\Pi_{\lambda k}}{\lambda^{\sum_j \alpha_j} f(L_{1k}, L_{2k}, \dots, L_{n-1k}, L_{nk})} = \left(\frac{1}{\lambda^{1 - \sum_j \alpha_j} \sum_j \alpha_j} - 1 \right) \quad (24)$$

Comparing (19) with (23) and (20) with (24), we find that:

$$\left(1 - \lambda^{1 - \sum_j \alpha_j} \sum_j \alpha_j \right) > (1 - \sum_j \alpha_j), \text{ and } \left(\lambda^{1 - \sum_j \alpha_j} \sum_j \alpha_j \right)^{-1} - 1 > \left(1 - \sum_j \alpha_j \right) \left(\sum_j \alpha_j \right)^{-1} \quad (25)$$

This means that the volume of profits will be lower for the individual firm and the profit rate greater than when maximizing profits function (15). Therefore, under free entry and exit, the higher profitability in terms of internal rate of return will attract new production units to the industry, at least to the extent that employs input volumes determined by profit-maximizing technique. The number of firms or production units entering through profitability effect caused by downscaling the k -th firm, to use the same amount of inputs determined by its profit maximizing plan will equal to λ^{-1} . So then, the magnitudes of the mass of and the rate of profit, respectively, at tiered prices depending on the size of the production units and the industry as a whole, will be:

$$\frac{1}{\lambda} \Pi_{\lambda k} = \lambda^{-1 + \sum_j \alpha_j} \left(1 - \sum_j \alpha_j \right) f(L_{1k}, L_{2k}, \dots, L_{n-1k}, L_{nk}) \quad (26)$$

and

$$\frac{\Pi_{\lambda k}}{\lambda^{\sum_j \alpha_j} \sum_j \alpha_j f(L_{1k}, L_{2k}, \dots, L_{n-1k}, L_{nk})} = \left(1 - \sum_j \alpha_j \right) \left(\sum_j \alpha_j \right)^{-1} \quad (27)$$

Note that the rate of return (27) of the most profitable options has dropped gradually to equal to that corresponding to the initial situation (that is, once it has been used the same amount of resources than when profits mass function was maximized by firms), only this time with superior results in terms of output, profits, wages and industry size, due to the entry of additional production units.

It is said that the prices are differentiated, indicating that payments to the factors once the scale of production of each company has declined and that the size of the industry has increased, corresponds to the following expression:

$$w_j^* = \alpha_j \frac{f(\lambda L_{1k}, \lambda L_{2k} \dots \lambda L_{n-1k}, \lambda L_{nk})}{\lambda L_{jk}} \quad (28)$$

In turn, production costs of the industry, once it has been used the same volume of inputs than in the situation of profit maximization, are given by:

$$\sum_j w_j^* L_{jk} = \lambda^{-1+\sum_j \alpha_j} \sum_j \alpha_j f(\lambda L_{1k}, \lambda L_{2k} \dots \lambda L_{n-1k}, \lambda L_{nk}) \quad (29)$$

These costs are already present in (26) and (27), expressions in which it is found that:

$$\frac{1}{\lambda^{1-\sum_j \alpha_j}} > 1 \quad (30)$$

What suffices to show outcomes that will exceed those corresponding to the profit-maximizing plan, using the exact same amount of input that in it, which involves inefficiency of such a plan.

For this demonstration, the case concerning prices that are invariable and determined by the maximization (15), now becomes trivial, since subtracting the same total costs to different quantities of product, will not change the sense of the difference between such amounts.

5. THE FIRST THEOREM OF WELFARE

While these demonstrations are relevant to the theory by the fact that they pose a problem of inefficiency about the economic behavior that the neoclassical tradition has attributed to firms, their deepest result is reached by the injury inflicted on the essential quality of all competitive equilibrium: the Pareto optimality. This quality, which is present in the first welfare theorem, in the analytical prose of Villar (1996),¹⁰ is expressed in the following propositions:

“Proposition 7.1 Let $\mathbf{Y} \equiv \sum_{j=1}^n \mathbf{Y}_j$ be the total production set of the economy, and let $\mathbf{p}^*$ be a price vector for which all supply correspondences are defined. Then $\mathbf{y}_j^* \in \eta_j(\mathbf{p}^*) \forall j$ if and only if $\mathbf{P}^* \mathbf{Y}^* \geq \mathbf{P}^* \mathbf{Y}, \forall \mathbf{y} \in \mathbf{Y}$.

Proposition 7.2 Let i be a consumer with a locally non-satiated utility function, defined over the consumption set X_i . Then, if for some price-wealth pair $(\mathbf{p}^*, w_i^*)$, $\mathbf{x}_i^*$ maximizes u_i in $\beta_i(\mathbf{p}^*, w_i^*)$, then it will verify that:

$$(a) \quad \mathbf{P}^* \mathbf{x}_i^* = w_i^*$$

¹⁰ Reproduced verbatim from Villar (1996: Chapter 7, p. 150), and here translated by the author of this paper.

(b) For all $\mathbf{X}'_i \in \mathbf{X}_i, u_i(\mathbf{X}'_i) \geq u_i(\mathbf{X}^*_i) \Rightarrow \mathbf{P}^* \mathbf{X}'_i \geq \mathbf{P}^* \mathbf{X}^*_i$. Particularly,

$$\mathbf{X}'_i \in \mathbf{X}_i, u_i(\mathbf{X}'_i) > u_i(\mathbf{X}^*_i) \Rightarrow \mathbf{P}^* \mathbf{X}'_i > \mathbf{P}^* \mathbf{X}^*_i.$$

Theorem 7.1 Let $\mathbf{E}_{pp}$ be a private property economy in which each consumer has a utility function that satisfies the assumption of being locally non-satiated. If $(\mathbf{P}^*, [\mathbf{X}^*_i], [\mathbf{Y}^*_i])$ is an equilibrium of this economy, then the allocation $[\mathbf{X}^*_i], [\mathbf{Y}^*_i]$ is efficient in the sense of Pareto."

After the necessary changes to make compatible this notation with that used in the analysis of the preceding paragraphs, by the statement of the theorem it is known that if there is a production plan $\mathbf{Q}^*_{\lambda k} = \frac{1}{\lambda} f(\lambda \mathbf{L}^*)$, $\lambda, \lambda \in (0,1)$ with which it is achieved a higher volume of product than with another $\mathbf{Q}^*_k = f(\mathbf{L}^*)$, employing for any of both cases a single and unique volume of inputs $\mathbf{L}^* = (L_{1k}, L_{2k}, \dots, L_{n-1k}, L_{nk})$, then the consumer incomes will be higher and also will their utility level, because the allocation previously considered as efficient in the sense of Pareto will be overcome. This will put the plan referred to $\mathbf{L}^*$ in the definition of Pareto inefficiency, and thus also the origin of the decision of the plan, which is none other than profit maximization under (15). The utility functions considered in the proofs of inefficiency, which correspond to (2), do fully satisfy the condition of local insatiability.

To generalize inefficiency demonstrations performed by us up to this point, we propose now a theorem in the following section.

6. THEOREM OF INEFFICIENCY

It is known that a function of n variables $f(L_1, L_2, \dots, L_{n-1}, L_n)$ defined on a domain $\tilde{\mathbf{L}}$, a convex subset of $\mathfrak{R}^n$, such that $\tilde{\mathbf{L}} = \mathfrak{R}^n_{0,+}$, $\tilde{\mathbf{L}} = \{\mathbf{L} \in \mathfrak{R}^n : \mathbf{L} \geq 0\}$, is strictly concave if, given a pure number $\lambda, \lambda \in (0,1)$, and any vectors $\mathbf{L}^*$ and $\mathbf{L}$, belonging to $\tilde{\mathbf{L}}$, $\mathbf{L}^* \neq \mathbf{L}$, it is verified that :

$$f(\lambda \mathbf{L}^* + (1-\lambda)\mathbf{L}) > \lambda f(\mathbf{L}^*) + (1-\lambda)f(\mathbf{L}) \quad (31)$$

Let Π_k be the profit function of the k -th firm of a competitive economy of private property, and $\mathbf{L}^*, \mathbf{L}^* > 0$, the input vector that maximizes this function at $\mathbf{w}^*$ prices:

$$\Pi_k^* = f_k(\mathbf{L}^*) - \mathbf{w}^* \mathbf{L}^* \quad (32)$$

The price vector $\mathbf{w}^*, \mathbf{w}^* > 0$, consist of marginal productivities of inputs, and function $f_k : \tilde{\mathbf{L}} \rightarrow \mathfrak{R}_{0,+}$ is strictly concave and homogeneous of degree $\mu_k, \mu_k \in (0,1)$ in its arguments.

The consumer's utility functions are quasi-concave and satisfy the condition of local insatiability, and budget constraints depend, by the revenue side, of their property rights on firms, same that determine a positive definite and stable relation with profits.

Under these conditions it is shown the following:

Proposition: In a system of free entry and production functions strictly concave and homogeneous of degree $\mu_k, \mu_k \in (0,1)$, wherein the k -th firm maximizes its profit function Π_k , with the vector of inputs $\mathbf{L}^*$, $\mathbf{L}^* > 0$, at prices $\mathbf{w}^*$, $\mathbf{w}^* > 0$, with $w_j^* = f_{jk}'$, there is at least one alternative plan referred to $\lambda \mathbf{L}^*$, $\lambda \in (0,1)$, more profitable than the inherent to $\mathbf{L}^*$, such that with a sufficient number of production units as for employing total inputs $\mathbf{L}^*$, it generates more product than $f_k(\mathbf{L}^*)$, a higher volume of profits than Π_k^* , and a more competitive size of the industry, thus implying the inefficiency of Π_k function and the violation of the first theorem of welfare.

Theorem: By (31) we know that:

$$f_k(\lambda \mathbf{L}^* + (1-\lambda)\mathbf{L}) > \lambda f_k(\mathbf{L}^*) + (1-\lambda)f_k(\mathbf{L}) \quad (33)$$

- Let $\mathbf{L} = 0$, so that inaction is a possibility. Then:

$$f_k(\lambda \mathbf{L}^*) > \lambda f_k(\mathbf{L}^*), \quad (34)$$

which implies that:

$$\lambda^{-1} f_k(\lambda \mathbf{L}^*) > f_k(\mathbf{L}^*) \quad , \quad (35)$$

$$\text{since: } \lambda^{\mu_k-1} > 1, \quad (36)$$

with which it is shown that with a number of productive units given by $\lambda^{-1} > 1$ for each profit-maximizing production unit similar to the k -th firm, will be generated a volume of product $\lambda^{\mu_k-1} - 1 > 0$ times greater, employing for it the same volume of inputs used in (32).

- Total costs for $f_k(\mathbf{L}^*)$ and for $\lambda^{-1} f_k(\lambda \mathbf{L}^*)$ are given by $\mu_k f_k(\mathbf{L}^*)$ and $\lambda^{\mu_k-1} \mu_k f_k(\mathbf{L}^*)$, respectively, which implies that:

$$\lambda^{-1} f_k(\lambda \mathbf{L}^*) - \lambda^{\mu_k-1} \mu_k f_k(\mathbf{L}^*) > f_k(\mathbf{L}^*) - \mu_k f_k(\mathbf{L}^*) \quad (37)$$

- Whence it follows that $\lambda^{-1} \Pi_{\lambda k} > \Pi_k^*$, due to $\lambda^{\mu_k-1} (1 - \mu_k) > 1 - \mu_k$ and to $\lambda^{\mu_k-1} (1 - \mu_k) \mu_k^{-1} > (1 - \mu_k) \mu_k^{-1}$, being $(1 - \mu_k) \mu_k^{-1}$ the profit rate which is the same for both cases, it is demonstrated that both the mass of profits as the factor payments will be higher in the alternative plan.

- So, being the alternative plan more competitive in terms of industry size, and of higher volume of product and of factor payments than the one referred to profit maximization, the proposition is fully demonstrated.

7. RETHINKING THE FUNDAMENTALS

The analytical error that has been revealed concerning the economic behavior of firms or producers: a market economy poorly explained in one of its foundations, involves the inability to effectively predict and control phenomena. However, from the very demonstration of inefficiency, emerge axiomatic resources to rebuild the foundations.

Profit rate maximization

The rate of profit or internal rate of return, which was used in previous paragraphs as a standard of comparison, has shown in the proposed analytical scenarios, be sufficient to identify a set of more efficient situations than those resulting from the profits function maximization. However, maximizing this function subject to standard production functions of the nature of those used in the previous sections, gives results of almost no interest for the theory. Therefore, it is necessary to rethink the notion of production functions as representative of the technological possibilities of a system.¹¹

In order to establish alternative bases, it will be convenient to place again the analysis in a simple scenario: a non-durable and non-cumulative product, and a single production factor: labor.¹²

Since maximizing the profit rate is equivalent to maximizing the average product, it is required a production function that makes it possible. That function will be:

$$q_s = f(L_d - L^*) , \quad (38)$$

defined for all $(L_d - L^*) > 0$.

In it, L^* corresponds to the flexible component of technology –i.e. the amount of labor employed for the organization of all production process–, which is determined by market size. It is not a rigidity nor corresponds to increasing returns, as will become apparent later.

Then, the firm's behavior is given by:

$$\begin{aligned} \text{Max } (1 + \pi) &= \frac{q_s}{wL_d} \\ \text{s.t. } q_s &= (L_d - L^*)^\alpha \end{aligned} \quad (39)$$

The first order conditions are:

$$\begin{aligned} \alpha(L_d - L^*)^{\alpha-1} &= \frac{(L_d - L^*)^\alpha}{L_d} \\ q_s &= (L_d - L^*)^\alpha \end{aligned} \quad (40)$$

The maximum profit rate is achieved at the point of the production function in which labor elasticity of product is equal to one, and comes to be a situation independent of prices

¹¹ Anderson & Ros (2005), propose an interesting alternative, although lies both in its initial conditions and results, in the traditional theory.

¹² Walker (2015), shows that, despite the demands of abstraction in theory, the methodological spaces must be opened to achieve the correspondence between fields such as institutional economics and the theory of prices. Aim underlying our approach.

and wages. Employment levels are now determined by the size of market, not by the real wage:

$$L_d = (1 + \alpha)^{-1} L^* \quad (41)$$

The condition of financial viability of the representative firm is:

$$\frac{q_s}{L_d} > w > 0 \quad (42)$$

This indicates that if wage is zero, the economy does not operate: the production level becomes zero and market activity, nonexistent; and if the real wage equals the average product, the profits will be zero and the firm may or may not operate. So, it turns out that the condition of existence of economic activity is a positive real wage, and the condition of financial viability for firms is that real wage also comes to be lower than the average product.

Under these circumstances the maximizing problem of the representative consumer will be:

$$\begin{aligned} &Max \ U(q_d, S), \\ &S = \tau - L_s, \\ &s.t. \ (1 + \pi)wL_s = q_d \end{aligned} \quad (43)$$

from which result the following product-demand and labor-supply functions, respectively:¹³

$$\begin{aligned} q_d &= \varphi(1 + \pi)w\tau \\ L_s &= \varphi\tau \end{aligned} \quad (44)$$

Once that the maximization problem of producers and consumers has been solved, the general equilibrium conditions will be as follows:

$$\begin{aligned} L_d - L_s &\leq 0 \\ q_d - q_s &= 0 \\ (L_d - L_s)w + (q_d - q_s) &= 0 \end{aligned} \quad (45)$$

So labor demand for the aggregate takes the form:

$$L_d = \alpha^{-1} [\varphi(1 + \pi)w\tau]^{\frac{1}{\alpha}}, \quad (46)$$

¹³ As before, the parameter τ , $\tau > 0$, refers to the maximum time biologically available to work; a natural initial endowment of any consumer in the system. Meanwhile, S represents leisure time demanded by the consumer. φ is a parameter resulting from preferences.

that comes to be a positive and increasing function of real wage, in deep contrast with the neoclassical expression (6), which is defined as a negative and increasing function of that same variable. According to (46), to a higher real wage will correspond a higher employment level.

The contrast between these results and those typical of the neoclassical tradition is evident, and it is a problem that must be solved in the same way that has been raised the inefficiency theorem: What technical reason exists to believe that the producers, if they could choose, they would choose to maximize the internal rate of return rather than the mass of profits?

These are the basic results of the theory of nonexistence of the labor market (TNLM). As shown in (45) and (46), the labor sector is not a market, and wages are a distributive variable expressed here as a degree of freedom in the system.

Note in (46) that employment level is determined by the scale of aggregated demand of product; which corresponds to the Keynesian hypothesis exhibited in chapter 20, “The employment function”, of *The General Theory of Employment, Interest and Money*,¹⁴ where Keynes postulates that employment level is determined by the size of effective demand. However, while in Keynes the real wage decreases when increasing the effective demand and thus the level of employment, in the TNLM the real wage increases and is the cause that the effective demand grows and expands the level of employment. This is a fundamental contrast of the TNLM with respect to the neoclassical theory and Keynes.¹⁵

Theorem of superiority

While the inefficiency theorem has already provided important elements about the neoclassical firm’s behavior hypothesis, it is essential to make an analytical comparison with the TNLM hypothesis. To that effect, it will immediately be demonstrated the following proposition:

Proposition: *If in a competitive system at least one firm maximizes rate of profit rather than the volume of profits, whatever the price vector, it will get the higher possible mass of profits and a Pareto superior situation for all consumers with respect to that would be achieved by maximizing the mass of profits function.*

Demonstration:

Lemma 1: *The consumer incomes, if firms maximize the rate of return would be higher than they would achieve by maximizing the difference between revenues and costs.*

This lemma is demonstrated in account of (2), and due to (37), which means that any rate of profit than that corresponding to the maximization of the mass-of-profits function, will be associated with a higher volume of revenue for consumers.

Lemma 2: *The product and the profits achieved by any firm through maximizing the profit rate are superior to those that would achieve in maximizing the difference between revenues and costs.*

¹⁴ Keynes (1936: 280-291)

¹⁵ Rauh (2014), Offers a treatment of the real wage-employment relationship, heterodox compared to the traditional method, despite inserting its analysis into the traditional methodological framework. This is possible basically because it establishes conditions of imperfect competition.

This lemma is demonstrated by the equations (33) to (37), because in them exists a non-produced factor (labor) that under hypothesis (38) enables production functions to allow the maximization of profit rate functions, also making possible to show that from all profit rates considered by the inefficiency theorem, one of them, the lowest, corresponds to the maximization of the mass of profits.

Theorem: *Let*
$$U=u[q_d,(\tau-L_s)] \quad (47)$$

be the utility function, strictly concave and differentiable, of any consumer in the system. Then, since the system is at full employment: $L_d = L_s = \tilde{L}$, and since the purchasing power of consumers, as well as the volume of product generated by the economy as a whole, is greater when at least one firm maximizes the rate profit rather than maximize the difference between revenues and costs, the consumer utility is also higher.

The theorem of superiority then turns out to be a logical implication of the theorem of inefficiency, with the difference that the latter does not require the reframing of analytical foundations, which derive in the theory of nonexistence of a labor market, as it has been shown in (41).

It is understood that the inefficiency theorem shows the inconsistency between the first welfare theorem and the neoclassical theory of producer in a competitive system. For its part, the superiority theorem proves the existence of an analytically superior theory of producer than the neoclassical. Thus, any application of the theorem of superiority will validate the results of the inefficiency theorem. It will be now provided an application of it in a dynamic discrete-time scenario.¹⁶

8. CONCLUSIONS

The explanatory power of the theory is its main but not unique attribute: efficient explanations are a prerequisite to efficient predictions, and the latter, necessary to establish efficient criteria to control explained phenomena. Thus, the evidence of explanatory inefficiencies that has been studied in this research, in addition to having limited the scope explanatory, predictive and of governance or control of neoclassical theory, have exposed the need to build efficient explanations of the functioning of a market economy. To that end seeks to contribute the theory of nonexistence of the labor market, whose pillars are the criticism and reconstruction of the theory of producer.¹⁷

One of the oldest theoretical errors in economics refers to the principle that the capitalist economy is composed by markets purely and simply; i.e. a system of goods and prices. It follows that the major pathologies that the economy should explain, predict and control, inevitably arise as market phenomena, whether they concern to imperfectly competitive markets or markets constrained by rigidities. Thus, the “labor market”, one of

¹⁶ The model described here is a refinement of that published in Noriega (2012: p. 19-45).

¹⁷ As Bradtke (2013) shows, for theory it is essential to build the methodological link with the real economy. The methodological difficulties should not be confused with the inconsistencies, which is the case that this author analyzes. What this means for our research is that correcting the errors of theory transcends at all levels of decision making.

all, is expected to operate according to the principles that govern the entire market, and to explain the problems of unemployment and wages as concerning to phenomena of disequilibrium. In the extreme case, typical of the new classical economics, the virtually instantaneous capacity of plans adjustment of agents to their calculation errors due to incomplete or incorrect information, simply cancels the existence of such pathologies.¹⁸ In any case, the general competitive equilibrium takes place as the final goal, due to its attribute of social efficiency. That is, no more no less, the core that has been injured by the results exposed inhere. Fortunately, the critique itself provides elements for reconstruction.

The demonstration that the inefficiency of the theory of firm in neoclassical tradition violates the first welfare theorem, injures the norm that guides all axiomatic deductions of this logical system, i.e. the perfectly competitive equilibrium. It then imposes the need to replace that norm by any other descriptive notion provided by a robust theory to orient the sense that the criteria of economic policy should follow, for the sake of a more desirable economic order than the current.¹⁹ Apparently, this concept should be to rethinking the demonstrations of existence of a general competitive equilibrium, this time based on the correction of the analytical error of neoclassical theory.

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¹⁸ See Hahn & Solow (1995, p. 3-9), making a dramatic statement on the matter, these two remarkable authors of the neoclassical tradition show a historical divergence of conscience more than of method. However, it has been inexplicably disregarded.

¹⁹ Bénassy (2011) and Bewley (2007) Bénassy (2011) and Bewley (2007) fully correspond in their analysis to the method and policy implications of mainstream economics, with explicit assumption of general competitive equilibrium as the best economic organization that can be achieved by modern societies.

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