

1 **Original Research Article**
 2 **Bayesian Approach for Bonus-Malus Systems with**
 3 **Gamma Distributed Claim Severities in Vehicles**
 4 **Insurance**

5 **ABSTRACT:**

Aims: This paper aims at developing a model to calculate the net premiums under Bonus-Malus System. This model depends on frequency, severity and policy year(s) component on an individual policyholders' basis in vehicle insurance.

Study Design: This study is an empirical research and was based on a secondary data in details about the number of claims and severities of vehicle insurance.

Place and Duration of Study: The study period ranged from 2013 to 2015. During the stated period, a random sample consisting of 3000 vehicle insurance policies was selected from a Saudi Insurance company. The detailed data collected included; number of claims and their severities.

Methodology: This study is based on the use of the Bayesian approach to conclude a compound model, which includes three variables; the number of claims, amount of claims and time. This model is used to calculate the net vehicle insurance premiums for individual policyholders, according to the policyholder's total number of claims and total amount of claims during a specific period of time. The data were analyzed using IBM SPSS Statistics 22 and MathCad 2001 professional software.

Results: This paper presents the design of optimal Bonus-Malus Systems using finite mixture models, probability distributions and considering the number of claims following Poisson distribution compound with time. Severities presented are in accordance with gamma distribution. The proposed model shows a directly proportional relationship between accident occurrence, increase in size of the loss incurred and increase in premium. This also reflects proportionately in situations, where premium decreases in the absence of accidents over a period of time

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 7 *Keywords: Vehicle insurance, Bonus-Malus System, Bayesian, Compound Poisson, Gamma*
 8 *distribution.*

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1. INTRODUCTION

1.1 An Overview of the Saudi Vehicle Insurance Market

Vehicles insurance is an important branch of non-life insurance. In Saudi Arabia, vehicle insurance ranks second after medical insurance by 21.42% of written premiums on average during the 2008 to 2015 period.

Gross written premium in 2015; 35.6 billion SAR progressed by a 19% increase over that of 2014; 29.8 billion SAR (7.9 – 9.5 billion USD). This was driven by the acceleration of growth in vehicle insurance with its gross premium surging by 29% to 10.3 billion SAR (2.7 billion USD) compared to 8 billion SAR (2.1 billion USD) in 2014. This surge however, was triggered by re-pricing of vehicle insurance policies. Table 1 shows the results of Net Incurred Claims for Vehicle insurers which was displayed at the end of 2015. The growth of Gross Written Premium for vehicle insurance amounted to 29% increase (10.3 billion SAR). Meanwhile, Net Written Premium edged up 24% coming in at 9.4 billion, while Net Earned Premium hit 8.7 billion SAR, surging 28% over 2014. Retention ratio stood at 91.33% compared to 94.85% at the end of 2014. Net incurred claims soared 26% to 7.9 billion SAR, thus trimming the claims ratio by 133 basis point to 89.98% compared to the 91.31% experienced in 2014. For Vehicle Insurance, seven companies captured 63% of the market. 15 other players shared 34%, and 5 insurers collectively accounted for the remaining 3% of the market. [13, 14]

Table 1. Results of Vehicle Insurance Processes

Vehicle insurance (SAR million)	2014	2015	Growth
Gross Premium Written	7,988	10,273	29%
Net Premium Written	7,577	9,382	24%
Net Premium Earned	6,809	8,729	28%
Net Claims Incurred	6,218	7,854	26%
Retention Ratio	94.5%	91%	-3.5%
Loss Ratio	91.3%	90%	-1.3%

In respect to the prices of Vehicle Insurance in Saudi Arabia, Specialists in insurance are generally asking a review of the decision to raise vehicle insurance rates. Where, in early 2016, insurance rates rose on vehicles by nearly 400% compared to 2014. On average, vehicle premium secured before 2014 did not exceed 590 SAR. During 2015, there was an increase in the price twice, first at the beginning of 2015 when insurance premium increased to the amount of 850 SAR, and the second at the end of 2015, when insurance premium increased again to 1500 SAR. At the moment, premiums for insurance can be deemed to be very expensive, and this has led to evasion of premium payment by policyholders.

Current BMS in Saudi insurance companies are unfair. A policyholder who had an accident with a small size of loss is penalized in the same way with a policyholder who had an accident with a big size of loss. In addition, bearing in mind the previous experience of the insurance policy.

This paper aims at designing the optimal net premiums under the Bonus-Malus System (BMS) concept in Saudi insurance companies, which can be applied in vehicle insurance and are based on the total number of claim of each policyholder and severities during a specific period of time.

1.2 Bonus-Malus Systems for Vehicles Insurance

When shopping for vehicle insurance, eligibility for one or more discounts should be determined. For high – risk drivers paying exorbitant premiums, improved driving records would result in a substantially reduced premium.[15] Vehicles insurance policies are typically offered by the insurers in differently bundled formats and some of which may be highly similar.[8]

Generally, in the insurance sector, a BMS is in vogue wherein adjustments are made to the premium paid by individual policyholders, according to claim history. Bonus usually is a discount in the premium which is given on the renewal of the policy if no claim is made in the previous year(s). Malus is an increase in the premium if there was a claim in the previous year(s). BMS are very common in vehicle insurance. This system is also called a No-Claim Discount (NCD) or no-claims bonus in Britain and Australia. The fundamental principle of BMS is that the higher the claim frequency of a policyholder, the higher the insurance costs that are charged on average to the policyholder.

The BMS proved optimal in reducing the frequency of reported accidents. The optimality criterion corresponds to that of the principal-agent model under moral hazard. Introducing the bonus-malus scheme could, in theory, be expected to create more incentives for safe driving, as it links individual premiums to past reported accidents.[12]

The use of a European style bonus-malus or no-claims discount system that provides percentage discounts based on the number of years driven without a claim, has gained favor among some insurers.[16]

Bonus-malus (or merit rating) systems are a more highly developed, safe driver discount system, and these are much more common outside of the United States. Under a bonus-malus system, drivers receive a fixed discount for a claim-free year and a fixed surcharge if an accident occurs. Empirical research on regulated bonus-malus rating systems in other countries confirms that they affect driving behavior in the manner predicted by economic theory (Dionne, 2002; Dionne and Ghali, 2005). These studies provide confirmatory evidence that driving, insuring, and claiming behaviors are sensitive to the price of insurance.[11]

The automobile insurance policy is usually a 1-year contract. An insured is free to move from one insurer to another carrying his bonus-malus record. The claim coefficient, an indicator of past driving record, is based on a bonus-malus scoring conversion formula.[19]

1.3 Advantages of BMS with a Frequency and a Severity Component

The following are the most important Advantages of BMS:

- In the beginning, all the policyholders who fall into similar degrees of risk pay the same premium.
- The more accidents caused and the more the size of loss that each claim incurred, the higher the premium.
- The system is fair for each insured, premium payments are proportional to total claim frequency and severities'.
- The net premium always decreases when no accidents are caused over time.
- BMS leads to reduction in the frequency of accidents.

2. LITERATURE REVIEW

Nicholas E. Frangos *et al.*, (2001) proposed a generalised BMS that takes into consideration simultaneously, the individual's characteristics, the number of his accidents and the exact level of severity for each accident. [1] Chong It Tan, (2016) derives the analytical formulae for the optimal linear relativities subject to a financial balanced inequality constraint. The results show that the prior risk segmentation is not a sensitive factor for the effectiveness of transition rules. [2] Xueyuan Wu, *et al.*, (2015) proposes a discrete-time risk model that has a certain type of correlation between premiums and claim amounts. The impact of the proposed correlation between premiums and claims on ruin probabilities is examined through numerical examples. [3] Weihong Ni *et al.*, (2014) focus on modelling the claim severity component as a Weibull distribution for a Negative Binomial number of claims and employ the Bayesian approach to derive the BM premiums for Weibull severities. They concluded by comparing explicit formulas and numerical results with those for Pareto severities that were introduced by Frangos & Vrontos.[4] Jean-Philippe Boucher and Rofick Inoussa, (2014) proposed a new way to deal with BMS when panel data are available. Where insurers have accumulated longitudinal information on their policyholders, and actuaries can now use many years of information for a single insured. For this kind of data, called panel or longitudinal data.[5] George Tzougas *et al.*, (2014) present the design of optimal BMS using finite mixture models. The researchers use a finite Negative Binomial mixture and a finite Pareto mixture updating the posterior mean. The generalized BMS proposed, integrate risk classification and experience rating by taking into account both the a priori and a posterior characteristics of each policyholder.[6] Park *et al.*, (2010) evaluated the toughness towards consumers of 16 Asian BMS and its correlation with cultural and economic variables. The researchers used Principal Components Analysis to define a Maturity Index of insurance markets and find supporting evidence for a conjecture that, as markets become more mature and policy-holders more sophisticated, countries adopt tougher BMS. In addition, we find, using regression analysis, that using a Common Law legal system is a crucial factor in BMS design. Cultural variables, such as uncertainty avoidance, also influence BMS.[7] Boucher *et al.*, (2009)

presented new models for panel count data based on the zero-inflated Poisson distribution. Which from the claims distributions, propose an approximation of the accident distribution, which can provide insight into the behaviour of insured.[10]

Dionne, G. & Ghali, O. (2005) indicate that BMS reduced the probability of reported accidents for good risks but had no effect on bad risks.[12]

Kim, Hyojoung *et.al.*, (2009) obtained only the accident records of the current contract year data. It however had bonus-malus rates that reflect past accidents. A bonus-malus rate ranges from 0.4 to 2.0, with a starting rate at 1.0. The current bonus-malus rate depends on the individual's prior rate. In the case of bonus rate, for example, having no accident during the last 3 years makes the rate 1.0 if the prior rate is higher than 1.1, and decreases the rate by 0.1 if it is 1.1 or below. The malus rate is determined by the number of accidents and the severity of each accident. Each accident increases the rate at least by 0.05. This way the bonus-malus rate reflects the insured's accident records for at least the past 3 years.[20]

The key element in the bonus-malus system, the claims coefficient, is obtained by the conversion of cumulative claim points, calculated as the sum of no-claim points and claim points in the past three years. In general, the marginal change in the premium for each additional claim is 20 per cent. In the case that a driver in a claim is not the policy-holder, the claim point is still counted against the policy-holder. Similarly, each no-claim year corresponds to a 20 per cent discount. The official loading factor permitted by Taiwan's insurance authority is 34.5 per cent.[17]

One possibility for the mixed results is variation in the underwriting process that can help limit the information asymmetry between the insurance company and the policyholder. Experience rating, often implemented through bonus-malus schemes such as the NCD in Australia, can be an important component of underwriting.[18]

Lemaire and Zi1 analysed 30 BMSBMS from all over the world and used Principal Components Analysis to create an Index of Toughness of BMS. They concluded that the following two conjectures were valid for their sample:[7]

- Conjecture 1: The design of a BMS is influenced by economic development. More advanced countries tend to enforce tougher BMS. As insurance markets become more mature and policyholders become more sophisticated, countries switch to more severe BMS. BMS in developing countries are often embryonic, with just a few classes and very simple transition rules.

- Conjecture 2: Culture also seems to influence BMS design, as countries with similar GDP per capita may adopt very different BMS. Strong geographical patterns emerge from the ranking of BMS. Countries from Northern and Central Europe use the toughest BMS. Asian countries for the most part adopt fairly mild BMS.

3. METHODOLOGY

A numerical illustration using at-fault and not-at-fault claims of a Saudi insurance company is included to support this discussion. Sample of 30000 case included number of claims and amount of claims to estimate parameters of model, according to the probability distributions fitted.

3.1 Mathematical Framework

It is assumed that the number of claims of each policyholder is independent from the severity of each claim, in order to deal with the frequency and the severity component separately.[1]

3.1.1 Frequency Distribution

In a bonus - malus system in car insurance, the bonus class of a customer is updated from one year to the next, as a function of the current class and the number of claims in the year (assumed Poisson), where the Poisson rate according to which claims are generated for a customer is the outcome of a random variable specific to the customer.[9]

Assuming that the random variable Z expresses the number of claims, given the parameter λ , assuming that it follows a Poisson distribution, where the probability distribution function is:

$$f(z|\lambda) = \frac{e^{-\lambda} \lambda^z}{z!}, \lambda > 0 \text{ and } z = 0, 1, 2, 3 \dots \quad (1)$$

The parameter λ is denoting the different underlying risk of each policyholder to have an accident. According to Bayesian approach, let us assume for the structure function that:

$$\lambda \sim \text{gamma}(\tau, \alpha)$$

The probability density function of λ is a prior distribution of the form:

$$\pi(\lambda) = \frac{\tau^\alpha \lambda^{\alpha-1} e^{-\tau\lambda}}{\Gamma(\alpha)}, \quad \lambda > 0, \tau > 0, \alpha > 0 \quad (2)$$

Where τ is the scale parameter and α is the shape parameter and the expected value and the variance as follows:

$$E(\lambda) = \alpha/\tau \quad (3) \quad 188$$

$$Var(\lambda) = \alpha/\tau^2 \quad (4)$$

The likelihood function is:

$$L(\lambda/z) \propto e^{-\sum_{i=1}^t \lambda} \lambda^{\sum_{i=1}^t z_i} \quad (5)$$

Assuming that: $\sum_{i=1}^t z_i = K \quad (6)$

Let us K expresses the total number of claims during the period t years.

$$L(\lambda/\underline{z}) \propto e^{-t\lambda} \lambda^K \quad (7)$$

The posterior structure function of λ for a policyholder. It is that

$$\pi(\lambda/\underline{z}) \propto \lambda^{K+\alpha-1} e^{-(t+\tau)\lambda} \quad (8)$$

This distribution is similar to gamma and completed as follows:

$$\pi(\lambda/\underline{z}) = \frac{(t+\tau)^{K+\alpha} \lambda^{K+\alpha-1} e^{-(t+\tau)\lambda}}{\Gamma(K+\alpha)} \quad (9)$$

Where $(t+\tau)$ is the scale parameter and $(K+\alpha)$ is the shape parameter and the expected value according the Bayes' estimation as follow:

$$E(\lambda/\underline{z}) = \frac{K+\alpha}{t+\tau} \quad (10)$$

3.1.2 Severity Distribution

Let x be the size of the claim of each insured and has a probability density function (PDF) given by:

$$X \sim \text{gamma}(\theta, \beta) \quad 207$$

$$f(X) = \frac{\theta^\beta X^{\beta-1} e^{-\theta X}}{\Gamma\beta}, \quad x > 0, \beta > 0, \theta > 0 \quad (11)$$

The probability density function of θ is prior distribution of the form:

$$\theta \sim \text{gamma}(\psi, \gamma) \quad 211$$

$$\pi(\theta) = \frac{\psi^\gamma \theta^{\gamma-1} e^{-\psi\theta}}{\Gamma\gamma}, \psi > 0, \gamma > 0, \theta > 0 \quad (12) \quad 212$$

Likelihood function of θ parameter

$$L(\theta/\underline{X}) \propto \theta^{\sum_{i=1}^K \beta} e^{-\theta \sum_{i=1}^K X_i} \quad (13)$$

Assuming that:

$$\sum_{i=1}^K X_i = S \quad (14)$$

Let us S denote the total amount of claims during the period t years.

$$\sum_{i=1}^K \beta = K \quad (15)$$

$$L(\theta/\underline{X}) \propto \theta^K e^{-\theta S} \quad (16)$$

The posterior structure function of θ for a policyholder. It is that

$$\pi(\theta/\underline{X}) \propto \theta^{K+\gamma-1} e^{-\theta(\psi+S)} \quad (17)$$

$$\pi(\theta/\underline{X}) = \frac{(\psi+S)^{K+\gamma} \theta^{K+\gamma-1} e^{-(\psi+S)\theta}}{\Gamma(K+\gamma)} \quad (18)$$

Where $(\psi+S)$ is the scale parameter and $(K+\gamma)$ is the shape parameter and the expected value according the Bayes' estimation as follow:

$$E(\theta/\underline{X}) = \frac{K+\gamma}{\psi+S} \quad (19)$$

where:

$$E(X/\theta) = \beta/\theta \quad (20)$$

$$E(X/\theta) = \frac{\beta(\psi+S)}{K+\gamma} \quad (21)$$

Thus, the net premium that must be paid from that of each policyholder or specific group of policyholders will be equal to the product of $E(\lambda/\underline{z})$ and $E(X/\theta)$ it will be equal to

$$Net\ Premium = E(\lambda/\underline{z}).E(X/\theta) \quad (22)$$

$$Net\ Premium = \frac{K+\alpha}{t+\tau} \cdot \frac{\beta(\psi+S)}{K+\gamma} \quad (23)$$

The net premiums when no claims are occurs with over time, given by:

$$Net\ Premium = \frac{\alpha}{t+\tau} \cdot \frac{\beta.\psi}{\gamma} \quad (24)$$

This premium decreases with overtime and increases with the increase of the total number of claims, and also increases with the total amount of claims.

4. RESULTS AND DISCUSSION

The maximum likelihood estimators of the parameters are $\hat{\alpha} = 1.4$, $\hat{t} = 10$, $\hat{\beta} = 0.368$, $\hat{\gamma} = 0.368$ and $\hat{\psi} = 100000$. The researcher computes the BMS net vehicles insurance premiums when no claims occurs with over time. Also using formula (23) to calculate the BMS net vehicles insurance premium, when at least an accident occurs. Thus the steps that must be followed in order to find the BMS net vehicles insurance premiums are:

- Determine the age of the policy t .
- Determine the total number of claims K by the policyholder in t years.
- Determine the total amount of claims by the policyholders S in t years.
- Computing the premium using (24) when no claims occurs.
- Computing the premium using (23) when at least an accident occurs.

Fig. 1 show the BMS net vehicles insurance premium when total claims equal 10000 SAR over time and total number of claims. Fig. 2 also show the BMS net vehicles insurance premium when total claims equal 100000 SAR over time and total number of claims. There is no difference in figures, where net premium increases with the increase in the total number of claims and the total amount of claims increase and decrease with over time on each level of the total losses.

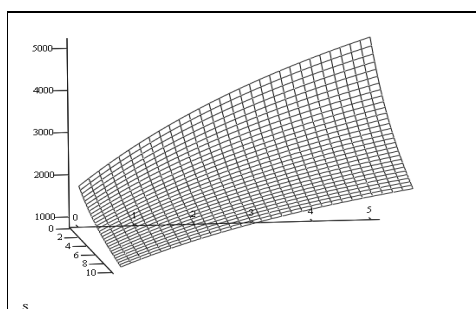


Fig. 1 The BMS net vehicles insurance premiums with total losses SAR 10000

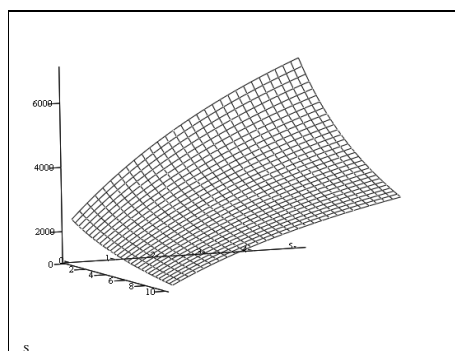


Fig. 2 The BMS net vehicles insurance premiums with total losses SAR 50000

Tables 2-10 shows the Net Vehicles Premiums with different total amount of claims. Where the net premium is higher increase in the total number of claims and the total amount of claims increase and decrease over time on each level of the total losses, as follow:

Table 2. The Net Vehicles Premiums with total claims size of 2500

Year		Number of claims				
t	0	1	2	3	4	5
0	1597			NA		
1	1451	2518	3236	3832	4335	4764
2	1330	2289	2942	3484	3941	4331
3	1228	2099	2697	3193	3612	3970
4	1140	1937	2489	2948	3334	3665
5	1064	1799	2312	2737	3096	3403
6	998	1679	2157	2555	2890	3176
7	939	1574	2023	2395	2709	2978
8	887	1481	1904	2254	2550	2803
9	840	1399	1798	2129	2408	2647
10	798	1325	1703	2017	2281	2508

Table 3. The Net Vehicles Premiums with total claims size of 5000

Year		Number of claims				
t	0	1	2	3	4	5
0	1597			NA		
1	1451	2580	3315	3926	4440	4881
2	1330	2345	3014	3569	4037	4437
3	1228	2150	2763	3271	3700	4067
4	1140	1984	2550	3020	3416	3754
5	1064	1843	2368	2804	3172	3486
6	998	1720	2210	2617	2960	3254
7	939	1612	2072	2454	2775	3050
8	887	1517	1950	2309	2612	2871

9	840	1433	1842	2181	2467	2711
10	798	1358	1745	2066	2337	2569

Table 4. The Net Vehicles Premiums with total claims size of 10000

Year		Number of claims				
t	0	1	2	3	4	5
0	1597			NA		
1	1451	2702	3473	4112	4652	5113
2	1330	2457	3157	3739	4229	4648
3	1228	2252	2894	3427	3877	4261
4	1140	2079	2671	3163	3578	3933
5	1064	1930	2481	2937	3323	3652
6	998	1802	2315	2742	3101	3409
7	939	1689	2171	2570	2907	3196
8	887	1590	2043	2419	2736	3008
9	840	1501	1929	2285	2584	2841
10	798	1422	1828	2164	2448	2691

Table 5. The Net Vehicles Premiums with total claims size of 25000

Year		Number of claims				
t	0	1	2	3	4	5
0	1597			NA		
1	1451	3071	3947	4673	5286	5810
2	1330	2792	3588	4248	4806	5282
3	1228	2559	3289	3894	4405	4842
4	1140	2362	3036	3595	4066	4469
5	1064	2194	2819	3338	3776	4150
6	998	2047	2631	3116	3524	3873
7	939	1919	2467	2921	3304	3631
8	887	1806	2321	2749	3110	3418
9	840	1706	2193	2596	2937	3228
10	798	1616	2077	2460	2782	3058

Table 6. The Net Vehicles Premiums with total claims size of 50000

Year		Number of claims				
t	0	1	2	3	4	5
0	1597					
1	1451	3685	4736	5608	6343	6972
2	1330	3350	4305	5098	5767	6338
3	1228	3071	3947	4673	5286	5810
4	1140	2835	3643	4314	4880	5363
5	1064	2632	3383	4006	4531	4980
6	998	2457	3157	3739	4229	4648
7	939	2303	2960	3505	3965	4358
8	887	2168	2786	3299	3731	4101

9	840	2047	2631	3116	3524	3873
10	798	1940	2493	2952	3339	3670

Table 7. The Net Vehicles Premiums with total claims size of 75000

Year		Number of claims				
t	0	1	2	3	4	5
0	1597			NA		
1	1451	4299	5525	6543	7401	8134
2	1330	3908	5023	5948	6728	7395
3	1228	3583	4604	5452	6167	6779
4	1140	3307	4250	5033	5693	6257
5	1064	3071	3947	4673	5286	5810
6	998	2866	3683	4362	4934	5423
7	939	2687	3453	4089	4625	5084
8	887	2529	3250	3849	4353	4785
9	840	2389	3070	3635	4112	4519
10	798	2263	2908	3443	3895	4281

Table 8. The Net Vehicles Premiums with total claims size of 100000

Year		Number of claims				
t	0	1	2	3	4	5
0	1597			NA		
1	1451	4913	6314	7477	8458	9296
2	1330	4467	5740	6798	7689	8451
3	1228	4095	5262	6231	7048	7747
4	1140	3780	4857	5752	6506	7151
5	1064	3510	4510	5341	6041	6640
6	998	3276	4210	4985	5639	6197
7	939	3071	3947	4673	5286	5810
8	887	2890	3714	4398	4975	5468
9	840	2730	3508	4154	4699	5165
10	798	2586	3323	3935	4452	4893

Table 9. The Net Vehicles Premiums with total claims size of 150000

Year		Number of claims				
t	0	1	2	3	4	5
0	1597			NA		
1	1451	6142	7893	9347	10570	11620
2	1330	5584	7175	8497	9611	10560
3	1228	5118	6578	7789	8810	9684
4	1140	4725	6072	7190	8133	8939
5	1064	4387	5638	6676	7552	8300
6	998	4095	5262	6231	7048	7747
7	939	3839	4933	5842	6608	7263
8	887	3613	4643	5498	6219	6835

9	840	3412	4385	5193	5874	6456
10	798	3233	4154	4919	5564	6116

Table 10. The Net Vehicles Premiums with total claims size of 200000

Year		Number of claims				
t	0	1	2	3	4	5
0	1597			NA		
1	1451	7370	9472	11220	12690	13940
2	1330	6700	8611	10200	11530	12680
3	1228	6142	7893	9347	10570	11620
4	1140	5669	7286	8628	9759	10730
5	1064	5264	6765	8011	9062	9960
6	998	4913	6314	7477	8458	9296
7	939	4606	5920	7010	7929	8715
8	887	4335	5572	6598	7463	8203
9	840	4095	5262	6231	7048	7747
10	798	3879	4985	5903	6677	7339

5. Conclusion

In conclusion, Bayesian approach to a compound model, where the number of claims with time follows a compound Poisson and severities follow gamma. This model is found to calculate the BMS net vehicles insurance premium for individual policyholders, according to total number of claims and total amount of claims during a specific period of time. The data were analyzed using IBM SPSS Statistics 22 and MathCad 2001 professional software.

This paper presents the design of optimal BMS using finite mixture models. The proposed model shows that the more accidents caused and size of loss that each claim incurred the higher the premium and the net premium always decreases when no accidents are caused with over time.

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