

Original Research Article

**A CRITICAL REVIEW OF COMPETITIVE FIRM'S
THEORY***

Abstract

The aim of this research is to demonstrate, through a proposed theorem, that economic behavior of competitive firms, as postulated by neoclassical tradition, is inefficient and violates the first welfare theorem. Once such calculation is rebuilt, it is also shown that the proposed reformulation is analytically superior.

ECONLIT Classification: D21, D30, D41, D50, D61

Key Words: Production, Efficiency, Welfare, Employment

1. INTRODUCTION

The behavior of any competitive firm in neoclassical theory is explained upon three hypotheses: first, that prices are publicly known and available data for everyone in the system, and the firm, being price-taker, can freely buy or sell any desired amount of inputs and outputs; second, that its objective is to maximize its profits (which are precisely defined as the total revenue minus the total cost), a goal that will pursue at current prices; and third, that it is subject to its technological constraint, which is given by the set of all production feasible plans. The firm will maximize its profits at the point of all its technical possibilities of production in which the difference between the total value of its output and the cost of its inputs is the highest, given the prices.

Note that the third hypothesis implies that there will be possible technical sets of production that will allow the competitive firm to obtain positive profits, and there will also be some others that will imply negative or zero profits, whatever the price vector is in a competitive environment. Profits will be determined, ultimately, by the properties of the technology available to the firm. If inputs are paid

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according to their marginal productivities and the set of technical possibilities of production is of increasing returns to scale, profits will be negative, whereas if the firm faces constant returns to scale, they will be zero, but with –and only with– decreasing returns to scale, profits will be strictly positive. This means that profits are a technical residue that at competitive prices will define its sign depending on the type of returns to scale.

So, in theorems of existence of a competitive general equilibrium it has become common to assume constant returns to scale, arguing that this is the implication of making existing conditions of additivity (which means free entry), and in action, both from which it follows that the optimal size of the industry in the long run is marked precisely by the existence of zero profits. This assumption is usually accepted as an implication of the *ad hoc* hypothesis that under free entry and exit there will be no incentives for admission nor expulsion of productive units. The general equilibrium analytical framework is thus completed assuming constant returns to scale, an exogenously determined positive and finite number of production units, and zero profits as well for the individual firms for the industry as a whole.¹

In contrast to the above, this research aims to demonstrate that maximizing profits as an objective function is inefficient for both the individual competitive firm as for the industry as an aggregate, in the sense that indicates neoclassical theory itself: with the same amount of resources that firms decide to demand at current prices, it is possible to reach a higher production level, a higher profits mass and a larger and more competitive industry size. The demonstration derives only and exclusively from the conditions of neoclassical theory, and even though it will be later related to the *superiority theorem* of the theory of nonexistence of the labor market (TNLM), it does not require of the latter in order to be performed.

The investigation closes by testing the superiority theorem and therefore the inefficiency theorem, in a discrete-time dynamic model.

2. THE OBJECTIVE FUNCTION

To explain in the very terms of neoclassical theory the reason that firms seek to maximize their profits mass in a fully decentralized and competitive system ruled by private property, suppose the existence of any firm of all those that constitute the industry, same that being $p, p > 0$, the price vector, q its production vector –with positive sign in its elements for outputs and negative for inputs– and Q its set of technical possibilities of production, which is assumed to be strictly convex, performs the following maximizing calculus:

$$(1) \quad \begin{aligned} & \underset{q}{\text{Max}} \quad p \cdot q \\ & \text{s.t.} \quad q \in Q \end{aligned}$$

In a system consisting only of only two goods, and being Π the amount of profits, its figure will be:

¹ In Arrow (1971: 2-405), we find fully exposed the methodological foundations and axiomatic proofs of existence of a competitive general equilibrium (CGE). Specifically, the Arrow-Debreu type equilibria are currently considered as the methodological basis of any microfounded analysis. Macroeconomic models that propose explanations of the great social pathologies on the basis of economic behavior of individual agents, do commonly refer to CGE as the norm that should guide the orientation of economic policy due to the qualitative properties of it in terms welfare.

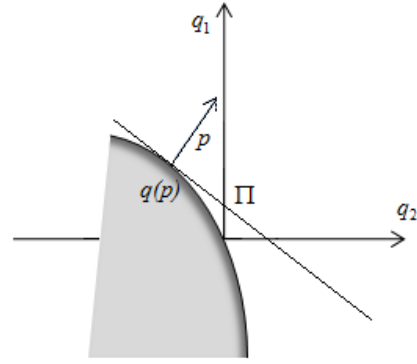


Figure 1

Competitive firm equilibrium

Similarly, the calculation of a representative consumer, who is a partly owner of any of all the existing firms in the industry, will show the reasons why the owners will indicate the managers to follow (1) as the main criterion of any firm in the system:

$$\begin{aligned} & \underset{x_i \geq 0}{\text{Max}} u_i(x_i) \\ & \text{s.t.} \\ & p \cdot x_i \leq w_i + \theta_i p \cdot q \end{aligned} \quad (2)$$

On the right side of the budget constraint in (2), we distinguish the revenues from the sales of factors to firms accomplished by the i -th consumer (w_i) and those from their property rights on firms ($\theta_i p \cdot q$) according to the following rule of partial ownership: $\sum_i \theta_i = 1$. So all the owners, who are also consumers, seek through the maximization of corporate profits, to obtain the highest possible budget to finance their consumption decisions.

3. INEFFICIENCY IN A SIMPLE SCENARIO

Suppose now, for simplicity, an economy in which there is just one non-durable product (q),² and labor as the only production factor (L), i.e. a scenario of two goods for consumers: the product and leisure time (S , $S = \tau - L_s$).³ In this expression, τ denotes the maximum time biologically available to work for every individual—an initial endowment naturally given to everyone—, L_s is the time supplied for labor, and subtraction between the two concepts, i.e. S , refers to leisure time. There is perfect divisibility, complete information and free entry and exit of production units, and every firm will be constituted by one or more of them, according to its maximizing behavior.

In adherence to the methodological guidelines of neoclassical theory, assume initially that there are n firms, $n > 0$, all price-takers, with production functions of diminishing marginal returns of the form: $q_s = L_d^\alpha$, $\alpha \in (0, 1)$

The nominal price of the product is equal to one, and the real wage (w) is a positive quantity equal to the marginal product of labor. The economy is at full employment.

Consumers, owners of all firms according to the rule of ownership set out for (2), prepare to compare the results achieved if instead of staying on the plan maximizing the mass of profits, seek an

² For every variable, the subscripts "s" and "d" denote demand and supply, respectively.

³ A similar case to that shown in Figure 1.

internal rate of return each time higher than that corresponding to that plan (1), using for it the same amount of labor or social effort determined in maximizing profits. The rate of profit or internal rate of return is defined as the ratio of the mass of profits between the total cost of each possible production plan. To make the comparison, consumers considered the properties of both functions: mass and rate of profit.

The *real mass of profits* Π of any of the firms, expressed as a function of the labor employed in it L_d , at the real wage w , is:⁴

$$\Pi(L_d) = L_d^\alpha - wL_d ; \quad \alpha \in (0,1) \quad (3)$$

By (3) it is known that the first and second derivatives of this function are given by:

$$\frac{\partial \Pi(L_d)}{\partial L_d} = \alpha L_d^{\alpha-1} - w \stackrel{?}{=} 0 \quad (3')$$

and

$$\frac{\partial^2 \Pi(L_d)}{\partial L_d^2} = -(1-\alpha)\alpha L_d^{\alpha-2} < 0 \quad (3'')$$

This means that it is a function that has an absolute maximum at the point where the first derivative is zero, which corresponds to the maximum mass of profits.

The rate of profit or internal rate of return of any technologically feasible production plan, denoted by π , is expressed as follows:

$$0 = -wL_d + \frac{L_d^\alpha}{1 + \pi(L_d)} \quad (4)$$

This means that:

$$\pi(L_d) = \frac{1}{wL_d^{(1-\alpha)}} - 1 \quad (5)$$

Note in (3') that the condition of *maximum mass of profits* is reached at the point of the function (3) which verifies that the marginal productivity of labor equals the real wage:

$$\alpha L_d^{-(1-\alpha)} = w \quad (6)$$

Thus, labor demand turns out to be a function of increasing negative slope in w , which means that the higher is the real wage, the lower the employment level:

⁴The properties of the profit function have been sufficiently developed, as highlighted by Varian (1992, pp. 49-58), by Hotelling (1932), Hicks (1946) and Samuelson (1947).

$$L_d = (\alpha^{-1} w)^{-(1-\alpha)^{-1}} \quad (7)$$

With these elements it will be shown right away the research problem: the inefficiency of traditional calculation represented in (1).

- From the maximum mass of profits (3) as the initial situation, consumers will evaluate the results of each production unit employing, in a second scenario, only a fraction of the amount of labor initially employed, retaining however full employment condition on the aggregate, due to the entry of enough additional productive units, up to the point of exhaustion of productive resources available at current prices. This means that the number of production units grow up attracted by the higher profitability, demanding, all units as a whole, the same amount of labor than in the initial situation. The evaluation will then consist in comparing levels of internal rate of return (5), and profits mass (3), between the two situations.

- Let λ , $0 < \lambda < 1$, be a pure number that makes it possible to determine the level of employment that every productive unit achieves in the second scenario. Then, if the initial level of employment per productive unit that guarantees the maximum mass of profits is L_d , the employment level in the second situation for each production unit will be λL_d ; magnitude that corresponds to just a fraction of the employment level reached by each at the initial situation. The number of incoming production units that will enable in the new situation to keep the same employment level per unit initially reached, will be λ^{-1} , which means that the economy will be, in the new situation, more competitive than in the previous one, as there will be more production units, each of them more profitable and smaller than before, thus preserving full employment in the aggregate.

- The comparison between the mass of profits of the first situation and of the second is given by the following inequality, in whose left member is shown the mass of profits of each productive unit, which are now smaller than in the initial situation, multiplied by the total number of these, and in whose right side is exhibited the situation already shown in (3):

$$\frac{1}{\lambda} [(\lambda L_d)^\alpha - (\lambda L_d) w] > L_d^\alpha - w L_d; \quad (8)$$

which means that:

$$\left(\frac{1}{\lambda^{(1-\alpha)}} - 1 \right) L_d^\alpha > 0 \quad (9)$$

- Note that in both cases the total costs are calculated at prices determined by the profit-maximizing plan, which means adopting the assumption that the new size of the industry would not change prices. This means that any position on the left of the maximum mass of profits for any of the n firms – and therefore also for the representative one – will be of higher internal rate of return, greater mass of profits and a larger number of production units, at current prices, that remain unchanged. This means that maximizing the mass of profits not really imply that there is a maximum mass, as usually assumed. There will be more efficient situations or scenarios with greater mass of profits and higher internal rate of return derived from the employment of a single, unchanging volume of resources, as shown in the following inequality:

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$$\frac{(\lambda L_{d1})^\alpha}{w(\lambda L_{d1})} - 1 > \frac{L_{d1}^\alpha}{w L_{d1}} - 1 \quad (10)$$

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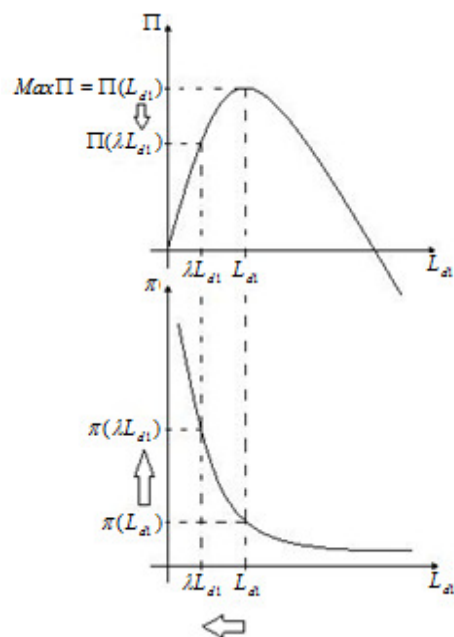
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- The following figure displays, for a single firm, the superiority of the internal rate of return (lower graph), in a situation in which each production unit uses only part of labor used in the initial situation, which is that in which maximizes the mass of profit:



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Figure 2

Mass of profits versus internal rate of return

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- Since each production unit that uses only a fraction of the labor available to the current real wage will reveal a higher rate of return than that which corresponds to the maximum mass of profit, and so a higher average product,⁵ once all labor is employed in productive units of smaller size and higher average productivity, the number of production units in the economy will increase, and thus the competitiveness; the aggregate volume of product and the mass of profits for the economy as a whole will be higher, even though the product and the profits generated by each unit will be lower.

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- The increase in the total amount of profits will result from the growth in the volume of output of the whole economy, due to the higher average labor productivity in each production unit, with the use of the same volume of social working effort than in the situation initial. This is shown in the following expression, wherein the left side the production function is multiplied by the inverse of the fraction of labor employed by each unit:

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$$\frac{1}{\lambda} L_d^\alpha > L_d^\alpha \quad (11)$$

⁵As shown in the graph following the arrows.

- This demonstrates, in a simple scenario with invariable prices, that the calculation the traditional theory attributed to competitive firms is technically inefficient: with any internal rate of return higher than the initial one and the same social working effort determined by the prices stated by the neoclassical theory, it is possible to produce more, which in turn means higher financing levels for consumers and therefore better situations in the sense of Pareto. It thus appears that the neoclassical theory explains the operation of a market economy in which firms operate inefficiently even though they could overcome their own results, that is acting irrationally, and they do so in a less competitive economy than that achieved under free entry and exit.

However, there remains the question: Does the outcome change if prices are modified for each of the situations compared, since the size of the industry and therefore the conditions for labor payment are different in each case?

To answer this question it is necessary to remove the condition that prices governing the comparison are those determined in the maximization of profit and allow price variation for the production units. So, each unit of those using lower amounts of labor and defining their production plans in the frontier points of technical possibilities in which the marginal productivity of labor is higher than in the profit-maximizing plan, will remunerate labor hired with a real wage w^* such that $w^* > w$. Formally, the real wage paid by each production unit or firm employing λL_{d1} of labor, is given by:

$$\alpha \lambda^{-(1-\alpha)} L_d^{-(1-\alpha)} = w^* \quad (12)$$

Calculating the total cost of production of all firms in this situation, and consequently redefining the inequality (8), we obtain:

$$\frac{1}{\lambda} [(\lambda L_{d1})^\alpha - (\lambda L_{d1}) w^*] > L_{d1}^\alpha - w L_{d1} \quad (13)$$

And replacing (6) and (12) into (13), it follows that:

$$\frac{1}{\lambda^{(1-\alpha)}} (1-\alpha) L_{d1}^\alpha > (1-\alpha) L_{d1}^\alpha \quad (14)$$

This proves once again, now with differentiated prices, that both the product and the total profits, as well as the number of production units will increase if consumers, as firms owners, seek higher rates of return than that corresponding to the profit maximization plan, using the same volume of resources than in that plan. However, the rate of return is now equal to that resulting from the maximization of the profits mass, equality that has been reached thanks to the free entry of new production units.⁶ This turns such a plan into an inefficient situation as long as

⁶ Its expression is: $(1-\alpha)\alpha^{-1}$, as can be seen by dividing each member (14) between its total costs. This is the technical result achieved by A. Pigeon for this analytical scenario and for the one referred to n inputs and one

the system allows free entry and exit. The consequence of this demonstration is that consumers committed ultimately with the efficient operation of firms, will not accept the mass function for profits as the objective to pursue on their part.

4. A SCENARIO OF n INPUTS AND ONE PRODUCT

Be a competitive economy composed of a large number m of firms, each of which uses non-negative amounts of the n existing productive inputs, $n-1$ of which are produced by the production system itself, and an n -th—labor—assumed homogeneous and perfectly divisible, not produced by firms, exclusively offered by consumers and indispensable into all production processes.

The production functions of all firms are homogeneous of positive degree greater than one and less than zero, i.e. decreasing returns to scale, ensuring that at competitive prices all reveal positive benefits. Thus, the maximizing behavior of the k -th firm, $k=1,2, \dots, m-1, m$, corresponds to the following expression:

$$\text{Max } \Pi_k = f(L_{1k}, L_{2k}, \dots, L_{n-1k}, L_{nk}) - \sum_j w_j L_{jk} \quad (15)$$

The degree of homogeneity of $f(\cdot)$ is given by $\sum_{j=1}^n \alpha_j \in (0,1)$, which is the sum of the elasticities of n inputs, and L_{jk} the amount of the j -th input used by the k -th firm.

Due to Euler's theorem for homogeneous functions it is known that competitive prices for any technique besides the maximizing one will be given by:

$$w_j = \alpha_j \frac{f(L_{1k}, L_{2k}, \dots, L_{n-1k}, L_{nk})}{L_{jk}}, \quad (16)$$

Full payment of the j -th factor will therefore be:

$$w_j L_{jk} = \alpha_j f(L_{1k}, L_{2k}, \dots, L_{n-1k}, L_{nk}) \quad (17)$$

Then substituting (17) into (15) we obtain:

$$\Pi_k = f(L_{1k}, L_{2k}, \dots, L_{n-1k}, L_{nk}) (1 - \sum_j \alpha_j), \quad (18)$$

which means that the profits-product ratio is a constant given by:

$$\frac{\Pi_k}{f(L_{1k}, L_{2k}, \dots, L_{n-1k}, L_{nk})} = (1 - \sum_j \alpha_j) \quad (19)$$

output. The implication of this is that any alternative plan more efficient than the original, being the latter inherent to profits maximizing firms, will reach the same internal rate of return of it but with higher levels of output, profits and factor payments.

This means that profits are a fixed proportion of the product, whatever the technique used. Since profit rate has been defined as the share of profits amount over total costs, according to (19) we have that its expression is given by:

$$\frac{\Pi_k}{\sum_j \alpha_j f(L_{1k}, L_{2k}, \dots, L_{n-1k}, L_{nk})} = (1 - \sum_j \alpha_j) \left(\sum_j \alpha_j \right)^{-1} \quad (20)$$

This means that the rate of profit also happens to be a constant, regardless of the technique chosen by the producer, as long as inputs are paid at competitive prices.

Suppose now, on the same grounds of the previous section referred to consumers into their role as owners of firms, that they decide to compare the results that would be obtained if instead of using the profit-maximizing technique, used any other in which will be employed only a fraction of inputs than in the aforementioned, being $\lambda, 1 > \lambda > 0$, this fraction. Then, analogous to the expression (15) is:

$$\Pi_{\lambda k} = f(\lambda L_{1k}, \lambda L_{2k}, \dots, \lambda L_{n-1k}, \lambda L_{nk}) - \sum_j w_j \lambda L_{jk}, \quad (21)$$

finally resulting into:

$$\Pi_{\lambda k} = \lambda^{\sum_j \alpha_j} f(L_{1k}, L_{2k}, \dots, L_{n-1k}, L_{nk}) \left(\lambda^{\sum_j \alpha_j} - \lambda \sum_j \alpha_j \right) \quad (22)$$

That is, analogously to (19), the mass of profits as a proportion of the product is given by the following expression:

$$\frac{\Pi_{\lambda k}}{\lambda^{\sum_j \alpha_j} f(L_{1k}, L_{2k}, \dots, L_{n-1k}, L_{nk})} = \left(1 - \lambda^{1 - \sum_j \alpha_j} \sum_j \alpha_j \right) \quad (23)$$

Meanwhile, the rate of profit is given by:

$$\frac{\Pi_{\lambda k}}{\lambda \sum_j \alpha_j f(L_{1k}, L_{2k}, \dots, L_{n-1k}, L_{nk})} = \left(\frac{1}{\lambda^{1 - \sum_j \alpha_j} \sum_j \alpha_j} - 1 \right) \quad (24)$$

Comparing (19) with (23) and (20) with (24), we find that:

$$\left(1 - \lambda^{1 - \sum_j \alpha_j} \sum_j \alpha_j \right) > (1 - \sum_j \alpha_j), \text{ and } \left(\lambda^{1 - \sum_j \alpha_j} \sum_j \alpha_j \right)^{-1} - 1 > \left(1 - \sum_j \alpha_j \right) \left(\sum_j \alpha_j \right)^{-1} \quad (25)$$

This means that the volume of profits will be lower for the individual firm and the profit rate greater than when maximizing the profit function (15). Therefore, under free entry and exit, the higher profitability in terms of internal rate of return will attract new production units to the industry, at least to the extent that employs input volumes determined by profit-maximizing technique. The number of firms or production units entering through profitability effect caused by downscaling the k -th firm, to use the same amount of inputs determined by its profit maximizing plan will equal to λ^{-1} . So then, the magnitudes of the mass of and the rate of profit, respectively, at tiered prices depending on the size of the production units and the industry as a whole, will be:

$$\frac{1}{\lambda} \Pi_{\lambda k} = \lambda^{-1 + \sum_j \alpha_j} \left(1 - \sum_j \alpha_j \right) f(L_{1k}, L_{2k}, \dots, L_{n-1k}, L_{nk}) \quad (26)$$

and

$$\frac{\Pi_{\lambda k}}{\lambda^{\sum_j \alpha_j} \sum_j \alpha_j f(L_{1k}, L_{2k}, \dots, L_{n-1k}, L_{nk})} = \left(1 - \sum_j \alpha_j \right) \left(\sum_j \alpha_j \right)^{-1} \quad (27)$$

Note that the rate of return (27) of the most profitable options has dropped gradually to equal to that corresponding to the initial situation (that is, once it has been used the same amount of resources than when the profit mass function was maximized by firms), only this time with superior results in terms of output, profits, wages and industry size, due to the entry of additional production units.

It is said that the prices are differentiated, indicating that payments to the factors once the scale of production of each company has declined and that the size of the industry has increased, corresponds to the following expression:

$$w_j^* = \alpha_j \frac{f(\lambda L_{1k}, \lambda L_{2k}, \dots, \lambda L_{n-1k}, \lambda L_{nk})}{\lambda L_{jk}} \quad (28)$$

In turn, production costs of the industry, once it has been used the same volume of inputs than in the situation of profit maximization, are given by:

$$\sum_j w_j^* L_{jk} = \lambda^{-1 + \sum_j \alpha_j} \sum_j \alpha_j f(\lambda L_{1k}, \lambda L_{2k}, \dots, \lambda L_{n-1k}, \lambda L_{nk}) \quad (29)$$

These costs are already present in (26) and (27), expressions in which it is found that:

$$\frac{1}{\lambda^{1 - \sum_j \alpha_j}} > 1 \quad (30)$$

What suffices to show outcomes that will exceed those corresponding to the profit-maximizing plan, using exactly the same amount of input that in it, which involves inefficiency of such a plan.

For this demonstration, the case concerning prices that are invariable and determined by the maximization (15), now becomes trivial, since subtracting the same total costs to different quantities of product, will not change the sense of the difference between such amounts.

5. THE FIRST THEOREM OF WELFARE

While these demonstrations are relevant to the theory by the fact that they pose a problem of inefficiency about the economic behavior that the neoclassical tradition has attributed to firms, their deepest result is reached by the injury inflicted on the essential quality of all competitive equilibrium: the Pareto optimality. This quality, which is present in the first welfare theorem, in the analytical prose of Villar (1996),⁷ is expressed in the following propositions:

“Proposition 7.1 Let $\mathbf{Y} \equiv \sum_{j=1}^n \mathbf{Y}_j$ be the total production set of the economy, and let $\mathbf{p}^*$ be a price vector for which all supply correspondences are defined. Then $\mathbf{y}_j^* \in \eta_j(\mathbf{p}^*) \forall j$ if and only if

$$\mathbf{p}^* \mathbf{y}^* \geq \mathbf{p}^* \mathbf{y}, \forall \mathbf{y} \in \mathbf{Y}.$$

Proposition 7.2 Let i be a consumer with a locally non-satiated utility function, defined over the consumption set X_i . Then, if for some price-wealth pair $(\mathbf{p}^*, w_i^*)$, $\mathbf{x}_i^*$ maximizes u_i in $\beta_i(\mathbf{p}^*, w_i^*)$, then it will verify that:

$$(a) \quad \mathbf{p}^* \mathbf{x}_i^* = w_i^*$$

$$(b) \quad \text{For all } \mathbf{x}_i' \in X_i, u_i(\mathbf{x}_i') \geq u_i(\mathbf{x}_i^*) \Rightarrow \mathbf{p}^* \mathbf{x}_i' \geq \mathbf{p}^* \mathbf{x}_i^*.$$

$$\mathbf{x}_i' \in X_i, u_i(\mathbf{x}_i') > u_i(\mathbf{x}_i^*) \Rightarrow \mathbf{p}^* \mathbf{x}_i' > \mathbf{p}^* \mathbf{x}_i^*.$$

Theorem 7.1 Let $\mathbf{E}_{pp}$ be a private property economy in which each consumer has a utility function that satisfies the assumption of being locally non-satiated. If $(\mathbf{p}^*, [\mathbf{x}_i^*], (\mathbf{y}_i^*))$ is an equilibrium of this economy, then the allocation $[(\mathbf{x}_i^*), (\mathbf{y}_i^*)]$ is efficient in the sense of Pareto.”

After the necessary changes to make compatible this notation with that used in the analysis of the preceding paragraphs, by the statement of the theorem it is known that if there is a production plan $Q_{\lambda k}^* = \frac{1}{\lambda} f(\lambda \mathbf{L}^*)$, $\lambda, \lambda \in (0,1)$ with which it is achieved a higher volume of product than with another $Q_k^* = f(\mathbf{L}^*)$, employing for any of both cases a single and unique volume of inputs $\mathbf{L}^* = (L_{1k}, L_{2k}, \dots, L_{n-1k}, L_{nk})$, then the consumer incomes will be higher and also will their utility level, because the allocation previously considered is efficient in the sense of Pareto will be overcome. This will put the plan referred to $\mathbf{L}^*$ in the definition of Pareto inefficiency, and thus also the origin of the decision of the plan, which is none other than profit maximization under (15). The utility functions considered in the proofs of inefficiency, which correspond to (2), do fully satisfy the condition of local insatiability.

To generalize inefficiency demonstrations performed in the previous sections, we propose the following section theorem.

6. THEOREM OF INEFFICIENCY

⁷Reproduced verbatim from Villar (1996: Chapter 7, p. 150), and here translated by the author of this paper.

It is known that a function of n variables $f(L_1, L_2, \dots, L_{n-1}, L_n)$ defined on a domain $\tilde{\mathbf{L}}$, a convex subset of $\mathfrak{R}^n$, such that $\tilde{\mathbf{L}} = \mathfrak{R}_{0,+}^n$, $\tilde{\mathbf{L}} = \{\mathbf{L} \in \mathfrak{R}^n : \mathbf{L} \geq 0\}$, is strictly concave if, given a pure number $\lambda, \lambda \in (0,1)$, and any vectors $\mathbf{L}^*$ and $\mathbf{L}$, belonging to $\tilde{\mathbf{L}}$, $\mathbf{L}^* \neq \mathbf{L}$, it is verified that:

$$f(\lambda \mathbf{L}^* + (1-\lambda)\mathbf{L}) > \lambda f(\mathbf{L}^*) + (1-\lambda)f(\mathbf{L}) \quad (31)$$

Let Π_k be the profit function of the k -th firm of a competitive economy of private property, and $\mathbf{L}^*, \mathbf{L}^* > 0$, the input vector that maximizes this function to the $\mathbf{w}^*$ prices.

$$\Pi_k^* = f_k(\mathbf{L}^*) - \mathbf{w}^* \mathbf{L}^* \quad (32)$$

The price vector $\mathbf{w}^*$, $\mathbf{w}^* > 0$, consist of marginal productivities of inputs, and function $f_k : \tilde{\mathbf{L}} \rightarrow \mathfrak{R}_{0,+}$ is strictly concave and homogeneous of degree $\mu_k, \mu_k \in (0,1)$ in its arguments.

The consumers utility functions are quasi-concave and satisfy the condition of local insatiability, and budget constraints depend, by the revenue side, of their property rights on firms, same that determine a positive definite and stable relation with the benefits.

Under these conditions it is shown the following:

Proposition: In a system of free entry and production functions strictly concave and homogeneous of degree $\mu_k, \mu_k \in (0,1)$, wherein the k -th firm maximizes its profit function Π_k , with the vector of inputs $\mathbf{L}^*, \mathbf{L}^* > 0$, to the prices $\mathbf{w}^*, \mathbf{w}^* > 0$, with $w_j^* = f_{jk}'$, there is at least one alternative plan referred to $\lambda \mathbf{L}^*, \lambda \in (0,1)$, more profitable than the inherent to $\mathbf{L}^*$, such that with a sufficient number of production units as for employing total inputs $\mathbf{L}^*$, it generates more product than $f_k(\mathbf{L}^*)$, a higher volume of profits than Π_k^* , and a more competitive size of the industry, thus implying the inefficiency of Π_k function and the violation of the first theorem of welfare.

Theorem: By (31) we know that:

$$f_k(\lambda \mathbf{L}^* + (1-\lambda)\mathbf{L}) > \lambda f_k(\mathbf{L}^*) + (1-\lambda)f_k(\mathbf{L}) \quad (33)$$

- Let $\mathbf{L} = 0$, so that inaction is a possibility. Then:

$$f_k(\lambda \mathbf{L}^*) > \lambda f_k(\mathbf{L}^*), \quad (34)$$

which implies that:

$$\lambda^{-1} f_k(\lambda \mathbf{L}^*) > f_k(\mathbf{L}^*), \quad (35)$$

$$\text{since: } \lambda^{\mu_k - 1} > 1, \quad (36)$$

with which it is shown that with a number of productive units given by $\lambda^{-1} > 1$ for each profit-maximizing production units similar to the k -th firm, will be generated a volume of product $\lambda^{\mu_k - 1} - 1 > 0$ times greater, employing for it the same volume of inputs used in (32).

- Total costs for $f_k(\mathbf{L}^*)$ and for $\lambda^{-1} f_k(\lambda \mathbf{L}^*)$ are given by $\mu_k f_k(\mathbf{L}^*)$ and $\lambda^{\mu_k - 1} \mu_k f_k(\mathbf{L}^*)$, respectively, which implies that:

$$\lambda^{-1} f_k(\lambda \mathbf{L}^*) - \lambda^{\mu_k - 1} \mu_k f_k(\mathbf{L}^*) > f_k(\mathbf{L}^*) - \mu_k f_k(\mathbf{L}^*) \quad (37)$$

- Whence it follows that $\lambda^{-1} \Pi_{\lambda k} > \Pi_k^*$, due to $\lambda^{\mu_k - 1} (1 - \mu_k) > 1 - \mu_k$ and to $\lambda^{\mu_k - 1} (1 - \mu_k) \mu_k^{-1} > (1 - \mu_k) \mu_k^{-1}$, being $(1 - \mu_k) \mu_k^{-1}$ the profit rate which is the same for both cases, it is demonstrated that both the mass of profits as the factor payments will be higher in the alternative plan.

- So, being the alternative plan more competitive in terms of industry size, and of higher volume of product and of factor payments than the one referred to profit maximization, the proposition is fully demonstrated.

7. RETHINKING THE FUNDAMENTALS

The analytical error that has been revealed concerning the economic behavior of firms or producers: a market economy poorly explained in one of its foundations, involves the inability to effectively predict and control phenomena. However, from the very demonstration of inefficiency, emerge axiomatic resources to rebuild the foundations.

Profit rate maximization

The rate of profit or internal rate of return, which was used in previous paragraphs as a standard of comparison, has shown in the proposed analytical scenarios, be sufficient to identify a set of more efficient situations than those resulting from the profits function maximization. However, maximizing this function subject to standard production functions of the nature of those used in the previous sections, gives results of almost no interest for the theory. Therefore, it is necessary to rethink the notion of production functions as representative of the technological possibilities of a system.

In order to establish alternative bases, it will be convenient to place again the analysis in a simple scenario: a non-durable and non-cumulative product, and a single production factor: labor.

Since maximizing the profit rate is equivalent to maximizing the average product, it is required a production function that makes it possible. That function will be:

$$q_s = f(L_d - L^*), \quad (38)$$

defined for all $(L_d - L^*) > 0$.

In it, L^* corresponds to the flexible component of technology –i.e. the amount of labor employed for the organization of all production process–, which is determined by market size. It is not a rigidity nor corresponds to increasing returns, as will become apparent later.

Then, the firm's behavior is given by:

$$Max (1 + \pi) = \frac{q_s}{wL_d} \quad (39)$$

$$s.t. q_s = (L_d - L^*)^\alpha$$

The first order conditions are:

$$\alpha(L_d - L^*)^{\alpha-1} = \frac{(L_d - L^*)^\alpha}{L_d} \quad (40)$$

$$q_s = (L_d - L^*)^\alpha$$

The maximum profit rate is achieved at the point of the production function in which labor elasticity of product is equal to one, and comes to be a situation independent of prices and wages. Employment levels are now determined by the size of the market, not by the real wage:

$$L_d = (1 + \alpha)^{-1} L^* \quad (41)$$

The condition of financial viability of the representative firm is:

$$\frac{q_s}{L_d} > w > 0 \quad (42)$$

This indicates that if wage is zero, the economy does not operate: the production level becomes zero and market activity, nonexistent; and if the real wage equals the average product, the profits will be zero and the firm may or may not operate. So, it turns out that the condition of existence of economic activity is a positive real wage, and the condition of financial viability for firms is that real wage also comes to be lower than the average product.

Under these circumstances the maximizing problem of the representative consumer will be:

$$\begin{aligned} &Max U(q_d, S), \\ &S = \tau - L_s, \quad (43) \\ &s.t. (1 + \pi)wL_s = q_d \end{aligned}$$

from which result the following product-demand and labor-supply functions, respectively:⁸

$$\begin{aligned} q_d &= \varphi(1 + \pi)w\tau \\ L_s &= \varphi\tau \end{aligned} \quad (44)$$

Once that the maximization problem of producers and consumers has been solved, the general equilibrium conditions will be as follows:

⁸ As before, the parameter τ , $\tau > 0$, refers to the maximum time biologically available to work; a natural initial endowment of any consumer in the system. Meanwhile, S represents leisure time demanded by the consumer.

$$\begin{aligned} L_d - L_s &\leq 0 \\ q_d - q_s &= 0 \\ (L_d - L_s)w + (q_d - q_s) &= 0 \end{aligned} \quad (45)$$

So labor demand for the aggregate takes the form:

$$L_d = \alpha^{-1} [\varphi(1 + \pi)w\tau]^\frac{1}{\alpha}, \quad (46)$$

that comes to be a positive and increasing function of real wage, in deep contrast with the neoclassical expression (6), which is defined as a negative and increasing function of that same variable. According to (46), to a higher real wage will correspond a higher employment level.

The contrast between their results and those typical of the neoclassical tradition is evident, and it is a problem that must be solved in the same way that has been raised in the inefficiency theorem: What technical reason exists to believe that the producers, if they could choose, they would choose to maximize the internal rate of return rather than the mass of profits?

Theorem of superiority

While the inefficiency theorem has already provided important elements to answer this question, it is essential to answer it in its own analytical context. To that effect, it will immediately be demonstrated the following proposition:

Proposition: *If in a competitive system at least one firm maximizes rate of profit rather than the volume of profits, whatever the price vector, it will get the higher possible mass of profits and a Pareto superior situation for all consumers with respect to that would be achieved by maximizing the mass of profits function.*

Demonstration:

Lemma 1: *The consumer incomes, if firms maximize the rate of return would be higher than they would achieve by maximizing the difference between revenues and costs.*

This lemma is demonstrated in account of (2), and due to (37), which means that any rate of profit than that corresponding to the maximization of the mass-of-profits function, will be associated with a higher volume of revenue for consumers.

Lemma 2: *The product and the profits achieved by any firm through maximizing the profit rate are superior to those that would achieve in maximizing the difference between revenues and costs.*

This lemma is demonstrated by the equations (33) to (37), because in them exists a non-produced factor (labor) that under hypothesis (38) enables production functions to allow the maximization of profit rate functions, also making possible to show that from all profit rates considered by the inefficiency theorem, one of them, the lowest, corresponds to the maximization of the mass of profits.

Theorem: *Let $U = u[q_d, (\tau - L_s)]$ (47)*

be the utility function, strictly concave and differentiable, of any consumer in the system. Then, since the system is at full employment: $L_d = L_s = \tilde{L}$, and since the purchasing power of consumers, as well as the volume of product generated by the economy as a whole, is greater when at least one firm maximizes the rate of profit rather than maximize the difference between revenues and costs, the consumer utility is also higher.

The theorem of superiority then turns out to be a logical implication of the theorem of inefficiency, with the difference that the latter does not require the reframing of analytical foundations, which derive in a theory of nonexistence of a labor market, as it has been shown in (41).

It is understood that the inefficiency theorem shows the inconsistency between the first welfare theorem and the neoclassical theory of production in a competitive system. For its part, the superiority theorem proves the existence of an analytically superior theory of production than the neoclassical. Thus, any application of the theorem of superiority will validate the results of the inefficiency theorem. It will be now provided an application of it in a dynamic discrete-time scenario. It is a Diamond-Samuelson overlapping generations model developed with strictly concave production functions.⁹

8. APPLICATION IN A DYNAMIC SCENARIO

Be defined a system consisting of two generations of agents: the young, incorporated into the system at time t , and the old, who entered the same in period $t-1$ and who will accomplish their last consumption plans in t and will be definitely out of the system at $t+1$. Each of these generations maximizes a separable-over-time utility function, which depends on their current consumption and their expected future consumption, under perfect foresight. The young evaluate their utility in present value, discounting the expected utility of future consumption at a subjective and strictly positive rate θ .¹⁰

The coexistence of young and old simultaneously implies that both sides share the product through an allocation system. It is assumed that this is a competitive, privately owned and fully decentralized system, in which the young are the only ones that possess the ability to work and the old ones the ownership of capital. However, it is also assumed that both participate in the ownership of firms through a participation rate of each of them in the profits resulting from production; a rate that is fixed ex ante through an agreement respected by both generations (i.e. through an institution). Participation rates, both belonging to nonnegative real numbers, are: ρ_w for the young and ρ_k for the old ones, such that $\rho_w + \rho_k = 1$. The subscript w refers to young people, whose characteristic in terms of revenue is that they earn wages added to their participation in profits, and the subscript k corresponds to the old ones, who, being owners of capital, perceive the rent that it generates, also added to its profit sharing. The young assign their income to their current

⁹The model described next is a refinement of that published in Noriega (2012: p.43-55).

¹⁰A more detailed treatment of the initial conditions, the accounting system and the basic results of overlapping generations models, initially developed by Allais (1947), Samuelson (1958) and Diamond (1965), are found in McCandless and Wallace (1991), although the formalization and variety of analytical scenarios offered in Chapter 3 of Blanchard and Fischer (1989), and far more profound and extensive in Bewley (2007), make it possible to show results of multiple equilibria and different institutional conditions linking the analysis with specific frameworks for macroeconomic policy assessment. A didactic and renewed exposition of this type of models is also found in Bénassy (2011).

consumption q_{c1t} as well as to their savings A_t , and expect to pay their future consumption q_{c2t+1} , with the returns provided by their savings at the prevailing rate of interest in $t+1$. The old ones, meanwhile, assign their total income to their consumption in t : q_{c2t} .

The economy generates in each period just one product, from which derive both consumption and savings, savings that will reenter into firms as capital for next period's production.

Each young consumer provides inelastically one unit of labor, and the sum of labor supply of all young consumers equals the employment level and the entire population of young people in the system, i.e. L_t . In order to simplify the model for the purposes of this application, it is assumed a zero population growth rate.

The firms, according to neoclassical theory, maximize the mass of profits, i.e. the difference between the total output and wages and capital costs, in contrast, according to the TNLM which has resulted in the previous sections, they maximize the rate of profit, which equals one plus the average product. The results of the maximization of one or another objective function differ substantially, as has become apparent in the previous section. In both cases, the maximization of firms is subject to a production function that, for purposes of comparing the two systems, should enable us to maximize the profit rate as well as the mass of profits, with economically significant results, i.e. with positive product and financial viability. This will be possible thanks again to the organizational costs L_t^* .

To begin, at first will be displayed the consumers' maximization, then, clearly marking the contrasts, will be analyzed the accounting differences between a system in which producers maximize the mass of profits, again denoted by the subscript Π , and another, in which firms maximize the profit rate, indicated by π . Later, based on accounting contrasts, we will proceed to analyze the differences in the results of firm's maximization.

Consumer's maximization

-Let:

$$\text{Max } u(q_{c1t}) + (1 + \theta)^{-1} u(q_{c2t+1}) \quad (48)$$

s.t.

$$q_{c1t} + A_t = w_t L_t + \rho_w \Pi_t \quad (49) \quad ,$$

$$q_{c2t+1} = (1 + r_{t+1}) A_t + \rho_k \Pi_{t+1} \quad (50)$$

be the aggregate calculation of consumers under the assumption of identity among all of them. On the right side of (49) are displayed total wage income and total profit earned by the young, and on the right side of (50) are exhibited savings and their expected return, added to the expected benefits of the young in their old age concerning the next period. The real interest rate for the next period is given by r_{t+1} .

- The intertemporal marginal rate of substitution is given by:

$$\frac{u'(q_{c1t})}{u'(q_{c2t+1})} = \frac{1 + r_{t+1}}{1 + \theta} \quad (51)$$

-Under the assumption that the time preference is the same for each individual during his youth as well as in old age, for example β such that $1 > \beta > 0$, that will come to be the consumption-elasticity of utility in each period according to (48), the expression (51) becomes equivalent to:

$$\left(\frac{q_{c1t}}{q_{c2t+1}} \right)^{1-\beta} = \frac{1+r_{t+1}}{1+\theta} \quad (52)$$

-Consumption and saving functions, respectively, are shown as follows:

$$q_{c1t} = \frac{(1+\theta)^{\frac{1}{1-\beta}}}{(1+r_{t+1})^{\frac{\beta}{1-\beta}}} A_t + \left(\frac{1+\theta}{1+r_{t+1}} \right)^{\frac{1}{1-\beta}} \rho_k \Pi_{t+1} \quad (53)$$

$$A_t = \left[\frac{(1+\theta)^{\frac{1}{1-\beta}}}{(1+r_{t+1})^{\frac{\beta}{1-\beta}}} + 1 \right]^{-1} \left[w_t T_t + \rho_w \Pi_t - \left(\frac{1+\theta}{1+r_{t+1}} \right)^{\frac{1}{1-\beta}} \rho_k \Pi_{t+1} \right] \quad (54)$$

-As it can be seen, the saving function has positive first derivative in wages, in benefits of young people and in the interest rate, and negative in the profits of the old ones. The second derivative is zero with respect to wages and to profits, and indefinite regarding the interest rate since the exact value of β is not determined.

- Functions (53) and (54) remain unchanged in its parametric structure whatever the objective function that producers decide to maximize.

- The effects of firms maximization will be felt in (53) and (54) through the benefits Π_t and Π_{t+1} . We will proceed under the assumption that wages and interest rates are determined by the marginal productivity of labor and capital, respectively, and so fully embracing the price vector corresponding to a system in which producers maximize profits volume.

Firm's maximization regarding neoclassical theory

These agents, under the traditional approach, maximize the total profit function Π_t , subject to a production function that, emulating Solow (1956), assumes constant returns to scale, and decreasing returns to factor, to represent the long term through the nullity of profits. That assumption, as usually invoked in dynamic models, undoubtedly represents a major simplification but significantly weakens the results. First, it is exhibiting a situation in which the number of firms has reached its maximum, and the nullity of the benefits implies that there is no reason why more firms will be attracted by the system, so in order to annul technically benefits, the methodological assumption of constant returns to scale is used. Second, it points to a long-term outlook in which the number of firms is a finite positive number known since before every process, thus eliminating the problem of indeterminacy in the number and size of firms in the long run, characteristic of neoclassical theory under non-increasing returns. Thus, the assumption of constant returns to scale eliminates unresolved problem analytically by neoclassical theory: the indeterminacy of the number of firms with positive size and sufficiently numerous to establish a competitive environment. Now instead, will propose a production function with organizational costs measured in terms of labor, in order to resolve the indeterminacy of long-term and also

allow so as to maximize the rate or the mass of profit. This will make it possible for producers to compare their results in both cases.¹¹

-Therefore, the calculation for the aggregate of these agents in the traditional approach, is given by:¹²

$$\text{Max } \Pi_t = q_{\Pi t} - w_t L_t - r_t K_{\Pi t} \quad (55)$$

s.t.

$$q_{\Pi t} = s_{\Pi} (L_t - L_t^*)^{\gamma} K_{\Pi t}^{\alpha} \quad (56)$$

$$\gamma, \alpha \in \mathfrak{R}^+; 1 > \alpha + \gamma > 0$$

-The parameter s_{Π} represents the number of production units in the system where the producers maximize the mass of profits. So, the first order conditions are:

$$\gamma s_{\Pi} (L_t - L_t^*)^{\gamma-1} K_{\Pi t}^{\alpha} = w_t \quad (57)$$

and $\alpha s_{\Pi} (L_t - L_t^*)^{\gamma} K_{\Pi t}^{\alpha-1} = r_t \quad (58)$

- As can be seen, these conditions do not differ from the standard.

Firm's maximization regarding TNLM

-According to the theory of nonexistence of the labor market, producers maximize the rate of profit, whose analytical expression for this scenario is:

$$\text{Max } (1 + \pi_t) = \frac{q_{\pi}}{w_t L_t + r_t K_{t\pi}} \quad (59)$$

s.t

$$q_{\pi} = s_{\pi} (L_t - L_t^*)^{\gamma} K_{t\pi}^{\alpha} \quad (60)$$

$$\gamma, \alpha \in \mathfrak{R}^+; 1 > \alpha + \gamma > 0$$

-The production function is identical for both cases. The first order conditions resulting from the maximization of profit rate function, are:

$$\gamma s_{\pi} (L_t - L_t^*)^{\gamma-1} K_{\pi}^{\alpha} = w_t \frac{s_{\pi} (L_t - L_t^*)^{\gamma} K_{\pi}^{\alpha}}{w_t L_t + r_t K_{\pi}} \quad (61)$$

and $\alpha s_{\pi} (L_t - L_t^*)^{\gamma} K_{\pi}^{\alpha-1} = r_t \frac{s_{\pi} (L_t - L_t^*)^{\gamma} K_{\pi}^{\alpha}}{w_t L_t + r_t K_{\pi}} \quad (62)$

¹¹ Maximizing mass of profits subject to function (38), does not change the first order conditions expected by neoclassical theory.

¹² Hereinafter, it will be made the distinction, by the subscript respective between the product or capital referred to the system in which firms maximize mass of profits and that in which maximize the rate of profit.

-The parameter s_π represents the number of production units that comprise the system corresponding to the firms that maximize profit rate, multiplied by the scale of production of each (which in this case is greater than zero and less than one). In this case, since the numerator of each of these expressions is superior to the denominator, the equilibrium conditions show that:

$$f'_{L\pi} > w_t \quad \text{y} \quad f'_{K\pi} > r_t \quad ; \quad (63)$$

i.e. the marginal productivity of each factor exceeds to its price in the production plan inherent to the maximum rate of profit for each productive unit; which does not occur in neoclassical theory, in which the marginal productivity of each factor necessarily equals to its price, at the point of production function where the producer decides to stand. This last is represented as follows:

$$f'_{L\Pi} = w_t \quad \text{y} \quad f'_{K\Pi} = r_t \quad (64)$$

-Note that, by dividing (61) by (62), we obtain the marginal rate of technical substitution:

$$\frac{\gamma}{\alpha} (L_t - L_t^*)^{-1} K_\pi = \frac{w_t}{r_t} \quad (65)$$

-When replaced it in (61) or (62), is reached the result that labor demand is independent of the interest rate and of the real wage, being this the basic result of the nonexistence of labor market theory, while demand of capital itself depends on those two variables:

$$L_t = \frac{1 - \alpha}{1 - \alpha - \gamma} L_t^* \quad (66)$$

$$K_\pi = \frac{\alpha}{1 - \alpha - \gamma} L_t^* \frac{w_t}{r_t} \quad (67)$$

Applying the theorem of superiority¹³

To compare the results of both systems, we will start from two conditions: first, that the competitive prices of neoclassical maximization will be the force in the two systems, and secondly, in both systems will be full employment and therefore the same, and the level of capital used at the time of the comparison will also be the same, although in future periods will differ because of the contrasts in the accumulation.

Financing possibilities

-Then let us compare the income-expenditure equations of the two systems:

$$q_\pi = (1 + \pi_t)(w_t L_t + r_t K_t) \quad (68)$$

$$q_{\Pi} = \Pi_{\Pi} + w_t L_t + r_t K_t \quad (69)$$

¹³ It is clear that the superiority theorem implies to the inefficiency theorem as well as the disability of the first theorem of welfare. So it shall be sufficient proving the first in order to validate also the theorem of inefficiency.

-Equating(68) and(69),it follows that themagnitudeofcapital whichsatisfies thesystemofequationsis given by:

$$K_t = \frac{\Pi_{\Pi t}}{\pi_t r_t} - \frac{w_t}{r_t} L_t \quad (70)$$

-Also, the amount of product that satisfies both equations is given by:

$$q_t = \Pi_{\Pi t} + \frac{\Pi_{\Pi t}}{\pi_t} \quad (71)$$

-Let ε be a positive real number as small as desired, then, leaving constant the amount of labor and prices, and adding that number to capital equation(70), and then replacing the result in(68) and(69), leads to:

$$q_{\pi} = \Pi_{\Pi t} + \frac{\Pi_{\Pi t}}{\pi_t} + (1 + \pi_t) r_t \varepsilon \quad (72)$$

and

$$q_{\Pi t} = \Pi_{\Pi t} + \frac{\Pi_{\Pi t}}{\pi_t} + r_t \varepsilon \quad (73)$$

-This implies that:

$$q_{\pi} > q_{\Pi t} \quad (74)$$

-That is, in the system which maximizes the rate of profit, it is possible to finance higher levels of consumption for the old and young of the current period, and greater savings for young people, above the system that maximizes the volume of profits. This is equivalent to **Lemma 1 of Theorem of Superiority**. There are levels of consumption and savings higher and possible, in the system which maximizes the rate of profit, to be made if the level of accumulation also happens to be higher than in the system in which is maximized the mass of profits.

Capital, product and profits

- By Euler's theorem, and admitting the possibility that the organizational costs are so small they tend to zero, will verify that:

$$(f'_{w\pi} \kappa L_t + f'_{k\pi} \kappa K_t) \cdot \kappa^{-1} = q_{\pi} (\gamma + \alpha) \quad (75)$$

and

$$f'_{w\Pi} L_t + f'_{k\Pi} K_t = q_{\Pi t} (\gamma + \alpha) \quad (76)$$

with $q_{\pi} > q_{\Pi t} \quad (77)$

being:

$$q_{\pi} = \frac{\kappa^{(\gamma+\alpha)}}{\kappa} q_{\Pi} \quad (77')$$

-This is because in both cases are used the same amount of capital and labor, and due to (63) and (64), assuming that $s_{\Pi} = 1$, and that in the system where is maximized π each production unit uses only a ratio κ , $1 > \kappa > 0$ of each of the factors. So, $s_{\pi} = \kappa^{(\gamma+\alpha)} \kappa^{-1}$, which means that the number of productive units multiplied by the scale of production $\kappa^{(\gamma+\alpha)}$, of each of them, is greater than in the system that maximizes the volume of profits. It means that there is a higher volume of product in the economy that maximizes profit rate, reflecting therein that the number of production units is also higher than in the other system ($\kappa^{-1} > 1$), as well as the scale of production. Consequently, after replacing in (77) expressions (56) and (60) specific to production functions, we arrive at the following inequality:

$$s_{\pi} > s_{\Pi} \quad (78)$$

-Since equation (75) and (76) correspond to the total costs that every economy covers, because both keep the condition of compensating factors according to their marginal productivity, the mass of profits in each one of them is given by the following expressions:¹⁴

$$\Pi_{\Pi} = (1 - \gamma - \alpha) q_{\Pi} \quad (79)$$

$$\text{and: } \Pi_{\pi} = \frac{\kappa^{(\gamma+\alpha)}}{\kappa} (1 - \gamma - \alpha) q_{\Pi} \quad (80)$$

which implies that:

$$\Pi_{\pi} > \Pi_{\Pi} \quad (81)$$

-However, if prices are the same for both economies, the total benefits resulting of profit rate maximization will be given by:

$$\Pi_{\pi} = \pi_t (w_t L_t + r_t K_t) \quad (82)$$

and profits of profit mass function maximization will be:

$$\Pi_{\Pi} = q_{\Pi} - (w_t L_t + r_t K_t) \quad (83)$$

by subtracting (83) of (82), is obtained the following:

¹⁴It has already been shown that that, even with different prices depending on the size of the production units and industry, these results are verified. To this effect, see equations (12), (13) and (14) of subtitle 3; (28), (29) and (30) of subtitle 4, and the statement of the theorem of inefficiency.

$$\Pi_{\pi} - \Pi_{\Pi_t} = (1 + \pi_t)w_t L_t + (1 + \pi_t)r_t K_t - q_{\Pi_t} \quad (84)$$

that due to (77) is known to be positive, and confirm (81).

Consumption and utility

-From equation (54) we have:

$$A_{\pi} = \left[\frac{(1 + \theta)^{\frac{1}{1-\beta}}}{(1 + r_{t+1})^{\frac{\beta}{1-\beta}}} + 1 \right]^{-1} \left[w_t L_t + \rho_w \Pi_{\pi} - \left(\frac{1 + \theta}{1 + r_{t+1}} \right)^{\frac{1}{1-\beta}} \rho_k \Pi_{\pi+1} \right] \quad (85)$$

-In view of (77') and (80), we have:

$$A_{\pi} > A_{\Pi_t} \quad (86)$$

-Substituting this in (53), we obtain the following:

$$q_{c1\pi} = \frac{(1 + \theta)^{\frac{1}{1-\beta}}}{(1 + r_{t+1})^{\frac{\beta}{1-\beta}}} A_{\pi} + \left(\frac{1 + \theta}{1 + r_{t+1}} \right)^{\frac{1}{1-\beta}} \rho_k \Pi_{\pi+1} \quad (87)$$

that implies:

$$q_{c1\pi} > q_{c1\Pi_t} \quad (88)$$

-Finally, since the consumption of the old ones in t is given by:

$$q_{c2\pi} = (1 + r_t)K_t + \rho_k \Pi_{\pi}, \quad (89)$$

which is obviously greater under profit rate maximization because of the superiority of capital and profits, and having been shown that:

$$A_{\pi} > A_{\Pi_t}, \quad (90)$$

and the expected consumption for $t+1$ by the young people in t , is also higher than would be expected by them from a maximization of profits.

-By (90), (87), (86) and (81), it is then clearly demonstrated that:

$$u(q_{c1\pi}) + (1 + \theta)^{-1} u(q_{c2\pi+1}) > u(q_{c1\Pi_t}) + (1 + \theta)^{-1} u(q_{c2\Pi_t+1}) \quad (91)$$

-This means that from the theorem of superiority becomes evident that to maximize profits mass involves dynamic inefficiency, not so maximizing profit rates.

9. CONCLUSIONS

The explanatory power of the theory is its main but not unique attribute: efficient explanations are a prerequisite to efficient predictions, and the latter, necessary to establish efficient criteria to control explained phenomena. Thus, the evidence of explanatory inefficiencies that has been studied in this research, in addition to having limited the scope explanatory, predictive and of governance or control of neoclassical theory, have exposed the need to build efficient explanations of the functioning of a market economy. To that end seeks to contribute the theory of nonexistence of the labor market, whose pillars are the criticism and reconstruction of the theory of producer.

The demonstration that the inefficiency of the theory of firm in neoclassical tradition violates the first welfare theorem, injures the norm that guides all axiomatic deductions of this logical system, i.e. the perfectly competitive equilibrium. It then imposes the need to replace that norm by any other descriptive notion provided of a robust theory to orient the sense that the criteria of economic policy should follow, for the sake of a more desirable economic order than the current. Apparently, this concept should be to rethinking the demonstrations of existence of a general competitive equilibrium, this time based on the correction of the analytical error of neoclassical theory.

The analytical implications of the application of the theorem to a dynamic context, are strong: First, it demonstrates the dynamic failure of neoclassical model because of its theory of firm. Capital accumulation and intertemporal consumption are Pareto suboptimal in its own logical framework. Secondly, the substitution of an inefficient producer by an efficient one, provides consistent and superior results in the sense of Pareto than those of the traditional theory. Moreover, the implications in the understanding of the functioning of a market economy differs significantly from the usual; now the labor market does not exist, the wage is not a price but a variable of distribution, and wages are not related in a regular and stable link with labor productivity in perfect competition. Despite diminishing returns, the profit rate may be zero and thus the mass of profits; the long term can be equally represented as zero profit situation or process of decreasing profit rate.

The bicentenary error alluded to in the title of this article refers to the principle that the capitalist economy is composed by markets purely and simply; i.e. a system of goods and prices. It follows that the major pathologies that the economy should explain, predict and control, inevitably arise as market phenomena, whether they concern to imperfectly competitive markets or markets constrained by rigidities. Thus, the labor market, one of all, is expected to operate according to the principles that govern the entire market, and to explain the problems of unemployment and wages as concerning to phenomena of disequilibrium. In the extreme case, typical of the new classical economics, the virtually instantaneous capacity of plans adjustment of the agents to their calculation errors due to incomplete or incorrect information, simply cancels the existence of such pathologies.¹⁵ And in any case, to ensure that the general competitive equilibrium takes place, is the ultimate goal, due to its attribute of social efficiency. That is, no more no less, the core that has been injured by the results exposed in here. Fortunately the critique itself provides elements for reconstruction.

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¹⁵ See Hahn & Solow (1995, p. 3-9), making a dramatic statement on the matter, these two remarkable authors of the neoclassical tradition show a historical divergence of conscience more than of method. However, it has been inexplicably disregarded.

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