Original Research Article

The Complementary Compound Truncated Poisson-Weibull Distribution for Pricing Catastrophic Bonds for Extreme Earthquakes

ABSTRACT:

Aims: This paper aims at bridging the gap or managing the level of overall losses between economic losses and insured losses that are usually caused by extreme earthquakes, by calculating the price of catastrophic bonds. While examining the frequency of the event, it was observed that, at least, one accident occurred periodically, which resulted in maximum losses.

Study Design: This study is an empirical research based on maximum losses that are due to earthquake events per year, as obtained from the International Disaster Database and Munich Re.

Place and Duration of Study: The study analyzed 80 extreme earthquake events in the world, between 1906 and 2015.

Methodology: The Complementary risk method used in calculating a mixed probability distribution expresses the number of earthquakes and the maximum losses realized. Zero truncated Poisson distribution is used for frequency distribution and Last order Weibull distribution for losses. The data were analyzed using IBM SPSS (Statistical Package for Social Sciences) Statistics 22 and MathCad 2001 professional software.

Results: It was discovered that the maximum losses of earthquake were fitted with compound truncated Poisson-Weibull distribution. Expected values have been calculated for extreme earthquake losses, which exceed the specific descriptive measures. The expected values of extreme losses caused by earthquakes are a net premium, present value or net price of the catastrophic bonds. It was observed that the present value of catastrophic bonds decreases as the retention increases. Risk management of natural disaster (by transferring the losses to the financial markets) is one of the derivative methods considered as an alternative or complement to traditional insurance. The process of transferring losses to capital markets through the catastrophic bonds (to cover natural disasters) leads to greater coverage of these losses, by maintaining that natural disaster losses are relative to the size of the capital markets, and are less than the ratio to the size of the insurance markets, in addition to the possibility of providing the necessary funds for reconstruction.

Keywords: CAT Bonds, Complementary Risk, Extreme Earthquakes, Pricing, Last Order Wiebull.

1. INTRODUCTION

A frequent occurrence of natural disasters poses a great risk to individuals and the state at large. The most important of these disasters are earthquakes, floods and storms. Sometimes, the potential capabilities of many countries and nations exceed the natural disaster losses. After the occurrence of the disaster, reconstruction begins; by way of compensating for the loss suffered by individuals. The government (which lost all or part of its infrastructure and the business of its institutions being interrupted), compensates people for their losses. Traditional insurance and reinsurance could not provide a full coverage of economic losses caused by natural disasters. Globally, a wide gap exists between the economic losses caused by earthquakes, and the losses covered by insurance. The importance of catastrophic bonds (CAT bonds) as one of the capital market tools to cover economic losses, is therefore illustrated here. Table 1. shows the events of losses of 10 costliest earthquakes, between 1980- 2015. Maximum Economic losses are US$ 250,000 million and insured losses are US$ 40,000 million, where the overall losses are US$ 210,000 million, without coverage by traditional insurance. This gap needs coverage (as CAT bonds) by securitization tools in the capital markets.[1]

Table 1. Comparison between overall losses and insured losses of earthquakes

Date	Overall losses In US$ m	Insured losses In US$ m
11.3.2011	210,000	40,000
17.1.1995	100,000	3,000
12.5.2008	85,000	300
17.1.1994	44,000	15,300
27.2.2010	30,000	8,000
23./24./27.10.2004	28,000	760
22.2.2011	24,000	16,500
20./29.5.2012	16,000	1,600
7.12.1988	14,000	Not available
21.9.1999	14,000	750

Source: Munich Re, NatCatSERVICE, 2016

In the nineties of the nineteenth century, while going back to the so-called reconstruction funds using CAT bonds, it was observed that CAT bonds guarantee return rates greater than the return rates for other types of bonds in the financial markets.

CAT bonds are the instruments that transform pure risks into speculative risks and allow the sponsor, for example, the insurer or reinsurer, to transfer risk to the capital markets. These insurances-linked securities provide the same protection as equity capital, but it is generated only if the triggering event occurs. They can provide a more efficient means of hedging some risks. [2]

Michel-kerjan, E., & Morlaye, F. (2008) discussed three types of Insurance-Linked Securities, ILS instruments provided by the capital markets: Industry Loss Warranties (ILW), CAT bonds and sidecars. The first two are similar to excess-of-loss reinsurance, while sidecars are more often, quota-share-like coverage and hence, are similar to proportional reinsurance. CAT bonds typically cover narrowly defined risks on an excess-of-loss basis. They are issued in the form of debt with high coupons.[3] The CAT bonds are considered in this case as prepaid payments, and are linked to the probability of one of the risks of natural disasters occurring in a specific area. It covers individual and institutions' losses in a manner different from the traditional insurance.

Traditional reinsurance operates efficiently in managing relatively small, uncorrelated risks and in facilitating efficient information sharing between cédants and reinsurers. However, when the magnitude of potential losses and the correlation of risks increase, the efficiency of the reinsurance model breaks down, and in turn, the cost of capital may become uneconomical. At this juncture, securitization has a role to play by passing the risks along to broader capital markets. Securitization also serves as a complement for reinsurance in other ways, such as facilitating regulatory arbitrage and collateralizing low-frequency risks.[4]

This paper aims to ensure that the price of the catastrophic bonds cover earthquake losses. A catastrophic bond is one of the most important securitization tools in the capital markets for risk management of natural disaster and it provides the necessary funding to cover the losses caused by the events' extreme earthquakes. It was observed from an examination of the frequency that at least, an accident occurred which resulted in maximum losses.

1.1 Research Problem

Globally, there is a gap or mismanagement in the level of overall losses (between economic losses and insured losses) caused by extreme earthquakes, which is causing higher losses compared to other earthquake losses. In most cases, a good number of countries cannot cover a lot of these losses.

1.2 Research Hypothesis

The best fit for total losses of extreme earthquakes will be the compound truncated Poisson-Weibull distribution.

2. LITERATURE REVIEW

The process of transferring losses to capital markets to cover natural disasters through the securitization process, leads to greater coverage of these losses: natural disaster losses relative to the size of the capital markets to the least of them, which is the size of the insurance markets. In addition to providing the necessary funds for reconstruction, it also entails the securitization of natural disaster system, requiring the participation of all parties to the beneficiaries or Stakeholders. The Stakeholders of CAT Bonds are: (i)

Home and business owners, who share in these bonds for compensation in the event of loss or decrease in their homes or companies. (ii) Insurers, providing these bonds to investors. (iii) Reinsurers, who protect insurers. (iv) The government, which regulates the relationship between the previous three Stakeholders.

Härdle et al. (2010) examines the calibration of a real parametric CAT bonds for earthquakes, sponsored by the Mexican government, which is of utmost interest, as it delivers several policy-relevant findings. The results demonstrate that a combination of reinsurance and CAT bond is optimal, in the sense that it provides coverage for a lower cost and lower exposure at default, than reinsurance itself. A hybrid CAT bond for earthquakes is also priced in order to reduce the basis and moral risk borne by the sponsor and to reflect the value of the loss by several variables.[5]

Although, the trend of insured losses and the trend of numbers of catastrophes are positive, reinsurance companies have to consider new ways of coping with the risk. One possibility is to transfer the risk from reinsurance markets to financial markets. Important financial instruments that are used for the transfer are catastrophe bonds. Due to an incomplete market for catastrophe risks and the lack of transparency in the CAT bond market, it is difficult to determine an accurate pricing model for the CAT bonds. For the same reason, the comparison of different CAT bond premium calculation models remains a challenging question. [6]

Hagendorff, B. et al (2013) examined changes in the market value of insurance and reinsurance firms which announce their engagement in insurance securitization by issuing catastrophe (CAT) bonds. They show that the wealth effects for shareholders in firms, which issue CAT bonds appear to be driven by explanations, according to which CAT bonds offer cost savings, relative to other forms of catastrophe risk management (and less by the potential of CAT bonds to hedge catastrophe risk).[7]

Jin-Ping, L., & Min-Teh, Y. (2002) computed default-free and default-risky CAT bond prices using the Monte Carlo method. The results showed that both moral hazard and risk decreased the bond prices substantially; these effects should not be ignored in pricing the CAT bonds.[8]

Michel-kerjan, E., & Morlaye, F. (2008) discussed some of the main drivers of the radical shift that occurred in the Insurance-Linked Securities (ILS) market, after the 2005 hurricane season in the Atlantic basin, which has rapidly become one of the world peak zones in terms of exposure. They introduced the concept of derivative solutions based on equity volatility dispersion.[3]

Bouriaux, S., & MacMinn, R. (2009) discussed the most recent developments in insurance securitization and assessed the potential for growth in the Insurance-linked Securities (ILS) market and in insurance-linked derivatives. They discussed the technical and regulatory issues that could be crucial to market growth and recommended new private and public initiatives, aimed at boosting the use and efficiency of CAT-linked securities and derivatives.[2]

Securitization can help to resolve reinsurance market inefficiencies in several ways: (1) Risks that are correlated within insurance and reinsurance markets may be uncorrelated with other risks in the economy. (2) In comparison with the total volume of securities traded in capital markets, the equity capital of insurers and reinsurers is miniscule. In addition, the largest projected insured loss events are also very large, relative to the total capitalization of the insurance industry. Modeling

firms have estimated that a $100 billion event in Florida or California has a probability of occurrence in the 1-2 percent range. Such events are large, relative to the capacity of the global reinsurance industry, but would be less than 0.5 of 1 percent of the value of stocks and bonds traded in the United States alone. Hence, transferring such risks directly to securities market appear to be much more efficient.(3) If properly structured, securitized financial instruments can significantly reduce or eliminate the credit risk (insolvency risk) inherent in reinsurance policies.[4]

The pricing of CAT bonds and other ILS, is perhaps, the most investigated area of research. Some researchers use an actuarial approach to model the yield paid on ILS. They usually start with the recognition that equilibrium models, do not explain why yields on CAT bonds consistently exceed actuarially fair levels, implying that disaster risks should yield an unbiased actuarial estimate of expected loss. Academics differ depending on the determinants of insurance-linked securities risk premium spreads. For CAT-linked instruments, the premium is most commonly determined as a fixed constant, times the volatility of loss (other higher loss distribution moments, such that skewness, may also partly determine the premium spread). Others, such as Major (1999), attributed high yields paid on ILS structures to the uncertainty associated with actuarial probabilities. On the other hand, Froot and Posner (2000) argued that the pricing of risks in ILS structures and therefore, the determination of risk premium spreads is determined by reinsurers, who, via the creation of Special Purpose Vehicles (SPV), facilitate the issuance of ILS.[3]

The analysis of comparative pricing of CAT bonds and reinsurance is even less developed than the analysis of CAT bond spreads. A comparison between CAT bond and reinsurance prices is difficult, due to the following reasons (among others): (1) CAT bonds have multiyear term, while reinsurance contracts are typically for 1 year; (2) CAT bonds are collateralized and thus, have lower counter-party risk than most reinsurance transactions; and (3) reinsurance contracts usually include reinstatement provisions, whereas, CAT bonds do not. The multiyear feature tends to increase CAT bond spreads relative to reinsurance because of the usually upward sloping term structure of interest rates. The lack of a reinstatement provision would also tend to increase CAT bond spreads relative to reinsurance, on the rationale that the reinsurer can count on another source of income following a loss, whereas, CAT bond investors cannot (GC Securities, 2008). Collateralization should reduce CAT bond prices relative to reinsurance.[4]

This paper helps the government in risk management of extreme cases of natural disasters, by proposing a model to calculate the present value of CAT bonds, in order to manage the gap or overall losses between economic losses and insured losses. It differs from most previous studies with regard to: (i) the application - the paper is concerned with analyzing the losses related to extreme earthquakes, in the sense that, if more than one earthquake occurs during a year, it will choose the largest losses within the data; (ii) the method of pricing of CAT bonds - compound probability distributions are used under the condition of at least, one occurrence of an earthquake during the year; (iii) flexible model - expected losses caused by extreme earthquake that exceed any certain value can be calculated.

3. METHODOLOGY

The Complementary risk method which is used to determine a mixed probability distribution, expresses the number of earthquakes and the maximum losses realized where zero truncated Poisson distribution represents frequency and Last order Weibull distribution for losses (Maximum Risk). Louzada, Francisco et al (2012) proposed a new three-parameter long-term lifetime distribution, induced by a latent complementary risk framework with decreasing, increasing and unimodal hazard function, and the long-term complementary exponential geometric distribution.[9]

3.1 Data Description

Data from the International Disaster Database (EM-DAT) reported that maximum losses of earthquakes occurred yearly around the globe during a time series, from 1906 to 2015.[10] The maximum loss within the research sample was selected from the earthquake losses recorded in each year. Table 2. shows some descriptive statistics of maximum losses resulting from earthquakes, since the maximum of maximum losses is equal to $ 210 billion, and the minimum of maximum losses is equal to 0.00045 billion dollars.

Table 2. Descriptive statistics of earthquakes maximum losses ($ Billion)

Minimum	1st Qu.	Median	Mean	3rd Qu.	Maximum	Percentiles	
						90%	95%
0.00045	0.05	0.575	8.21657	4.043	210	19.58	30

3.2 Zero truncated Poisson distribution for frequency distribution

Conceição, Katiane S. et al (2014) modified poison by truncated zero.[11] N which is the random variable expresses the number of earthquakes, and follows a Poisson distribution, where the probability distribution is given by:

$$P(N) = \frac{e^{-\lambda}\lambda^{N}}{N!} \ , \ N = 0,1, \dots and \ \lambda > 0 \qquad (1)$$

Where the probability of an earthquake occurring at least, is given by:

$$\sum_{N=1}^{\infty} P(N) = 1 - P(0) = 1 - e^{-\lambda} \qquad (2)$$

So the sum of probabilities which is equal to one is divided by the two sides on a $1 - e^{-\lambda}$, as follow:

$$\frac{\sum_{N=1}^{\infty} P(N)}{1 - e^{-\lambda}} = 1 \qquad (3)$$

Thus, the truncated Poisson probability distribution is given by:

$$P(z) = \frac{e^{-\lambda}\lambda^{z}}{z!\,(1-e^{-\lambda})}\ ,\quad z = 1,2,\ldots,\infty \qquad (4)$$

This probability distribution means the occurrence of one earthquake at least, without none or zero earthquakes.

3.3 Last order Weibull distribution for losses

Y which is the random variable expresses the maximum losses of earthquakes, and follows the Last order Weibull (Maximum Risk), and has a probability density function (pdf) given by:

$$f(y) = \alpha\,(\frac{1}{\theta})^{\alpha}\,y^{\alpha-1}\,e^{-(\frac{y}{\theta})^{\alpha}} \qquad (5)$$

Where θ is the scale parameter and α is the shape parameter and let:

$$Y = \max(y_1, y_2, \ldots, y_n)$$

In general, the last distribution for any continuous variable is given by:

$$f_1(y/z) = z\,f(y)[F(y)]^{z-1} \qquad (6)$$

Where f (y) is the probability density function PDF, and F(y) is the cumulative density function, which is the CDF function.

The joint distribution between y and z are obtained by multiplying formulas 4 and 6, as follow:

$$f_2(y,z) = P(z)f_1(y/z) \qquad (7)$$

$$f_2(y,z) = \frac{e^{-\lambda}\lambda^{z}}{z!\,(1-e^{-\lambda})} \times z\,f(y)[F(y)]^{z-1} \qquad (8)$$

$$f_2(y,z) = \frac{\lambda e^{-\lambda} f(y)[\lambda F(y)]^{z-1}}{(z-1)!\,(1-e^{-\lambda})} \qquad (9)$$

The Marginal distribution for Y is given by:

$$g(y) = \frac{\lambda e^{-\lambda} f(y)}{(1-e^{-\lambda})} \times \sum_{z=1}^{\infty} \frac{[\lambda F(y)]^{z-1}}{(z-1)!} \qquad (10)$$

Where:

$$\sum_{z=1}^{\infty} \frac{[\lambda F(y)]^{z-1}}{(z-1)!} = e^{\lambda F(y)} \qquad (11)$$

$$g(y) = \frac{\lambda\, e^{\ \lambda} f(y)\, e^{\lambda F(y)}}{(1-e^{-\lambda})}, y > 0 \qquad (12)$$

Where:

$$f(y) = \alpha \left(\frac{1}{\theta}\right)^{\alpha} y^{\alpha-1} e^{-\left(\frac{y}{\theta}\right)^{\alpha}} \quad (13)$$

$$F(y) = 1 - e^{-\left(\frac{y}{\theta}\right)^{\alpha}} \quad (14)$$

Thus, the PDF of compound truncated Poisson Weibull distribution is given by:

$$g(y) = \frac{\lambda \alpha \left(\frac{1}{\theta}\right)^{\alpha} y^{\alpha-1} e^{-\left(\frac{y}{\theta}\right)^{\alpha}} e^{-\lambda\left(\frac{y}{\theta}\right)^{\alpha}}}{(1 - e^{-\lambda})} \quad (15)$$

Where θ is the scale parameter and λ, α are the shape parameters.

In general, the CDF is defined as follow:

$$G(y) = \frac{e^{-\lambda[1-F(y)]} - e^{-\lambda}}{(1 - e^{-\lambda})} \quad (16)$$

Where:

$$1 - F(y) = e^{-\left(\frac{y}{\theta}\right)^{\alpha}} \quad (17)$$

$$G(y) = \frac{e^{-\lambda e^{-\left(\frac{y}{\theta}\right)^{\alpha}}} - e^{-\lambda}}{(1 - e^{-\lambda})} \quad (18)$$

Where $G(0) = 0$ and $G(\infty) = 1$

$$\mathrm{E}(y) = \int_0^{\infty} y.g(y).d_y \quad (19)$$

$$Var(y) = \int_0^{\infty} [y - \mathrm{E}(y)]^2.g(y).d_y \quad (20)$$

4. RESULTS AND DISCUSSION

In this section, the proposed model for estimating the expected values that exceed the specific values is discussed and applied. The expected value is the present value of the CAT bonds, which is also a similar net premium of excess-of-loss reinsurance.

4.1 Estimation of parameters

The Method of moments used to estimate the parameters, where θ is the scale parameter equal 1000000000 and λ, α are the shape parameters, their estimates are as follow:

$$\mathrm{E}(y) = \int_0^{\infty} y.g(y).d_y \; = Mean(y) = 8.21657 \quad (21)$$

$$Var(y) = \int_0^{\infty} [y - \mathrm{E}(y)]^2.g(y).d_y \; = Var(y) = \; 764.866 \quad (22)$$

This equation system had been solved by Mathcad software, and the following values were obtained:

$$\begin{cases} \lambda = 0.95672 \\ \alpha = 0.33875 \end{cases}$$

4.2 Goodness of fit

Kolmogorov-Smirnov test sample was used to test the following hypothesis:

H_0: Maximum of maximum losses fit with Compound Truncated Poisson-Weibull Distribution.

We found the statistic test equals 0.088, Critical Value equals 0.785998 and P-Value equals 0.567082, thus, we fail to reject the null hypothesis. (Appendix)

4.3 Application of Model

The model is applied on the expected value that exceeds some descriptive statistics or any other values calculated by formula No. 19. Table 3 shows the present value of CAT bonds in accordance with the specific values and the cost of full coverage for extreme earthquake losses of $ 8.281 billion, while the cost of cover losses in excess of 95% equals $ 4.013 billion.

Table 3. The present value of CAT bonds ($Billion)

Classes of maximum of maximum losses		Present value of CAT bonds
More than minimum (full coverage)		8.281
More than 1st Qu.		7.088
More than Median		7.032
More than 3rd Qu.		6.544
More than mean		5.994
More than Percentiles	90%	4.814
	95%	4.013

By the proposed model, we can calculate the cost of extreme earthquake losses for any specified level of coverage. Table 3 demonstrates that the present value of CAT bonds decreases with increase in the retention of losses.

For example, the proportion of losses from extreme earthquakes in any country is equal to 15% of extreme earthquake losses globally. Par adventure this country wants to cover all of these losses, the expected losses will be 1.24215 billion dollars (.15 * 8.281), or 1,242,150,000 dollars. This amount is the present value of the CAT bonds that must be issued to investors. If this country decides to issue

1,000,000 CAT bonds with interest rates of 9% and worth at the end of the year, the price of the CAT bond will be equal to 1,353.94 dollars, which is calculated by the following formula:

$$[1{,}242{,}150{,}000/1{,}000{,}000(1.09)^{-1}] \qquad (23)$$

Thus if the country decides to cover losses that exceed the mean of losses, according to formula 23, the price of CAT bond will be equal to 980.18 dollars. Similarly we can calculate the price of CAT bonds for any amount of coverage of losses.

5. Conclusion

The proposed model was based on complementary risk for pricing CAT bonds, where zero truncated Poisson distribution represents the frequency and the Last order Weibull distribution represents losses. The compound truncated Poisson-Weibull distribution was found to be the best fit for total losses of extreme earthquakes. Present values have been calculated for extreme earthquake losses, which exceed specific measures and the model was used to calculate the expected loss for losses that exceed any value. This is similar to the net premium for excess-of-loss reinsurance in traditional insurance. Any country may choose to calculate its share of the relatively extreme earthquake losses. Thereafter, if this country specifies the number of CAT bonds (for instance, 1 million bonds) for investors, it is possible to determine the price of the CAT bond, according to a specific interest rate and time duration. On the other hand, if the country specifies the price of the CAT bond, the country can determine the amount of CAT bonds needed by investors to cover losses of probable extreme earthquakes. The author observed that the present value of CAT bonds decreases as the retention increases. One of the derivative methods that is considered as an alternative or complement to traditional insurance is the risk management of natural disaster, which transfers the losses to the financial markets The process of transferring losses to capital markets to cover natural disasters through the catastrophic bonds leads to greater coverage of these losses by maintaining natural disaster losses relative to the size of the capital markets to be less than the ratio to the size of the insurance markets, in addition to the possibility of providing the necessary funds for reconstruction. Insurance and reinsurance companies issue CAT bonds for investors and the governments are supervising and developing legislations.

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Appendix: Goodness of fit with Compound Truncated Poisson-Weibull Distribution

Kolmogorov-Smirnov test

$i := 0 .. n - 1$

$Y1 := \mathrm{sort}(Y)$

$$pv(x) := 2 \cdot \sum_{k=1}^{100000} (-1)^{k-1} \cdot e^{-2 \cdot k^2 \cdot x^2}$$

$$Z_i := \frac{i+1}{n}$$

$$F(i) := \frac{e^{-\lambda 1 \cdot (1 - \mathrm{pweibull}(Y1_i, \alpha 1))} - e^{-\lambda 1}}{1 - e^{-\lambda 1}}$$

F(i)=

0.04375	0.21058	0.45107	0.71823
0.10456	0.22336	0.45107	0.73181
0.11511	0.24496	0.49555	0.74552
0.12399	0.24496	0.49867	0.75086
0.1244	0.24583	0.50695	0.76085
0.14787	0.26298	0.51824	0.76085
0.16338	0.2756	0.51892	0.76175

0.16792	0.28215	0.52094	0.80723
0.16792	0.34936	0.54332	0.8105
0.16792	0.36902	0.55405	0.86092
0.16792	0.4063	0.5606	0.87071
0.1701	0.41564	0.57462	0.88282
0.17429	0.4284	0.57462	0.90451
0.17631	0.43018	0.5924	0.90868
0.17828	0.43416	0.61565	0.93098
0.17828	0.43556	0.61565	0.93575
0.18393	0.43603	0.6477	0.93575
0.19343	0.43673	0.6546	0.98306
0.20052	0.44016	0.68105	0.98674
0.21058	0.44016	0.70964	0.99659

347

348 $d_i := \left| Z_i - F(i) \right|$

349 $D := \max(d)$

350 $D = 0.088$

351 $K := D \cdot \sqrt{n}$

352 $K = 0.785998$

353 $pv(K) = 0.567082$