

Original Research Article

Formation Damage Analysis due to Filtrate Invasion in Deviated Wells: A Numerical Approach

ABSTRACT

During production, drilling, completion, stimulation and work-over operations, the formation is exposed to several types of fluids which have high tendencies to impair and impact gas and oil productivity by reducing pore throat size and relative permeability. Therefore, it is desirable to accurately characterize filtration process during these operations. A mathematical model was developed describing the mechanism of formation damage in deviated wells. Rate of mud cake build up, cumulative filtrate volume and depth of invasion into the formation at different angles of inclination were determined. The model employed numerical approach rather than the conventional analytical method to solve the equations developed using the fully implicit discretization scheme. The effect of filtration on the angle of deviation was determined and this should help in planning and allocation of drilling time in order to reduce formation damage. Also the analysis shows the result of infiltration on cake thickness and filtrate volume at the early, middle and late time of permeation into the formation. The model was validated with the previous analytical model. Results showed that the previous conventional analytical models developed overestimate the formation damage when compared to the numerical model.

Keywords: Deviated wells, Formation damage, Filtration, Mud cake, Numerical model.

1. INTRODUCTION

The pressure differential between the hydrostatic pressure of the mud column and the formation pressure gives rise to filtration from drilling fluids. The hydrostatic pressure in the borehole is always greater than the formation pressure, fluid filters into the porous medium, depositing a porous, permeable and compressible cake of mud particles on the wall of the borehole. Typically, any unintended impedance to the flow of fluids into and out of the wellbore is referred to as formation damage. This broad definition includes flow restrictions caused by a reduction in permeability in the near-wellbore region, changes in relative permeability to the hydrocarbon phase, and unintended flow restriction in the completion itself. Any formation damage potential is only realized during drilling, completion and production operations. In their work [1] submitted that mud-filtrate invasion takes place in permeable rock formations penetrated by a well that is hydraulically overbalanced by mud circulation. This condition is of interest in numerous oilfield applications including drilling engineering, reservoir simulation, reservoir stimulation, and well-log interpretation.

One of the relevant technical problems often considered in mud filtrate invasion studies is the description of mudcake buildup. The development and thickening of mudcake on the borehole wall causes the permeability of mudcake to decrease which in turn causes the flow rate of filtrate to decrease. These led to the determination of the rate at which solids accumulate on the borehole wall and the thickness and effective permeability of the mudcake by chemical and fluid mechanical processes. In 1952, [2] investigated filtration and mud cake formation under high temperatures and pressures. Their investigation established the fact that measurements at surface temperatures cannot be used to predict bottom hole filter loss behaviour. [3] investigated filtration under both dynamic and static conditions and established that cake accumulation depends upon whether or not the mud is circulated. He observed that muds having high filtration rates deposited thick filter cakes in spite of very high eroding velocities; and that mud cakes deposited under either static or dynamic filtration possess different properties in terms of permeability, thickness and erodibility. [4] in 1953 simulated filtration under different bottom hole geometry and hydrodynamic conditions using a model

oil well. Three classes of filtration were established: static filtration, dynamic filtration and filtration from beneath the bit during drilling. In 1984, [5] showed that under dynamic condition higher annular velocity and shear rate give rise to an increase fluid loss while in 2004, [1] showed that six mechanical and petrophysical parameters and two mud parameters (filtrate viscosity and solids content) are required to calculate the flow rate of mud-filtrate invasion into permeable rock formations.

In 1963, [6] presented an analytical approach to the filtration theory. The mechanism of filtration was described by a theoretical-empirical non-linear equation which was linearized and solved explicitly under certain conditions. The study demonstrated the effects of mud properties like viscosity on filtration rate with a major conclusion that several quantities that affect dynamic filtration have no counterpart in static filtration and therefore static filtration cannot be relied on as a measure of dynamic filtration and vice versa. In his unpublished work, [7] used the nonlinear equation developed by Outmans to study the filtration through a compressible cake. He derived a relationship between the solid pressure in the cake to the fine and the cake thickness build up for both static and dynamic filtrate. He also derived an expression for the cumulative volume filtrate and radius of investigation for the static filtration process. He concluded that mud viscosity and specific cake volume can be controlled to achieve reduced filtrate volume, and that by the combinations of the times used for static and dynamic filtration, it is possible to estimate the total cumulative volume of filtrate and the expected radius of invasion during the whole period of drilling. In 2013, [8] developed a mathematical model for analyzing mud filtration in deviated wells in an effort to accurately characterize filtration process during oil well drilling. The model determined solid pressure distribution within cake, cake thickness and the effect of filtration on the angle of deviation. They also confirmed that mud filtration tend to be higher in deviated than in vertical wells.

2. MODEL DESIGN

The filtration through a compressible mud cake can be described by combining the law of conservation of mass, equation of state and Darcy's equation [8] to give:

$$q = -\frac{KA}{\mu} \left[\frac{dp}{dx} + \rho g \sin \theta \right] \quad 1$$

Applying equation (1) to horizontal and vertical directions gives:

$$\frac{\partial}{\partial y} \left[\frac{\rho K}{\mu} \frac{dp}{dy} \right] dv + \frac{\partial}{\partial y} \left[\frac{\rho^2 g \sin \theta K}{\mu} \right] dv + \frac{\partial}{\partial z} \left[\frac{\rho K}{\mu} \frac{dp}{dz} \right] dv = \phi \frac{\partial}{\partial t} [\rho dv] \quad 2$$

Writing equation (2) in terms of Terzaghi's stress theory [11], cake compressibility and differentiating gives:

$$\frac{1}{\mu} \frac{\partial}{\partial y} \left[K \frac{dp_s}{dy} \right] dv + \frac{g \sin \theta}{\mu} \frac{\partial}{\partial y} [\rho_s K] dv + \frac{1}{\mu} \frac{\partial}{\partial z} \left[K \frac{dp_s}{dz} \right] dv = \phi c dv \frac{\partial p_s}{\partial t} \quad 3$$

Introducing empirical relationship of [9] to equation (3), we have:

$$D_1 \frac{\partial}{\partial y} \left[\epsilon^\lambda \frac{\partial \epsilon}{\partial y} \right] + D_2 g \sin \theta \frac{\partial}{\partial y} [\epsilon^\lambda] + D_1 \frac{\partial}{\partial z} \left[\epsilon^\lambda \frac{\partial \epsilon}{\partial z} \right] = \phi \mu \frac{\partial \epsilon}{\partial t} \quad 4$$

Setting,

$$\lambda = \frac{n-f-1}{m+f}, \quad \gamma = \frac{m+n}{m+f} \quad 5a$$

$$D_1 = \frac{1}{f\phi_o a_o \rho_s} \left[\frac{-B(m+f)}{f\phi_o} \right]^\lambda, \quad D_2 = \frac{1}{a_o B} \left[\frac{-B(m+f)}{f\phi_o} \right]^\gamma \quad 5b$$

Equation (4) defines the diffusivity equation for the process/ mechanism of filtration in inclined wells across porous and permeable compressible cake medium. The model can be solved for various conditions of the filtration process at different angles of inclination.

2.1 Rate of Mud Cake Build Up

The rate of mud cake build up is directly proportional to the flow rate of filtrate [8,10] and applying Terzaghi's stress theory [11], the mud cake build up equation is given as:

$$\frac{dh}{dt} = b \left[\frac{1}{\mu} D_3 \varepsilon^\sigma \frac{d\varepsilon}{dy} + D_4 \right] + b \left[\frac{1}{\mu} D_3 \varepsilon^\sigma \frac{d\varepsilon}{dz} \right] \quad 6$$

2.2 Cumulative Volume of Filtrate

In their work [8, 10], showed that the general equation that describes cumulative filtrate volume at a given time, t, with rate q at the filtrate through put in an inclined well is expresses as:

$$Q = \frac{D_3}{\mu} \int_0^t \varepsilon^\sigma \frac{d\varepsilon(y,t)}{dy} dt + D_4 t + \frac{D_4}{\mu} \int_0^t \varepsilon^\sigma \frac{d\varepsilon(z,t)}{dy} dt \quad 7a$$

$$\varpi = \frac{-(m+f+1)}{m+f}, \quad D_3 = \frac{-KA}{m+f} \left(\frac{-B(m+f)}{f\phi_o} \right)^{\frac{-1}{m+f}}, \quad D_4 = \frac{-KA}{m+f} \rho g \sin\theta \quad 7b$$

2.3 Depth of Invasion

The depth of invasion of filtrate is influenced by both conditions within the wellbore and formation. The depth of invasion, otherwise known as spatial space or extent of invasion can be estimated from the filtrate volume using the material balance approach.

i.e. Volume of filtrate = pore volume invaded

$$Q_t = \pi h \phi (r_a^2 - r_w^2) E \quad 8$$

This gives:

$$r_a = \sqrt{\frac{Q_t}{\pi h \phi E} + r_w^2} \quad 9$$

3.0 NUMERICAL SOLUTION TO DEVELOPED MODELS

Inner boundary, outer boundary and moving boundary conditions are the boundary conditions for the solution of the equation governing the system of filtration process in an inclined well include.

3.1 Inner Boundary Condition

When filtration process begins, some cake is deposited at the wall of the borehole. The fluid pressure takes a constant value of zero, i.e. $P_f = 0$ $P_t = P_f + P_s$ then, $P_t = P_s$.

120 Therefore the stress condition becomes:

$$121 \quad \varepsilon(0, t) = \varepsilon_o \forall t \geq 0 \quad 10a$$

$$122 \quad \varepsilon_o = \frac{f\phi_o P_i^{-(m+f)}}{-B(m+f)} \quad 10b$$

124 3.2 Outer Boundary Condition

125 As the drilling of the deviated well goes on, the well is exposed to drilling fluids, however, before the
126 filtration commences, no cake is formed. This implies that there is no solid pressure; we therefore left
127 with fluid pressure (i.e., $P_t = P_f$ as $P_s = 0$)

$$128 \quad \varepsilon(h, t) = \varepsilon_i \forall t \geq 0 \quad 11$$

130 Substituting the boundary conditions into equation (4) and rearranging, gives:
131
132

$$133 \quad \frac{\partial \theta}{\partial t} = J \left[\frac{\partial^2 \theta}{\partial y^2} + \frac{\partial^2 \theta}{\partial z^2} \right] \quad 12$$

$$134 \quad \text{Where: } J = \frac{D_i \varepsilon^{\lambda}}{\mu \phi} \quad 13$$

135 Equation (12) which describes the filtration process in deviated wells is 2-D nonlinear partial
136 differential in nature, analytical method only provides an exact solution to simplified problems while
137 numerical method provides solution at discrete points within the system. The partial differential
138 equation was discretized employing the fully implicit finite difference discretization scheme.

139 3.3 Discretization Process

140 Discretizing the right hand side of equation (13) using Taylor's series,

$$141 \quad \theta_{m+1} = \theta(y_m + h) = \theta(y_m) + h\theta'(y_m) + \frac{h^2}{2}\theta''(y_m) + \frac{h^3}{6}\theta'''(y_m) + o(h^4) \quad 14$$

$$142 \quad \theta_{m-1} = \theta(y_m - h) = \theta(y_m) - h\theta'(y_m) + \frac{h^2}{2}\theta''(y_m) - \frac{h^3}{6}\theta'''(y_m) + o(h^4) \quad 15$$

143 By adding equations (14) and (15) gives an expression of second derivative at point:

$$144 \quad \theta''(y_m) = \frac{\theta_{m+1} - 2\theta_m + \theta_{m-1}}{h^2} + o(h^2) \quad 16a$$

145 Getting the same relation for z variable

$$146 \quad \theta''(z_m) = \frac{\theta_{m+1} - 2\theta_m + \theta_{m-1}}{h^2} + o(h^2) \quad 16b$$

$$147 \quad \text{Setting } h_y = \frac{\text{size}(y)}{N_y}, \quad h_z = \frac{\text{size}(z)}{N_z} \quad 17a$$

$$148 \quad \text{And with convention, } \theta(z_m, y_m) = \theta(i, j) \quad 17b$$

149 Right and side of equation (12), becomes:

$$\frac{\partial^2 \theta}{\partial z^2} + \frac{\partial^2 \theta}{\partial y^2} = \frac{\theta_{i+1,j} - 2\theta_{i,j} + \theta_{i-1,j}}{h_z^2} + \frac{\theta_{i,j+1} - 2\theta_{i,j} + \theta_{i,j-1}}{h_y^2} \quad 18$$

151

152 Discretizing the left hand side of equation (12),

153

$$\frac{\partial \theta_n(z_m, y_m)}{\partial t} = \frac{\theta_{i,j}^{n+1} + \theta_{i,j}^n}{\Delta t} \quad 19$$

155

156 Equation (12) in discretized form becomes:

157

$$\frac{\theta_{i,j}^{n+1} + \theta_{i,j}^n}{\Delta t} = J \left[\frac{\theta_{i+1,j}^{n+1} - 2\theta_{i,j}^{n+1} + \theta_{i-1,j}^{n+1}}{h_z^2} + \frac{\theta_{i,j+1}^{n+1} - 2\theta_{i,j}^{n+1} + \theta_{i,j-1}^{n+1}}{h_y^2} \right] \quad 20$$

159

$$\theta_{i,j}^n = s_z \theta_{i+1,j}^{n+1} + s_z \theta_{i-1,j}^{n+1} - (1 + 2s_z + 2s_y) \theta_{i,j}^{n+1} + s_y \theta_{i,j+1}^{n+1} + s_y \theta_{i,j-1}^{n+1} \quad 21a$$

161

$$s_z = \frac{J\Delta t}{h_z^2}, \quad s_y = \frac{J\Delta t}{h_y^2} \quad 21b$$

163

Equation (21a) is the Implicit Pentadiagonal Matrix Scheme of equation (20)

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Expressing stress in dimensionless form, $\mathcal{E} = \theta \mathcal{E}_o$ 22

165

Applying initial and outer boundary conditions to dimensionless form

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Initial boundary condition:

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$$\mathcal{E}(0, t) = \mathcal{E}_o \forall t \geq 0; \quad \theta(0, t) = \frac{\mathcal{E}(0, t)}{\mathcal{E}_o} = \frac{\mathcal{E}_o}{\mathcal{E}_o} = 1 \quad 23$$

169

Outer boundary condition:

$$\mathcal{E}(h, t) = \mathcal{E}_1 \forall t \geq 0; \quad \theta(1, t) = \frac{\mathcal{E}(h, t)}{\mathcal{E}_o} = \frac{\mathcal{E}_1}{\mathcal{E}_o} = \left(\frac{p_f}{p_t} \right)^{-(m+f)} \quad 24$$

171

3.3.1 Discretized Mud Cake Thickness

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Integrating equation (6), gives:

$$h = \left[\frac{b}{\mu} D_3 \mathcal{E}^\sigma \mathcal{E}_o \frac{\theta_{i+1,j}^{n+1} - \theta_{i,j}^{n+1}}{h_z^2} + b D_4 + \frac{b}{\mu} D_3 \mathcal{E}^\sigma \mathcal{E}_o \frac{\theta_{i,j+1}^{n+1} - \theta_{i,j}^{n+1}}{h_z^2} \right] t \quad 25$$

174

3.3.2 Discretized Cumulative Filtrate Volume

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Integrating equation (7), gives:

$$Q = \frac{1}{\mu h_z^2} D_3 \mathcal{E}^\sigma \mathcal{E}_o (\theta_{i+1,j}^{n+1} - \theta_{i,j}^{n+1}) + D_4 + \frac{1}{\mu h_y^2} D_3 \mathcal{E}^\sigma \mathcal{E}_o (\theta_{i,j+1}^{n+1} - \theta_{i,j}^{n+1}) \quad 26$$

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3.4 SOLID PRESSURE VARIATION IN THE CAKE

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Equation (10b) relates the solid pressure within the cake to the stress function distribution, in dimensionless form, we have

181

$$p_s = \left(\frac{-B(m+f)}{f\phi_o} \right)^{\frac{-1}{m+f}} (\theta \epsilon_o)^{\frac{-1}{m+f}}$$

27

4. RESULTS AND DISCUSSION

Figure 1 which is a plot of Cake thickness versus Time at deviation angle of 30° for three different viscosities shows the relationship between cake thickness, viscosity and time. It can be deduced that mud cake build up increases as time increases. This means a linear relationship exist between mud cake build up and time.

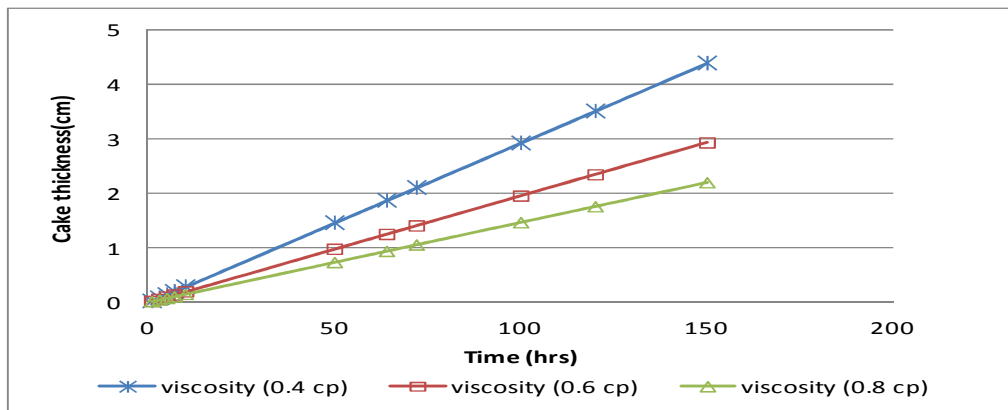


Figure 1: Cake thickness versus Time at deviation angle of 300 for three different viscosities

Figure 2 which is a plot of Cake thickness versus Deviation Angle at 10, 50 and 150 hrs show the relationship between cake thickness, deviation angle and time. The plot shows that as deviation angle increases the cake thickness also increases until a certain angle is reached where cake thickness begins to reduce as the deviation angle keeps increasing. The plot also shows cake thickness against deviation angle when just three time range was considered. The plot showed that for a given set of deviation angles as time increases, the cake thickness also increases.

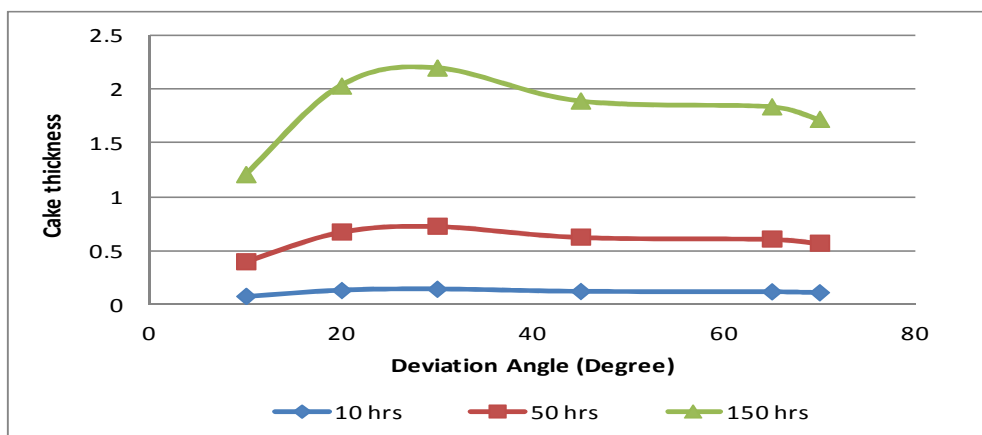
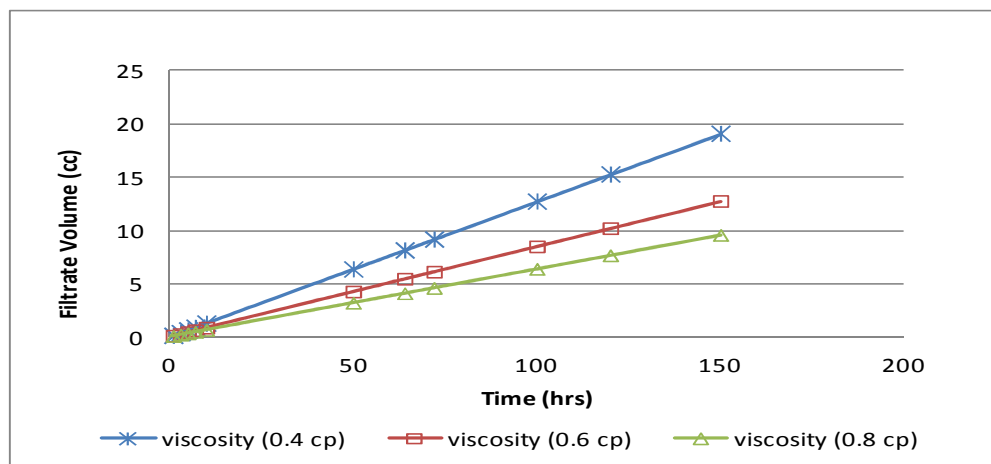


Figure 2: Cake thickness versus deviation angle at different time

Figure 3 which is a plot of filtrate volume versus Time at an angle of 30° for three different viscosities shows the relationship between filtrate volume, time and viscosity. The linear relationship implies that the cumulative filtrate increases linearly with time at different values of cake compressibility. Also the higher the viscosity of the filtrate, the lower the filtrate volume formed. This can be seen in figure 3

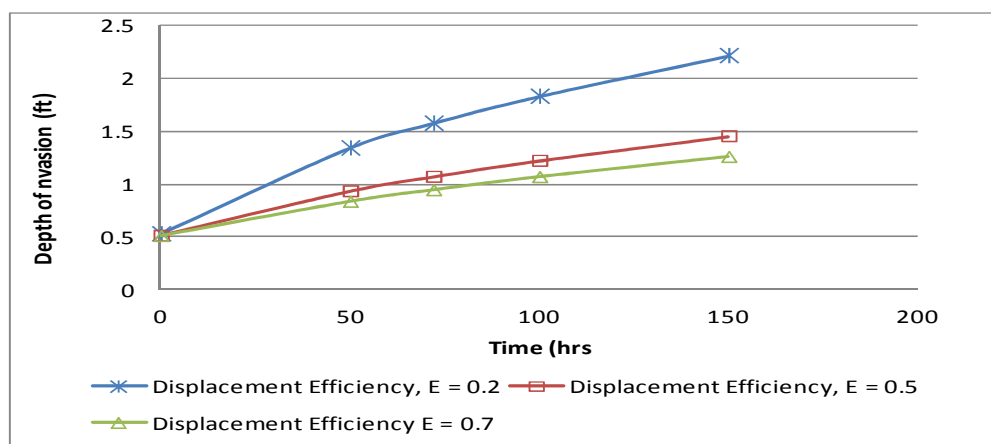
211 where three different viscosities were used in the plot of filtrate volume against time. This implies that
212 cumulative filtrate volume has an inverse relationship with viscosity of the filtrate.



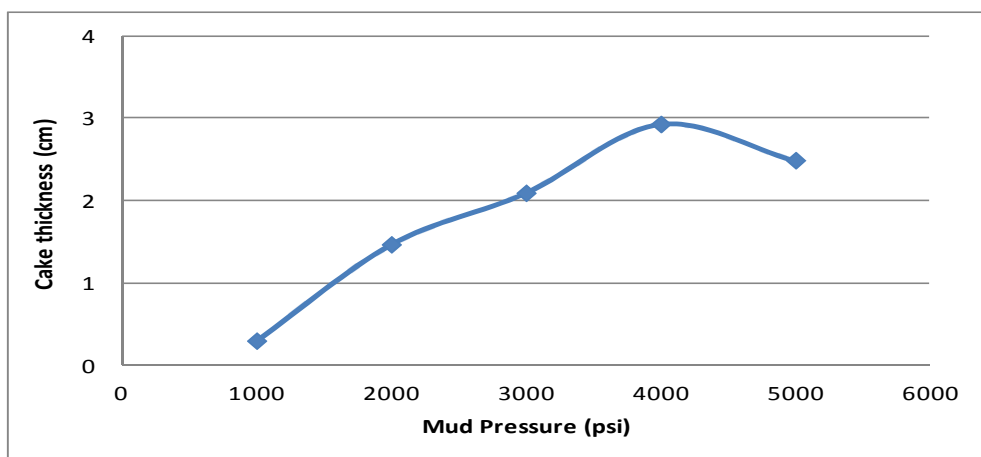
213 Figure 3: Filtrate volume versus Time at an angle of 30° for three different viscosities

214
215
216 Figure 4 which is a plot of depth of invasion versus time for deviated wells at different fluid
217 displacement Efficiencies, E, show the relationship between depth of invasion, time and displacement
218 efficiency. It was shown that depth of invasion of the filtrate into the formation increases as time
219 increases. At different fluid displacement efficiencies, the depth of invasion differs. At a lower value of
220 displacement efficiency the depth of invasion of mud filtrate into the formation is higher when
221 compared to a higher value of displacement efficiency.

222 Figure 5 shows the plot of cake thickness versus solid pressure at constant of nonlinearity



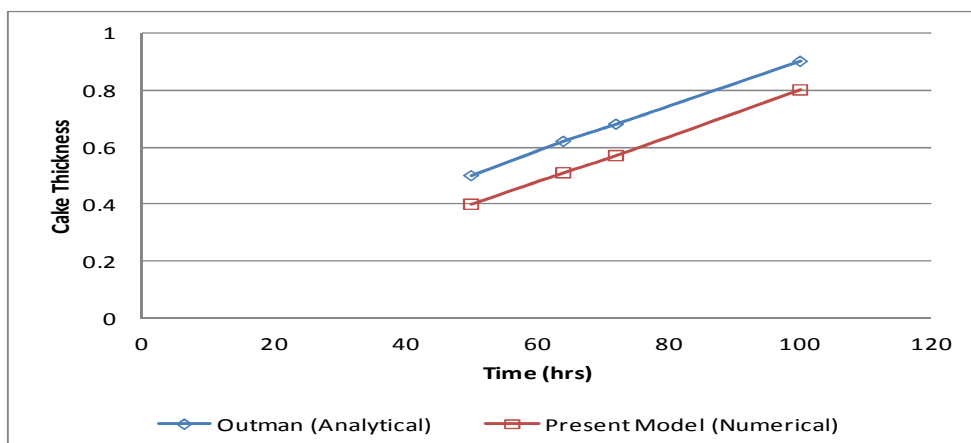
226 Figure 4: Depth of invasion versus time for deviated wells at different Efficiency



228

229 Figure 5: Cake thickness as a function of Mud Pressure.

230 Figure 6 shows compare cake thickness versus time for both present numerical model and analytical
 231 model. The value of cake thickness as time increases is higher for the analytical model when
 232 compared to the numerical model developed.



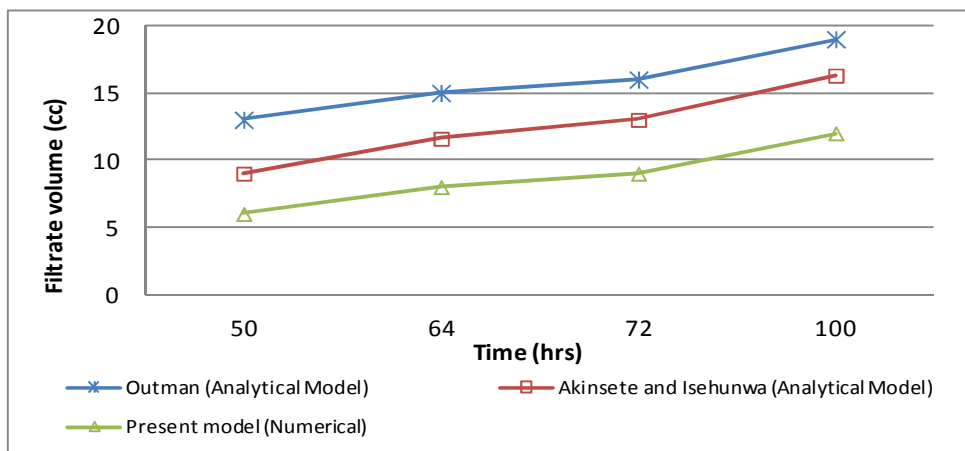
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234 Figure 6: Cake thickness versus Time (Comparing Numerical Model to Analytical Model)

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236 Figure 7 shows a plot of filtrate volume versus time for both numerical model and analytical model.
 237 The value of filtrate volume as time increases is higher for the analytical models when compared to
 238 the present numerical model developed.

239



240

241 Figure 7: Filtrate Volume versus Time (Comparing Numerical and Analytical Models)

242 5. CONCLUSION

243 A numerical approach was applied to solve the filtration equation for a compressible filter cake. The
 244 analysis showed that the increasing cake build up under the static condition considered can adversely
 245 affect the formation permeability, causing a zone of reduced permeability which is equivalent to
 246 severe damage around the wellbore. The solution shows that the reduction of solid pressure plays an
 247 important part in the minimizing of formation damage. The analysis also shows that increased
 248 viscosity of the filtrate also gives rise to reduced volume of filtrate. The effect of filtration on the angle
 249 of deviation was determined and this should help in planning and allocation of drilling time in order to
 250 reduce formation damage.

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274 275 DEFINITIONS, ACRONYMS, ABBREVIATIONS

277 A	Cross-sectional area, L^2
278 α_z	Local specific filtration resistance at $y, z = 0$, LM^{-1}
279 α_y	Local specific filtration at y , LM^{-1}
280 B	Solid fraction in the filter cake at 1 psi of solid pressure
281 B	Specific cake volume, L^3M^{-1}
282 C	Cake compressibility, $M^{-1}LT^2$
283 D1,D2,D3,D4	Expressions in terms of constants and exponents
284 D_p	Change in pressure, $ML^{-1}T^{-2}$
285 D	Change in stress
286 E	Fluid displacement efficiency
287 $\epsilon(y,z,t)$	Stress function in terms of solid pressure
288 ϵ_0, ϵ_1	Stress at $y, z = 0$ and $y, z = h$ respectively
289 f,m,n	Tiller exponents
290 G	Gravitational constant, LT^{-2}
291 H	Thickness of cake medium, L
292 i,j	Position and time element respectively
293 K	Cake permeability, L^2
294 Pf	Fluid pressure, $ML^{-1}T^{-2}$

295	P_s	Solid pressure, $ML^{-1}T^{-2}$
296	P_t	Total pressure, $ML^{-1}T^{-2}$
297	$q, q(y), q(z)$	Volume flow rate, L^3T^{-1}
298	Q	Cumulative volume of filtrate, L^3
299	r_a	Depth of invasion, L
300	r_w	Wellbore radius, L
301	ω	Exponent Expression
302	μ	Drilling fluid viscosity
303	γ	Exponent expression
304	Φ	Formation porosity