

Central Limit Theorem to Approximate Aggregate Risk of Portfolio: Using the ModelRisk Software

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Abstract. In this note, the non-sampling information in portfolio management is considered. These information may be the past belief of investor about a special asset. They are characterized as the correlated binary random variables. Then, the Monte Carlo is applied to derive the posterior distribution of binary variables given the past returns which indicates the tendency of investor to keep or drop a portfolio via using the non-sampling and sampling information simultaneously. The posterior distribution of belief of investor and the accuracy of Bayesian method are shown via plotting histograms.

Keywords: Copula; Dirichlet distribution; Mixture distribution; ModelRisk software

1 Introduction. Modern portfolio management (MPT) is highly related to theoretical contributions of Markovitz (1952). The MPT has been extended substantially by the Capital Asset Pricing Model (CAPM) of Sharp (1964). However, in portfolio management, to reduce the unsystematic risk, a diversified different financial assets is held in a portfolio. However, from a Bayesian point of view to the problem, investors have prior information about including or excluding a special asset in a specified portfolio. These are important non-sampling information which should be considered in determining weights of assets. These information may be the past belief of investor about a special asset. Therefore, let $(J_1, \dots, J_{n-1})$ is a vector of Binary random variables comes from the Dirichlet distribution. Here, J_i is one if i -th asset exists in portfolio and zero, otherwise. Let $J_n = n - \sum_{i=1}^{n-1} J_i$.

Let R_i be the return of i -th asset. If an asset exists in the portfolio its share is π_i . Then, the portfolio return is given by

$$R_p = (1/n) \sum_{i=1}^n J_i \pi_i R_i,$$

such that $(1/n) \sum_{i=1}^n J_i \pi_i = 1$. Obviously, the distribution of R_p is mixture, which its exact functional form is too difficult. Let $R_i^* = \pi_i R_i$.

By implementing the Markovitz mean-variance portfolio selection on R_i^* 's, weights π_i are derived by minimizing the $var(R_p)$, for a predetermined level of $E(R_p)$. Thus, in this paper, it is assumed that π_i 's are known. A useful instrument in this problem is the posterior distribution of $(J_1, \dots, J_{n-1})$ given observing the historical data R_p . More precisely, what the posterior probability of configuration of $(j_{i_1}, \dots, j_{i_{n-1}})$ is?, i.e.,

$$P(J_1 = j_{i_1}, \dots, J_{n-1} = j_{i_{n-1}} | R_p).$$

Kass *et al.* (2005) studied the normal approximation and normal power (NP) approximation for mixture distribution and concluded that NP approximation is too useful. They used the Monte Carlo simulation to obtain the accuracy of approximation. ModelRisk (Vose, 2015) is an Excel-based software for Monte Carlo simulation, sensitivity analysis and optimization in financial problems. It has many applications in advanced risk analysis. ModelRisk is applied by researchers in financial literatures. For example, Ye and Tiong (2000) introduced the concept of Net present value at risk (NPVaR) for financed infrastructure projects. Duffey and Van Dorp (2002) applied the Monte Carlo

method for risk analysis for large engineering projects by modeling cost uncertainty for ship production activities. Yuwen (2012) applied Monte Carlo simulation of investment returns for toll projects. Nowak and Hnilica (2012) studied the business valuation under uncertainty (considering dependency between cash flows) using copula concept. Laurentiu and Gabriela (2014) considered sensitivity assessment modeling in European funded projects proposed by Romanian companies.

The rest of paper is organized as follows. In this paper, to find the marginal distribution of R_p and posterior $(J_1, \dots, J_{n-1})$ given R_p , the Monte Carlo is used. First, returns of each asset are assumed to be independent, mutually, however, several types of correlations based on copula function is considered in the simulation section.

2 Simulations. In this section, two examples are considered. The first example studies the central limit theorem approximation for aggregate risk of a portfolio. The second example considers the effect of correlation between assets and the use of copula function in modeling the correlation.

Example 1. The data is taken from Excel sheet entitled "CLT_risk_portfolio_approximation" downloadable from "Example models and tutorial videos - Vose Software¹". The portfolio contains 132 risky assets with Lognormal, PERT, Normal, ModPERT and Triangle distributions. Investor belief on each risk assets comes from independent Bernoulli random variable. The following histogram (Figure 1) shows the marginal distribution of portfolio return, R_p . The number of simulation is 1000.

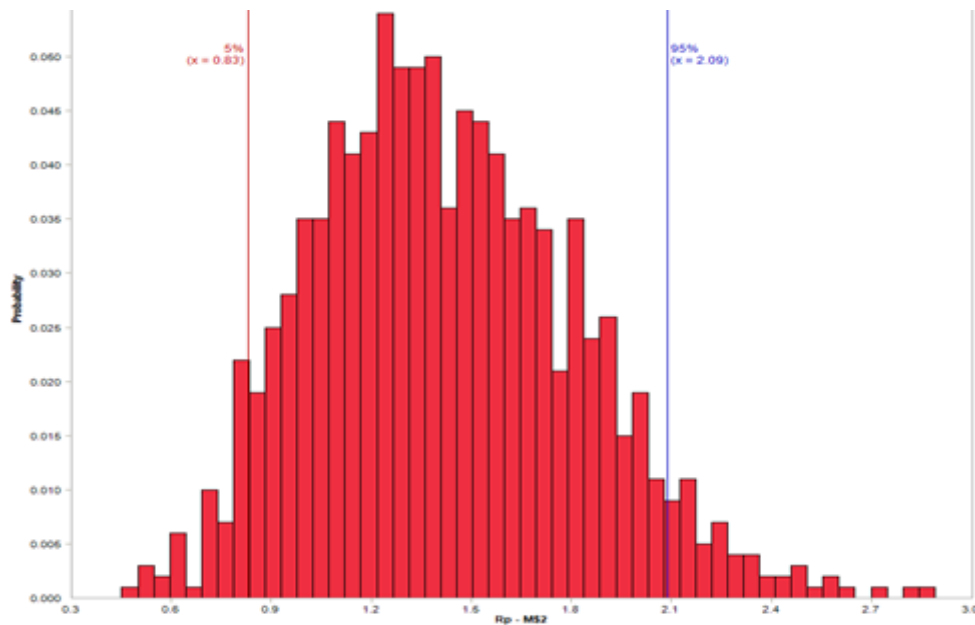


Figure 1: Marginal density of R_p

¹ http://www.vosesoftware.com/vosesoftware/ModelRiskHelp/Resources/Example_models/Example_models_introduction.htm

Some summaries statistics are given as follows

Table 1: summaries statistics

Min	Mean	Max	St.dev	Variance	CV	Skew	Kurt.
0.45	1.42	2.84	0.29	0.153	0.275	0.406	3.04

The value at risk (VaR) for some significance level α are given as follows (V_0 is the initial value of portfolio).

Table 2: VaR for some significance level α

α	0.005	0.01	0.05	0.1
VaR	$0.58V_0$	$0.62V_0$	$0.83V_0$	$0.94V_0$

However, if the belief of investor isn't included to the problem, then the density of R_p is as follows (Figure 2). Again, some summaries statistics are given as follows. In this special case, the performance of Markovitz portfolio and the R_p are the same.

Table 3: Summaries statistics

Min	Mean	Max	St.dev	Variance	CV	Skew	Kurt.
0.35	1.43	2.99	0.404	0.163	0.28	0.5	3.15

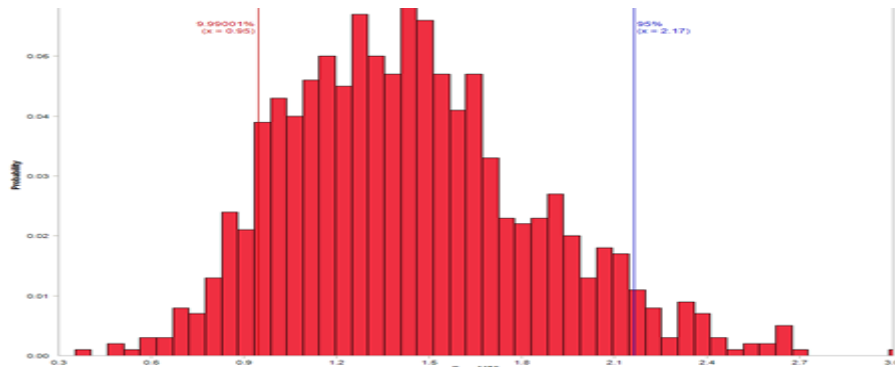


Figure 2: Density of Usual R_p

Example 2. In this example, the Gaussian copula is considered between all financial assets. The following histogram (Figure 3) shows the probability density function of R_p .

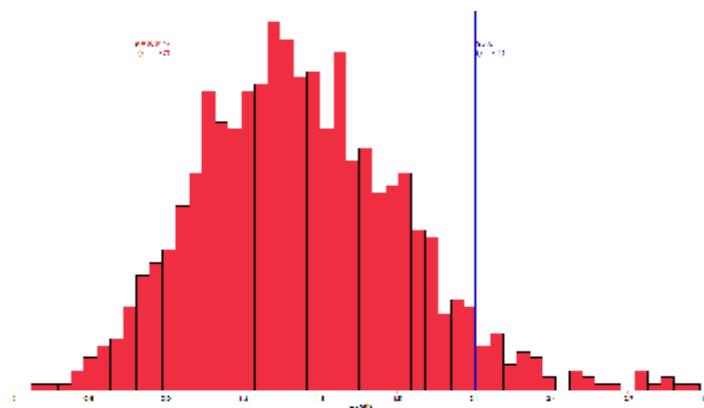


Figure 3: Marginal density of R_p

Table 4: Summaries statistics

Min	Mean	Max	St.dev	Variance	CV	Skew	Kurt.
0.35	1.43	2.99	0.294	0.087	0.206	0.9	4.1

It is seen that the use of copula improves the skewness and kurtosis of marginal distribution of portfolio return.

3 Two concepts. In this section, two important concepts are studied. The first is the posterior distribution of investor belief and the second is the stochastic programming applied in this paper.

3.1 Posterior distribution. An MCMC method is really suitable for estimating the posterior distribution of the investor belief sequence. The reversible jump algorithm is based on the fact that the dimension of the model can change, according to the number of segments. Unfortunately, this algorithm converges slowly, and many iterations are needed for estimating correctly the posterior distribution. Therefore, the Hastings Metropolis algorithm with the Gibbs sampler is applied. The following Table give some posterior values for investor belief for Example 1. In the table, for example, (95,125) means the investor has selected the assets numbered 95 and 125.

Table 5: Posterior belief of investor

(1,2,3)	(3,4,75)	(60,61,62)	(95,125)	(1,...,132)	(1,...,66)	(67,...,132)	(131,132)
0.09	0.1	0.05	0.09	0.1	0.51	0.60	0.55

3.2 Stochastic programming. Consider the portfolio management in which the return of portfolio by considering the pre-information of investor is maximized as well as the diversified risk should be in controlled. That is, to

$$\max (E(R_p))$$

s.t. $var(R_p) \leq LB$. The following Table gives some values of weights for the first asset by this method.

Table 6: The weight of the first asset

LB	0.05	0.1	0.15	0.2	0.5	0.75	0.9
Weight	0.023	0.051	0.055	0.063	0.064	0.081	0.1

4 Conclusions. The portfolio management is an important part of finance. There are many well-known techniques such as efficient frontier method to find the best combinations of assets in a portfolio. In the regular methods, only the statistical properties of returns of assets are considered. However, sometimes, investors have belief about the returns. This fact motivates the use of Bayesian methods in portfolio management. This paper considers this concept and uses the computational tools such as MCMC and stochastic programming methods to perform the portfolio management using posterior belief of investor distribution. This paper also uses the ModelRisk software.

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References

[1] Duffey, M. R. and Van Dorp, J. R. (2002). Risk analysis for large engineering projects: modeling cost uncertainty for ship production activities. *Journal of Engineering Valuation and Cost Analysis* 2, 285-301.

[2] Kass, R., Goovaerts, M., Dhaene, J., and Denuit, M. (2005). *Modern Actuarial Risk Theory: Using R*. Springer.

[3] Laurentiu, D. and Gabriela, D. (2014). Sensitivity assessment modeling in European funded projects proposed by Romanian companies. *Tech. Repo.* Faculty of Economics. University of Oradea. Oradea, Romania.

[4] Markowitz, H. (1952). Portfolio Selection. *The Journal of Finance* **1**, 77-91.

[5] Nowak, O. and Hnilica, J. (2012). Business valuation under uncertainty. *Tech. Repo.* Department of Business Economics. University of Economics Prague. Prague. Czech.

[6] Sharpe, W. (1964). Capital asset prices: A theory of market equilibrium under conditions of risk. *Journal of Finance* **19**, 425-442.

[7] Vose Risk Analysis (2015). <http://www.vosesoftware.com>.

[8] Ye, S. and Tiong, R. L. K. (2000). NNPV-at-risk method in infrastructure project investment evaluation. *Journal of construction engineering and management* **7**, 227-233.

[9] Yuwen, W. (2012). Simulation of Investment Returns for Toll Projects. Unpublished Thesis. Master of Science in Engineering. University of Texas at Austin.