

Original Research Article

Dualities Between "Kamal & Mahgoub Integral Transforms" and "Some Famous Integral Transforms"

Abstract :

In this paper, Abdelilah Kamal and Mohand Mahgoub recently introduced integral transforms, that are closely studied in the relation to the some existing famous integral transforms , which are defined in the time domain. The study also tried to establish the duality relations existing between these newly introduced integral transforms and Laplace, Sumudu, Elzaki and Aboodh integral transforms as an interest. Further, supporting illustrations obtained from some test functions as examples are presented.

Keywords: Kamal , Mahgoub ,Aboodh ,Elzaki ,Sumudu, Laplace Transforms.

1. Introduction :

Integral transforms are mathematical tools for solving differential and integral equations for centuries. However, this old area has recently got a center stage among many researchers by introducing many integral transforms among which are [1-6]. Furthermore, these transforms are studied and analyzed in order to establish relationships and connections existing among them among which are [7-9]. However, in this paper, the recently introduced integral transforms by Abdelilah Kamal [10] and Mohand Mahgoub [11] are closely studied in relation to the some existing famous integral transforms that are defined in the time domain. The study also tried to establish the duality relations existing between these newly introduced integral transforms and Laplace, Sumudu , Elzaki and Aboodh integral transforms as particular cases and further some test functions are examined as examples.

The paper is organized as follows: In Section 2, we present the basic concept of Kamal and Mahgoub transforms. Section 3 presents Kamal transform dualities, while Section 4 presents Mahgoub transform dualities and Section 5 gives the conclusion.

2. Kamal and Mahgoub Transforms :

In this section, we present the basic concept of Kamal and Mahgoub transforms.

2.1. Kamal Transform :

Kamal transform is a newly introduced integral transform similar to Laplace transform [12] and other integral transforms [1-6] that are defined in the time domain $t \geq 0$, and for functions of exponential order.

We consider functions in the set B defined by:

$$B = \{ f: |f(t)| < Q e^{\frac{|t|}{k_j}} \text{ if } t \in (-1)^j \times [0, \infty), j = 1, 2; (Q, k_1, k_2 > 0) \}$$

For a given function in the set B , the constant Q must be finite number; k_1, k_2 may be finite or infinite.

Kamal integral transform denoted by the operator $K(.)$ is defined (by [10]) by the integral equation:

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$$G(u) = K[f(t)] = \int_0^\infty e^{\frac{-t}{u}} f(t) dt, \quad u \in (k_1, k_2) \quad (1)$$

39

40 2.1.1. Kamal Transform for Derivatives :

42 For any given function $f(t)$ defined over the set B , Kamal transform of derivatives are given as
43 follows:

44

$$K[f^{(n)}(t)] = u^{(-n)}K[f(t)] - \sum_{k=0}^{n-1} u^{k-n+1} f^{(k)}(0), \quad \text{for } n = 0, 1, 2, \dots$$

45

46 2.2 . Mahgoub Transforms :

47 The newly introduced integral transform by Mahgoub [11] is an integral transform that has the same
48 setting with Kamal transform [10]. Thus, Mahgoub integral transform denoted by the operator $M(\cdot)$ is
49 defined by:

50

$$H(u) = M[f(t)] = u \int_0^\infty e^{-ut} f(t) dt, \quad u \in (k_1, k_2) \quad (2)$$

52

53 2.2.1. Mahgoub Transform for Derivatives :

55 For any given function $f(t)$ defined over the set B , Mahgoub transform of derivatives are given as
56 follows:

$$M[f^{(n)}(t)] = u^{(n)}M[f(t)] - \sum_{k=0}^{n-1} u^{n-k} f^{(k)}(0), \quad \text{for } n = 0, 1, 2, \dots$$

57 2.3. Connection Between Kamal and Mahgoub Transforms :

58 To examine the connection between Kamal and Mahgoub transforms, we give the following
59 theorem:

60 **Theorem 2.3.1 :** Let $f(t) \in B$ with Kamal transform $G(u)$ and Mahgoub transform $H(u)$, then

$$G(u) = u H\left(\frac{1}{u}\right) \quad (3)$$

63 and

$$H(u) = u G\left(\frac{1}{u}\right) \quad (4)$$

66 **Proof :** To prove Eq. (3), hence $f(t) \in B$, then from Eq. (1)

$$G(u) = \int_0^\infty e^{\frac{-t}{u}} f(t) dt = u \left(\frac{1}{u} \int_0^\infty e^{\frac{-t}{u}} f(t) dt \right)$$

67 Thus, from Eq. (2) : $G(u) = u H\left(\frac{1}{u}\right)$

68 To prove Eq. (4), from the Eq. (2), we have

$$H(u) = u \int_0^{\infty} e^{-ut} f(t) dt = u \left(\int_0^{\infty} e^{\frac{-t}{u}} f(t) dt \right)$$

Thus, from Eq. (1), we get : $H(u) = uG\left(\frac{1}{u}\right)$

Example 2.3.2 :

Consider the function $f(t) = t^2$. Kamal and Mahgoub transforms are obtained respectively as follows: $G(u) = 2u^3$ and $H(u) = \frac{2}{u^2}$, and the graph is given in figure 1:

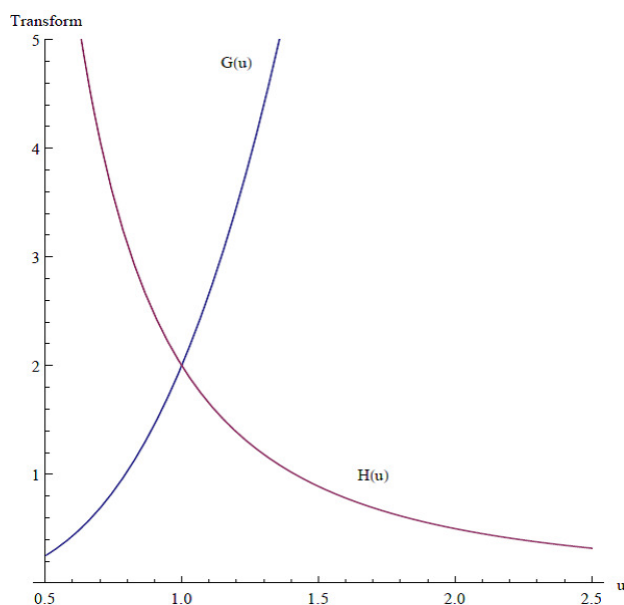


Figure 1: Plots for Kamal and Mahgoub Transforms of $f(t) = t^2$

3 . Kamal Transform Dualities :

In this section, we present Kamal transform dualities starting with Laplace , Sumudu , Elzaki and Aboodh transforms.

3.1 . Duality of "Kamal - Laplace ":

Laplace transform of $f(t)$ for $t \geq 0$, denoted by (u) , is defined by the equation

$$F(u) = L[f(t)] = \int_0^{\infty} e^{-ut} f(t) dt \quad (5)$$

Theorem 3.1.1 : Let $f(t) \in B$ with Kamal transform $G(u)$ and Laplace transform $F(u)$, then

$$G(u) = F\left(\frac{1}{u}\right) \quad (6)$$

and

$$F(u) = G\left(\frac{1}{u}\right) \quad (7)$$

92 **Proof:** To prove Eq. (6) , hence $f(t) \in B$, then from Eq. (1)

$$G(u) = \int_0^{\infty} e^{-\frac{t}{u}} f(t) dt = \int_0^{\infty} e^{-\left(\frac{1}{u}\right)t} f(t) dt.$$

93 Thus , from Eq. (5):

$$G(u) = F\left(\frac{1}{u}\right)$$

94 To prove Eq. (7) , from the Eq. (5), we have

$$F(u) = \int_0^{\infty} e^{-ut} f(t) dt = \int_0^{\infty} e^{-\frac{t}{\left(\frac{1}{u}\right)}} f(t) dt$$

95 Thus, from Eq. (1), we get :

$$F(u) = G\left(\frac{1}{u}\right)$$

96 **3.2 . Duality of "Kamal - Sumudu" :**

97 Sumudu transform of $f(t)$ for $t \geq 0$ denoted by $N(u)$, is defined by the equation

98

$$99 N(u) = S[f(t)] = \int_0^{\infty} e^{-t} f(ut) dt, \quad k_1, k_2 > 0$$

100

(8)

101

102 **Theorem 3.2.1 :** Let $f(t) \in B$ with Kamal transform $G(u)$ and Sumudu transform $N(u)$, then

103

$$104 G(u) = uN(u)$$

105

(9)

106 and

107

$$108 N(u) = \frac{1}{u} G(u)$$

109

(10)

110 **Proof :** To prove Eq. (9), hence $f(t) \in B$, then from Eq. (1)

111

$$G(u) = \int_0^{\infty} e^{-\frac{t}{u}} f(t) dt$$

112

113 Let , $w = \frac{t}{u} \Rightarrow t = uw$ and $dt = u dw$. Then ,

$$G(u) = \int_0^{\infty} e^{-w} f(uw) u dw = u \left(\int_0^{\infty} e^{-w} f(uw) dw \right)$$

114

115 Hence, on comparing with Eq. (8) , we thus obtain :

$$G(u) = uN(u)$$

116

To prove Eq. (10), from Eq. (8) ,

$$N(u) = \int_0^{\infty} e^{-t} f(ut) dt.$$

117 Let , $w = ut \Rightarrow t = \frac{w}{u}$ and $dt = \frac{dw}{u}$. Then ,

$$N(u) = \int_0^{\infty} e^{-\frac{w}{u}} f(w) \frac{dw}{u} = \frac{1}{u} \left(\int_0^{\infty} e^{-\frac{w}{u}} f(w) dw \right)$$

118 Hence, on comparing with Eq. (1) , we thus obtain :

$$N(u) = \frac{1}{u} G(u)$$

3.3 . Duality of "Kamal - Elzaki " :

Elzaki transform of $f(t)$ for $t \geq 0$, denoted by $T(u)$, is defined by the equation

$$T(u) = E[f(t)] = u^2 \int_0^\infty e^{-t} f(ut) dt, \quad u \in (k_1, k_2)$$

Theorem 3.3.1 : Let $f(t) \in B$ with Kamal transform $G(u)$ and Elzaki transform $T(u)$, then

$$G(u) = \frac{1}{u} T(u)$$

and

$$T(u) = uG(u)$$

Proof : To prove Eq. (12), hence $f(t) \in B$, then from Eq. (1)

$$G(u) = \int_0^\infty e^{-\frac{t}{u}} f(t) dt$$

Let, $w = \frac{t}{u} \Rightarrow t = uw$ and $dt = u dw$. Then,

$$G(u) = \int_0^\infty e^{-w} f(uw) u dw = u \left(\int_0^\infty e^{-w} f(uw) dw \right) = \frac{1}{u} \left(u^2 \int_0^\infty e^{-w} f(uw) dw \right)$$

Hence, on comparing with Eq. (11), we thus obtain :

$$G(u) = \frac{1}{u} T(u)$$

To prove Eq. (13), from Eq. (11),

$$T(u) = u^2 \int_0^\infty e^{-t} f(ut) dt$$

Let, $t = \frac{w}{u} \Rightarrow dt = \frac{dw}{u}$. Then,

$$T(u) = u^2 \int_0^\infty e^{-\frac{w}{u}} f(w) \frac{dw}{u} = u \left(\int_0^\infty e^{-\frac{w}{u}} f(w) dw \right).$$

Hence, on comparing with Eq. (1), we thus obtain

$$T(u) = uG(u)$$

3.4 . Duality of "Kamal - Aboodh " :

Aboodh transform of $f(t)$ for $t \geq 0$, denoted by $K(u)$, is defined by the equation

$$K(u) = A[f(t)] = \frac{1}{u} \int_0^\infty e^{-ut} f(t) dt, \quad u \in (k_1, k_2)$$

Theorem 3.4.1 : Let $f(t) \in B$ with Kamal transform $G(u)$ and Aboodh transform $K(u)$, then

$$G(u) = \frac{1}{u} K\left(\frac{1}{u}\right)$$

and

$$K(u) = \frac{1}{u} G\left(\frac{1}{u}\right)$$

Proof : To prove Eq. (15), hence $f(t) \in B$, then from Eq. (1)

$$G(u) = \int_0^\infty e^{-\frac{t}{u}} f(t) dt = \frac{1}{u} \left(\frac{1}{\frac{1}{u}} \int_0^\infty e^{-\left(\frac{1}{u}\right)t} f(t) dt \right)$$

Thus, from Eq. (14) :

$$G(u) = \frac{1}{u} K\left(\frac{1}{u}\right)$$

To prove Eq. (16), from the Eq. (14), we have

$$K(u) = \frac{1}{u} \int_0^{\infty} e^{-ut} f(t) dt = \frac{1}{u} \left(\int_0^{\infty} e^{\frac{-t}{u}} f(t) dt \right)$$

Thus , from Eq. (1), we get :

$$K(u) = \frac{1}{u} G\left(\frac{1}{u}\right)$$

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Example 3.5 :

Consider the function $f(t) = e^{-2t}$. We obtain the following transforms:

$$F(u) = \frac{1}{u+2}, N(u) = \frac{1}{1+2u}, T(u) = \frac{u^2}{1+2u}, K(u) = \frac{1}{u^2+2u} \text{ and } G(u) = \frac{u}{1+2u}$$

Figure 2 gives the illustration of the above transforms:

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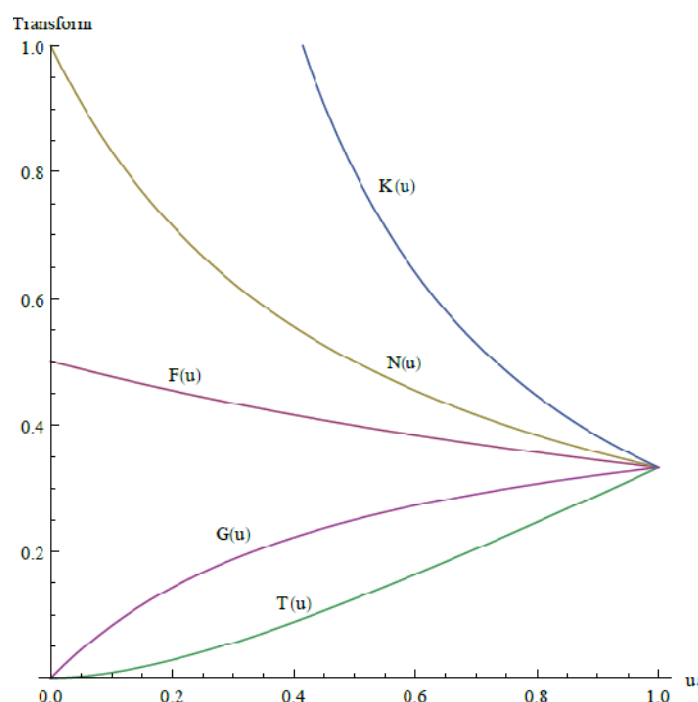


Figure 2: Plots for Laplace, Sumudu , Elzaki , Aboodh and Kamal Transforms of $f(t) = e^{-2t}$.

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4. Mahgoub Transform Dualities :

In the same pattern, we present Mahgoub transform dualities, together with the definitions of the other integral transforms presented in section 3.

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4.1 . Duality of "Mahgoub - Laplace " :

Theorem 4.1.1 : Let $f(t) \in B$ with Mahgoub transform $H(u)$ and Laplace transform $F(u)$, then

167

168

$$H(u) = uF(u)$$

169

$$(17)$$

and

$$F(u) = \frac{1}{u} H(u)$$

(18)

Proof : To prove Eq. (17) , hence $f(t) \in B$, then from Eq. (2)

$$H(u) = u \int_0^{\infty} e^{-ut} f(t) dt = u \left(\int_0^{\infty} e^{-ut} f(t) dt \right)$$

Thus, from Eq. (5) :

$$H(u) = uF(u)$$

To prove Eq. (18), from the Eq. (5), we have

$$F(u) = \int_0^{\infty} e^{-ut} f(t) dt = \frac{1}{u} \left(u \int_0^{\infty} e^{-ut} f(t) dt \right)$$

Thus, from Eq. (2), we get :

$$F(u) = \frac{1}{u} H(u)$$

4.2 . Duality of "Mahgoub - Sumudu :

Theorem 4.2.1 : Let $f(t) \in B$ with Mahgoub transform $H(u)$ and Sumudu transform $N(u)$, then

$$H(u) = N\left(\frac{1}{u}\right)$$

(19)

and

$$N(u) = H\left(\frac{1}{u}\right)$$

(20)

Proof : To prove Eq. (19) , hence $f(t) \in B$, then from Eq. (2)

$$H(u) = u \int_0^{\infty} e^{-ut} f(t) dt$$

Let , $w = ut \Rightarrow t = \frac{w}{u}$ and $dt = \frac{dw}{u}$.Then ,

$$H(u) = \int_0^{\infty} e^{-w} f\left(\frac{w}{u}\right) dw = \int_0^{\infty} e^{-w} f\left(\frac{1}{u}w\right) dw$$

Hence, on comparing with Eq. (8) , we thus obtain : $H(u) = N\left(\frac{1}{u}\right)$

To prove Eq. (20), from Eq. (8) ,

$$N(u) = \int_0^{\infty} e^{-t} f(ut) dt$$

Let , $w = ut \Rightarrow t = \frac{w}{u}$ and $dt = \frac{dw}{u}$. Then ,

$$N(u) = \int_0^{\infty} e^{-\frac{w}{u}} f(w) \frac{dw}{u} = \frac{1}{u} \left(\int_0^{\infty} e^{-\left(\frac{1}{u}\right)w} f(w) dw \right)$$

Hence, on comparing with Eq. (2) , we thus obtain : $N(u) = H\left(\frac{1}{u}\right)$

4.3 . Duality of "Mahgoub - Elzaki " :

Theorem 4.3.1 : Let $f(t) \in B$ with Mahgoub transform $H(u)$, and Elzaki transform $T(u)$, then

$$H(u) = u^2 T(u)$$

(21)

and

$$T(u) = u^2 H\left(\frac{1}{u}\right) \quad (22)$$

Proof : To prove Eq. (21) , hence $f(t) \in B$, then from Eq. (2)

$$H(u) = u \int_0^\infty e^{-ut} f(t) dt$$

203

204 Let , $w = ut \Rightarrow t = \frac{w}{u}$ and $dt = \frac{dw}{u}$.Then ,

205

$$H(u) = \int_0^\infty e^{-w} f\left(\frac{w}{u}\right) dw = u^2 \left(\left(\frac{1}{u}\right)^2 \int_0^\infty e^{-w} f\left(\frac{1}{u} w\right) dw \right)$$

206 Hence, on comparing with Eq. (11) , we thus obtain : $H(u) = u^2 T\left(\frac{1}{u}\right)$

207 To prove Eq. (22), from Eq. (11)

$$T(u) = u^2 \int_0^\infty e^{-t} f(ut) dt$$

208 Let , $w = ut \Rightarrow t = \frac{w}{u}$ and $dt = \frac{dw}{u}$. Then ,

$$T(u) = u \int_0^\infty e^{-\frac{w}{u}} f(w) dw = u^2 \left(\frac{1}{u} \int_0^\infty e^{-\left(\frac{1}{u}\right)w} f(w) dw \right)$$

209 Hence, on comparing with Eq. (2) , we thus obtain : $T(u) = u^2 H\left(\frac{1}{u}\right)$

210 4.4 . Duality of "Mahgoub - Aboodh " :

211 **Theorem 4.4.1 :** Let $f(t) \in B$ with Mahgoub transform $H(u)$ and Aboodh transform $K(u)$, then

212

$$H(u) = u^2 K(u)$$

214 (23)

215 and

$$K(u) = \frac{1}{u^2} H(u)$$

217 (24)

218 **Proof :** To prove Eq. (23) , hence $f(t) \in B$, then from Eq. (2)

$$H(u) = u \int_0^\infty e^{-ut} f(t) dt = u^2 \left(\frac{1}{u} \int_0^\infty e^{-ut} f(t) dt \right)$$

219 And , we obtain from Eq. (14) : $H(u) = u^2 K(u)$

220 To prove Eq. (24), from the Eq. (14), we have

$$K(u) = \frac{1}{u} \int_0^\infty e^{-ut} f(t) dt = \frac{1}{u^2} \left(u \int_0^\infty e^{-ut} f(t) dt \right)$$

221 Thus, from Eq. (2), we get $K(u) = \frac{1}{u^2} H(u)$

222 Example 4.5 :

223 Consider the function $f(t) = t^2$. We obtain the following transforms:

$$224 F(u) = \frac{2}{u^3} , N(u) = 2u^2 , T(u) = 2u^4 , K(u) = \frac{2}{u^4} \text{ and } H(u) = \frac{2}{u^2}.$$

225 Figure 3 gives the illustrations of the connections between the transforms under consideration as
226 follows:
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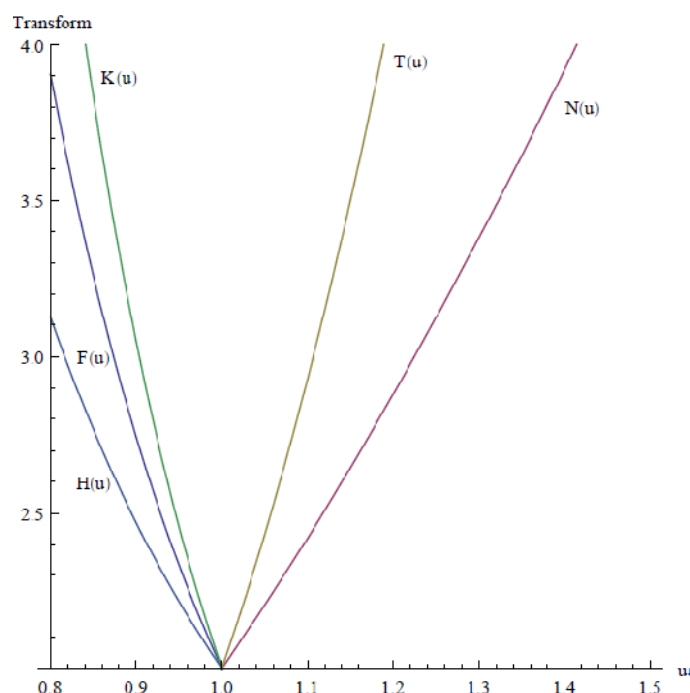


Figure 3: Plots for Laplace, Sumudu, Elzaki, Aboodh and Mahgoub Transforms of $f(t) = t^2$.

5. Conclusion :

In conclusion, the duality relations between Kamal and Mahgoub integral transforms with Laplace, Sumudu, Elzaki and Aboodh transforms are established together with their converses. The dualities are presented in forms of theorems which were later proved in a friendly manner. Some test functions are considered as examples and illustrated graphically with the help of Mathematica software.

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