

Original Research Article
**Bayesian Approach for Bonus-Malus Systems with
Gamma Distributed Claim Severities in Vehicles
Insurance**

ABSTRACT:

Aims: This paper aims at developing a model to calculate the net premiums under Bonus-Malus System. This model depends on frequency, severity and policy year(s) component on an individual policyholders' basis in vehicle insurance.

Study Design: This study is an empirical research and was based on a secondary data and details about the number of claims and severities of vehicle insurance.

Place and Duration of Study: The study period ranged from 2013 to 2015. During the stated period, a random sample consisting of 3000 vehicle insurance policies was selected from a Saudi Insurance company. The detailed data collected included number of claims and their severities.

Methodology: This study is based on the use of the Bayesian approach to arrive at a compound model, which includes three variables: the number of claims, amount of claims and time. This model is used to calculate the net vehicle insurance premiums for individual policyholders, according to the policyholder's total number of claims and total amount of claims during a specific period of time. The data was analyzed using IBM SPSS Statistics 22 and MathCad 2001 professional software.

Results: This paper presents the design of optimal Bonus-Malus Systems using finite mixture models, probability distributions and considering the number of claims following Poisson distribution compound with time. Severities presented are in accordance with gamma distribution. The proposed model shows a directly proportional relationship between accident occurrence, increase in size of the loss incurred and increase in premium. This also reflects proportionately in situations, where premium decreases in the absence of accidents over a period of time. The proposed model achieves a stability and fairness in the premium of vehicle insurance for all policyholders under different levels of total losses and different levels of total number of claims realized during a given period of time. Currently this fairness is lacking in the Saudi vehicle insurance market.

Keywords: Vehicle insurance, Bonus-Malus System, Bayesian, Compound Poisson, Gamma distribution.

1. INTRODUCTION

1.1 An Overview of the Saudi Vehicle Insurance Market

Vehicles insurance is an important branch of non-life insurance. In Saudi Arabia, vehicle insurance ranks second after medical insurance by 21.42% of written premiums on average during the 2008 to 2015 period.

In respect to the prices of Vehicle Insurance in Saudi Arabia, Specialists in insurance are generally asking a review of the decision to raise vehicle insurance rates. Where, in early 2016, insurance rates rose on vehicles by nearly 400% compared to 2014. On average, vehicle premium secured before 2014 did not exceed 590 SAR. During 2015, there was an increase in the price twice, first at the beginning of 2015 when insurance premium increased to the amount of 850 SAR, and the second at the end of 2015, when insurance premium increased again to 1500 SAR. At the moment, premiums for insurance can be deemed to be very expensive, and this has led to evasion of premium payment by policyholders.[13,14]

The prices in the Saudi vehicle insurance market is unfair , where the premiums are increasing every year and don't apply Bonus-Malus System (BMS) or any stratigy to distinguish between policyholders of vehicle insurance according to their history. The increase in premiums is applied for all policyholders regardless of their history concerning the total number of claims and the total losses of these claims.

A policyholder who had an accident with a small size of loss is penalized in the same way with a policyholder who had an accident with a big size of loss. In addition, bearing in mind the previous experience of the insurance policy.

This paper aims at designing the optimal net premiums under the Bonus-Malus System (BMS) concept in Saudi insurance companies, which can be applied in vehicle insurance and are based on the total number of claim of each policyholder and severities during a specific period of time.

This paper contributes to calculate the fair vehicle insurance premium. This type of insurance, which only allows the bonus or malus depends on policyholders' history of the total number of claims and the total losses of these claims, as is the case in most countries of the world in the field of insurance. Under BMS vehicle insurance, premium is fair for both the policyholders and the insurer. The objective is to distinguish between the policyholders, where a policyholder who has good history of the total number of claims and total losses of these claims should pay less premium than the policyholder who has bad history .

1.2 Advantages of BMS with a Frequency and a Severity Component

The following are the most important Advantages of BMS:

- In the beginning, all the policyholders who fall into similar degrees of risk pay the same premium.
- The more accidents caused and the more the size of loss that each claim incurred, the higher the premium.
- The system is fair for each insured, premium payments are proportional to total claim frequency and severities'.
- The net premium always decreases when no accidents are caused over time.
- BMS leads to reduction in the frequency of accidents.

2. LITERATURE REVIEW

Generally, in the insurance sector, a BMS is in vogue wherein adjustments are made to the premium paid by individual policyholders, according to claim history. Bonus usually is a discount in the premium which is given on the renewal of the policy if no claim is made in the previous year(s). Malus is an increase in the premium if there was a claim in the previous year(s). BMS are very common in vehicle insurance. This system is also called a No-Claim Discount (NCD) or no-claims bonus in Britain and Australia. The fundamental principle of BMS is that the higher the claim frequency of a policyholder, the higher the insurance costs that are charged on average to the policyholder.

The BMS proved optimal in reducing the frequency of reported accidents. The optimality criterion corresponds to that of the principal-agent model under moral hazard. Introducing the bonus-malus scheme could, in theory, be expected to create more incentives for safe driving, as it links individual premiums to past reported accidents.[12]

The use of a European style bonus-malus or no-claims discount system that provides percentage discounts based on the number of years driven without a claim, has gained favor among some insurers.[16]

Bonus-malus (or merit rating) systems are a more highly developed, safe driver discount system, and these are much more common outside of the United States. Under a bonus-malus system, drivers receive a fixed discount for a claim-free year and a fixed surcharge if an accident occurs. Empirical research on regulated bonus-malus rating systems in other countries confirms that they affect driving behavior in the manner predicted by economic theory (Dionne, 2002; Dionne and Ghali, 2005). These studies provide confirmatory evidence that driving, insuring, and claiming behaviors are sensitive to the price of insurance.[11]

The automobile insurance policy is usually a 1-year contract. An insured is free to move from one insurer to another carrying his bonus-malus record. The claim coefficient, an indicator of past driving record, is based on a bonus-malus scoring conversion formula.[19]

Nicholas E. Frangos *et al.*, (2001) proposed a generalised BMS that takes into consideration simultaneously, the individual's characteristics, the number of his accidents and the exact level of severity for each accident. [1] Chong It Tan, (2016) derives the analytical formulae for the optimal linear relativities subject to a financial balanced inequality constraint. The results show that the prior risk segmentation is not a sensitive factor for the effectiveness of transition rules. [2] Xueyuan Wu, *et*

al., (2015) proposes a discrete-time risk model that has a certain type of correlation between premiums and claim amounts. The impact of the proposed correlation between premiums and claims on ruin probabilities is examined through numerical examples. [3] Weihong Ni *et al.*, (2014) focus on modelling the claim severity component as a Weibull distribution for a Negative Binomial number of claims and employ the Bayesian approach to derive the BM premiums for Weibull severities. They concluded by comparing explicit formulas and numerical results with those for Pareto severities that were introduced by Frangos & Vrontos.[4] Jean-Philippe Boucher and Rofick Inoussa, (2014) & Boucher *et al.*, (2009) proposed a new way to deal with BMS when panel data are available.[5, 10] George Tzougas *et al.*, (2014) present the design of optimal BMS using finite mixture models. The researchers use a finite Negative Binomial mixture and a finite Pareto mixture updating the posterior mean. The generalized BMS proposed, integrate risk classification and experience rating by taking into account both the a priori and a posterior characteristics of each policyholder.[6] Park *et al.*, (2010) evaluated the toughness towards consumers of 16 Asian BMS and its correlation with cultural and economic variables. The researchers used Principal Components Analysis to define a Maturity Index of insurance markets and find supporting evidence for a conjecture that, as markets become more mature and policy-holders more sophisticated, countries adopt tougher BMS. In addition, using regression analysis, it was found that a Common Law legal system is a crucial factor in BMS design and cultural variables such as uncertainty avoidance, also influence BMS.[7] Dionne, G. & Ghali, O. (2005) indicate that BMS reduced the probability of reported accidents for good risks but had no effect on bad risks.[12] Kim, Hyojoung *et.al.*, (2009) obtained only the accident records of the current contract year data. It however had bonus-malus rates that reflect past accidents. Each accident increases the rate at least by 0.05. [20] The key element in the bonus-malus system, the claims coefficient, is obtained by the conversion of cumulative claim points, calculated as the sum of no-claim points and claim points in the past three years.[17] One possibility for the mixed results is variation in the underwriting process that can help limit the information asymmetry between the insurance company and the policyholder. Experience rating, often implemented through bonus-malus schemes such as the NCD in Australia, can be an important component of underwriting.[18] Lemaire and Zi1 analysed 30 BMS from all over the world and used Principal Components Analysis to create an Index of Toughness of BMS. [7] When shopping for vehicle insurance, eligibility for one or more discounts should be determined. For high – risk drivers paying exorbitant premiums, improved driving records would result in a substantially reduced premium.[15] Vehicles insurance policies are typically offered by the insurers in differently bundled formats and some of which may be highly similar.[8]

3. METHODOLOGY

A numerical illustration using at-fault and not-at-fault claims of a Saudi insurance company is included to support this discussion. Sample of 3000 case included number of claims and amount of claims to estimate parameters of model, according to the probability distributions fitted.

3.1 Mathematical Framework

It is assumed that the number of claims of each policyholder is independent from the severity of each claim, in order to deal with the frequency and the severity component separately.[1]

3.1.1 Frequency Distribution

In a bonus - malus system in car insurance, the bonus class of a customer is updated from one year to the next, as a function of the current class and the number of claims in the year (assumed Poisson), where the Poisson rate according to which claims are generated for a customer is the outcome of a random variable specific to the customer.[9]

Assuming that the random variable Z expresses the number of claims, given the parameter λ , assuming that it follows a Poisson distribution, where the probability distribution function is:

$$f(z|\lambda) = \frac{e^{-\lambda} \lambda^z}{z!}, \lambda > 0 \text{ and } z = 0, 1, 2, 3 \dots \quad (1)$$

The parameter λ is denoting the different underlying risk of each policyholder to have an accident. According to Bayesian approach, let us assume for the structure function that:

$$\lambda \sim \text{gamma}(\tau, \alpha)$$

The probability density function of λ is a prior distribution of the form:

$$\pi(\lambda) = \frac{\tau^\alpha \lambda^{\alpha-1} e^{-\tau\lambda}}{\Gamma(\alpha)}, \quad \lambda > 0, \tau > 0, \alpha > 0 \quad (2)$$

Where τ is the scale parameter and α is the shape parameter and the expected value and the variance as follows:

$$E(\lambda) = \alpha/\tau \quad (3) \quad 157$$

$$Var(\lambda) = \alpha/\tau^2 \quad (4)$$

The likelihood function is:

$$L(\lambda/\underline{z}) \propto e^{-\sum_{i=1}^I \lambda} \lambda^{\sum_{i=1}^I z_i} \quad (5)$$

161 Assuming that: $\sum_{i=1}^t z_i = K$ (6)

162 Let us K expresses the total number of claims during the period t years.

163
$$L(\lambda/\underline{z}) \propto e^{-\tau\lambda} \lambda^K \quad (7)$$

164 The posterior structure function of λ for a policyholder. It is that

165
$$\pi(\lambda/\underline{z}) \propto \lambda^{K+\alpha-1} e^{-(t+\tau)\lambda} \quad (8)$$

166 This distribution is similar to gamma and completed as follows:

167
$$\pi(\lambda/\underline{z}) = \frac{(t+\tau)^{K+\alpha} \lambda^{K+\alpha-1} e^{-(t+\tau)\lambda}}{\Gamma(K+\alpha)} \quad (9)$$

168 Where $(t+\tau)$ is the scale parameter and $(K+\alpha)$ is the shape parameter and the expected
169 value according the Bayes' estimation as follow:

170

171
$$E(\lambda/\underline{z}) = \frac{K+\alpha}{t+\tau} \quad (10)$$

172 3.1.2 Severity Distribution

173

174 Let x be the size of the claim of each insured and has a probability density function (PDF)
175 given by:

$$X \sim \text{gamma}(\theta, \beta) \quad 176$$

177
$$f(X) = \frac{\theta^\beta x^{\beta-1} e^{-\theta x}}{\Gamma\beta}, \quad x > 0, \beta > 0, \theta > 0 \quad (11)$$

178 The probability density function of θ is prior distribution of the form:

179

$$\theta \sim \text{gamma}(\psi, \gamma) \quad 180$$

181
$$\pi(\theta) = \frac{\psi^\gamma \theta^{\gamma-1} e^{-\psi\theta}}{\Gamma\gamma}, \psi > 0, \gamma > 0, \theta > 0 \quad (12)$$

182 Likelihood function of θ parameter

183

184
$$L(\theta/\underline{X}) \propto \theta^{\sum_{i=1}^K \beta} e^{-\theta \sum_{i=1}^K X_i} \quad (13)$$

185 Assuming that:

$$186 \quad \sum_{i=1}^K X_i = S \quad (14)$$

187 Let us S denote the total amount of claims during the period t years.

$$\sum_{i=1}^K \beta = K \quad (15)$$

188

$$L(\theta/\underline{X}) \propto \theta^K e^{-\theta S} \quad (16) \quad 189$$

190 The posterior structure function of θ for a policyholder. It is that

$$191 \quad \pi(\theta/\underline{X}) \propto \theta^{K+\gamma-1} e^{-\theta(\psi+S)} \quad (17)$$

$$192 \quad \pi(\theta/\underline{X}) = \frac{(\psi+S)^{K+\gamma} \theta^{K+\gamma-1} e^{-(\psi+S)\theta}}{\Gamma(K+\gamma)} \quad (18)$$

193 Where $(\psi+S)$ is the scale parameter and $(K+\gamma)$ is the shape parameter and the expected
194 value according the Bayes' estimation as follow:

195

$$196 \quad E(\theta/\underline{X}) = \frac{K+\gamma}{\psi+S} \quad (19)$$

197 where:

$$198 \quad E(X/\theta) = \beta/\theta \quad (20)$$

$$199 \quad E(X/\theta) = \frac{\beta(\psi+S)}{K+\gamma} \quad (21)$$

200 Thus, the net premium that must be paid from that of each policyholder or specific group of
201 policyholders will be equal to the product of $E(\lambda/\underline{z})$ and $E(X/\theta)$ it will be equal to

202

$$203 \quad \text{Net Premium} = E(\lambda/\underline{z}).E(X/\theta) \quad (22)$$

$$204 \quad \text{Net Premium} = \frac{K+\alpha}{1+\tau} \cdot \frac{\beta(\psi+S)}{K+\gamma} \quad (23)$$

205 Advantages of this model are: (1) achieves a fairness in the premium of vehicle insurance for all
206 policyholders under all different levels of total losses and all different levels of the total number of

claims realized during a given period of time. (2) The present model based on Poisson and Gamma distributions will ensure that the premium increases with the total number of claims and the amount of claims. This model overcomes the deficiency of the model proposed by Weihong Ni *et al.* (2014) based on Weibull distribution and a Negative Binomial distributions, where the premium increases with the increase in total number of claims only to a certain limit and then the premium decrease with increase in the total number of claims, which is contrary to fact [4].

The net premiums when no claims are occurs with over time, given by:

$$Net\ Premium = \frac{\alpha}{t + \tau} \cdot \frac{\beta \cdot \psi}{\gamma} \quad (24)$$

This premium decreases with overtime and increases with the increase of the total number of claims, and also increases with the total amount of claims.

4. RESULTS AND DISCUSSION

The maximum likelihood estimators of the parameters are $\hat{\alpha} = 1.4$, $\hat{\tau} = 10$, $\hat{\beta} = 0.368$, $\hat{\psi} = 3.227$ and $\hat{\psi} = 100000$. The researcher computes the BMS net vehicles insurance premiums when no claims occurs with over time. Also using formula (23) to calculate the BMS net vehicles insurance premium, when at least an accident occurs. Thus the steps that must be followed in order to find the BMS net vehicles insurance premiums are:

- Determine the age of the policy t.
- Determine the total number of claims K by the policyholder in t years.
- Determine the total amount of claims by the policyholders S in t years.
- Computing the premium using (24) when no claims occurs.
- Computing the premium using (23) when at least an accident occurs.

Fig. 1 show the BMS net vehicles insurance premium when total claims equal 10000 SAR over time and total number of claims. Fig. 2 also show the BMS net vehicles insurance premium when total claims equal 100000 SAR over time and total number of claims. There is no difference in figures, where net premium increases with the increase in the total number of claims and the total amount of claims increase and decrease with over time on each level of the total losses. This shows the fairness of premiums under the different total number of claims and different total amount of claims. This property is also the most important characteristic of the proposed model.

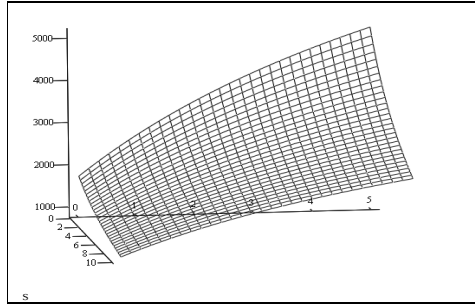


Fig. 1 The BMS net vehicles insurance premiums with total losses 10000 SAR

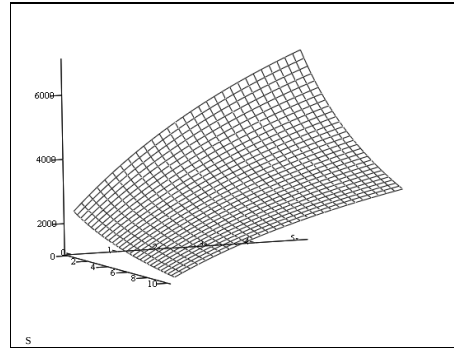


Fig. 2 The BMS net vehicles insurance premiums with total losses 100000 SAR

Tables 1-9 shows the Net Vehicles Premiums with different total amount of claims. Where the net premium is higher increase in the total number of claims and the total amount of claims increase and decrease over time on each level of the total losses, as follow:

Table 1. The Net Vehicles Premiums with total claims size of 2500

Year		Number of claims					
t		0	1	2	3	4	5
0	1597				NA		
1	1451		1947	2231	2423	2562	2668
2	1330		1785	2045	2221	2349	2445
3	1228		1647	1887	2050	2168	2257
4	1140		1530	1753	1904	2013	2096
5	1064		1428	1636	1777	1879	1956
6	998		1339	1533	1666	1762	1834
7	939		1260	1443	1568	1658	1726
8	887		1190	1363	1481	1566	1630
9	840		1127	1291	1403	1483	1544
10	798		1071	1227	1333	1409	1467

Table 2. The Net Vehicles Premiums with total claims size of 5000

Year	Number of claims
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t	0	1	2	3	4	5
0	1597			NA		
1	1451	1994	2285	2482	2625	2733
2	1330	1828	2095	2275	2406	2505
3	1228	1688	1933	2100	2221	2312
4	1140	1567	1795	1950	2062	2147
5	1064	1463	1676	1820	1925	2004
6	998	1371	1571	1706	1804	1879
7	939	1291	1478	1606	1698	1768
8	887	1219	1396	1517	1604	1670
9	840	1155	1323	1437	1520	1582
10	798	1097	1257	1365	1444	1503

Table 3. The Net Vehicles Premiums with total claims size of 10000

Year		Number of claims				
t	0	1	2	3	4	5
0	1597			NA		
1	1451	2089	2394	2600	2750	2863
2	1330	1915	2194	2384	2521	2624
3	1228	1768	2025	2200	2327	2422
4	1140	1642	1881	2043	2160	2249
5	1064	1532	1755	1907	2016	2099
6	998	1436	1646	1788	1890	1968
7	939	1352	1549	1683	1779	1852
8	887	1277	1463	1589	1680	1749
9	840	1210	1386	1505	1592	1657
10	798	1149	1317	1430	1512	1575

Table 4. The Net Vehicles Premiums with total claims size of 25000

Year		Number of claims				
t	0	1	2	3	4	5
0	1597			NA		
1	1451	2374	2720	2955	3125	3253
2	1330	2176	2493	2709	2864	2982
3	1228	2009	2302	2500	2644	2753
4	1140	1866	2137	2322	2455	2556
5	1064	1741	1995	2167	2291	2386
6	998	1632	1870	2031	2148	2237
7	939	1536	1760	1912	2022	2105
8	887	1451	1662	1806	1910	1988
9	840	1375	1575	1711	1809	1883
10	798	1306	1496	1625	1719	1789

Table 5. The Net Vehicles Premiums with total claims size of 50000

Year		Number of claims				
t	0	1	2	3	4	5
0	1597					
1	1451	2849	3264	3546	3750	3904
2	1330	2612	2992	3250	3437	3578
3	1228	2411	2762	3000	3173	3303
4	1140	2239	2565	2786	2946	3067
5	1064	2089	2394	2600	2750	2863
6	998	1959	2244	2438	2578	2684
7	939	1844	2112	2294	2426	2526
8	887	1741	1995	2167	2291	2386
9	840	1650	1890	2053	2171	2260
10	798	1567	1795	1950	2062	2147

Table 6. The Net Vehicles Premiums with total claims size of 75000

Year		Number of claims				
t	0	1	2	3	4	5
0	1597			NA		
1	1451	3324	3808	4137	4375	4554
2	1330	3047	3491	3792	4010	4175
3	1228	2813	3222	3500	3702	3854
4	1140	2612	2992	3250	3437	3578
5	1064	2438	2793	3034	3208	3340
6	998	2285	2618	2844	3007	3131
7	939	2151	2464	2677	2831	2947
8	887	2031	2327	2528	2673	2783
9	840	1924	2205	2395	2533	2637
10	798	1828	2095	2275	2406	2505

Table 7. The Net Vehicles Premiums with total claims size of 100000

Year		Number of claims				
t	0	1	2	3	4	5
0	1597			NA		
1	1451	3799	4352	4728	4999	5205
2	1330	3482	3990	4334	4583	4771
3	1228	3215	3683	4000	4230	4404
4	1140	2985	3420	3715	3928	4090
5	1064	2786	3192	3467	3666	3817
6	998	2612	2992	3250	3437	3578
7	939	2458	2816	3059	3235	3368
8	887	2322	2660	2889	3055	3181
9	840	2199	2520	2737	2894	3013
10	798	2089	2394	2600	2750	2863

Table 8. The Net Vehicles Premiums with total claims size of 150000

Year		Number of claims				
t	0	1	2	3	4	5
0	1597			NA		
1	1451	4749	5440	5910	6249	6506
2	1330	4353	4987	5417	5729	5964
3	1228	4018	4603	5001	5288	5505
4	1140	3731	4275	4643	4910	5112
5	1064	3482	3990	4334	4583	4771
6	998	3265	3740	4063	4296	4473
7	939	3073	3520	3824	4044	4210
8	887	2902	3325	3612	3819	3976
9	840	2749	3150	3421	3618	3767
10	798	2612	2992	3250	3437	3578

Table 9. The Net Vehicles Premiums with total claims size of 200000

Year		Number of claims				
t	0	1	2	3	4	5
0	1597			NA		
1	1451	5698	6528	7092	7499	7808
2	1330	5224	5984	6501	6874	7157
3	1228	4822	5524	6001	6345	6606
4	1140	4477	5129	5572	5892	6135
5	1064	4179	4787	5201	5499	5726
6	998	3918	4488	4876	5156	5368
7	939	3687	4224	4589	4852	5052
8	887	3482	3990	4334	4583	4771
9	840	3299	3780	4106	4342	4520
10	798	3134	3591	3900	4125	4294

As can be observed from Table 1, in the beginning of policy year (policy years equal to zero) if no claims occurs, all the policyholders should pay the expected value which is equal to 1597 SAR, being the minimum premium in the beginning. If there are no accidents, the premium decreases every year up to 798 SAR in the tenth year, the premium decreases nearly by 50% from the basic premium. However, the contrary is happening in Saudi motor insurance market, where premium is increasing each year up to 5 times more during the period 2005 to 2015, even if there are no claims.

For example, as shown in Table 5, if one claim occurred during the first year with a total loss 50000 SAR, the policyholder pays the insurance premium of 2849 SAR. This premium increase to 3904 SAR if the total number of claims increases to 5, and decreases over time to 1567 SAR in the tenth year of the insurance policy if there are no further claims, which ensures fairness in values of premiums. Similarly the stability in fairness of premiums is demonstrated for different values of net vehicle premiums in Tables 1 to 9. These results demonstrate advantages of the proposed model in achieving fairness of premiums for all policyholders according to their history.

5. Conclusion

In conclusion, Bayesian approach to a compound model, where the number of claims with time follows a compound Poisson and severities follow gamma. This model is found to calculate the BMS net vehicles insurance premium for individual policyholders, according to total number of claims and total amount of claims during a specific period of time. The data were analyzed using IBM SPSS Statistics 22 and MathCad 2001 professional software.

This paper presents the design of optimal BMS using finite mixture models. The proposed model shows that the more accidents caused and size of loss that each claim incurred the higher the premium and the net premium always decreases when no accidents are caused with over time. The proposed model also explained stability and fairness of premiums for different total number of claims and different total amount of claims. The fair premium of vehicle insurance, which is calculated by the proposed model leads to improved profits for the insurance organization, keeping in view the loss ratio of vehicle insurance. The model ensures the adequacy of the premium charged to the insurer in order to improve the ability of the organization to pay the required compensation to the insured. Further the fair premium protects the insured from the insolvency risk, its inability to pay its financial obligations to insured and others in the due date. Optimal BMS proposed contributes mainly in vehicle insurance in Saudi Arabia's development, as nearly 29 insurance companies among the 35 companies in the Saudi insurance market are making losses in the insurance activity in 2015.

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